ECEN 667
Power System Stability

Lecture 17: Transient Stability Solutions, Load Models

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Announcements

- Read Chapter 7
- Homework 5 is due today
- Homework 6 is assigned today, due on Nov 9
- Final is as per TAMU schedule. That is, Friday Dec 8 from 3 to 5pm

Nonlinear Network Equations

- With constant impedance loads the network equations can usually be written with **I** independent of **V**, then they can be solved directly (as we've been doing)

$$\mathbf{V} = \mathbf{Y}^{-1}\,\mathbf{I}(\mathbf{x})$$

- In general this is not the case, with constant power loads one common example
- Hence a nonlinear solution with Newton's method is used
- We'll generalize the dependence on the algebraic variables, replacing **V** by **y** since they may include other values beyond just the bus voltages

Nonlinear Network Equations

- Just like in the power flow, the complex equations are rewritten, here as a real current and a reactive current
  YV – I(x,y) = 0
- The values for bus i are

$$g_{Di}(\mathbf{x},\mathbf{y}) = \sum_{k=1}^{n}\left(G_{ik}V_{Dk} - B_{ik}V_{QK}\right) - I_{NDi} = 0$$

$$g_{Qi}(\mathbf{x},\mathbf{y}) = \sum_{k=1}^{n}\left(G_{ik}V_{Qk} + B_{ik}V_{DK}\right) - I_{NQi} = 0$$

This is a rectangular formulation; we also could have written the equations in polar form

- For each bus we add two new variables and two new equations
- If an infinite bus is modeled then its variables and equations are omitted since its voltage is fixed

Nonlinear Network Equations

- The network variables and equations are then

In general there are no slack buses

$$\mathbf{y} = \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ \vdots \\ V_{Dn} \\ V_{Qn} \end{bmatrix} \qquad \mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{bmatrix} \sum_{k=1}^{n} \left(G_{1k} V_{Dk} - B_{1k} V_{QK} \right) - I_{ND1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{ik} V_{Qk} + B_{ik} V_{DK} \right) - I_{NQ1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{2k} V_{Dk} - B_{2k} V_{QK} \right) - I_{ND2}(\mathbf{x},\mathbf{y}) = 0 \\ \vdots \\ \sum_{k=1}^{n} \left(G_{nk} V_{Dk} - B_{nk} V_{QK} \right) - I_{NDn}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{nk} V_{Qk} + B_{nk} V_{DK} \right) - I_{NQn}(\mathbf{x},\mathbf{y}) = 0 \end{bmatrix}$$

Nonlinear Network Equation Newton Solution

The network equations are solved using a similar procedure to that of the Netwon-Raphson power flow

Set $v = 0$; make an initial guess of $\mathbf{y}$, $\mathbf{y}^{(v)}$

While $\left\| \mathbf{g}(\mathbf{y}^{(v)}) \right\| > \varepsilon$ Do

$$
\begin{aligned}
\mathbf{y}^{(v+1)} &= \mathbf{y}^{(v)} - \mathbf{J}(\mathbf{y}^{(v)})^{-1}\mathbf{g}(\mathbf{y}^{(v)}) \\
v &= v+1
\end{aligned}
$$

End While

Network Equation Jacobian Matrix

- The most computationally intensive part of the algorithm is determining and factoring the Jacobian matrix, $\mathbf{J}(\mathbf{y})$

$$\mathbf{J}(\mathbf{y}) = \begin{bmatrix} \dfrac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \dfrac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \dfrac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \\ \dfrac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \dfrac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \dfrac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \\ \vdots & \ddots & \ddots & \vdots \\ \dfrac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \dfrac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \dfrac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \end{bmatrix}$$

Network Jacobian Matrix

- The Jacobian matrix can be stored and computed using a 2 by 2 block matrix structure
- The portion of the 2 by 2 entries just from the $\mathbf{Y}_{\text{bus}}$ are

$$\begin{bmatrix} \dfrac{\partial \hat{g}_{Di}(\mathbf{x},\mathbf{y})}{\partial V_{Dj}} & \dfrac{\partial \hat{g}_{Di}(\mathbf{x},\mathbf{y})}{\partial V_{Qj}} \\ \dfrac{\partial \hat{g}_{Qi}(\mathbf{x},\mathbf{y})}{\partial V_{Dj}} & \dfrac{\partial \hat{g}_{Qi}(\mathbf{x},\mathbf{y})}{\partial V_{Qj}} \end{bmatrix} = \begin{bmatrix} G_{ij} & -B_{ij} \\ B_{ij} & G_{ij} \end{bmatrix}$$

The "hat" was added to the g functions to indicate it is just the portion from the $\mathbf{Y}_{\text{bus}}$

- The major source of the current vector voltage sensitivity comes from non-constant impedance loads; also dc transmission lines

Example: Constant Current and Constant Power Load

- As an example, assume the load at bus k is represented with a ZIP model

$$P_{Load,k} = P_{BaseLoad,k}\left(P_{z,k}\left|\bar{V}_k\right|^2 + P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(Q_{z,k}\left|\bar{V}_k\right|^2 + Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right)$$

The base load values are set from the power flow

- The constant impedance portion is embedded in the $\mathbf{Y}_{\text{bus}}$

$$\hat{P}_{Load,k} = P_{BaseLoad,k}\left(P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right) = \left(P_{BL,i,k}\left|\bar{V}_k\right| + P_{BL,p,k}\right)$$

$$\hat{Q}_{Load,k} = Q_{BaseLoad,k}\left(Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right) = \left(Q_{BL,i,k}\left|\bar{V}_k\right| + Q_{BL,p,k}\right)$$

- Usually solved in per unit on network MVA base

Example: Constant Current and Constant Power Load

- The current is then

$$\bar{I}_{Load,k} = I_{D,Load,k} + jI_{Q,Load,k} = \left(\frac{\hat{P}_{Load,k} + j\hat{Q}_{Load,k}}{\bar{V}_k} \right)^*$$

$$= \left(\frac{\left(P_{BL,i,k}\sqrt{V_{DK}^2 + V_{QK}^2} + P_{BL,p,k} \right) - j\left(Q_{BL,i,k}\sqrt{V_{DK}^2 + V_{QK}^2} + Q_{BL,p,k} \right)}{V_{Dk} - jV_{Qk}} \right)$$

- Multiply the numerator and denominator by $V_{DK}+jV_{QK}$ to write as the real current and the reactive current

Example: Constant Current and Constant Power Load

$$I_{D,Load,k} = \frac{V_{Dk}P_{BL,p,k} + V_{QK}Q_{BL,p,k}}{V_{DK}^2 + V_{QK}^2} + \frac{V_{Dk}P_{BL,i,k} + V_{QK}Q_{BL,i,k}}{\sqrt{V_{DK}^2 + V_{QK}^2}}$$

$$I_{Q,Load,k} = \frac{V_{Qk}P_{BL,p,k} - V_{DK}Q_{BL,p,k}}{V_{DK}^2 + V_{QK}^2} + \frac{V_{Qk}P_{BL,i,k} - V_{DK}Q_{BL,i,k}}{\sqrt{V_{DK}^2 + V_{QK}^2}}$$

