ECEN 667
Power System Stability

Lecture 9: Synchronous Machine Models

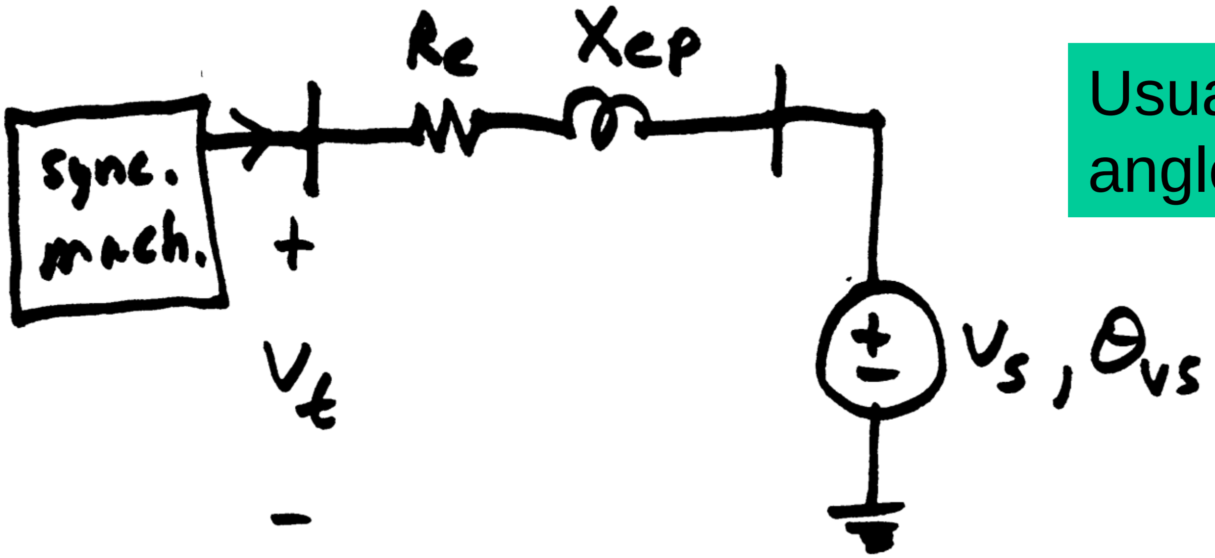
Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University, overbye@tamu.edu

TEXAS A&M UNIVERSITY

Announcements

- Read Chapter 5 and Appendix A
- Homework 3 is posted, due on Thursday Oct 5
- Midterm exam is Oct 17 in class; closed book, closed notes, one 8.5 by 11 inch hand written notesheet allowed; calculators allowed

Chapter 5, Single Machine, Infinite Bus System (SMIB)



Usually infinite bus angle, θ_{vs}, is zero

Book introduces new variables by combining machine values with line values

$$\psi_{de} = \psi_d + \psi_{ed}$$

$$X_{de} = X_d + X_{ep}$$

$$R_{se} = R_s + R_e$$

etc

Introduce New Constants

$$\omega_t = T_s(\omega - \omega_s)$$

"Transient Speed"

$$T_s = \sqrt{\frac{2H}{\omega_s}}$$

Mechanical time constant

$$\varepsilon = \frac{1}{\omega_s}$$

A small parameter

We are ignoring the exciter and governor for now; they will be covered in much more detail later

Stator Flux Differential Equations

$$\varepsilon \frac{d\psi_{de}}{dt} = R_{se} I_d + \left(1 + \frac{\varepsilon}{T_s} \omega_t \right) \psi_{qe} + V_s \sin\left(\delta - \theta_{vs} \right)$$

$$\varepsilon \frac{d\psi_{qe}}{dt} = R_{se} I_q - \left(1 + \frac{\varepsilon}{T_s} \omega_t \right) \psi_{de} + V_s \cos\left(\delta - \theta_{vs} \right)$$

$$\varepsilon \frac{d\psi_{oe}}{dt} = R_{se} I_o$$

Elimination of Stator Transients

- If we assume the stator flux equations are much faster than the remaining equations, then letting ε go to zero allows us to replace the differential equations with algebraic equations

$$0 = R_{se} I_d + \psi_{qe} + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$0 = R_{se} I_q - \psi_{de} + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$0 = R_{se} I_o$$

Impact on Studies

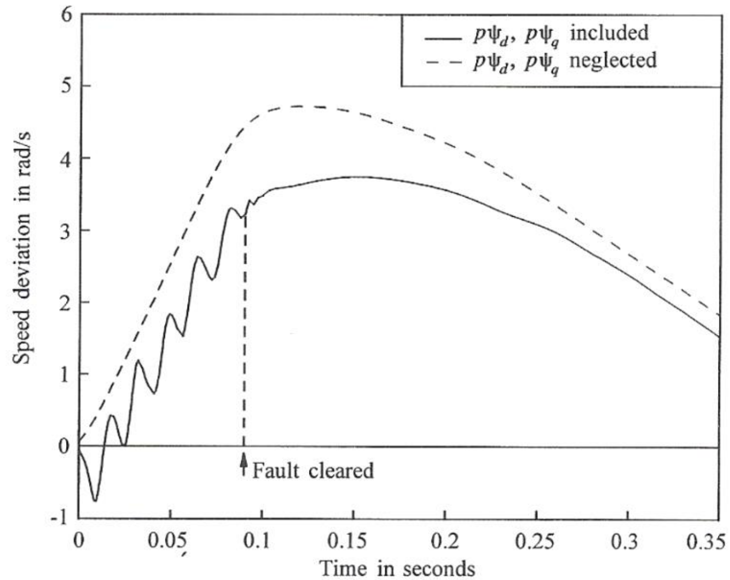


Figure 5.3 Effect of neglecting stator transients on speed deviation

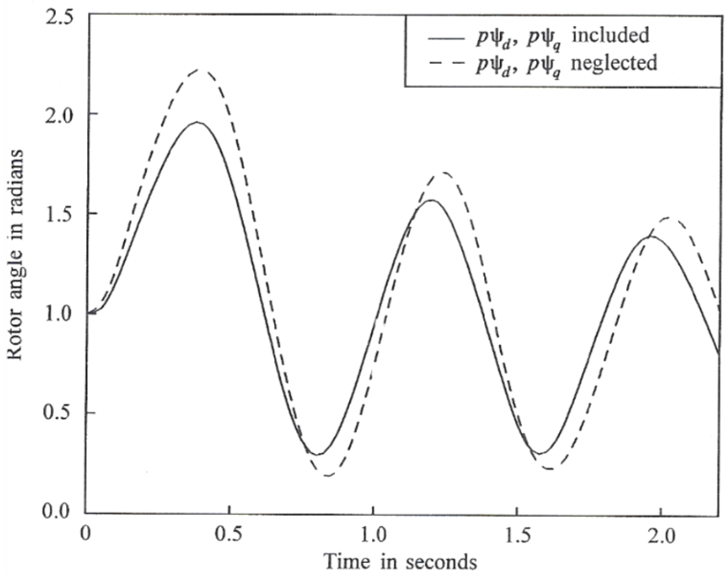


Figure 5.4 Effect of neglecting stator transients on rotor angle swings

Stator transients are not considered in transient stability

Image Source: P. Kundur, *Power System Stability and Control, EPRI, McGraw-Hill, 1994*

