ECEN 667
Power System Stability

Lecture 23:Measurement Based Modal Analysis, FFT

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Announcements

- Read Chapter 8
- Homework 7 is posted; due on Thursday Nov 30
  - Extended two days due to break
- Final is as per TAMU schedule. That is, Friday Dec 8 from 3 to 5pm

Measurement Based Modal Analysis

- With the advent of large numbers of PMUs, measurement based SSA is increasing used
  - The goal is to determine the damping associated with the dominant oscillatory modes in the system
  - Approaches seek to approximate a sampled signal by a series of exponential functions (usually damped sinusoidals)
- Several techniques are available with Prony analysis the oldest
  - Method, which was developed by Gaspard Riche de Prony, dates to 1795; power system applications from about 1980's
- Here we'll consider a newer alternative, based on the variable projection method

Some Useful References

- J.F. Hauer, C.J. Demeure, and L.L. Scharf, "Initial results in Prony analysis of power system response signals," *IEEE Trans. Power Systems*, vol.5, pp 80-89, Feb 1990
- D.J. Trudnowski, J.M. Johnson, and J.F. Hauer, "Making Prony analysis more accurate using multiple signals," *IEEE Trans. Power Systems*, vol.14, pp.226-231, Feb 1999
- A. Borden, B.C. Lesieutre, J. Gronquist, "Power System Modal Analysis Tool Developed for Industry Use," *Proc. 2013 North American Power Symposium*, Manhattan, KS, Sept. 2013

Variable Projection Method (VPM)

- Idea of all techniques is to approximate a signal, $y_{org}(t)$, by the sum of other, simpler signals (basis functions)
  - Basis functions are usually exponentials, with linear and quadratic functions also added to detrend the signal
  - Properties of the original signal can be quantified from basis function properties (such as frequency and damping)
  - Signal is considered over an interval with t=0 at the beginning
- Approaches work by sampling the original signal $y_{org}(t)$
- Vector **y** consists of m uniformly sampled points from $y_{org}(t)$ at a sampling value of ΔT, starting with t=0, with values y_j for j=1…m
  - Times are then $t_j = (j-1)\Delta T$

Variable Projection Method (VPM)

- At each time point j, where $t_j = (j-1)\Delta T$ the approximation of y_j is

$$\hat{y}_j(\boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}$ is a vector with the real and imaginary

eigenvalue components, with $\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j}$ for

a_i corresponding to a real eigenvalue, and

$\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j} \cos(\alpha_{i+1} t_j)$ and $\phi_{i+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \sin(\alpha_{i+1} t_j)$

for a complex eigenvector value

Variable Projection Method (VPM)

- Error (residual) value at each point j is

$$r_j(t_j, \boldsymbol{\alpha}) = y_j - \hat{y}_j(t_j, \boldsymbol{\alpha})$$

  – α is the vector containing the optimization variables

- Function being minimized is

$$\frac{1}{2}\sum_{j=1}^{m}(y_j - \hat{y}_j(t_j, \boldsymbol{\alpha}))^2 = \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\alpha})\right\|_2^2$$

r(α) is the residual vector

Method iteratively changes α to reduce the minimization function

Variable Projection Method (VPM)

- A key insight of the variable projection method is that $\hat{\mathbf{y}}(\boldsymbol{\alpha}) = \boldsymbol{\Phi}(\boldsymbol{\alpha})\mathbf{b}$
  And then the residual is minimized by selecting
  $\mathbf{b} = \boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\mathbf{y}$
  where $\boldsymbol{\Phi}(\boldsymbol{\alpha})$ is the m by n matrix with values
  $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ if α_{i} corresponds to a real eigenvalue,
  and $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{i+1} t_j)$ and $\Phi_{ji+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{i+1} t_j)$
  for a complex eigenvalue; $t_j = \left(j-1\right)\Delta T$
  Finally, $\boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}$ is the pseudoinverse of $\boldsymbol{\Phi}(\boldsymbol{\alpha})$