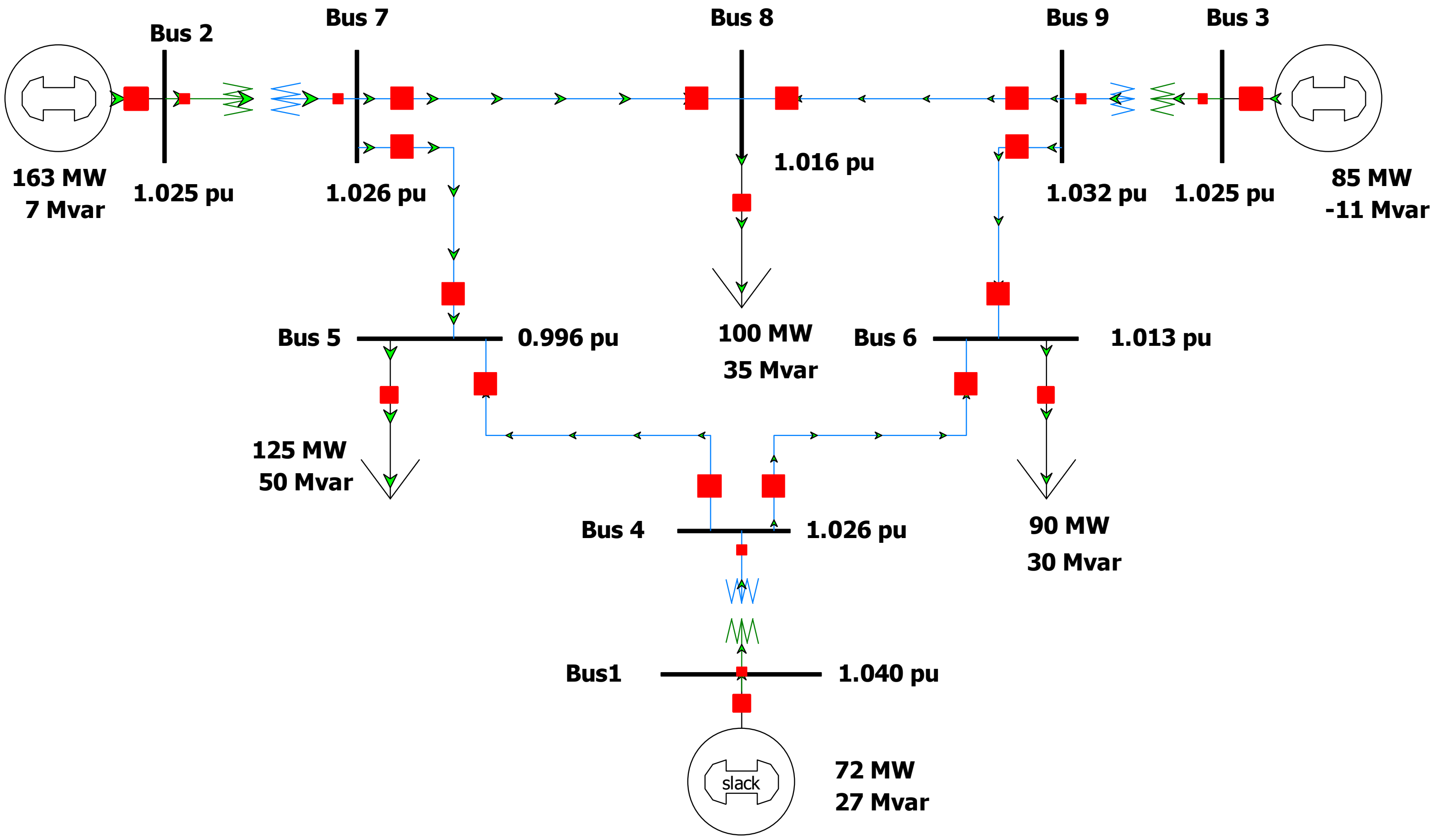
- The Jacobian entries are then found by differentiating with respect to V_{DK} and V_{QK}
  - Only affect the 2 by 2 block diagonal values
- Usually constant current and constant power models are replaced by a constant impedance model if the voltage goes too low, like during a fault

Example: 7.4.ZIP Case

- Example 7.4 is modified so the loads are represented by a model with 30% constant power, 30% constant current and 40% constant impedance
  - In PowerWorld load models can be entered in a number of different ways; a tedious but simple approach is to specify a model for each individual load
    - Right click on the load symbol to display the Load Options dialog, select Stability, and select WSCC to enter a ZIP model, in which p1&q1 are the normalized about of constant impedance load, p2&q2 the amount of constant current load, and p3&q3 the amount of constant power load

Case is Example_7_4_ZIP

Example 7.4.ZIP One-line



Example 7.4.ZIP Bus 8 Load Values

- As an example the values for bus 8 are given (per unit, 100 MVA base)

$$1.00 = P_{BaseLoad,8}\left(0.4\times 1.016^2 + 0.3\times 1.016 + 0.3\right)$$

$$\rightarrow P_{BaseLoad,8} = 0.983$$

$$0.35 = Q_{BaseLoad,8}\left(0.4\times 1.016^2 + 0.3\times 1.016 + 0.3\right)$$

$$\rightarrow Q_{BaseLoad,8} = 0.344$$

$$I_{D,Load,8} + jI_{Q,Load,8} = \left(\frac{1 + j0.35}{1.0158 + j0.0129}\right)^{*} = 0.9887 - j0.332$$

Example: 7.4.ZIP Case

- For this case the 2 by 2 block between buses 8 and 7 is

$$\begin{bmatrix} -1.155 & 9.784 \\ -9.784 & -1.155 \end{bmatrix}$$

- And between 8 and 9 is

$$\begin{bmatrix} -1.617 & 13.698 \\ -13.698 & -1.617 \end{bmatrix}$$

These entries are easily checked with the $\mathbf{Y}_{bus}$

- The 2 by 2 block for the bus 8 diagonal is

$$\begin{bmatrix} 2.876 & -23.352 \\ 23.632 & 3.745 \end{bmatrix}$$

The check here is left for the student

Additional Comments

- When coding Jacobian values, a good way to check that the entries are correct is to make sure that for a small perturbation about the solution the Newton's method has quadratic convergence
- When running the simulation the Jacobian is actually seldom rebuilt and refactored
  - If the Jacobian is not too bad it will still converge
- To converge Newton's method needs a good initial guess, which is usually the last time step solution
  - Convergence can be an issue following large system disturbances, such as a fault

Simultaneous Implicit

- The other major solution approach is the simultaneous implicit in which the algebraic and differential equations are solved simultaneously
- This method has the advantage of being numerically stable

Simultaneous Implicit

- Recalling the second lecture, we covered two common implicit integration approaches for solving $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
  - Backward Euler $\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t \mathbf{f}\left(\mathbf{x}(t+\Delta t)\right)$

    For a linear system we have

    $$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\,\mathbf{A}\right]^{-1} \mathbf{x}(t)$$

  - Trapezoidal $$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\left[\mathbf{f}\left(\mathbf{x}(t)\right) + \mathbf{f}\left(\mathbf{x}(t+\Delta t)\right)\right]$$

    For a linear system we have

    $$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\,\mathbf{A}\right]^{-1}\left[I + \frac{\Delta t}{2}\mathbf{A}\right]\mathbf{x}(t)$$

- We'll just consider trapezoidal, but for nonlinear cases

Nonlinear Trapezoidal

- We can use Newton's method to solve $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ with the trapezoidal

$$-\mathbf{x}(t+\Delta t)+\mathbf{x}(t)+\frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)\right)+\mathbf{f}\left(\mathbf{x}(t)\right)\right)=\mathbf{0}$$

- We are solving for **x**(t+Δt); **x**(t) is known
- The Jacobian matrix is

$$\mathbf{J}\left(\mathbf{x}(t+\Delta t)\right)=\frac{\Delta t}{2}\begin{bmatrix}\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n}\\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n}\end{bmatrix}-\mathbf{I}$$

