ECEN 667
Power System Stability

Lecture 18: Voltage Stability, Load Models

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Announcements

- Read Chapter 7

- Homework 6 is due on Nov 9

- Final is as per TAMU schedule. That is, Friday Dec 8 from 3 to 5pm

Small Disturbance Voltage Stability

- Small disturbance voltage stability can be assessed using a power flow (maximum loadability)

- Depending on the assumed load model, the power flow can have multiple (or now solutions)

- PV curve is created by plotting power versus voltage
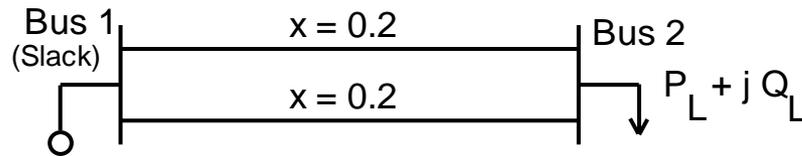
Assume $V_{slack}=1.0$

$$P_L - BV \sin \theta = 0$$

$$Q_L + BV \cos \theta - BV^2 = 0$$

Where B is the line susceptance =-10, $V\angle\theta$ is the load voltage

3

Small Disturbance Voltage Stability

- Question: how do the power flow solutions vary as the load is changed?

- A Solution: Calculate a series of power flow solutions for various load levels and see how they change

- Power flow Jacobian

$$\mathbf{J}(\theta, V) = \begin{bmatrix} -BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta - 2BV \end{bmatrix}$$

PV and QV Curves

- PV curves can be traced by plotting the voltage as the real power is increased; QV curves as reactive power is increased
  - At least the upper portion
- Two bus example PV and QV curves
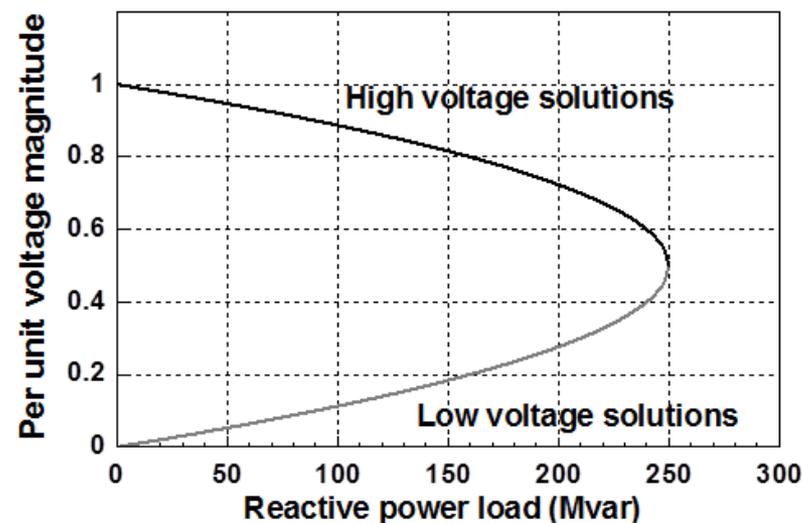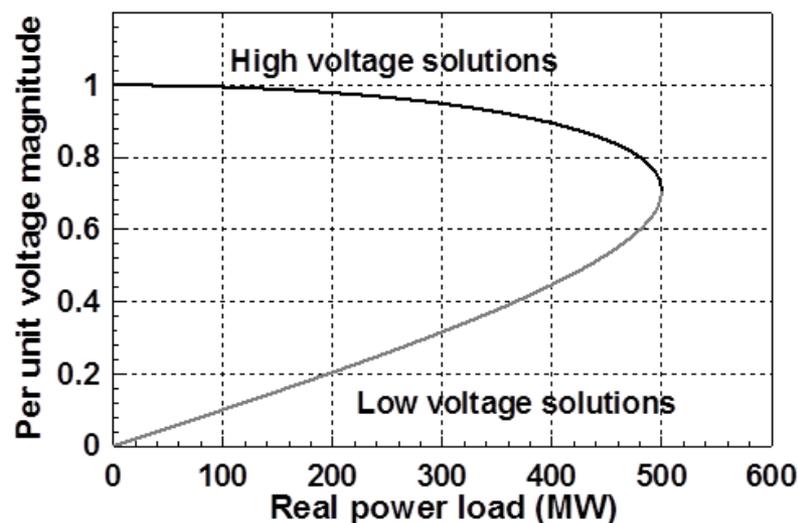
Load Parameter Space Representation