Pseudoinverse of a Matrix

- The pseudoinverse of a matrix generalizes concept of a matrix inverse to an m by n matrix, in which m >= n
  - Specifically talking about a Moore-Penrose Matrix Inverse
- Notation for the pseudoinverse of **A** is $\mathbf{A}^+$
- Satisfies $\mathbf{A}\mathbf{A}^+\mathbf{A} = \mathbf{A}$
- If **A** is a square matrix, then $\mathbf{A}^+ = \mathbf{A}^{-1}$
- Quite useful for solving the least squares problem since the least squares solution of $\mathbf{A}\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = \mathbf{A}^+\mathbf{b}$
- Can be calculated using an SVD

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$$

$$\mathbf{A}^+ = \mathbf{V}\boldsymbol{\Sigma}^+\mathbf{U}^T$$

Simple Least Squares Example

- Assume we wish to fix a line ($mx + b = y$) to three data points: (1,1), (2,4), (6,4)
- Two unknowns, m and b; hence $\mathbf{x} = [m \;\; b]^T$
- Setup in form of $\mathbf{Ax} = \mathbf{b}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} \quad \text{so} \;\; \mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 6 & 1 \end{bmatrix}$$

We are trying to select m and b to minimize the error in this overdetermined problem

Simple Least Squares Example

- Doing an economy SVD

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \begin{bmatrix} -0.182 & -0.765 \\ -0.331 & -0.543 \\ -0.926 & 0.345 \end{bmatrix} \begin{bmatrix} 6.559 & 0 \\ 0 & 0.988 \end{bmatrix} \begin{bmatrix} -0.976 & -0.219 \\ 0.219 & -0.976 \end{bmatrix}$$

- Computing the pseudoinverse

$$\mathbf{A}^+ = \mathbf{V}\,\mathbf{\Sigma}^+\mathbf{U}^T = \begin{bmatrix} -0.976 & 0.219 \\ -0.219 & -0.976 \end{bmatrix} \begin{bmatrix} 0.152 & 0 \\ 0 & 1.012 \end{bmatrix} \begin{bmatrix} -0.182 & -0.331 & -0.926 \\ -0.765 & -0.543 & 0.345 \end{bmatrix}$$

$$\mathbf{A}^+ = \mathbf{V}\,\mathbf{\Sigma}^+\mathbf{U}^T = \begin{bmatrix} -0.143 & -0.071 & 0.214 \\ 0.762 & 0.548 & -0.310 \end{bmatrix}$$

In an economy SVD the Σ matrix has dimensions of m by m if m < n or n by n if n < m

Simple Least Squares Example

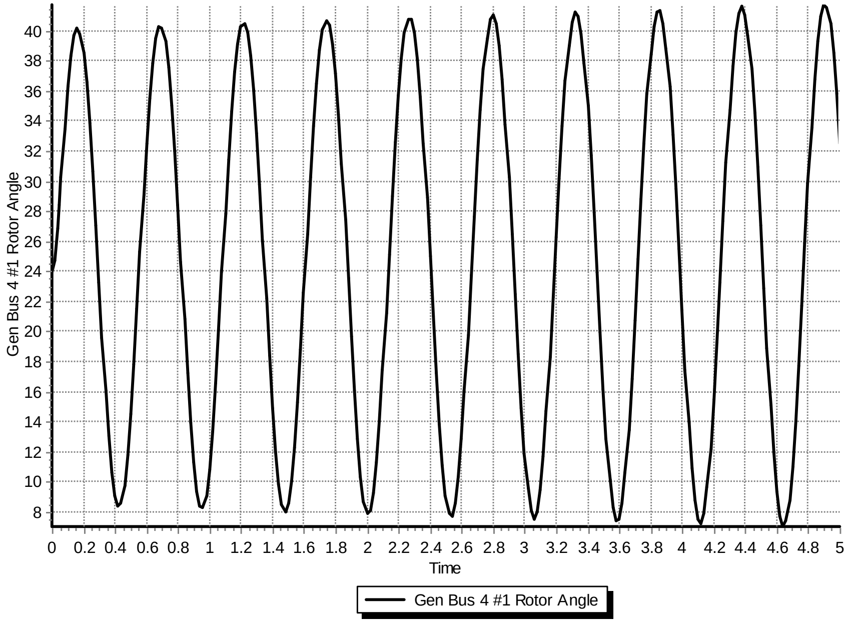
- Computing $\mathbf{x} = [m\ b]^T$ gives