Right now we are just considering the differential equations; we'll introduce the algebraic equations shortly

The **–I** comes from differentiating -**x**(t+Δt)

Nonlinear Trapezoidal using Newton's Method

- The full solution would be at each time step
  - Set the initial guess for **x**(t+Δt) as **x**(t), and initialize the iteration counter k = 0
  - Determine the mismatch at each iteration k as

$$\mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) \triangleq -\mathbf{x}(t+\Delta t)^{(k)} + \mathbf{x}(t) + \frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) + \mathbf{f}\left(\mathbf{x}(t)\right)\right)$$

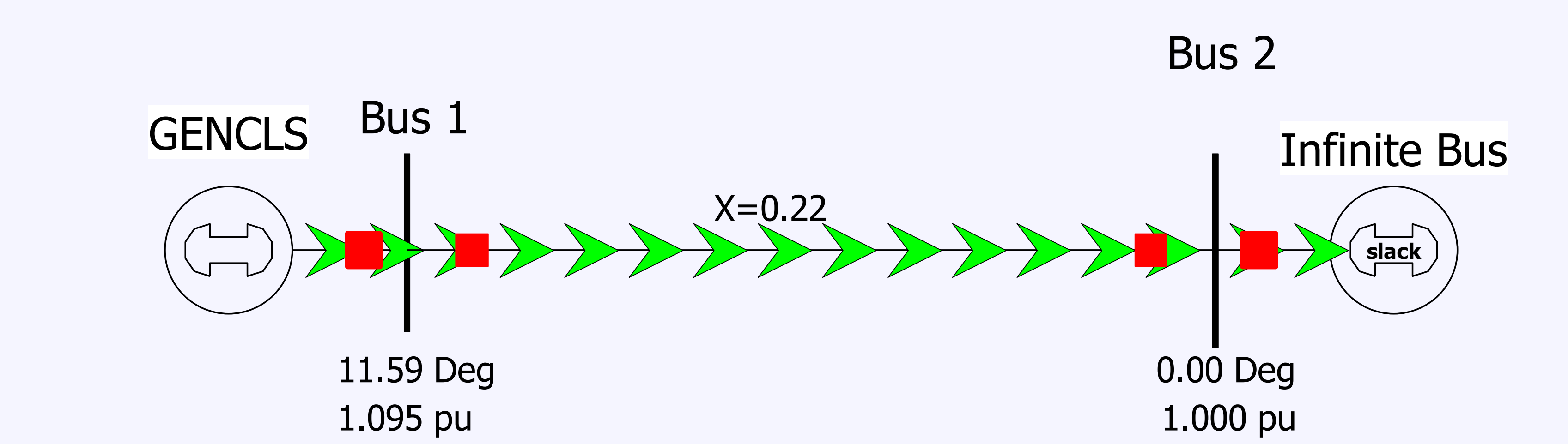
  - Determine the Jacobian matrix
  - Solve

$$\mathbf{x}(t+\Delta t)^{(k+1)} = \mathbf{x}(t+\Delta t)^{(k)} - \left[\mathbf{J}(\mathbf{x}(t+\Delta t)^{(k)}\right]^{-1}\mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right)$$

  - Iterate until done

Infinite Bus GENCLS Example

- Use the previous two bus system with gen 4 again modeled with a classical model with X_d'=0.3, H=3 and D=0



In this example Xth = (0.22 + 0.3), with the internal voltage $\bar{E}'_1 = 1.281\angle 23.95°$ giving E'_1=1.281 and δ_1= 23.95°

Infinite Bus GENCLS Implicit Solution

- Assume a solid three phase fault is applied at the bus 1 generator terminal, reducing P_{E1} to zero during the fault, and then the fault is self-cleared at time T^{clear}, resulting in the post-fault system being identical to the pre-fault system
  - During the fault-on time the equations reduce to

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times 3}(1-0)$$

That is, with a solid fault on the terminal of the generator, during the fault $P_{E1} = 0$

Infinite Bus GENCLS Implicit Solution

- The initial conditions are

$$\mathbf{x}(0) = \begin{bmatrix} \delta(0) \\ \omega_{pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

- Let $\Delta t = 0.02$ seconds
- During the fault the Jacobian is

$$\mathbf{J}\left(\mathbf{x}(t+\Delta t)\right) = \frac{0.02}{2}\begin{bmatrix} 0 & \omega_s \\ 0 & 0 \end{bmatrix} - \mathbf{I} = \begin{bmatrix} -1 & 3.77 \\ 0 & -1 \end{bmatrix}$$

- Set the initial guess for **x**(0.02) as **x**(0), and

$$\mathbf{f}\left(\mathbf{x}(0)\right) = \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- Then calculate the initial mismatch

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(0)}\right) \triangleq -\mathbf{x}(0.02)^{(0)} + \mathbf{x}(0) + \frac{0.02}{2}\left(\mathbf{f}\left(\mathbf{x}(0.02)^{(0)}\right) + \mathbf{f}\left(\mathbf{x}(0)\right)\right)$$

- With $\mathbf{x}(0.02)^{(0)} = \mathbf{x}(0)$ this becomes

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(0)}\right) = -\begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{0.02}{2}\left(\begin{bmatrix} 0 \\ 0.167 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.167 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0.00334 \end{bmatrix}$$

- Then

$$\mathbf{x}(0.02)^{(1)} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 & 3.77 \\ 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0.00334 \end{bmatrix} = \begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- Repeating for the next iteration

$$\mathbf{f}\left(\mathbf{x}\left(0.02\right)^{(1)}\right)=\begin{bmatrix} 1.259 \\ 0.1667 \end{bmatrix}$$

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(1)}\right)=-\begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix}+\begin{bmatrix} 0.418 \\ 0 \end{bmatrix}+\frac{0.02}{2}\left(\begin{bmatrix} 1.259 \\ 0.167 \end{bmatrix}+\begin{bmatrix} 0 \\ 0.167 \end{bmatrix}\right)$$
$$=\begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

- Hence we have converged with $\mathbf{x}(0.02)=\begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix}$

Infinite Bus GENCLS Implicit Solution

- Iteration continues until t = T^{clear}, assumed to be 0.1 seconds in this example

$$\mathbf{x}(0.10) = \begin{bmatrix} 0.7321 \\ 0.0167 \end{bmatrix}$$

- At this point, when the fault is self-cleared, the equations change, requiring a re-evaluation of $\mathbf{f}(\mathbf{x}(T^{clear}))$

$$\frac{d\delta}{dt} = \Delta\omega_{pu}\omega_s$$

$$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{6}\left(1 - \frac{1.281}{0.52}\sin\delta\right)$$

$$\mathbf{f}\left(\mathbf{x}\left(0.1^{+}\right)\right) = \begin{bmatrix} 6.30 \\ -0.1078 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- With the change in **f**(**x**) the Jacobian also changes

$$\mathbf{J}\left(\mathbf{x}(0.12^{(0)})\right)=\frac{0.02}{2}\begin{bmatrix}0 & \omega_s\\ -0.305 & 0\end{bmatrix}-\mathbf{I}=\begin{bmatrix}-1 & 3.77\\ -0.00305 & -1\end{bmatrix}$$