- With a constant power model there is a maximum loadability surface, Σ
  - Defined as point in which the power flow Jacobian is singular
 - For the lossless two bus system it can be determined as

$$-\frac{P_L^2}{B} + Q_L + \frac{1}{4}B = 0$$
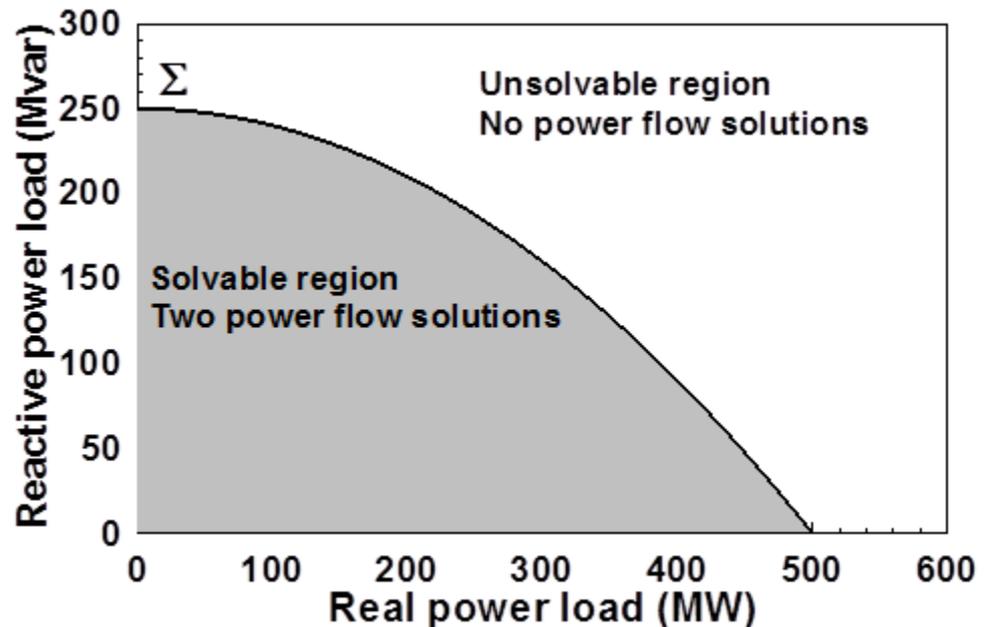
Load Model Impact

- With a static load model regardless of the voltage dependency the same PV curve is traced
  - But whether a point of maximum loadability exists depends on the assumed load model
    - If voltage exponent is > 1 then multiple solutions do not exist (see B.C. Lesieutre, P.W. Sauer and M.A. Pai "Sufficient conditions on static load models for network solvability,"NAPS 1992, pp. 262-271)