Machine Variable Summary

3 fast dynamic states, now eliminated

$\psi_{de}, \psi_{qe}, \psi_{oe}$

7 not so fast dynamic states

$$E'_q, \psi_{1d}, E'_d, \psi_{2q}, \delta, \omega_t E_{fd}$$

8 algebraic states

$$I_d, I_q, I_o, V_d, V_q, V_t, \psi_{ed}, \psi_{eq}$$

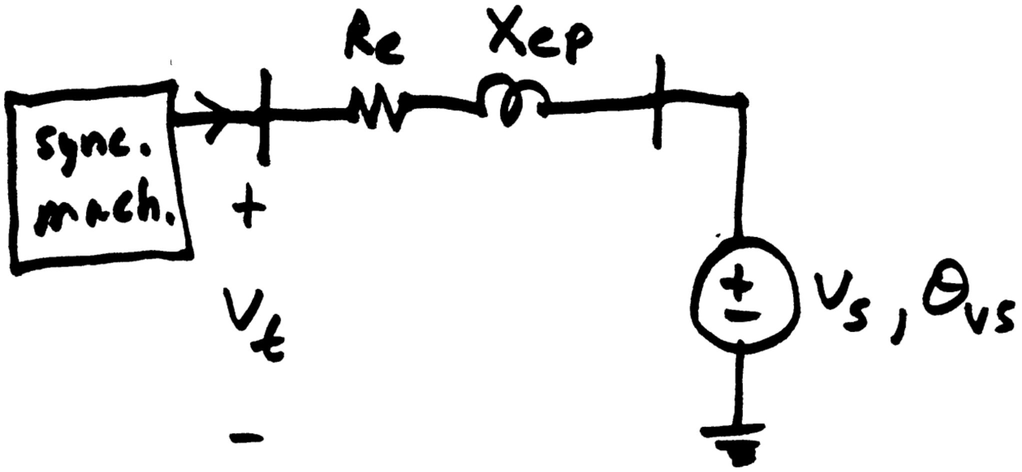
We'll get to the exciter and governor shortly

Network Expressions

$$V_t = \sqrt{V_d^2 + V_q^2}$$

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos\left(\delta - \theta_{vs}\right)$$



Network Expressions

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos\left(\delta - \theta_{vs}\right)$$

These two equations can be written as one complex equation.

$$\left(V_d + jV_q\right) e^{j(\delta - \pi/2)} = \left(R_e + jX_{ep}\right)\left(I_d + jI_q\right) e^{j(\delta - \pi/2)}$$

$$+ V_s e^{j\theta_{vs}}$$

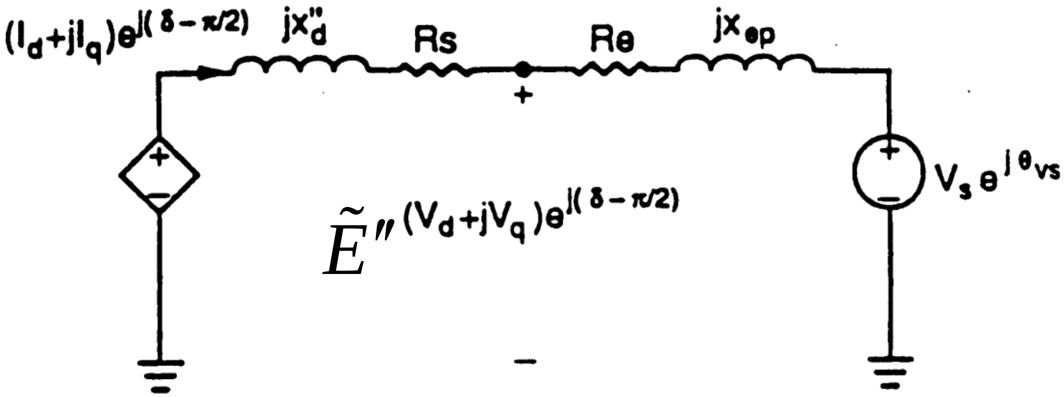
Stator Flux Expressions

$$\psi_{de} = -X''_{de} I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$\psi_{qe} = -X''_{qe} I_q - \frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$\psi_{oe} = -X_{oe} I_o$$

Subtransient Algebraic Circuit



$$\tilde{E}'' = \left[\left(\frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d - \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q} + \left(X''_q - X''_d\right) I_q \right) \right.$$

$$\left. + j \left(\frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d} \right) \right] e^{j(\delta - \pi/2)}$$

Network Reference Frame

- In transient stability the initial generator values are set from a power flow solution, which has the terminal voltage and power injection
  - Current injection is just conjugate of Power/Voltage
- These values are on the network reference frame, with the angle given by the slack bus angle

$$\bar{V}_j = V_{r,j} + jV_{i,j} \quad \text{In book } \bar{V}_i = V_{Di} + jV_{Qi}$$

- Voltages at bus j converted to d-q reference by

$$\begin{bmatrix} V_{d,j} \\ V_{q,j} \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_{r,j} \\ V_{i,j} \end{bmatrix} \quad \begin{bmatrix} V_{r,j} \\ V_{i,j} \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_{d,j} \\ V_{q,j} \end{bmatrix}$$

Similar for current; see book 7.24, 7.25

Network Reference Frame

- Issue of calculating δ, which is key, will be considered for each model
- Starting point is the per unit stator voltages (3.215 and 3.216 from the book)

$$V_d = -\psi_q \omega - R_s I_d$$

$$V_q = \psi_d \omega - R_s I_q$$

Equivalently, $\left(\mathrm{V_d} + \mathrm{jV_q}\right) + R_s\left(I_d + \mathrm{jI_q}\right) = \omega\left(-\psi_q + j\psi_d\right)$

- Sometimes the scaling of the flux by the speed is neglected, but this can have a major solution impact
- In per unit the initial speed is unity

Simplified Machine Models

- Often more simplified models were used to represent synchronous machines
- These simplifications are becoming much less common but they are still used in some situations and can be helpful for understanding generator behavior
- Next several slides go through how these models can be simplified, then we'll cover the standard industrial models

Two-Axis Model

- If we assume the damper winding dynamics are sufficiently fast, then T''_{do} and T''_{qo} go to zero, so there is an integral manifold for their dynamic states

$$\psi_{1d} = E'_q - \left(X'_d - X_{\ell s} \right) I_d$$

$$\psi_{2q} = -E'_d - \left(X'_q - X_{\ell s} \right) I_q$$

Two-Axis Model

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right)I_d = 0$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)\times$$

$$\left[I_d - \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left(\psi_{1d} + \left(X'_d - X_{\ell s}\right)I_d - E'_q\right)\right] + E_{fd}$$

Note this term becomes zero

Which can be simplified to

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)I_d + E_{fd}$$

Two-Axis Model

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right)I_q = 0$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\times$$

$$\left[I_q - \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

Note this term becomes zero

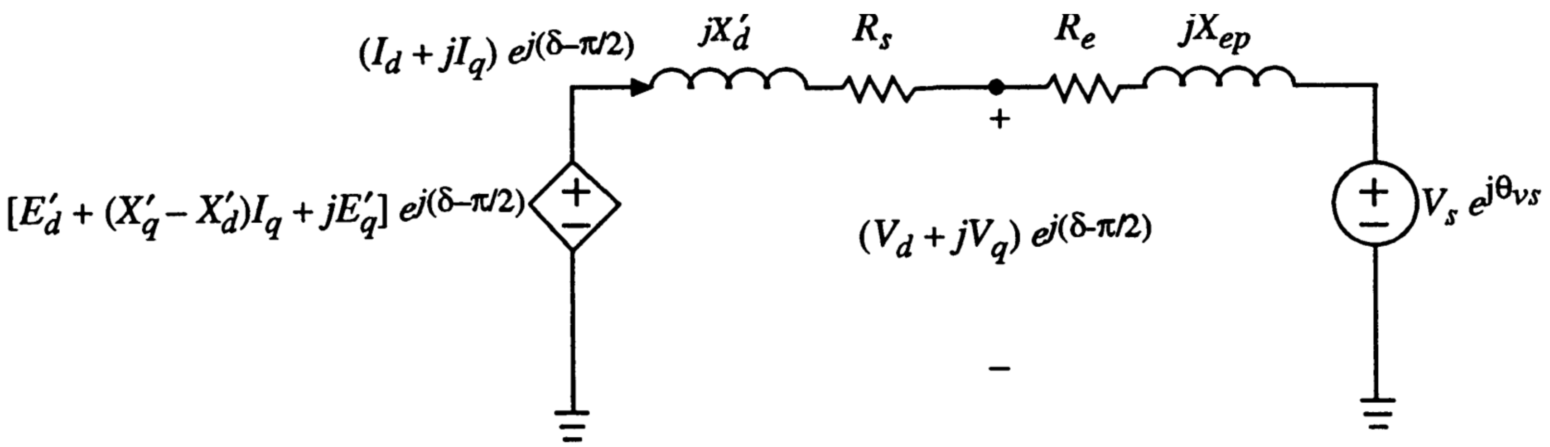
Which can simplified to

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + I_q\left(X_q - X'_q\right)$$