- Iteration for **x**(0.12) is as before, except using the new function and the new Jacobian

$$\mathbf{h}\left(\mathbf{x}(0.12)^{(0)}\right)\triangleq-\mathbf{x}(0.12)^{(0)}+\mathbf{x}(0.01)+\frac{0.02}{2}\left(\mathbf{f}\left(\mathbf{x}(0.12)^{(0)}\right)+\mathbf{f}\left(\mathbf{x}(0.10^{+})\right)\right)$$

$$\mathbf{x}(0.12)^{(1)}=\begin{bmatrix}0.7321\\ 0.0167\end{bmatrix}-\begin{bmatrix}-1 & 3.77\\ -0.00305 & -1\end{bmatrix}^{-1}\begin{bmatrix}0.1257\\ -0.00216\end{bmatrix}=\begin{bmatrix}0.848\\ 0.0142\end{bmatrix}$$

This also converges quickly, with one or two iterations

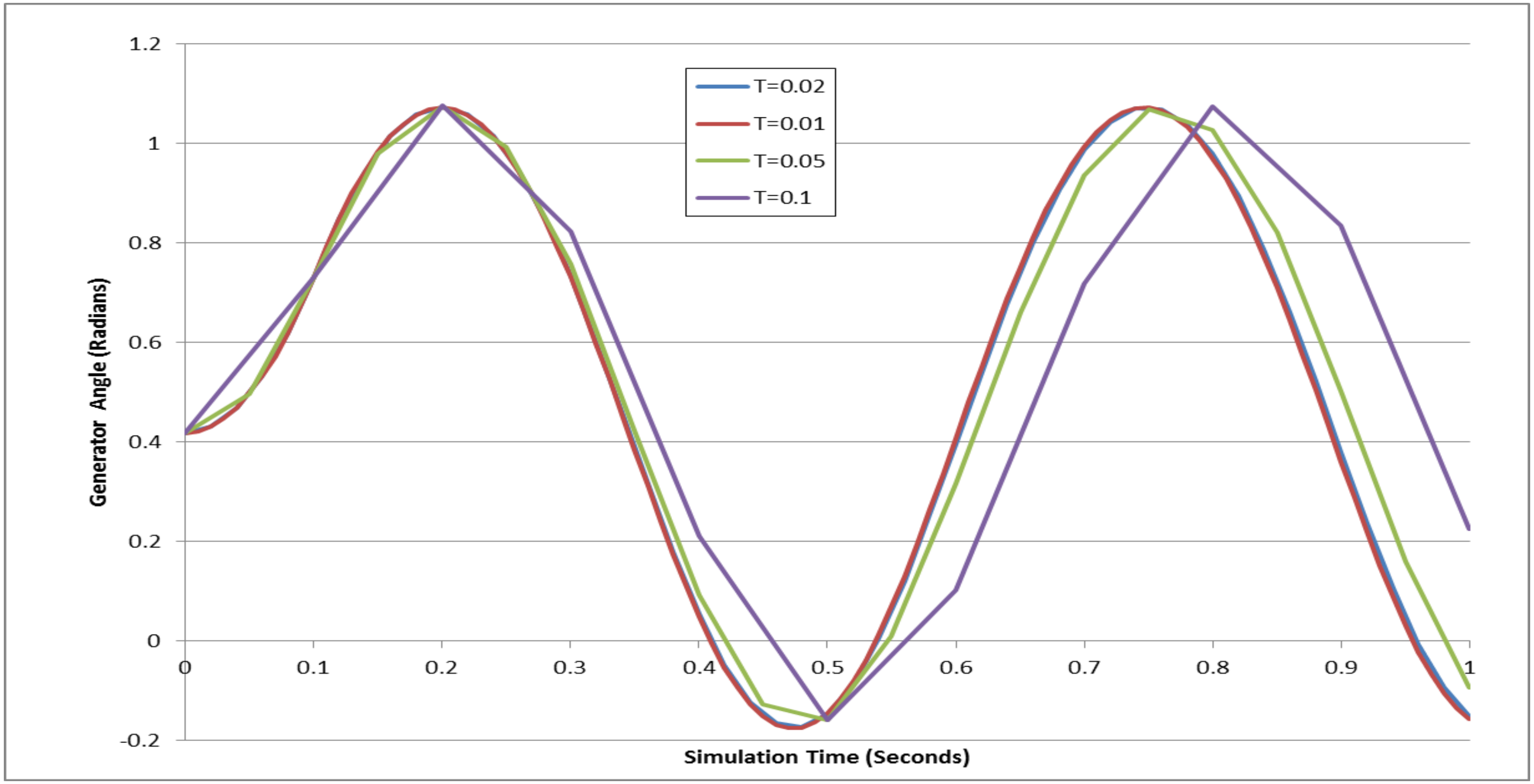
Computational Considerations

- As presented for a large system most of the computation is associated with updating and factoring the Jacobian. But the Jacobian actually changes little and hence seldom needs to be rebuilt/factored
- Rather than using **x**(t) as the initial guess for **x**(t+Δt), prediction can be used when previous values are available

$$\mathbf{x}(t+\Delta t)^{(0)} = \mathbf{x}(t) + \left(\mathbf{x}(t) - \mathbf{x}(t-\Delta t)\right)$$

Two Bus Results

- The below graph shows the generator angle for varying values of Δt; recall the implicit method is numerically stable



Adding the Algebraic Constraints

- Since the classical model can be formulated with all the values on the network reference frame, initially we just need to add the network equations
- We'll again formulate the network equations using the form

$$\mathbf{I}(\mathbf{x},\mathbf{y}) = \mathbf{Y}\,\mathbf{V} \quad \text{or} \quad \mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x},\mathbf{y}) = \mathbf{0}$$

- As before the complex equations will be expressed using two real equations, with voltages and currents expressed in rectangular coordinates

Adding the Algebraic Constraints

- The network equations are as before

$$
\mathbf{y} = \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ \vdots \\ V_{Dn} \\ V_{Qn} \end{bmatrix} \qquad
\mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{bmatrix}
\sum_{k=1}^{n} \left(G_{1k} V_{Dk} - B_{1k} V_{QK} \right) - I_{ND1}(\mathbf{x},\mathbf{y}) = 0 \\
\sum_{k=1}^{n} \left(G_{ik} V_{Qk} + B_{ik} V_{DK} \right) - I_{NQ1}(\mathbf{x},\mathbf{y}) = 0 \\
\sum_{k=1}^{n} \left(G_{2k} V_{Dk} - B_{2k} V_{QK} \right) - I_{ND2}(\mathbf{x},\mathbf{y}) = 0 \\
\vdots \\
\sum_{k=1}^{n} \left(G_{nk} V_{Dk} - B_{nk} V_{QK} \right) - I_{NDn}(\mathbf{x},\mathbf{y}) = 0 \\
\sum_{k=1}^{n} \left(G_{nk} V_{Qk} + B_{nk} V_{DK} \right) - I_{NQn}(\mathbf{x},\mathbf{y}) = 0
\end{bmatrix}
$$