Bus 1

Bus 2

X=0.2

X=0.2

slack

1.000 pu
11.59 Deg

0.995 pu
5.82 Deg

100 MW

0 Mvar
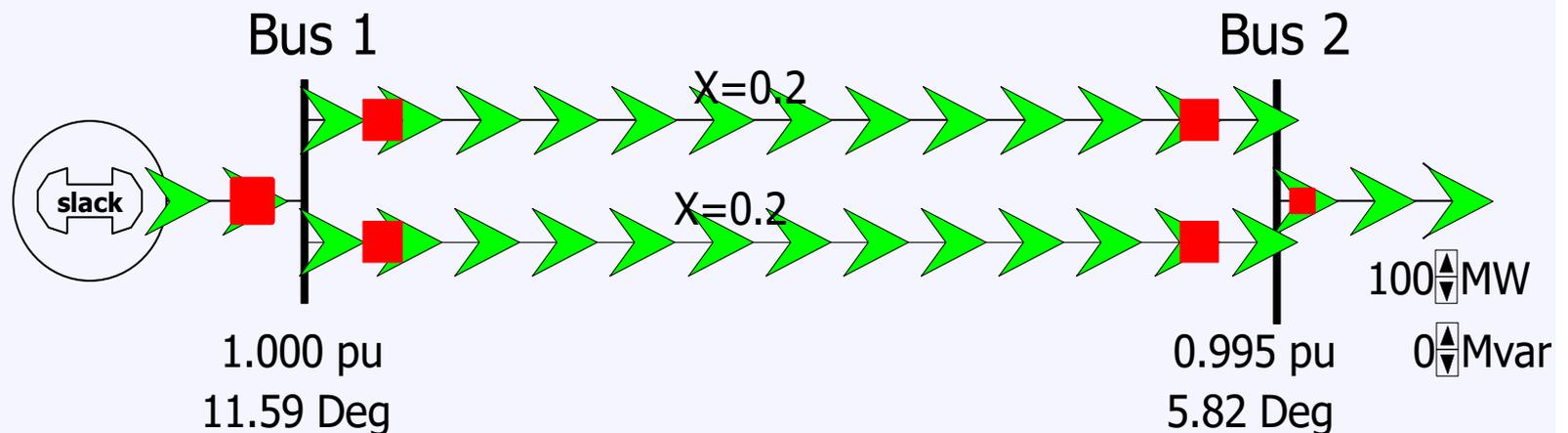
Power Flow Region of Convergence

Convergence regions with P=100 MW, Q=0 Mvar
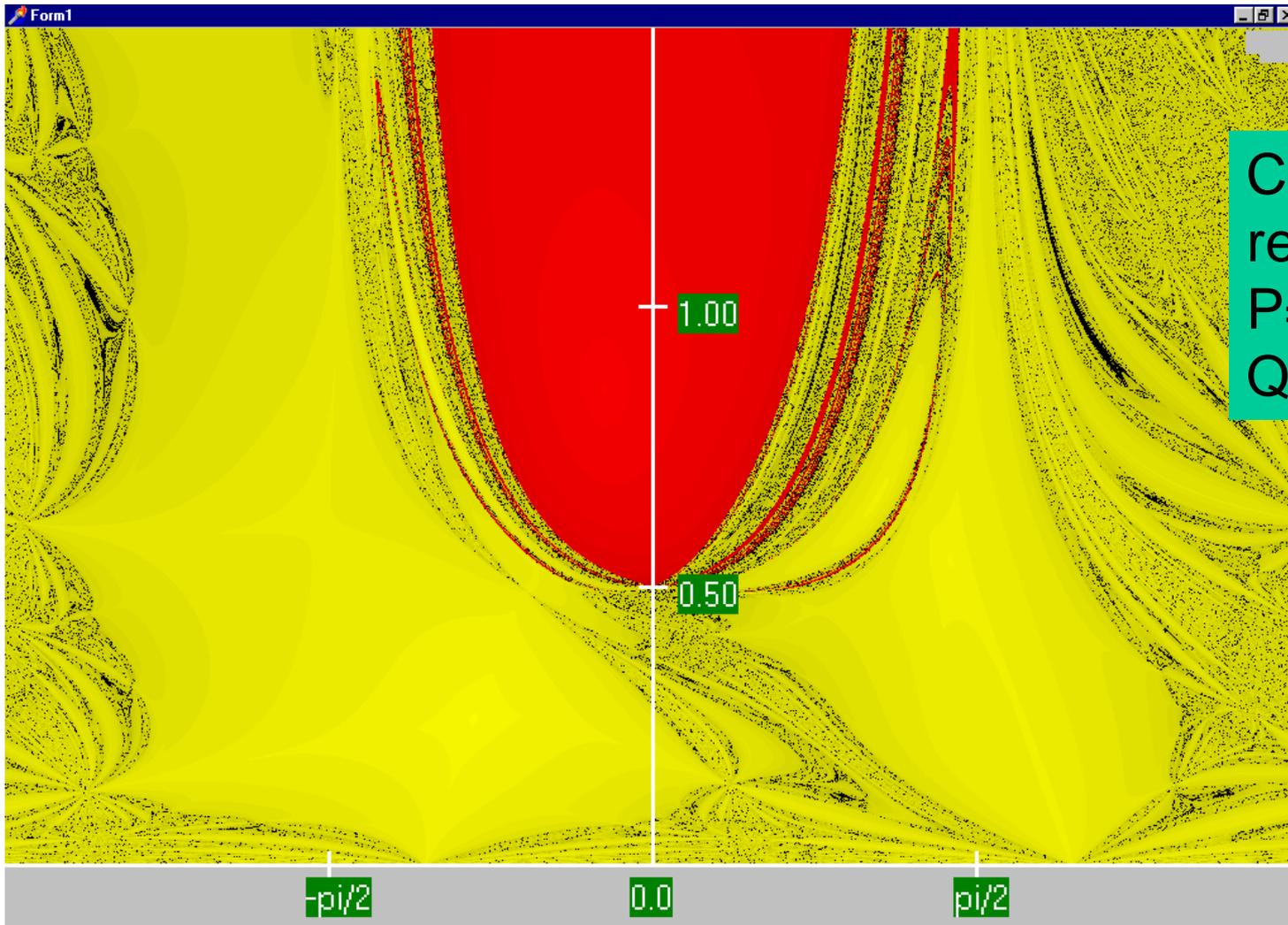
End Aside, Induction Motor Models

- Induction motors, both three phase and single phase, make up a very large percentage of the load

- Next several slides describe how induction motors are modeled in transient stability
  - This model would not apply to induction motors controlled by ac drives, since the converter in the drive will make the motor's behavior independent of the source voltage (up to a point); it will look more like a constant power load

- Originally invented independently by Galileo Ferraris (1885) and Nikola Tesla (1887)
  - Tesla received the US patent in 1888
  - Key to growth of ac, as opposed to dc, electric systems