Two-Axis Model

$$0=(R_s+R_e)I_d-(X'_q+X_{ep})I_q-E'_d+V_s\sin(\delta-\theta_{vs})$$

$$0=(R_s+R_e)I_q+(X'_d+X_{ep})I_d-E'_q+V_s\cos(\delta-\theta_{vs})$$



Two-Axis Model

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)I_d + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

No saturation effects are included with this model

Two-Axis Model

$$0 = \left(R_s + R_e\right) I_d - \left(X'_q + X_{ep}\right) I_q - E'_d + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$0 = \left(R_s + R_e\right) I_q + \left(X'_d + X_{ep}\right) I_d - E'_q + V_s \cos\left(\delta - \theta_{vs}\right)$$

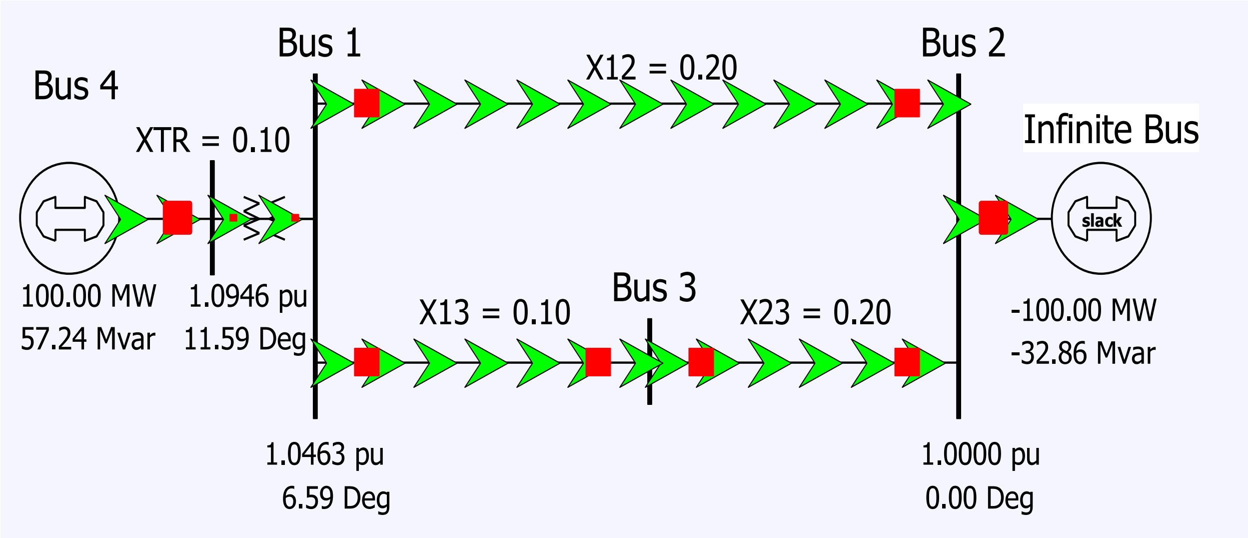
$$V_d = R_e I_d - X_{ep} I_q + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$V_t = \sqrt{V_d^2 + V_q^2}$$

Example (Used for All Models)

- Below example will be used with all models. Assume a 100 MVA base, with gen supplying 1.0+j0.3286 power into infinite bus with unity voltage through network impedance of j0.22
  - Gives current of 1.0 - j0.3286 = $1.0526\angle -18.19°$
  - Generator terminal voltage of 1.072+j0.22 = $1.0946\angle 11.59°$



Sign convention on current is out of the generator is positive

Two-Axis Example

- For the two-axis model assume H = 3.0 per unit-seconds, R_s=0, X_d = 2.1, X_q = 2.0, X'_d= 0.3, X'_q = 0.5, T'_{do} = 7.0, T'_{qo} = 0.75 per unit using the 100 MVA base.
- Solving we get

$$\bar{E} = 1.0946\angle 11.59^\circ + (j2.0)(1.0526\angle -18.19^\circ) = 2.81\angle 52.1^\circ$$

$$\rightarrow \delta = 52.1^\circ$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

Sign convention on current is out of the generator is positive

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

Two-Axis Example

- And

$$E'_q = 0.8326 + (0.3)(0.9909) = 1.130$$

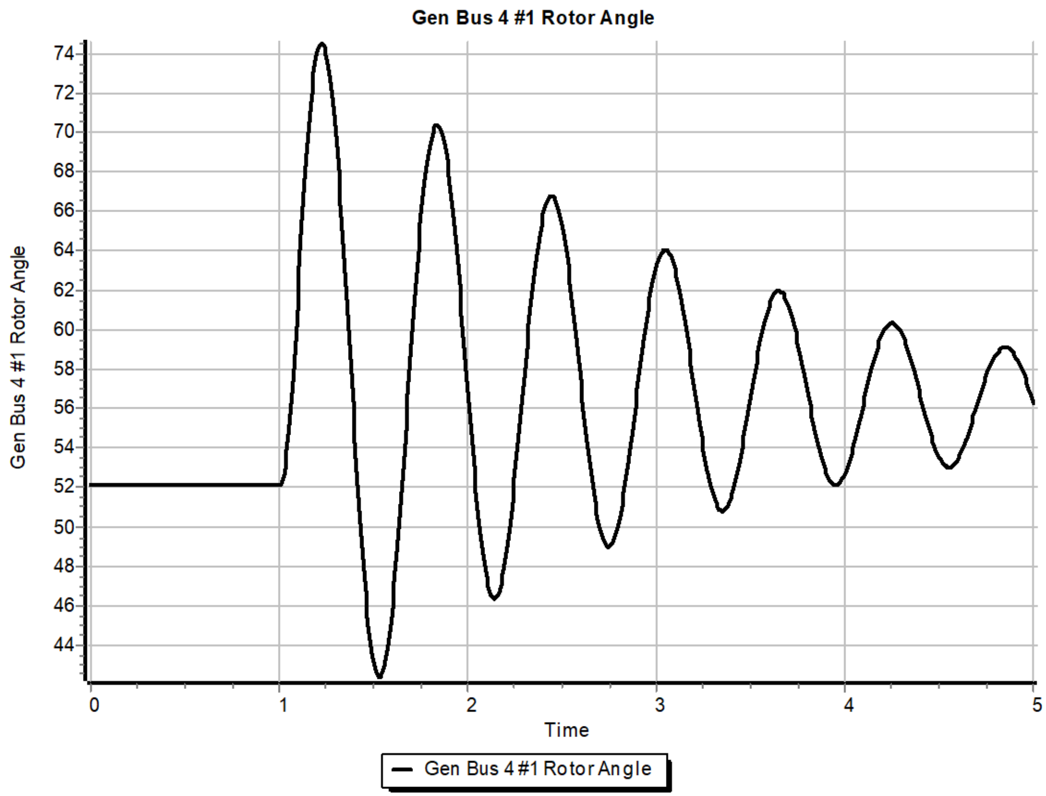
$$E'_d = 0.7107 - (0.5)(0.3553) = 0.533$$