Classical Model Coupling of x and y

- In the simultaneous implicit method **x** and **y** are determined simultaneously; hence in the Jacobian we need to determine the dependence of the network equations on **x**, and the state equations on **y**
- With the classical model the Norton current depends on **x** as

$$\bar{I}_{Ni} = \frac{E_i' \angle \delta_i}{R_{s,i} + jX_{d,i}'}, \quad G_i + jB_i = \frac{1}{R_{s,i} + jX_{d,i}'}$$

$$\bar{I}_{Ni} = I_{DNi} + jI_{QNi} = E_i'\left(\cos\delta_i + j\sin\delta_i\right)\left(G_i + jB_i\right)$$

$$E_{Di} + jE_{Qi} = E_i'\left(\cos\delta_i + j\sin\delta_i\right)$$

$$I_{DNi} = E_{Di}G_i - E_{Qi}B_i$$

$$I_{QNi} = E_{Di}B_i + E_{Qi}G_i$$

Recall with the classical model E_i' is constant

Classical Model Coupling of x and y

- In the state equations the coupling with **y** is recognized by noting

$$P_{Ei} = E_{Di} I_{Di} + E_{Qi} I_{Qi}$$

$$I_{Di} + jI_{Qi} = \left(\left(E_{Di} - V_{Di} \right) + j \left(E_{Qi} - V_{Qi} \right) \right) \left(G_i + jB_i \right)$$

$$I_{Di} = \left(E_{Di} - V_{Di} \right) G_i - \left(E_{Qi} - V_{Qi} \right) B_i$$

$$I_{Qi} = \left(E_{Di} - V_{Di} \right) B_i + \left(E_{Qi} - V_{Qi} \right) G_i$$

$$P_{Ei} = E_{Di} \left(\left(E_{Di} - V_{Di} \right) G_i - \left(E_{Qi} - V_{Qi} \right) B_i \right) + E_{Qi} \left(\left(E_{Di} - V_{Di} \right) B_i + \left(E_{Qi} - V_{Qi} \right) G_i \right)$$

$$P_{Ei} = \left(E_{Di}^2 - E_{Di} V_{Di} \right) G_i + \left(E_{Qi}^2 - E_{Qi} V_{Qi} \right) G_i + \left(E_{Di} V_{Qi} - E_{Qi} V_{Di} \right) B_i$$

Variables and Mismatch Equations

- In solving the Newton algorithm the variables now include **x** and **y** (recalling that here **y** is just the vector of the real and imaginary bus voltages
- The mismatch equations now include the state integration equations

$$\mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) =$$

$$-\mathbf{x}(t+\Delta t)^{(k)} + \mathbf{x}(t) + \frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)^{(k)}, \mathbf{y}(t+\Delta t)^{(k)}\right) + \mathbf{f}\left(\mathbf{x}(t), \mathbf{y}(t)\right)\right)$$

- And the algebraic equations

$$\mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)}, \mathbf{y}(t+\Delta t)^{(k)}\right)$$

Jacobian Matrix

- Since the $\mathbf{h}(\mathbf{x},\mathbf{y})$ and $\mathbf{g}(\mathbf{x},\mathbf{y})$ are coupled, the Jacobian is

$$J\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)$$

$$=\begin{bmatrix} \dfrac{\partial \mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{x}} & \dfrac{\partial \mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{y}} \\ \dfrac{\partial \mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{x}} & \dfrac{\partial \mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{y}} \end{bmatrix}$$

  - With the classical model the coupling is the Norton current at bus i depends on δ_i (i.e., **x**) and the electrical power (P_{Ei}) in the swing equation depends on V_{Di} and V_{Qi} (i.e., **y**)

Jacobian Matrix Entries

- The dependence of the Norton current injections on δ is

$$I_{DNi} = E_i' \cos\delta_i G_i - E_i' \sin\delta_i B_i$$

$$I_{QNi} = E_i' \cos\delta_i B_i + E_i' \sin\delta_i G_i$$

$$\frac{\partial I_{DNi}}{\partial \delta_i} = -E_i' \sin\delta_i G_i - E_i' \cos\delta_i B_i$$

$$\frac{\partial I_{QNi}}{\partial \delta_i} = -E_i' \sin\delta_i B_i + E_i' \cos\delta_i G_i$$

  – In the Jacobian the sign is flipped because we defined

$$\mathbf{g}(\mathbf{x},\mathbf{y}) = \mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x},\mathbf{y})$$

Jacobian Matrix Entries

- The dependence of the swing equation on the generator terminal voltage is

$$\dot{\delta}_i = \Delta\omega_{i.pu}\omega_s$$

$$\Delta\dot{\omega}_{i,pu} = \frac{1}{2H_i}\left(P_{Mi} - P_{Ei} - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

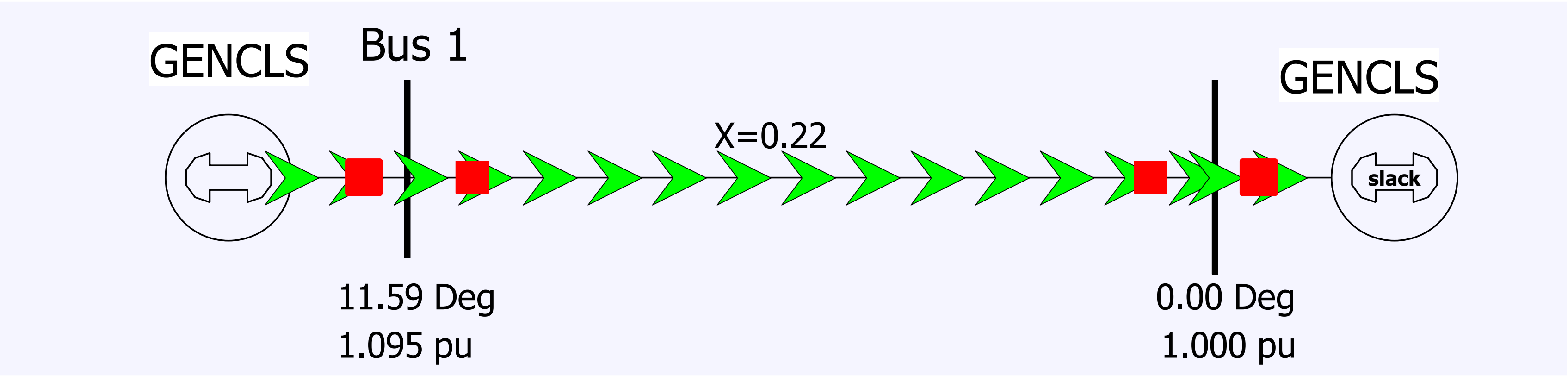
$$\mathrm{P}_{Ei} = \left(E_{Di}^2 - E_{Di}V_{Di}\right)G_i + \left(E_{Qi}^2 - E_{Qi}V_{Qi}\right)G_i + \left(E_{Di}V_{Qi} - E_{Qi}V_{Di}\right)B_i$$

$$\frac{\partial\Delta\dot{\omega}_{i,pu}}{\partial V_{Di}} = \frac{1}{2H_i}\left(E_{Di}G_i + E_{Qi}B_i\right)$$

$$\frac{\partial\Delta\dot{\omega}_{i,pu}}{\partial V_{Qi}} = \frac{1}{2H_i}\left(E_{Qi}G_i - E_{Di}B_i\right)$$

Two Bus, Two Gen GENCLS Example

- We'll reconsider the two bus, two generator case from Lecture 16; fault at Bus 1, cleared after 0.06 seconds
  - Initial conditions and $\mathbf{Y}_{bus}$ are as covered in Lecture 16