Induction Machines

- Term induction machine is used to indicate either generator or motor; most uses are as motors

- Induction machines have two major components
  - A stationary stator, which is supplied with an ac voltage; windings in stator create a rotating magnetic field
  - A rotating rotor, in which an ac current is induced (hence the name)

- Two basic design types based on rotor design
  - Squirrel-cage: rotor consists of shorted conducting bars laid into magnetic material in a cage structure
  - Wound-rotor: rotor has windings similar to stator, with slip rings used to provide external access to the rotor windings

Induction Machine Overview

- Speed of rotating magnetic field (synchronous speed) depends on number of poles

$$N_s = f_s \frac{120}{p}$$ where N_s is the synchronous speed in RPM, f_s is

the stator electrical frequency (e.g., 60 or 50Hz) and p is the number of poles

- Frequency of induced currents in rotor depends on frequency difference between the rotating magnetic field and the rotor

$$\omega_r = \omega_s - \left(\frac{p}{2} \right) \omega_m$$

where ω_s is the stator electrical frequency, ω_m is mechanical speed, and ω_r is the rotor electrical frequency

Induction Machine Slip

- Key value is slip, s, defined as

$$s = \frac{N_s - N_{act}}{N_s}$$

  wherer N_s is the synchronous speed, and

  N_{act} is the actual speed (in RPM)

- As defined, when operating as a motor an induction machine will have a positive slip, slip is negative when operating as a generator
  - Slip is zero at synchronous speed, a speed at which no rotor current is induced; s=1 at stand still

Basic Induction Machine Model

- A basic (single cage) induction machine circuit model is given below
  - Model is derived in an undergraduate machines class
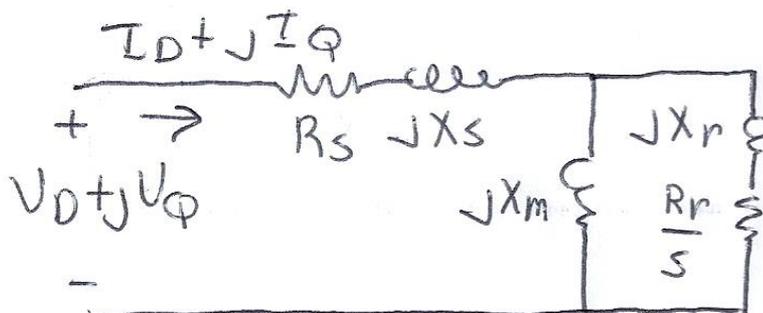
$$\frac{R_r}{s} = R_r + \frac{(1-s)}{s} R_r$$

- Circuit is useful for understanding the static behavior of the machine

- Effective rotor resistance (R_r/s) models the rotor electrical losses (R_r) and the mechanical power $R_r(1-s)/s$

Induction Machine Dynamics

- Expressing all values in per unit (with the base covered later), the mechanical equation for a machine is

$$\frac{ds}{dt} = \frac{1}{2H}\left(T_M - T_E\right)$$

  where H is the inertia constant, T_M is the mechanical torque and T_E is the electrical torque (to be defined)

- Similar to what was done for a synchronous machine, the induction machine can be modeled as an equivalent voltage behind a stator resistance and transient reactance (later we'll introduce, but not derive, the subtransient model)

Induction Machine Dynamics

- Define

$$X' = X_s + \frac{X_r X_m}{X_r + X_m}$$

$$X = X_s + X_m$$

where X' is the apparent reactance seen when the rotor is locked (s=1) and X is the synchronous reactance

- Also define the open circuit time constant

$$T_o' = \frac{\left(X_r + X_m \right)}{\omega_s R_r}$$

15

Induction Machine Dynamics

- Electrically the induction machine is modeled similar to the classical generator model, except here we use the "motor convention" in which $I_D + jI_Q$ is assumed positive into the machine

$$V_D = E_D' + R_s I_D - X' I_Q$$

$$V_Q = E_Q' + R_s I_Q + X' I_D$$

$$\frac{dE_D'}{dt} = \omega_s s E_Q' - \frac{1}{T_o'}\left(E_D' + (X - X')I_Q\right)$$

$$\frac{dE_Q'}{dt} = -\omega_s s E_D' - \frac{1}{T_o'}\left(E_Q' - (X - X')I_D\right)$$

All calculations are done on the network reference frame

Induction Machine Dynamics

- The induction machine electrical torque, T_E, and terminal electrical load, P_E, are then

$$T_E = \frac{\left(E'_D I_D + E'_Q I_Q\right)}{\omega_s}$$

$$P_E = V_D I_D + V_Q I_Q$$

Recall we are using the motor convention so positive P_E represents load

- Similar to a synchronous machine, once the initial values are determined the differential equations are fairly easy to simulate
  - Key initial value needed is the slip