$$E_{fd} = 1.1299 + (2.1 - 0.3)(0.9909) = 2.913$$

Saved as case B4_TwoAxis

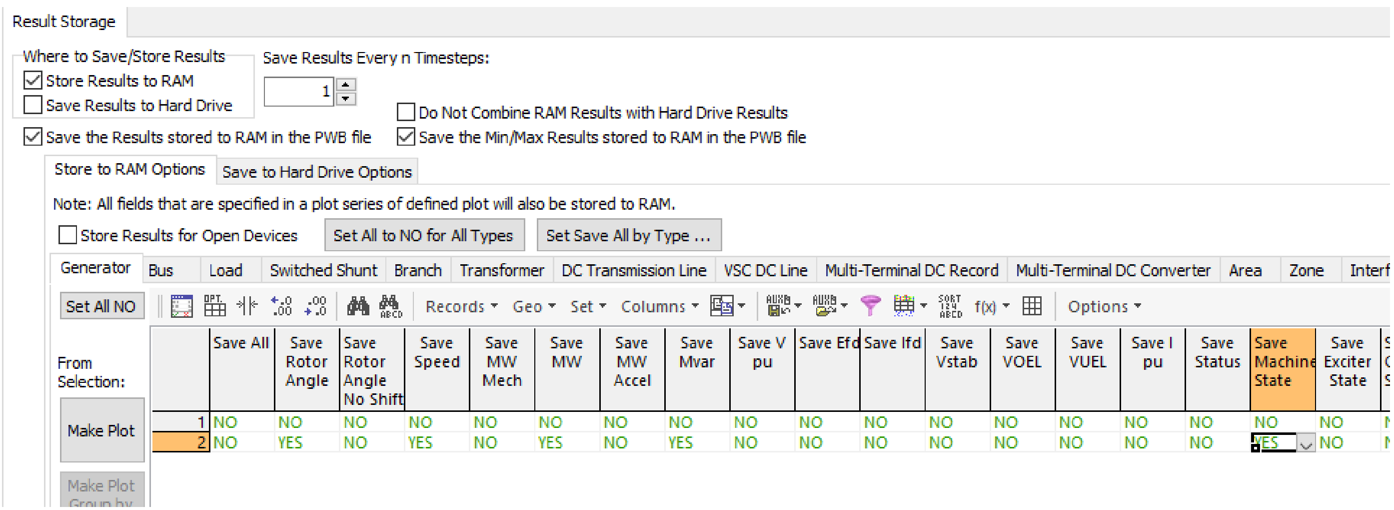
Two-Axis Example

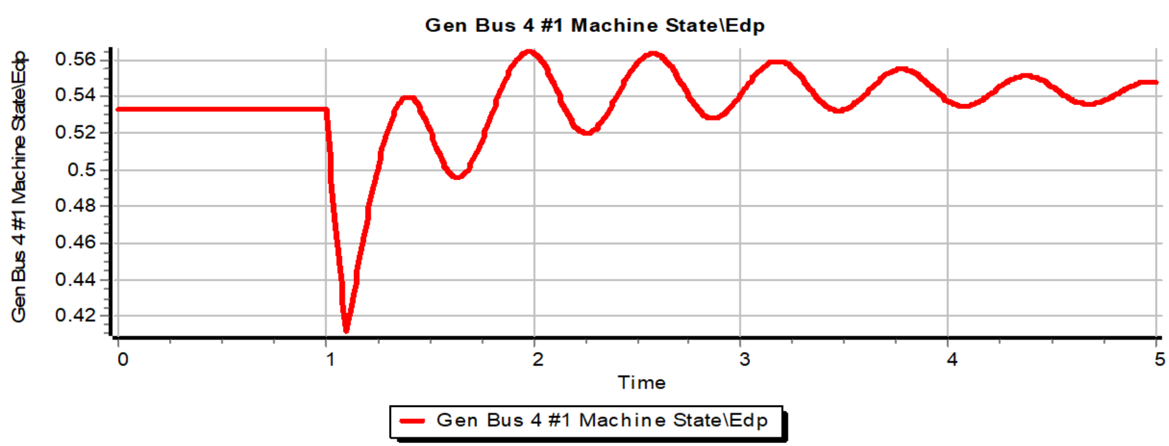
- Assume a fault at bus 3 at time t=1.0, cleared by opening both lines into bus 3 at time t=1.1 seconds



Two-Axis Example

- PowerWorld allows the gen states to be easily stored.





Graph shows variation in E_d'

Flux Decay Model

- If we assume T'_{qo} is sufficiently fast then

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q = 0$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)I_d + E_{fd}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

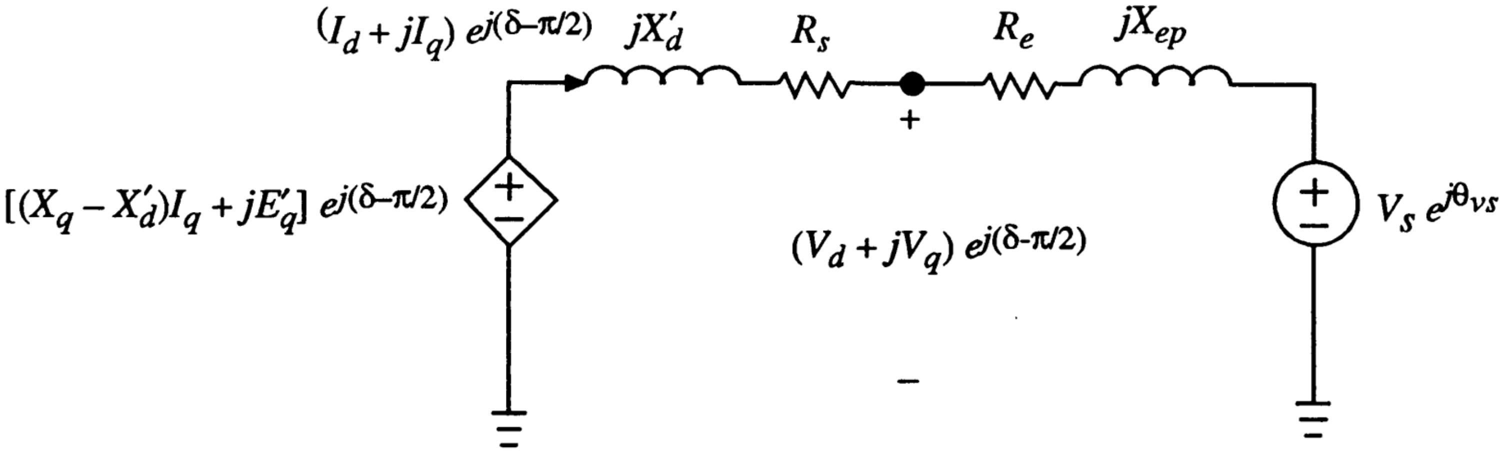
$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

$$= T_M - \left(X_q - X'_q\right)I_q I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