PowerWorld Case B2_CLS_2Gen

Two Bus, Two Gen GENCLS Example

- Initial terminal voltages are

$$V_{D1} + jV_{Q1} = 1.0726 + j0.22, \quad V_{D2} + jV_{Q2} = 1.0$$

$$\bar{E}_1 = 1.281\angle 23.95°, \quad \bar{E}_2 = 0.955\angle -12.08$$

$$\bar{I}_{N1} = \frac{1.1709 + j0.52}{j0.3} = 1.733 - j3.903$$

$$\bar{I}_{N2} = \frac{0.9343 - j0.2}{j0.2} = -1 - j4.6714$$

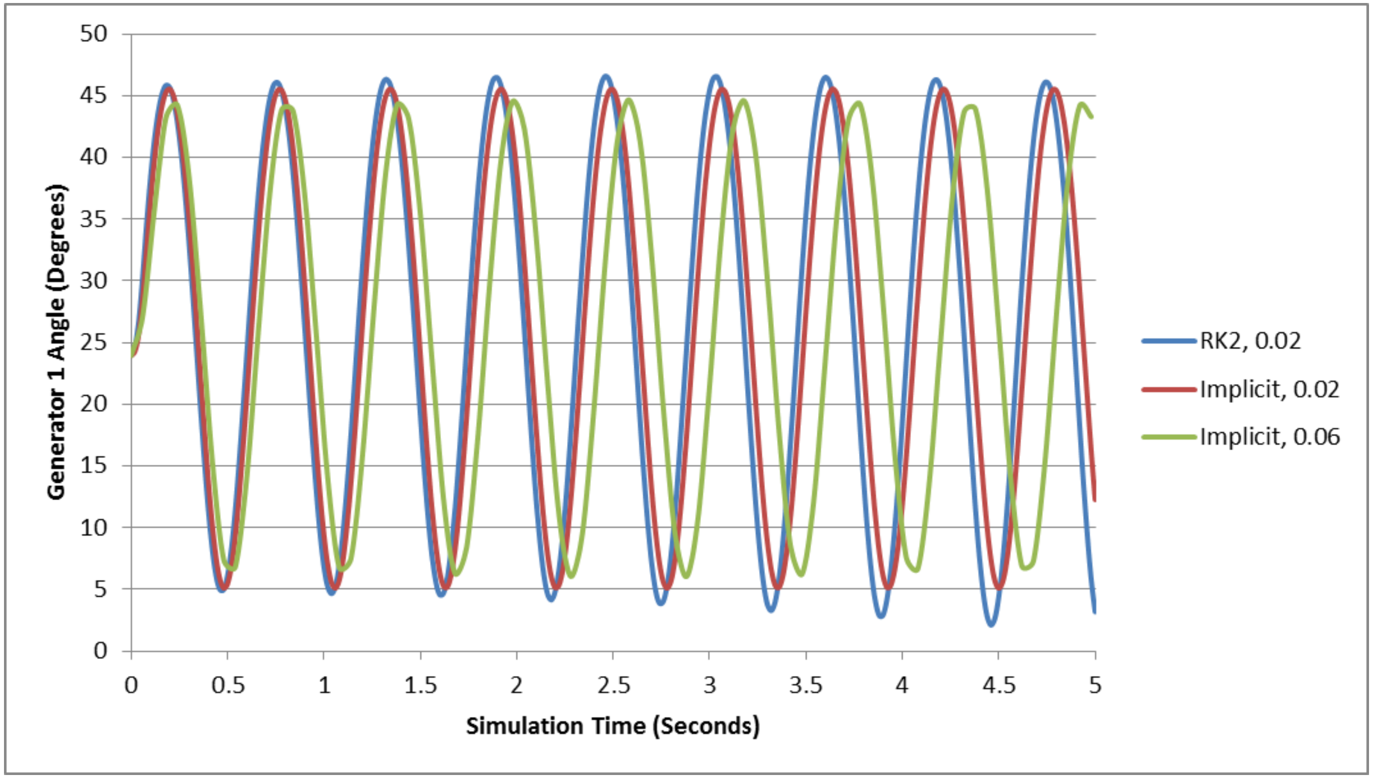
$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.333} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}$$

Two Bus, Two Gen Initial Jacobian

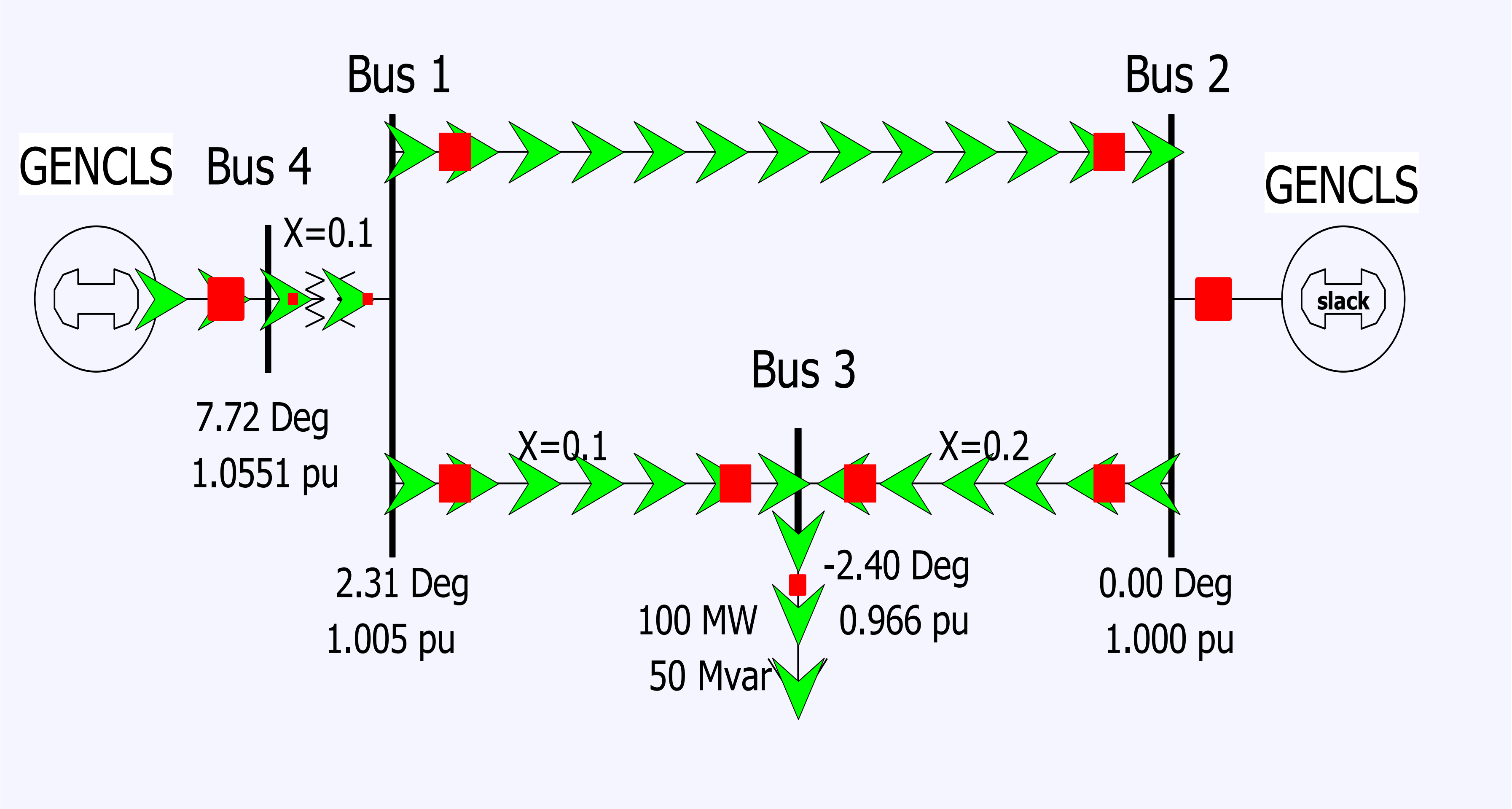
$$
\begin{array}{cc}
 & \begin{array}{cccccccc} \delta_1 & \Delta\omega_1 & \delta_2 & \Delta\omega_1 & V_{D1} & V_{Q1} & V_{D2} & V_{Q2} \end{array} \\
\begin{array}{c} \dot{\delta}_1 \\ \Delta\dot{\omega}_1 \\ \dot{\delta}_2 \\ \Delta\dot{\omega}_2 \\ I_{D1} \\ I_{Q1} \\ I_{D2} \\ I_{Q2} \end{array} &
\begin{bmatrix}
-1 & 3.77 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0076 & -1 & 0 & 0 & -0.0029 & 0.0065 & 0 & 0 \\
0 & 0 & -1 & 3.77 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.0039 & -1 & 0 & 0 & 0.0008 & 0.0039 \\
-3.90 & 0 & 0 & 0 & 0 & 7.879 & 0 & -4.545 \\
-1.73 & 0 & 0 & 0 & -7.879 & 0 & 4.545 & 0 \\
0 & 0 & -4.67 & 0 & 0 & -4.545 & 0 & 9.545 \\
0 & 0 & 1.00 & 0 & 4.545 & 0 & -9.545 & 0
\end{bmatrix}
\end{array}
$$