$$\mathbf{A}^{+}\mathbf{b} = \begin{bmatrix} -0.143 & -0.071 & 0.214 \\ 0.762 & 0.548 & -0.310 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.429 \\ 1.71 \end{bmatrix}$$

- With the pseudoinverse approach we immediately see the sensitivity of the elements of **x** to the elements of **b**
  - New values of m and b can be readily calculated if **y** changes
- Computationally the SVD is order m^2n+n^3 (with $n < m$)

VPM Example

- Assume we'd like to determine the characteristics of the below SMIB angle response



The signal itself is given from 0 to 5 seconds; we will be sampling it over a shorter time period

- For simplicity we'll just consider this signal from 1 to 2 seconds, and work with m=6 samples (ΔT=0.2, from 1 to 2 seconds); hence we'll set our t=0 as 1.0 seconds

VPM Example

- Assume we know a good approximation of this signal (over the desired range) is

$$y_{org}(t) = e^{0.047t}\left(-13.58\cos(11.97t) + 8.34\sin(11.97t)\right) + 24.42e^{0.0065t}$$

- Hence the zero error values would be

How we got the initial α will be shown in a few slides

$$\boldsymbol{\alpha} = \begin{bmatrix} 0.047 \\ 11.97 \\ 0.0065 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -13.58 \\ 8.34 \\ 24.42 \end{bmatrix}$$

- With ΔT=0.2, m=6 then

$$\mathbf{y} = \begin{bmatrix} 10.85 \\ 40.23 \\ 14.94 \\ 22.60 \\ 37.13 \\ 7.90 \end{bmatrix}$$

VPM Example

- To verify

First row is t=0, second t=0.2, etc

$$\boldsymbol{\Phi}(\boldsymbol{\alpha}) = \begin{bmatrix} 1 & 0 & 1 \\ -0.741 & 0.685 & 1.001 \\ 0.079 & -1.016 & 1.003 \\ 0.637 & 0.807 & 1.004 \\ -1.026 & -0.161 & 1.005 \\ 0.871 & -0.583 & 1.006 \end{bmatrix}$$

$$\Phi_{2,1}(\boldsymbol{\alpha}) = e^{0.047\times 0.2}\cos(11.97\times 0.2) = -0.741$$

$$\mathbf{b} = \boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\mathbf{y} \rightarrow$$

$$\boldsymbol{\Phi}^{+}(\boldsymbol{\alpha}) = \begin{bmatrix} 0.242 & -0.213 & -0.057 & 0.175 & -0.328 & 0.181 \\ 0.055 & 0.259 & -0.396 & 0.366 & -0.096 & -0.187 \\ 0.135 & 0.206 & 0.156 & 0.159 & 0.207 & 0.134 \end{bmatrix}$$

- Giving $\mathbf{b} = \boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\mathbf{y} = \begin{bmatrix} -13.57 \\ 8.34 \\ 24.43 \end{bmatrix}$

Which matches the known values!

VPM, cont.