$$= T_M - E'_q I_q - \left(X_q - X'_d\right)I_d I_q - T_{FW}$$

This model assumes that E_d' stays constant. In previous example T_{q0}'=0.75

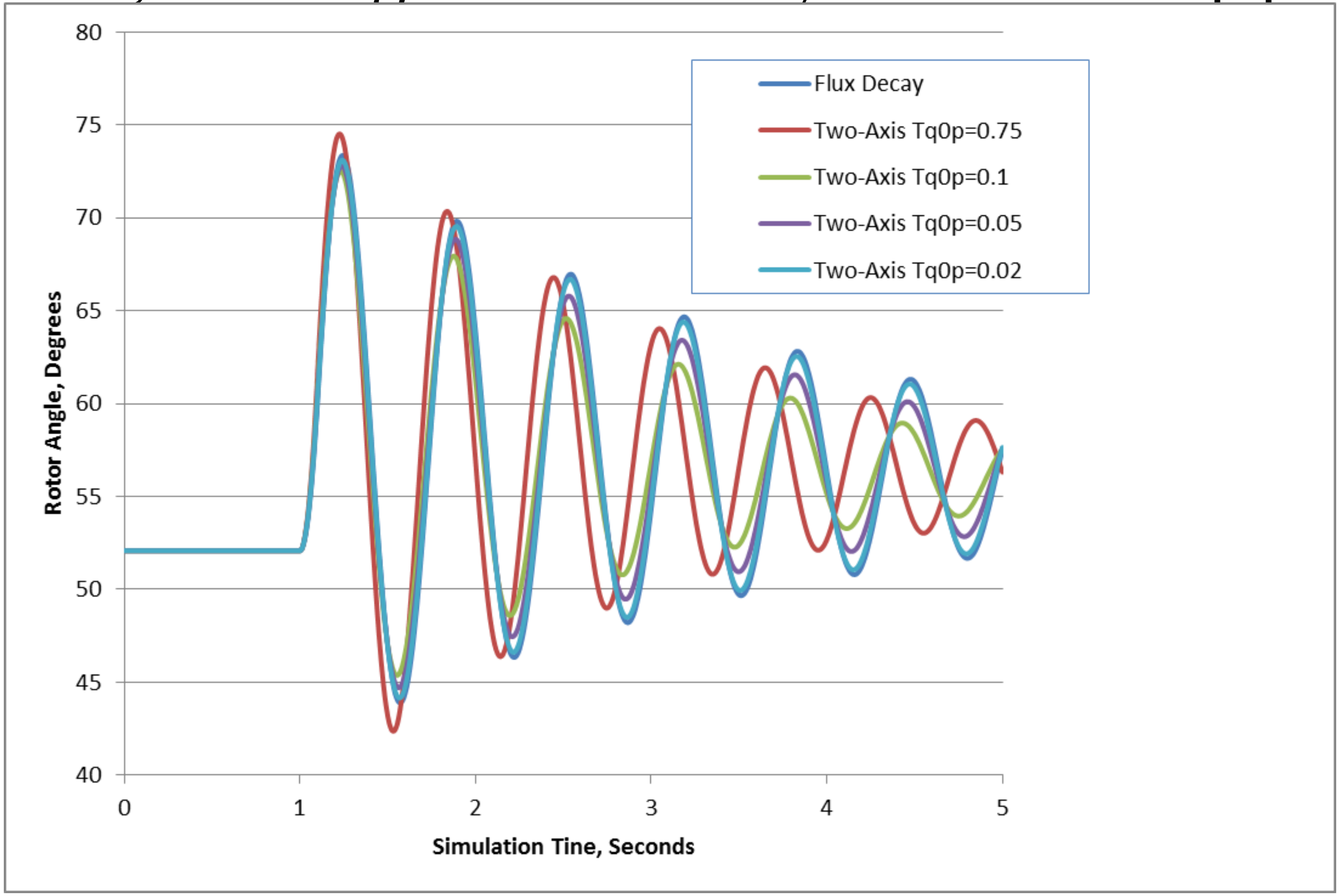
Flux Decay Model



This model is no longer common

Rotor Angle Sensitivity to Tqop

- Graph shows variation in the rotor angle as Tqop is varied, showing the flux decay is same as Tqop = 0



Classical Model

- Has been widely used, but most difficult to justify
- From flux decay model $X_q = X'_d \qquad T'_{do} = \infty$

$$E' = E'_q \qquad \delta'^0 = 0$$

- Or go back to the two-axis model and assume

$$X'_q = X'_d \qquad T'_{do} = \infty \qquad T'_{qo} = \infty$$

$$(E'_q = \text{const} \quad E'_d = \text{const})$$

$$E' = \sqrt{E'^{0\,2}_q + E'^{0\,2}_d}$$

$$\delta'^0 = \tan^{-1}\left(E'^0_q \big/ E'^0_d \right) - \pi/2$$

Classical Model

Or, argue that an integral manifold exists for

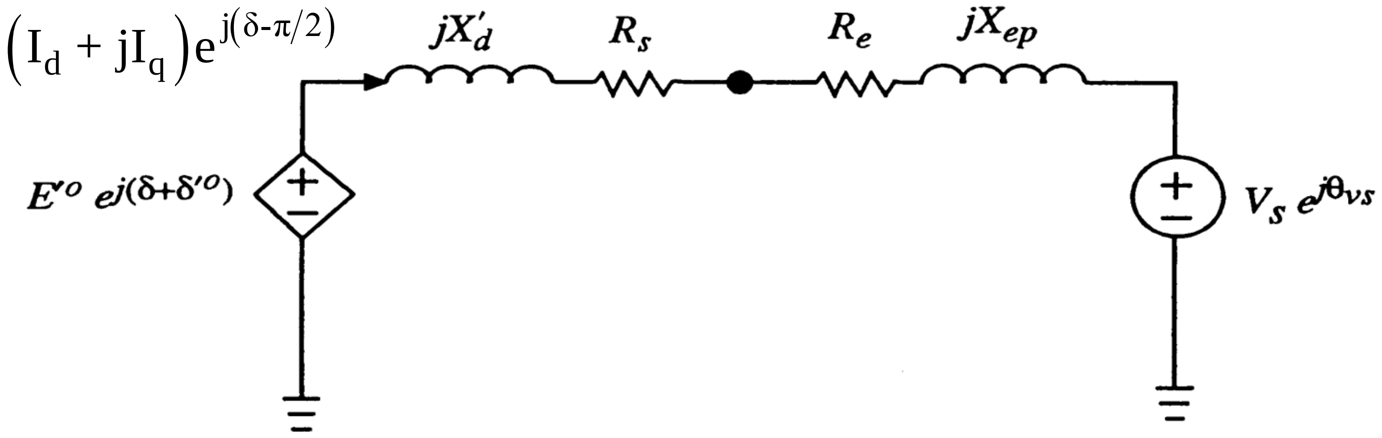
$E'_q, E'_d, E_{fd}, R_f, V_R$ such that $E'_q = \text{const.}$

$$E'_d + \left(X'_q - X'_d\right) I_q = \text{const}$$

$$E'^0 = \sqrt{\left(E'^0_d + \left(X'_q - X'_d\right) I^0_q\right)^2 + E'^{0^2}_q}$$