Results Comparison

- The below graph compares the angle for the generator at bus 1 using Δt=0.02 between RK2 and the Implicit Trapezoidal; also Implicit with Δt=0.06

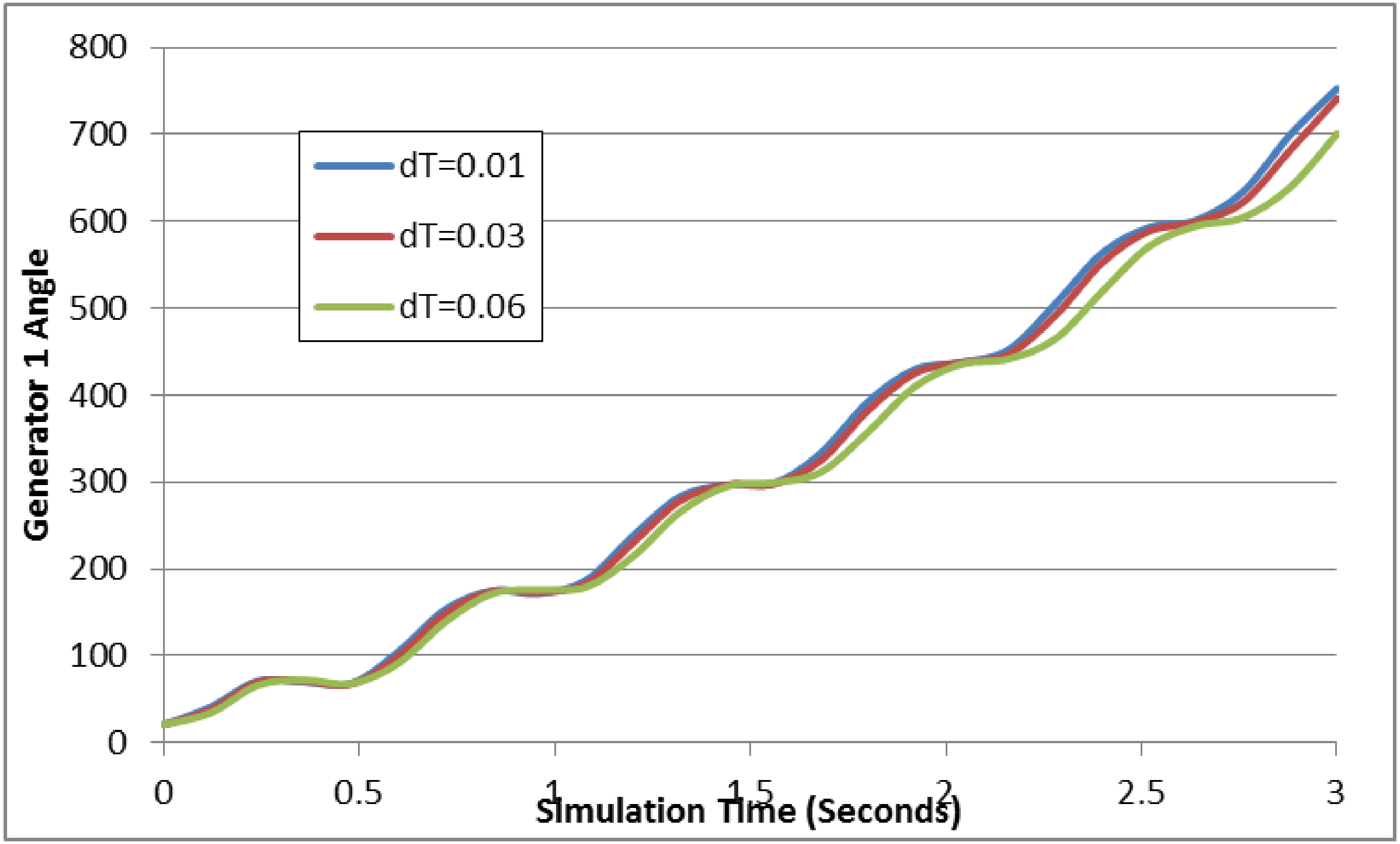


Four Bus Comparison



Four Bus Comparison

Fault at Bus 3 for 0.12 seconds; self-cleared



Done with Transient Stability Solutions: On to Load Modeling

- Load modeling is certainly challenging!
- For large system models an aggregate load can consist of many thousands of individual devices
- The load is constantly changing, with key diurnal and temperature variations
  - For example, a higher percentage of lighting load at night, more air conditioner load on hot days
- Load model behavior can be quite complex during the low voltages that may occur in transient stability
- Testing aggregate load models for extreme conditions is not feasible – we need to wait for disturbances!

Load Modeling

- Traditionally load models have been divided into two groups
  - Static: load is a algebraic function of bus voltage and sometimes frequency
  - Dynamic: load is represented with a dynamic model, with induction motor models the most common
- The simplest load model is a static constant impedance
  - Has been widely used
  - Allowed the $\mathbf{Y}_{bus}$ to be reduced, eliminating essentially all non-generator buses
  - Presents no issues as voltage falls to zero
  - Is rapidly falling out of favor

Load Modeling References

- Many papers and reports are available!
- A classic reference on load modeling is by the IEEE Task Force on Load Representation for Dynamic Performance, "Load Representation for Dynamic Performance Analysis," IEEE Trans. on Power Systems, May 1993, pp. 472-482
- A more recent report that provides a good overview is "Final Project Report Loading Modeling Transmission Research" from Lawrence Berkeley National Lab, March 2010

ZIP Load Model

- Another common static load model is the ZIP, in which the load is represented as

$$P_{Load,k} = P_{BaseLoad,k}\left(P_{z,k}\left|\bar{V}_k\right|^2 + P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(Q_{z,k}\left|\bar{V}_k\right|^2 + Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right)$$