Specifying Induction Machine Parameters

- In transient stability packages induction machine parameters are specified in per unit
    - If unit is modeled as a generator in the power flow (such as CIMTR1 or GENWRI) then use the generator's MVA base (as with synchronous machines)
    - With loads it is more complicated.
        - Sometimes an explicit MVA base is specified. If so, then use this value. But this can be cumbersome since often the same per unit machine values are used for many loads
        - The default is to use the MW value for the load, often scaled by a multiplier (say 1.25)

Determining the Initial Values

- To determine the initial values, it is important to recognize that for a fixed terminal voltage there is only one independent value: the slip, s

  – For a fixed slip, the model is just a simple circuit with resistances and reactances

- The initial slip is chosen to match the power flow real power value. Then to match the reactive power value (for either a load or a generator), the approach is to add a shunt capacitor in parallel with the induction machine

- We'll first consider torque-speed curves, then return to determining the initial slip
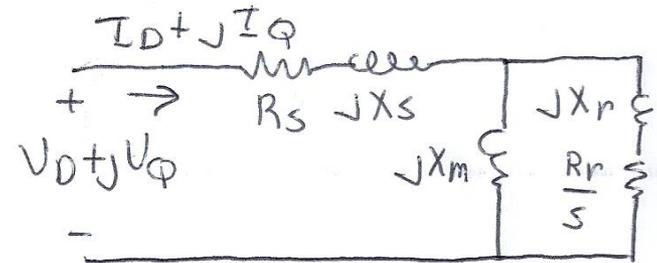
Torque-Speed Curves

- To help understand the behavior of an induction machine it is useful to plot various values as a function of speed (or equivalently, slip)
  – Solve the equivalent circuit for a specified terminal voltage, and varying values of slip
  – Plot results
  – Recall torque times speed = power
    - Here speed is the rotor speed
  – When using per unit, the per unit speed is just 1-s

$$P_E = T_E \left(1 - s \right)$$

Induction Motor Example

- Assume the below 60 Hz system, with the entire load modeled as a single cage induction motor with per unit values on a 125 MVA base of H=1.0, R_s=0.01, X_s=0.06, X_m=4.0, R_r=0.03, X_r=0.04
 - In the CIM5 model $R_1=R_r$ and $X_1=X_r$

Signal Generator

Bus 1

Bus 2

CIM5

slack

X=0.1

1.000 pu
0.00 Deg

0.995 pu
-5.77 Deg

100 MW

0 Mvar
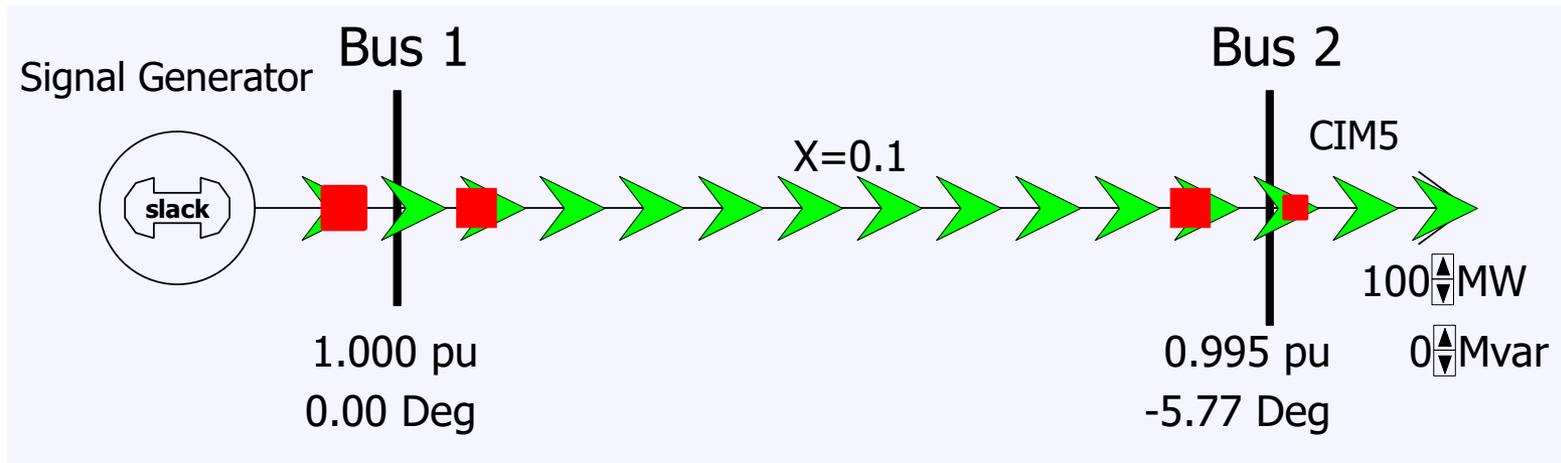
PowerWorld case B2_IndMotor

Induction Motor Example

- With a terminal voltage of $0.995\angle 0°$ we can solve the circuit for specified values of s