- This is an iterative process, requiring an initial guess of $\boldsymbol{\alpha}$, and then a method to update $\boldsymbol{\alpha}$ until the residual vector, **r**, is minimized

$$\min_{\boldsymbol{\alpha}} \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\alpha})\right\|_2^2 = \min_{\boldsymbol{\alpha}} \frac{1}{2}\left\|\left(\mathbf{I} - \boldsymbol{\Phi}(\boldsymbol{\alpha})\boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\right)\mathbf{y}\right\|_2^2$$

  - Solved with a gradient method, with the details on finding the gradient direction given in the Borden, Lesieutre, Gronquist 2013 NAPS paper
  - Iterative until a minimum is reached
- Like any iterative method, its convergence depends on the initial guess, in this case of $\boldsymbol{\alpha}$

VPM: Initial Guess of α

- The initial guesses for $\boldsymbol{\alpha}$ are calculated using a Matrix Pencil method
- First, with m samples, let L=m/2
- Then form a Hankel matrix, **Y** such that

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{L+1} \\ y_2 & y_3 & \cdots & y_{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L} & y_{m-L+1} & \cdots & y_m \end{bmatrix}$$

The computational complexity increases with the cube of the number of measurements!

- And calculate its singular values with an economy SVD

$$\mathbf{Y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$$

VPM: Initial Guess of α

- The ratio of each singular value is then compared to the largest singular value σ_c; retain the ones with a ratio > than a threshold (e.g., 0.16)
  - This determines the modal order, M
  - Assuming $\mathbf{V}$ is ordered by singular values (highest to lowest), let $\mathbf{V}_p$ be then matrix with the first M columns of $\mathbf{V}$
- Then form the matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ such that
  - $\mathbf{V}_1$ is the matrix consisting of all but the last row of $\mathbf{V}_p$
  - $\mathbf{V}_2$ is the matrix consisting of all but the first row of $\mathbf{V}_p$
  - NAPS paper equation is incorrect on this
- Discrete-time poles are found as the generalized eigenvalues of the pair $\{\mathbf{V}_2^T\mathbf{V}_1, \mathbf{V}_1^T\mathbf{V}_1\}$

Generalized Eigenvalues

- Generalized eigenvalue problem for a matrix pair (**A**,**B**) consists of determining values α_k, β_k and $\mathbf{x}_k$ such that

$$\beta_k \mathbf{A}\mathbf{x}_k = \alpha_k \mathbf{B}\mathbf{x}_k$$

If **B** = **I** then this gives the regular eigenvalues

- The generalized eigenvalues are then α_k/β_k
- If **B** is nonsingular than these are the eigenvalues of $\mathbf{B}^{-1}\mathbf{A}$
  - That is the situation here
- These eigenvalues are the discrete-time poles, z_i ,with the modal eigenvalues then

$$\lambda_i = \frac{\ln(z_i)}{\Delta T}$$

Recall the log of a complex number $z=r\angle\theta$ is $\ln(r) + j\theta$

Returning to Example

- With m=6, L=3, and

$$\mathbf{y} = \begin{bmatrix} 10.85 \\ 40.23 \\ 14.94 \\ 22.60 \\ 37.13 \\ 7.90 \end{bmatrix} \rightarrow \mathbf{Y} = \begin{bmatrix} 10.85 & 40.23 & 14.94 & 22.60 \\ 40.23 & 14.94 & 22.60 & 37.13 \\ 14.94 & 22.60 & 37.13 & 7.90 \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{U\Sigma V}^T \rightarrow diag(\mathbf{\Sigma}) = \begin{bmatrix} 83.49 \\ 29.90 \\ 22.61 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} -0.4877 & 0.5731 & 0.1241 & -0.6468 \\ -0.5146 & -0.6759 & -0.4375 & -0.2948 \\ -0.5015 & -0.2497 & 0.7702 & 0.3048 \\ -0.4958 & 0.3904 & -0.4471 & 0.6339 \end{bmatrix}$$

- In this example we retain all three singular values

Example

- Which gives

$$\mathbf{V}_1 = \begin{bmatrix} -0.4877 & 0.5731 & 0.1241 \\ -0.5146 & -0.6759 & -0.4375 \\ -0.5015 & -0.2497 & 0.7702 \end{bmatrix}, \quad \mathbf{V}_2 = \begin{bmatrix} -0.5146 & -0.6759 & -0.4375 \\ -0.5015 & -0.2497 & 0.7702 \\ -0.4958 & 0.3904 & -0.4471 \end{bmatrix}$$

$$\mathbf{V}_2^T\mathbf{V}_1 = \begin{bmatrix} 0.758 & 0.168 & -0.226 \\ 0.262 & -0.316 & 0.326 \\ 0.041 & -0.660 & -0.736 \end{bmatrix}, \quad \mathbf{V}_1^T\mathbf{V}_1 = \begin{bmatrix} 0.754 & 0.194 & -0.222 \\ 0.194 & 0.848 & 0.175 \\ -0.222 & 0.175 & 0.800 \end{bmatrix}$$