$$\delta'^0 = \tan^{-1}\left(\ \right) - \pi/2$$

Classical Model



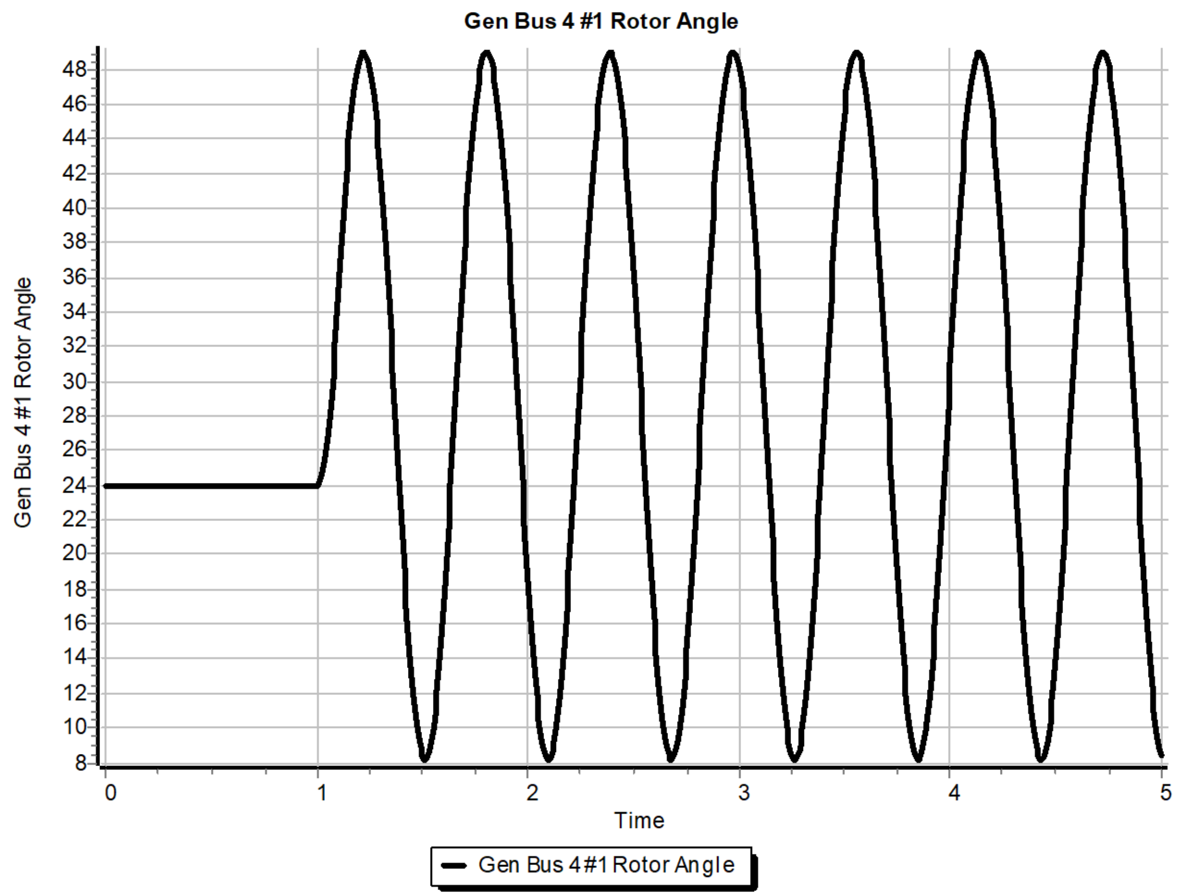
$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_0}\frac{d\omega}{dt} = T_M^0 - \frac{E'^0 V_s}{X'_d + X_{ep}}\sin\left(\delta - \theta_{vs}\right) - T_{FW}$$

This is a pendulum model

Classical Model Response

- Rotor angle variation for same fault as before



Notice that even though the rotor angle is quite different, its initial increase (of about 24 degrees) is similar. However there is no damping

Subtransient Models

- The two-axis model is a transient model
- Essentially all commercial studies now use subtransient models
- First models considered are GENSAL and GENROU, which require $X''_d = X''_q$
- This allows the internal, subtransient voltage to be represented as

$$\bar{E}'' = \bar{V} + (R_s + jX'')\bar{I}$$

$$E''_d + jE''_q = \left(-\psi''_q + j\psi''_d\right)\omega$$

Subtransient Models

- Usually represented by a Norton Injection with

$$I_d + jI_q = \frac{E''_d + jE''_q}{R_s + jX''} = \frac{\left(-\psi''_q + j\psi''_d\right)\omega}{R_s + jX''}$$

- May also be shown as

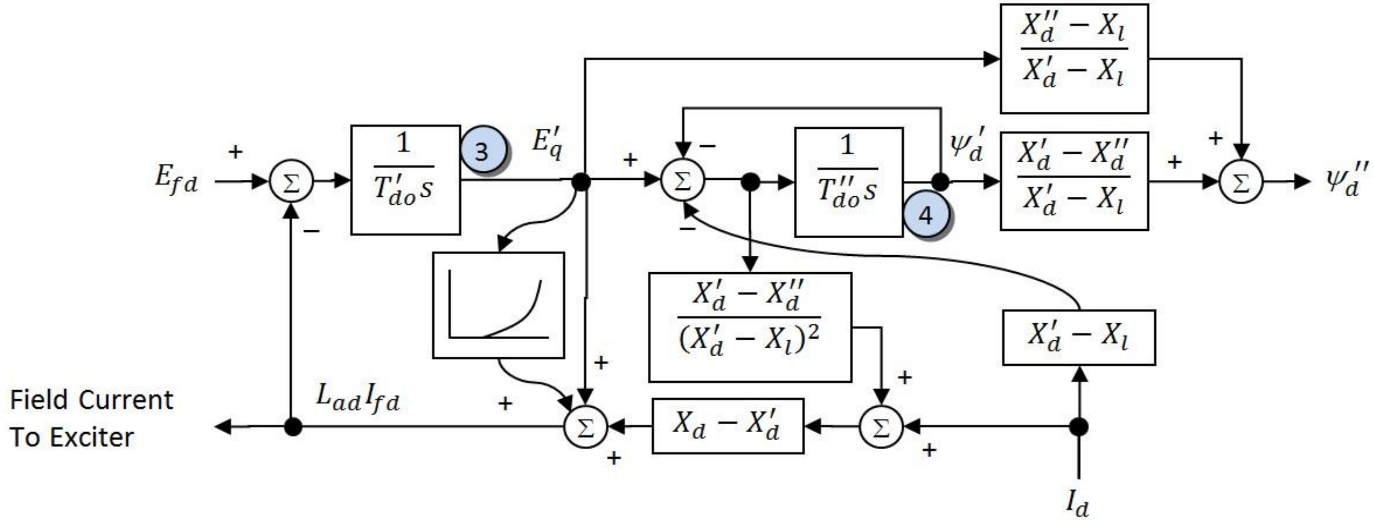
$$-j\left(I_d + jI_q\right) = I_q - jI_d = \frac{-j\left(-\psi''_q + j\psi''_d\right)\omega}{R_s + jX''} = \frac{\left(\psi''_d + j\psi''_q\right)\omega}{R_s + jX''}$$

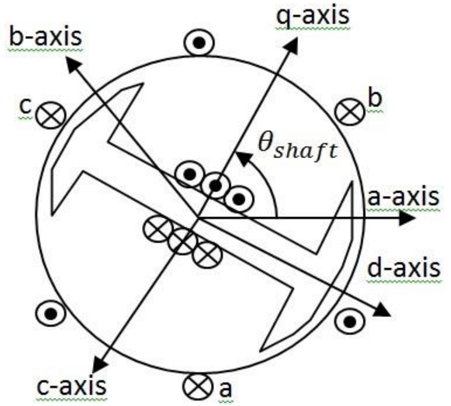
In steady-state $\omega = 1.0$

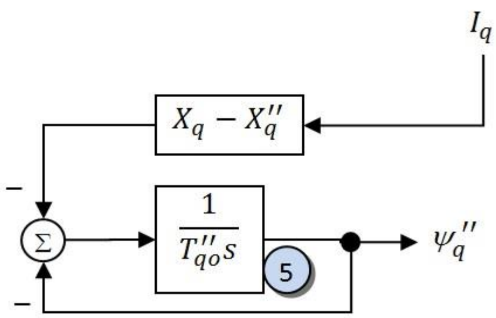
GENSAL

- The GENSAL model has been widely used to model salient pole synchronous generators
  - In the 2010 WECC cases about 1/3 of machine models were GENSAL; in 2013 essentially none are, being replaced by GENTPF or GENTPJ
  - A 2014 series EI model had about 1/3 of its machines models set as GENSAL
- In salient pole models saturation is only assumed to affect the d-axis

GENSAL Block Diagram







A quadratic saturation function is used. For initialization it only impacts the E_{fd} value