- The input impedance and current are
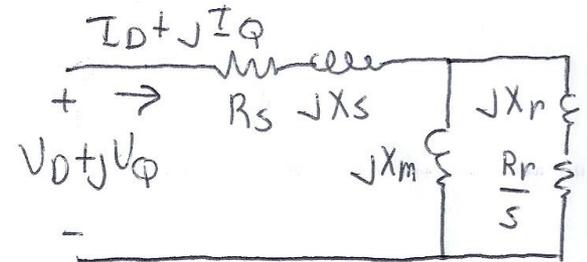
$$Z_{in} = (R_s + jX_s) + \cfrac{jX_m\left(\cfrac{R_r}{s} + jX_r\right)}{\cfrac{R_r}{s} + j\left(X_r + X_m\right)}, \qquad \bar{I} = \frac{\bar{V}}{Z_{in}} = \frac{0.995\angle 0°}{Z_{in}}$$

- Then with s=1 we get

Note, values are per unit on a 125 MVA base

$$\bar{I} = \frac{0.995}{0.0394 + j0.0998} = 3.404 - j8.624 \rightarrow S = 3.387 + j8.581$$

Induction Motor Example

- PowerWorld allows for display of the variation in various induction machine values with respect to speed
  - Right click on load, select Load Information Dialog, Stability
  - On bottom of display click Show Torque Speed Dialog
  - Adjust the terminal voltage and pu scalar as desired; set v=0.995 and the pu scalar to 1.0 to show values on the 125 MVA base used in the previous solution
  - Right click on column and select Set/Toggle/Columns, Plot Column to plot the column

Induction Motor Example Torque-Speed Curves

- The below graph shows the torque-speed curve for this induction machine; note the high reactive power consumption on starting (which is why the lights may dim when starting a cloth dryer!)
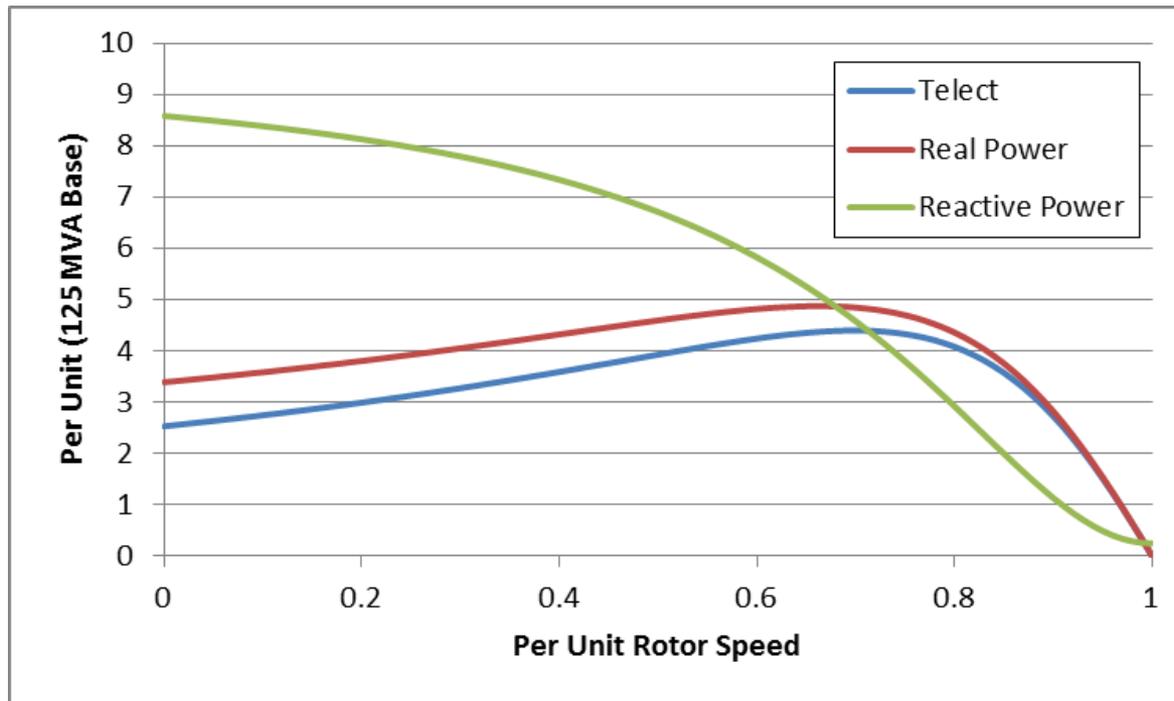
From the graph you can see with a 100 MW load (0.8 pu on the 125 MW base), the slip is about 0.025