- And generalized eigenvalues of 1.0013, -0.741±j0.6854
- Then with ΔT=0.2

$$\lambda_1 = \frac{\ln(1.0013)}{0.2} = 0.0065$$

$$\lambda_{2,3} = \frac{\ln(1.0094\angle\pm 137.2°)}{0.2} = 0.047 \pm j11.97$$

Example

- This initial guess of $\boldsymbol{\alpha}$ is very close to the final solution
- The iteration works by calculating the Jacobian, $\mathbf{J}(\boldsymbol{\alpha})$ (with details in the paper), and the gradient

$$\nabla \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\alpha})\right\|_2^2 = \mathbf{J}(\boldsymbol{\alpha})^T \mathbf{r}(\boldsymbol{\alpha})$$

- A gradient search optimization (such as Golden Section) is used to determine the distance to move in the negative gradient direction
- For the example

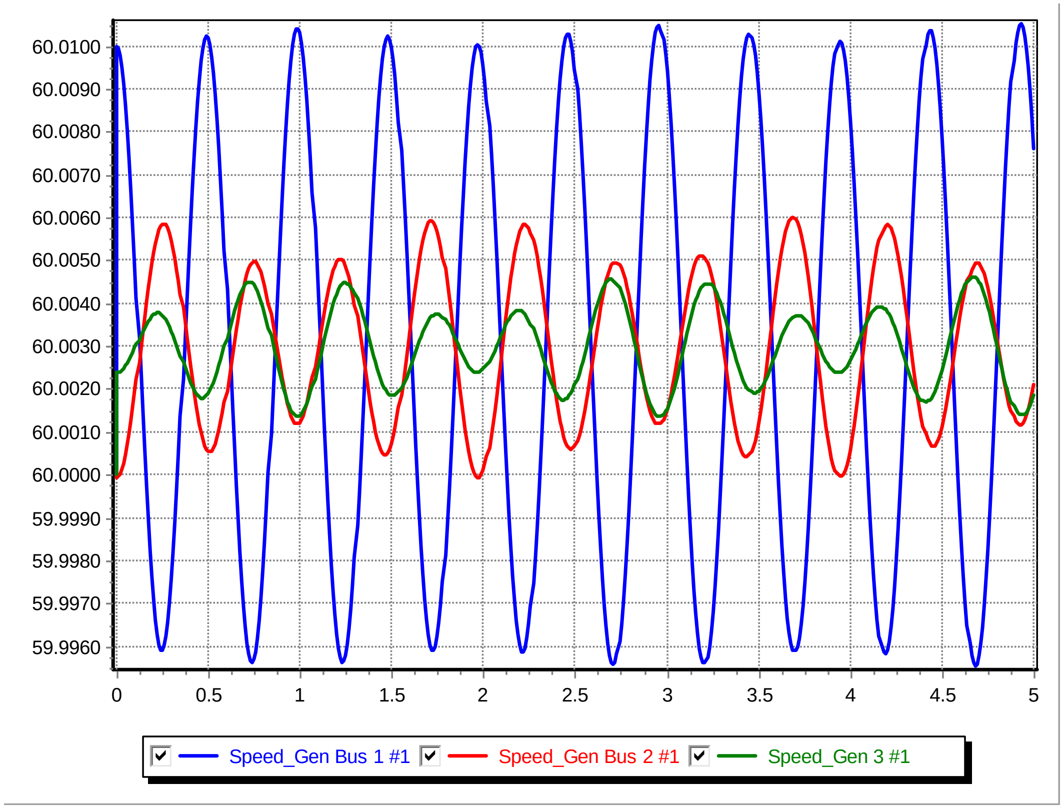
$$\mathbf{J}(\boldsymbol{\alpha})^T = \begin{bmatrix} -7.25 & 4.52 & -1.14 & -0.29 & -3.84 & 7.96 \\ 3.56 & -0.38 & -2.27 & -2.40 & 1.81 & -0.32 \\ 11.21 & 5.55 & 6.14 & -6.08 & -5.72 & -11.00 \end{bmatrix}$$