- Some models allow more general voltage dependence

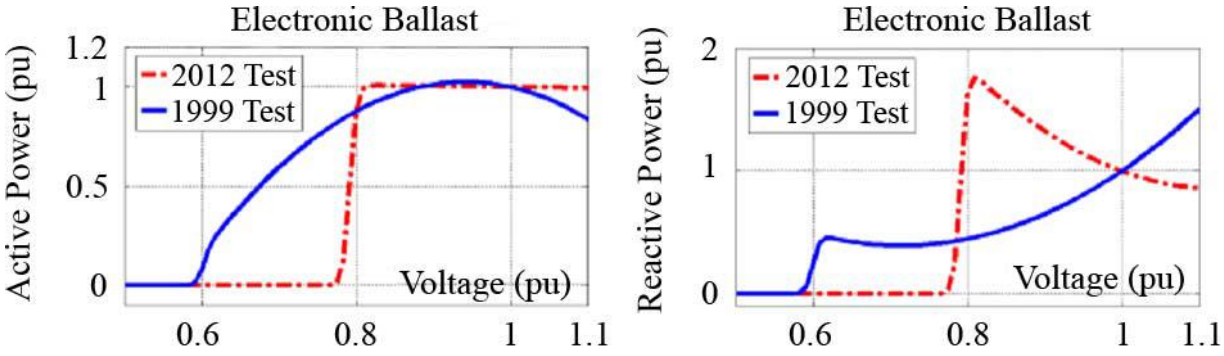
$$P_{Load,k} = P_{BaseLoad,k}\left(a_{1,k}\left|\bar{V}_k\right|^{n1} + a_{2,k}\left|\bar{V}_k\right|^{n2} + a_{3,k}\left|\bar{V}_k\right|^{n3}\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(a_{4,k}\left|\bar{V}_k\right|^{n4} + a_{5,k}\left|\bar{V}_k\right|^{n5} + a_{6,k}\left|\bar{V}_k\right|^{n6}\right)$$

The voltage exponent for reactive power is often > 2

ZIP Model Coefficients

- An interesting paper on the experimental determination of the ZIP parameters is A. Bokhari, et. al., "Experimental Determination of the ZIP Coefficients for Modern Residential and Commercial Loads, and Industrial Loads," IEEE Trans. Power Delivery, 2014
  - Presents test results for loads as voltage is varied; also highlights that load behavior changes with newer technologies
    - Below figure (part of fig 4 of paper), compares real and reactive behavior of light ballast



ZIP Model Coefficients

TABLE VII
ACTIVE AND REACTIVE ZIP MODEL. FIRST HALF OF THE ZIPS WITH 100-V CUTOFF VOLTAGE. SECOND HALF REPORTS THE ZIPS WITH ACTUAL CUTOFF VOLTAGE

Equipment/ component	No. tested	V_{cut}	V_o	P_o	Q_o	Z_p	I_p	P_p	Z_q	I_q	P_q
Air compressor 1 Ph	1	100	120	1109.01	487.08	0.71	0.46	-0.17	-1.33	4.04	-1.71
Air compressor 3 Ph	1	174	208	1168.54	844.71	0.24	-0.23	0.99	4.79	-7.61	3.82
Air conditioner	2	100	120	496.33	125.94	1.17	-1.83	1.66	15.68	-27.15	12.47
CFL bulb	2	100	120	25.65	37.52	0.81	-1.03	1.22	0.86	-0.82	0.96
Coffeemaker	1	100	120	1413.04	13.32	0.13	1.62	-0.75	3.89	-6	3.11
Copier	1	100	120	944.23	84.57	0.87	-0.21	0.34	2.14	-3.67	2.53
Electronic ballast	3	100	120	59.02	5.06	0.22	-0.5	1.28	9.64	-21.59	12.95
Elevator	3	174	208	1381.17	1008.3	0.4	-0.72	1.32	3.76	-5.74	2.98
Fan	2	100	120	163.25	83.28	-0.47	1.71	-0.24	2.34	-3.12	1.78
Game consol	3	100	120	60.65	67.61	-0.63	1.23	0.4	0.76	-0.93	1.17
Halogen	3	100	120	97.36	0.84	0.46	0.64	-0.1	4.26	-6.62	3.36
High pressure sodium HID	4	100	120	276.09	52.65	0.09	0.7	0.21	16.6	-28.77	13.17
Incandescent light	2	100	120	87.16	0.85	0.47	0.63	-0.1	0.55	0.38	0.07
Induction light	1	100	120	44.5	4.8	2.96	-6.04	4.08	1.48	-1.29	0.81
Laptop charger	1	100	120	35.94	71.64	-0.28	0.5	0.78	-0.37	1.24	0.13
LCD Television	1	100	120	208.03	-20.58	0.11	-0.17	1.06	1.58	-1.72	1.14
LED light	1	100	120	3.38	5.85	0.58	1.13	-0.71	1.78	-0.8	0.02
Magnetic ballast	1	100	120	81.23	8.2	-1.58	3.79	-1.21	36.18	-67.78	32.6
Mercury vapor HID light	2	100	120	268.27	77.66	0.52	1.02	-0.54	-1.33	2.4	-0.07
Metal halide HID electronic ballast	2	100	120	113.7	26.37	1	-2.02	2.02	8.8	-18.64	10.84
Metal halide HID magnetic ballast	2	100	120	450	102.94	0.86	-0.66	0.8	32.54	-59.83	28.29
Microwave	2	100	120	1365.53	451.02	1.39	-1.96	1.57	50.07	-93.55	44.48
Minibar	1	100	120	90.65	126.94	2.5	-4.1	2.6	2.56	-2.76	1.2
PC (Monitor & CPU)	1	100	120	118.9	172.79	0.2	-0.3	1.1	0	0.6	0.4

A portion of Table VII from Bokhari 2014 paper

The Z,I,P coefficients sum to zero; note that for some models the absolute values of the parameters are quite large, indicating a difficult fit

Discharge Lighting Models

- Discharge lighting (such as fluorescent lamps) is a major portion of the load (10-15%)
- Discharge lighting has been modeled for sufficiently high voltage with a real power as constant current and reactive power with a high voltage dependence
  - Linear reduction for voltage between 0.65 and 0.75 pu
  - Extinguished (i.e., no load) for voltages below

$$P_{DischargeLighting} = P_{Base}\left(\left|\bar{V}_k\right|\right)$$

$$Q_{DischargeLighting} = Q_{Base}\left(\left|\bar{V}_k\right|^{4.5}\right)$$

May need to change with newer electronic ballasts – e.g., reactive power increasing as the voltage drops!

Static Load Model Frequency Dependence

- Frequency dependence is sometimes included, to recognize that the load could change with the frequency

$$P_{Load,k} = P_{BaseLoad,k}\left(P_{z,k}\left|\bar{V}_k^2\right| + P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right)\left(1 + P_{f,k}\left(f_k - 1\right)\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(Q_{z,k}\left|\bar{V}_k^2\right| + Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right)\left(1 + Q_{f,k}\left(f_k - 1\right)\right)$$

- Here f_k is the per unit bus frequency, which is calculated as

$$\theta_k \rightarrow \boxed{\frac{s}{1+sT}} \rightarrow f_k$$

A typical value for T is about 0.02 seconds. Some models just have frequency dependence on the constant power load

- Typical values for P_f and Q_f are 1 and -1 respectively

Aside: Voltage Stability

- Next few slides are an aside on static voltage stability
- **Voltage Stability**: The ability to maintain system voltage so that both power and voltage are controllable. System voltage responds as expected (i.e., an increase in load causes proportional decrease in voltage).
- **Voltage Instability**: Inability to maintain system voltage. System voltage and/or power become uncontrollable. System voltage does not respond as expected.
- **Voltage Collapse**: Process by which voltage instability leads to unacceptably low voltages in a significant portion of the system. Typically results in loss of system load.

Voltage Stability

- Two good references are
  - P. Kundur, et. al., “Definitions and Classification of Power System Stability,” *IEEE Trans. on Power Systems*, pp. 1387-1401, August 2004.
  - T. Van Cutsem, “Voltage Instability: Phenomena, Countermeasures, and Analysis Methods,” *Proc. IEEE*, February 2000, pp. 208-227.
- Classified by either size of disturbance or duration
  - Small or large disturbance: small disturbance is just perturbations about an equilibrium point (power flow)
  - Short-term (several seconds) or long-term (many seconds to minutes)