Calculating the Initial Slip

- One way to calculate the initial slip is to just solve the below five equations for five unknowns (s, I_D, I_Q, E'_D, E'_Q) with P_E, V_D and V_Q inputs

$$P_E = V_D I_D + V_Q I_Q$$

$$V_D = E'_D + R_s I_D - X' I_Q$$

$$V_Q = E'_Q + R_s I_Q + X' I_D$$

$$\frac{dE'_D}{dt} = 0 = \omega_s s E'_Q - \frac{1}{T'_o}\left(E'_D + (X - X') I_Q\right)$$

$$\frac{dE'_Q}{dt} = 0 = \omega_s s E'_Q - \frac{1}{T'_o}\left(E'_Q - (X - X') I_D\right)$$

These are nonlinear equations that can have multiple solutions so use Newton's method, with an initial guess of s small (say 0.01)

Initial slip in example is 0.0251

25

Double Cage Induction Machines

- In the design of induction machines, there are various tradeoffs, such as between starting torque (obviously one needs enough to start) and operating efficiency
  - The highest efficiency possible is 1-slip, so operating at low slip is desirable
- A common way to achieve high starting torque with good operating efficiency is to use a double cage design
  - E.g., the rotor has two embedded squirrel cages, one with a high R and lower X for starting, and one with lower R and higher X for running
  - Modeled by extending our model by having two rotor circuits in parallel; add subtransient values X'' and T''_o

Example Double Cage Model

- Double cage rotors are modeled by adding two additional differential equations
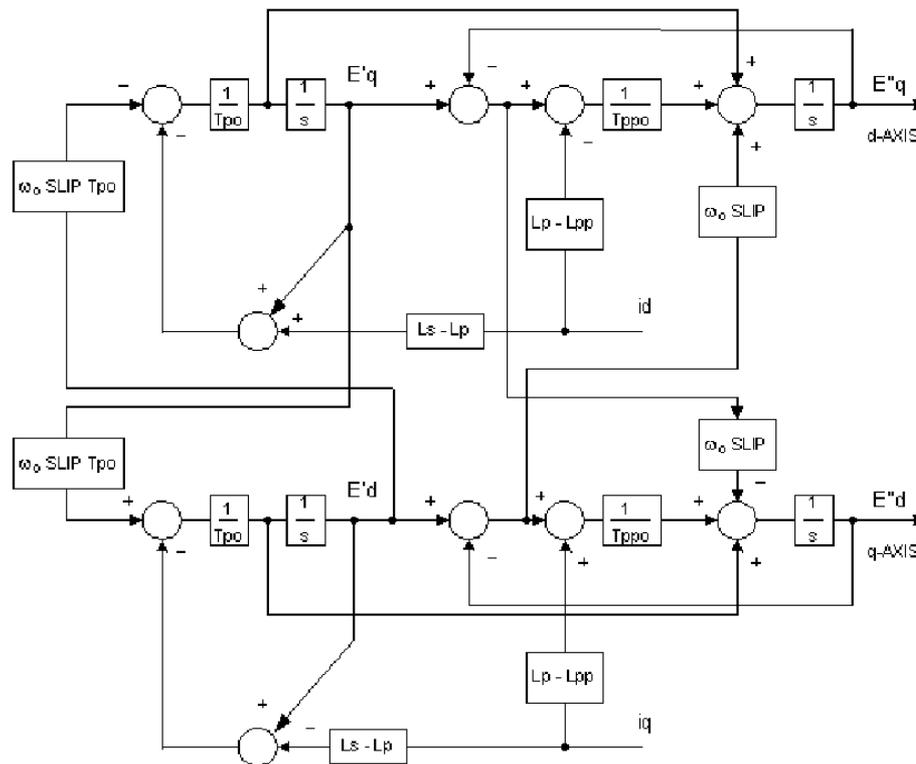