Comments

- These techniques only match the signals at the sampled time points
- The sampling frequency must be at least twice the highest frequency of interest
  - A higher sampling rate is generally better, but there is a computational limitation associated with the size of the Hankel matrix
  - Aliasing is a concern since we are dealing with a time limited signal
- Detrending can be used to remove a polynomial offset
- Method can be extended to multiple signals

VPM: Example 2

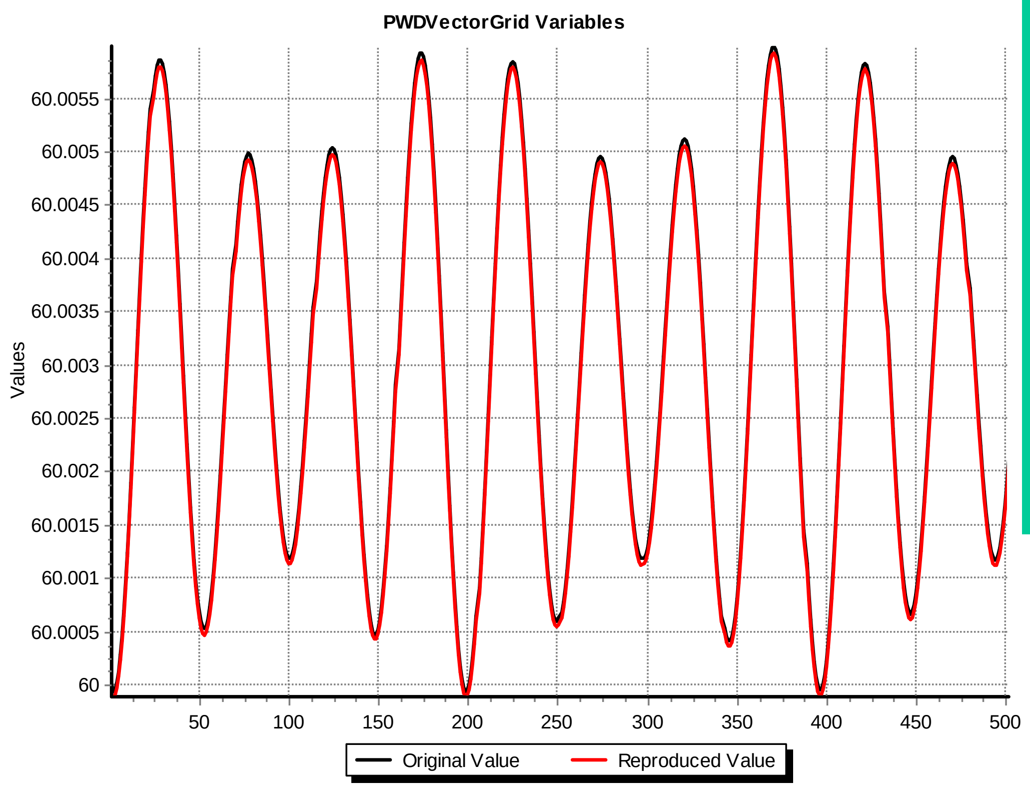
- Do VPM on speed for generator 2 from previous three bus small signal analysis case
  - Calculated modes were at 1.51 and 2.02 Hz