GENSAL Example

- Assume same system as before with same common generator parameters: H=3.0, D=0, $R_a = 0$, $X_d = 2.1$, $X_q = 2.0$, $X'_d = 0.3$, $X''_d=X''_q=0.2$, $X_l = 0.13$, $T'_{do} = 7.0$, $T''_{do} = 0.07$, $T''_{qo} =0.07$, S(1.0) =0, and S(1.2) = 0.
- Same terminal conditions as before
  - Current of 1.0-j0.3286 and generator terminal voltage of 1.072+j0.22 = 1.0946 $\angle$11.59 °
- Use same equation to get initial δ

$$|E|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$

$$= 1.072 + j0.22 + (0.0 + j2)(1.0 - j0.3286)$$

$$= 1.729 + j2.22 = 2.81\angle 52.1°$$

Same delta as with the other models

GENSAL Example

- Then as before

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

And $$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

$$\begin{aligned} &\bar{V} + (R_s + jX'')\bar{I} \\ &= 1.072 + j0.22 + (0 + j0.2)(1.0 - j0.3286) \\ &= 1.138 + j0.42 \end{aligned}$$

GENSAL Example

- Giving the initial fluxes (with ω = 1.0)

$$\begin{bmatrix} -\psi''_q \\ \psi''_d \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.138 \\ 0.420 \end{bmatrix} = \begin{bmatrix} 0.6396 \\ 1.031 \end{bmatrix}$$

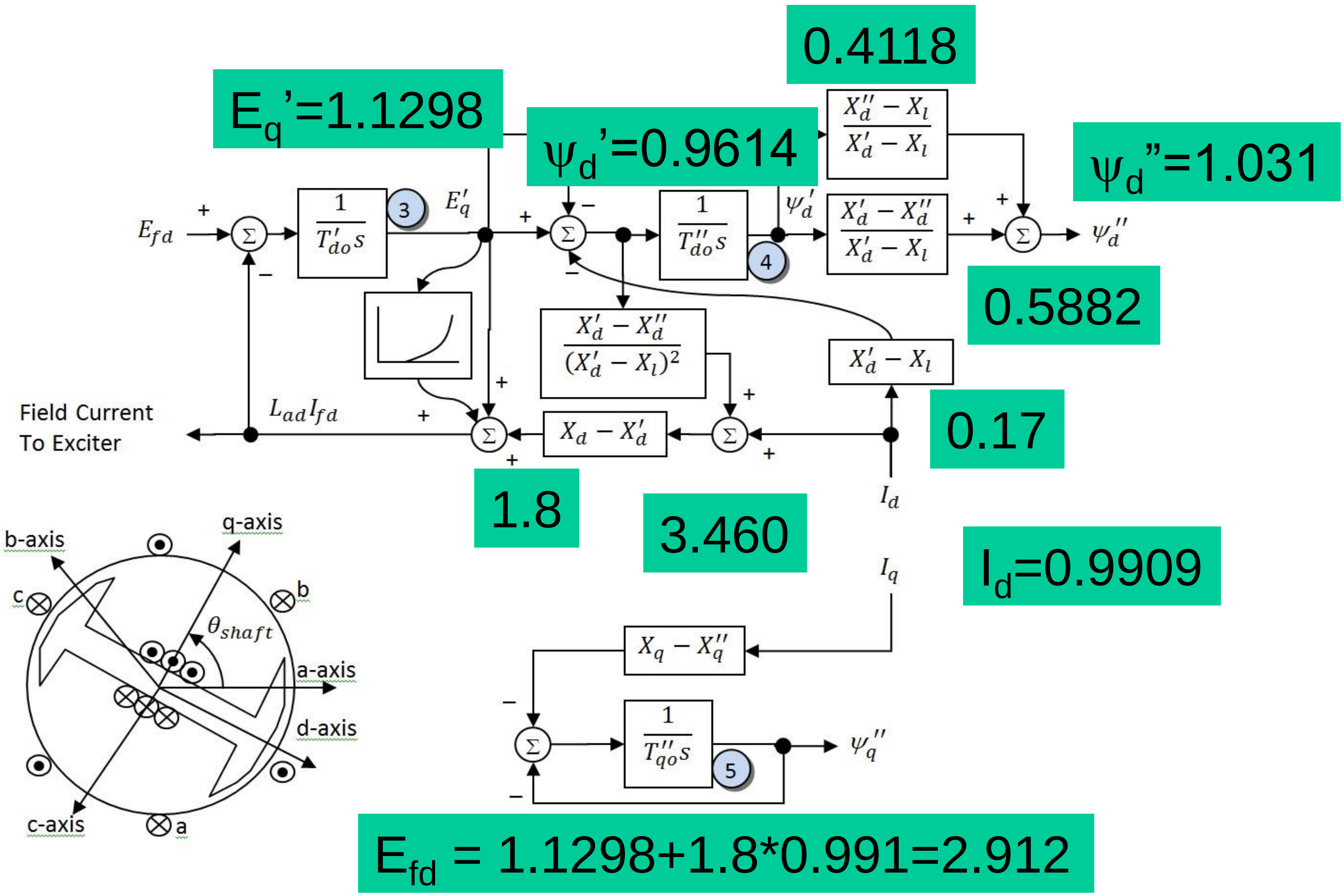
- To get the remaining variables set the differential equations equal to zero, e.g.,

$$\psi''_q = -\left(X_q - X''_q\right) I_q = -(2-0.2)(0.3553) = -0.6396$$

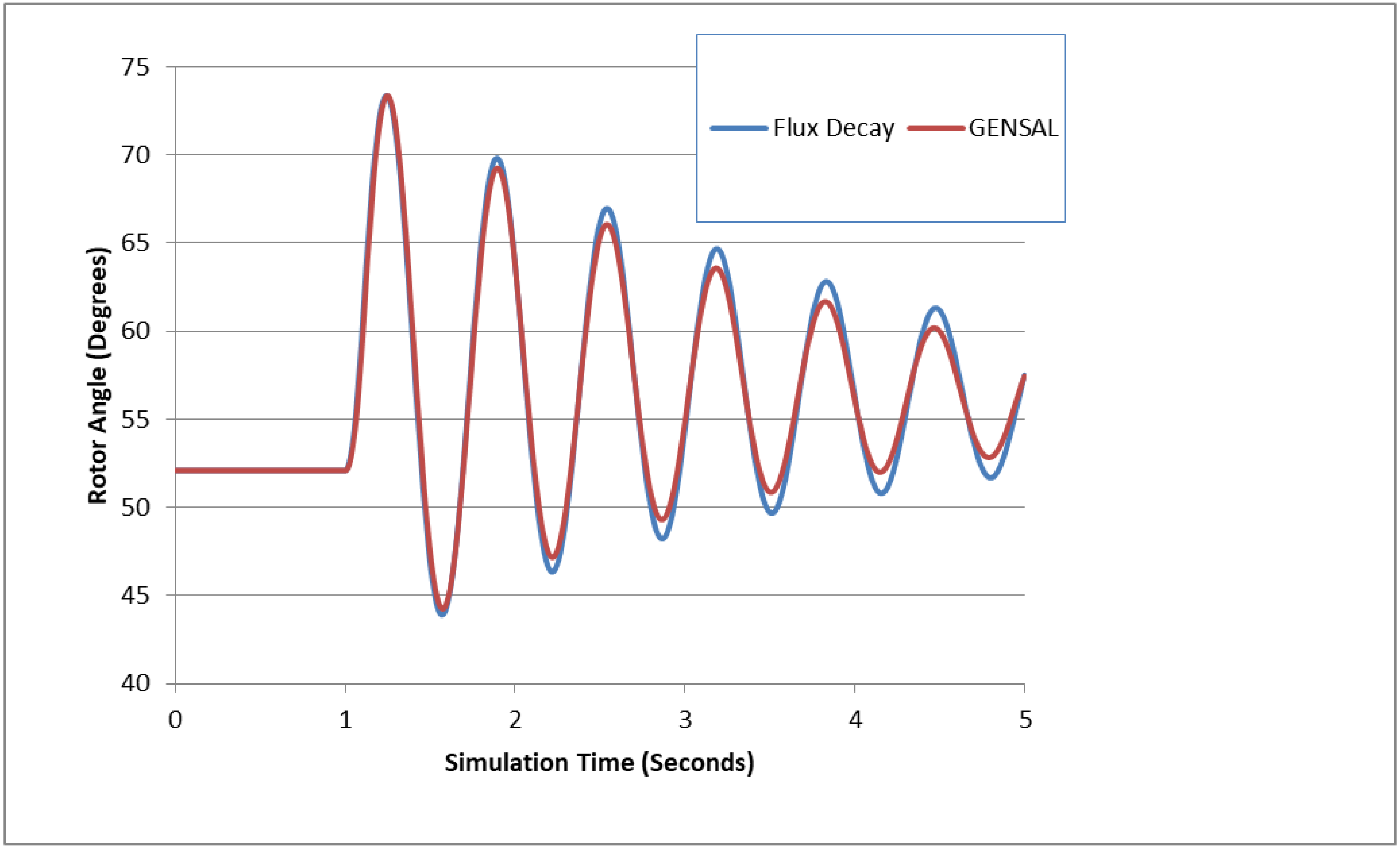
$$E'_q = 1.1298, \quad \psi'_d = 0.9614$$