Some models also include saturation, a topic that we will skip

Image source: PSLF Manual, version 18.1_02; MotorW

Double Cage Induction Motor Model

- The previous example can be extended to model a double cage rotor by setting $R_2=0.01$, $X_2=0.08$
  - The below graph shows the modified curves, notice the increase in the slope by s=0, meaning it is operating with higher efficiency (s=0.0063 now!)
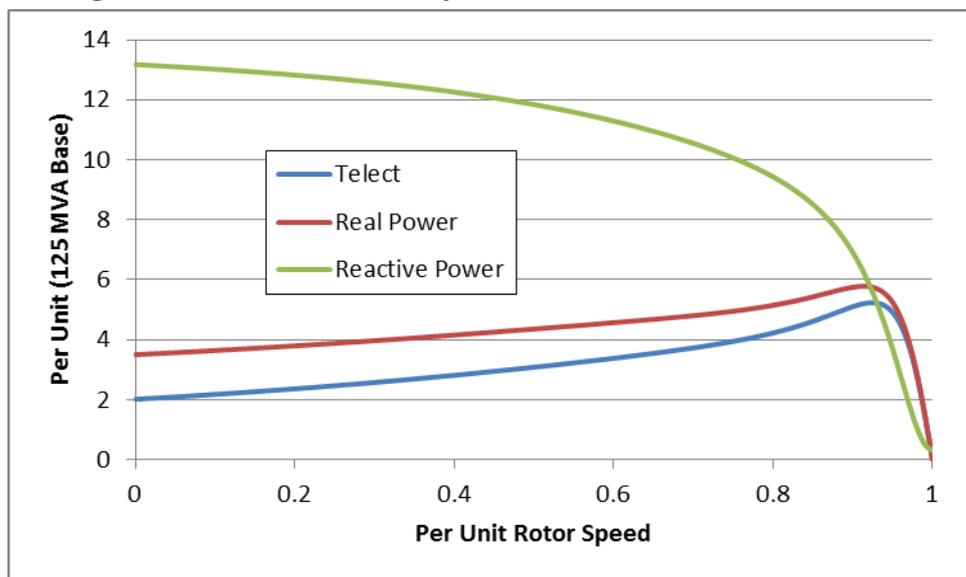
The additional winding does result in lower initial impedance and hence a higher starting reactive power

PowerWorld case B2_IndMotor_DoubleCage

Induction Motor Mechanical Load

- An induction motor is operating in steady-state when the electrical torque is equal to the mechanical torque

- Mechanical torque depends on the type of load
  - Usually specified as function of speed, $T_M = T_{base}(\omega_r)^m$
  - Torque of fans and pumps varies with the square of the speed, conveyors and hoists tend to have a constant torque

- Total power supplied to load is equal to torque times speed
  - Hence the exponent is m+1, with $P_M = P_{base}(\omega_r)^m$

Induction Motor Classes

- Four major classes of induction motors, based on application. Key values are starting torque, pull-out torque, full-load torque, and starting current
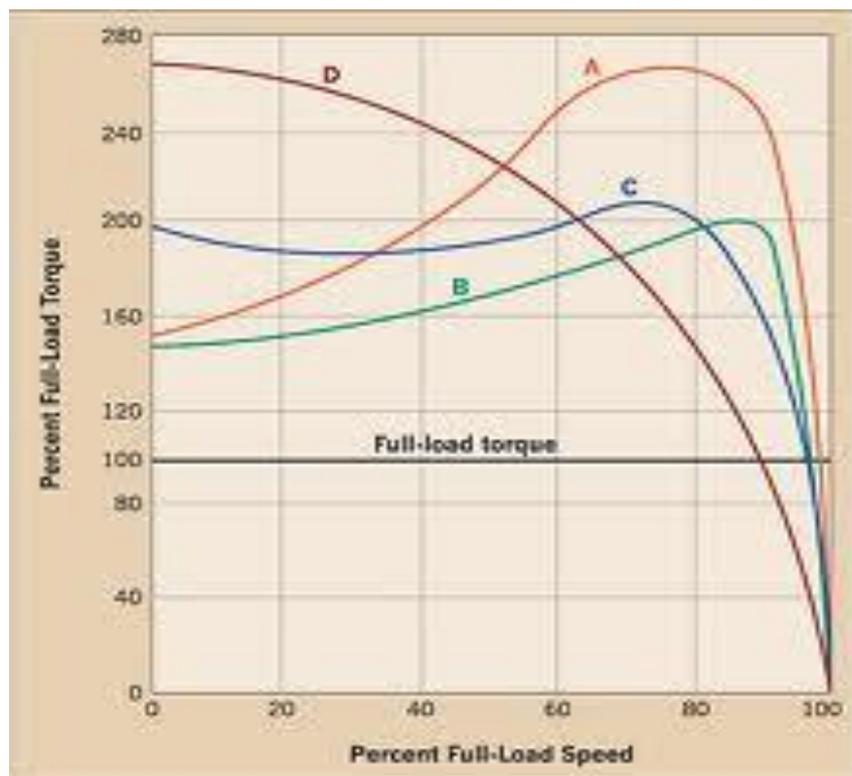
In steady-state the motor will operate on the right side of the curve at the point at which the electrical torque matches the mechanical torque

Image source: ecmweb.com/motors/understanding-induction-motor-nameplate-information