Input data here is the red curve

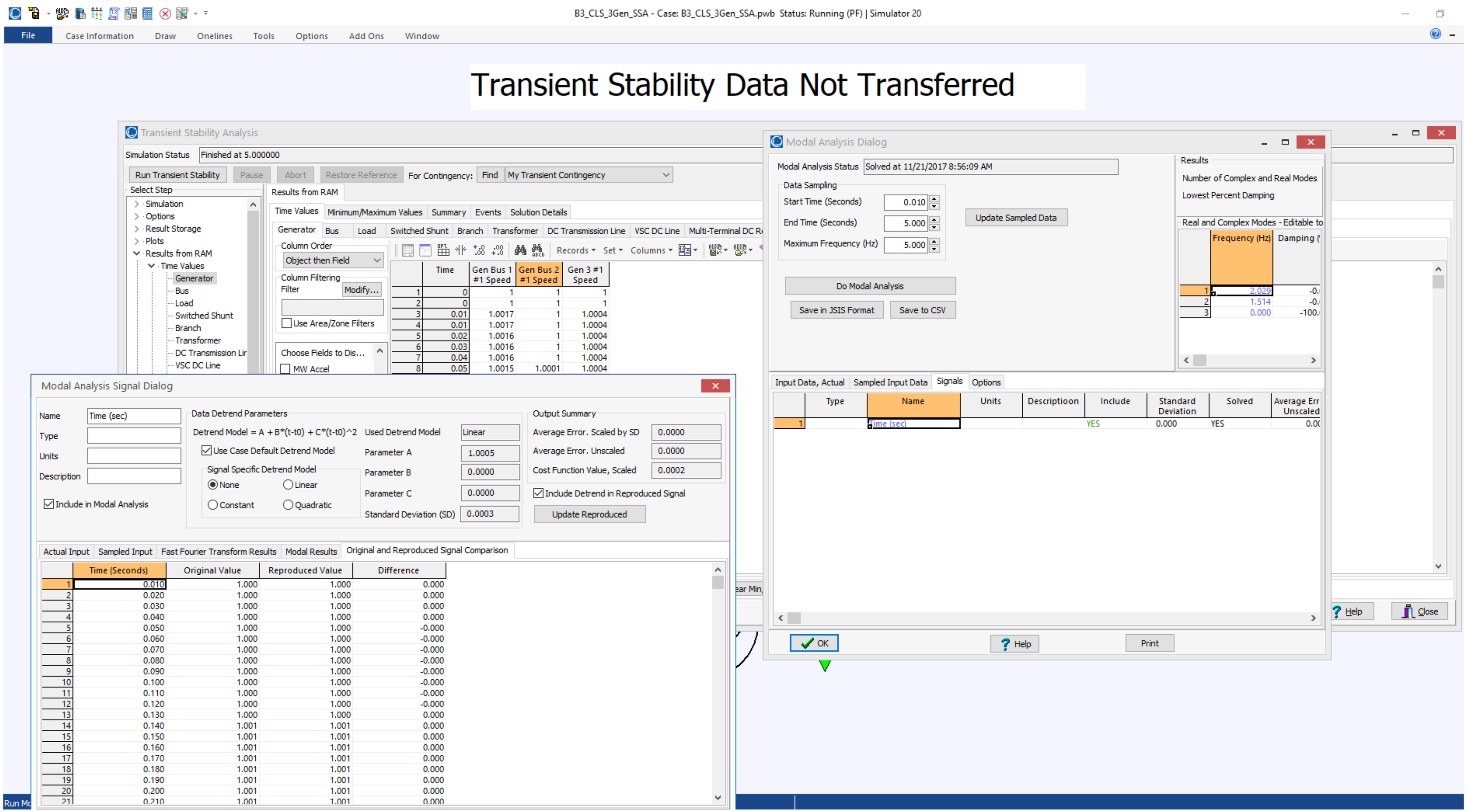
VPM: Example 2

- Below results were obtained from sampling the input data every 0.1 seconds (10 Hz)