Solving the d-axis requires solving two linear equations for two unknowns

GENSAL Example



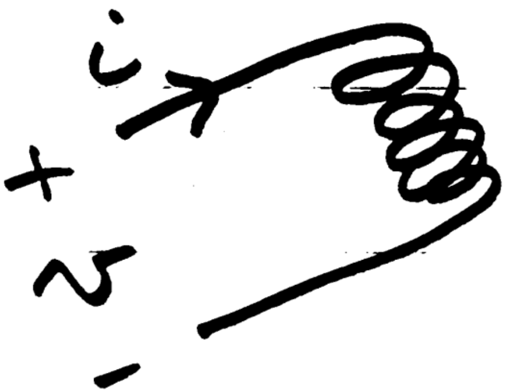
Comparison Between Gensal and Flux Decay



Nonlinear Magnetic Circuits

- Nonlinear magnetic models are needed because magnetic materials tend to saturate; that is, increasingly large amounts of current are needed to increase the flux density

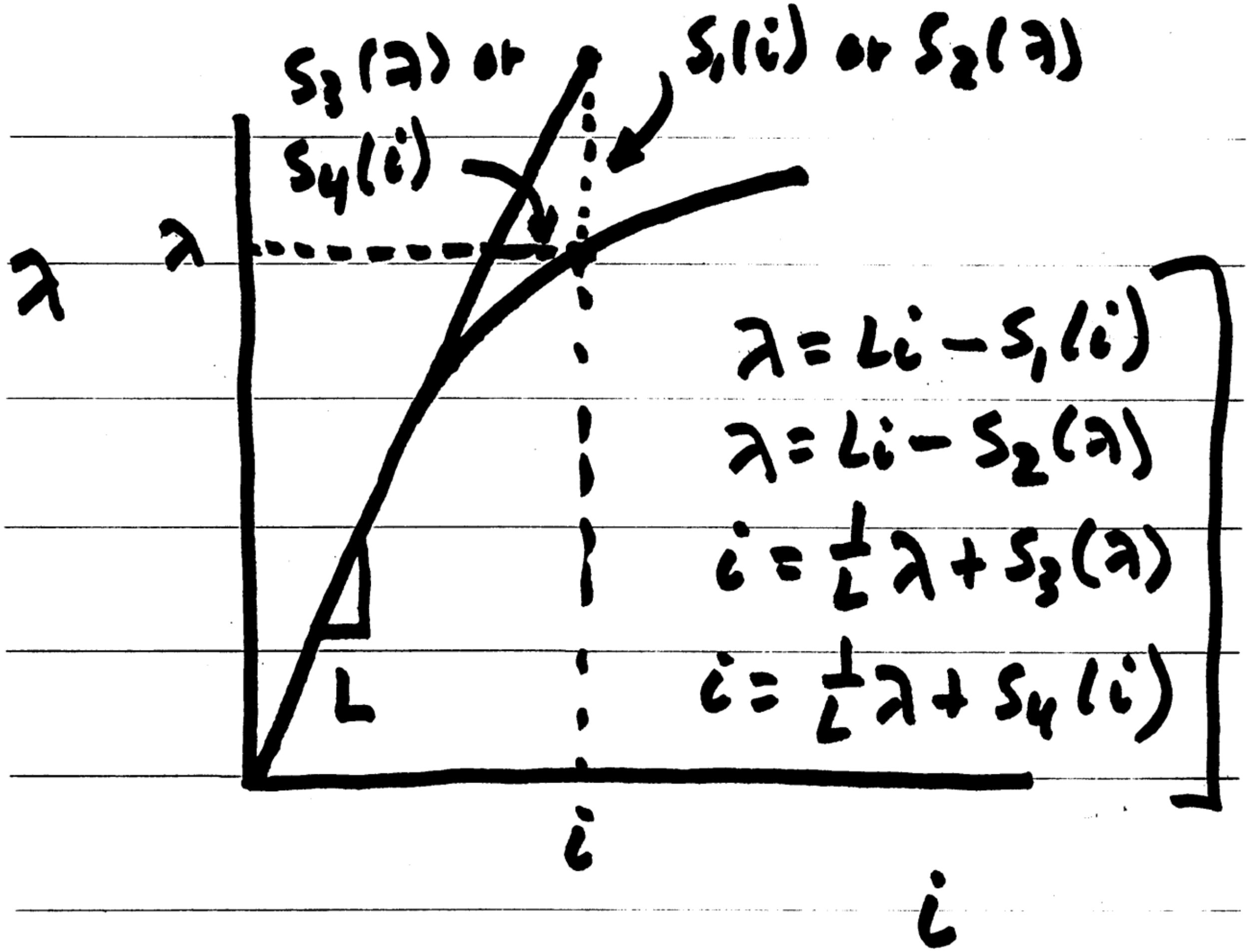
$$R = 0$$



$$v = \frac{d\lambda}{dt} = N\frac{d\phi}{dt}$$

Linear $\lambda = Li$

Saturation


$S_3(\lambda)$ or $S_4(i)$
$S_1(i)$ or $S_2(\lambda)$
λ
λ
L
i
i
$\lambda = Li - S_1(i)$
$\lambda = Li - S_2(\lambda)$
$i = \frac{1}{L}\lambda + S_3(\lambda)$
$i = \frac{1}{L}\lambda + S_4(i)$

Saturation Models

- Many different models exist to represent saturation
  - There is a tradeoff between accuracy and complexity
- Book presents the details of fully considering saturation in Section 3.5
- One simple approach is to replace

$$\frac{dE_q'}{dt} = \frac{1}{T_{do}'}\left(-E_q' - (X_d - X_d')I_d + E_{fd}\right)$$

- With

$$\frac{dE_q'}{dt} = \frac{1}{T_{do}'}\left(-E_q' - (X_d - X_d')I_d - Se(E_q') + E_{fd}\right)$$

Saturation Models

- In steady-state this becomes

$$E_{fd} = E_q' + (X_d - X_d')I_d + Se(E_q')$$

- Hence saturation increases the required E_{fd} to get a desired flux
- Saturation is usually modeled using a quadratic function, with the value of Se specified at two points (often at 1.0 flux and 1.2 flux)

$$Se = B(E_q' - A)^2$$

An alternative model is $Se = \dfrac{B(E_q' - A)^2}{E_q'}$

A and B are determined from the two data points