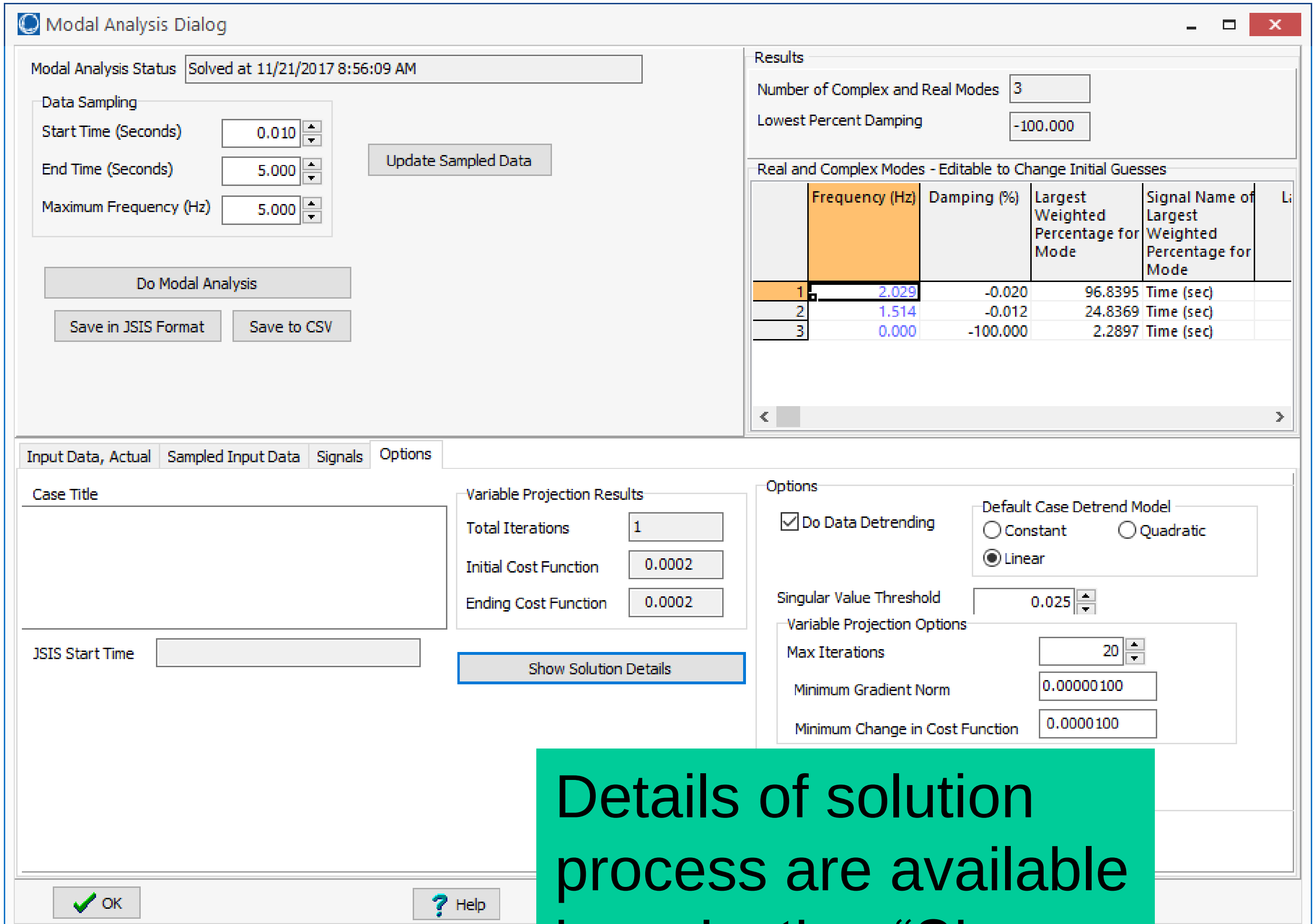
Calculated frequencies were 2.03 and 1.51 Hz with a dc offset; the 2.03 frequency has a value of almost 4 times that of the 1.51 Hz

Example using PowerWorld Modal Analysis Dialog



Case is earlier B3_CLS_3Gen_SSA

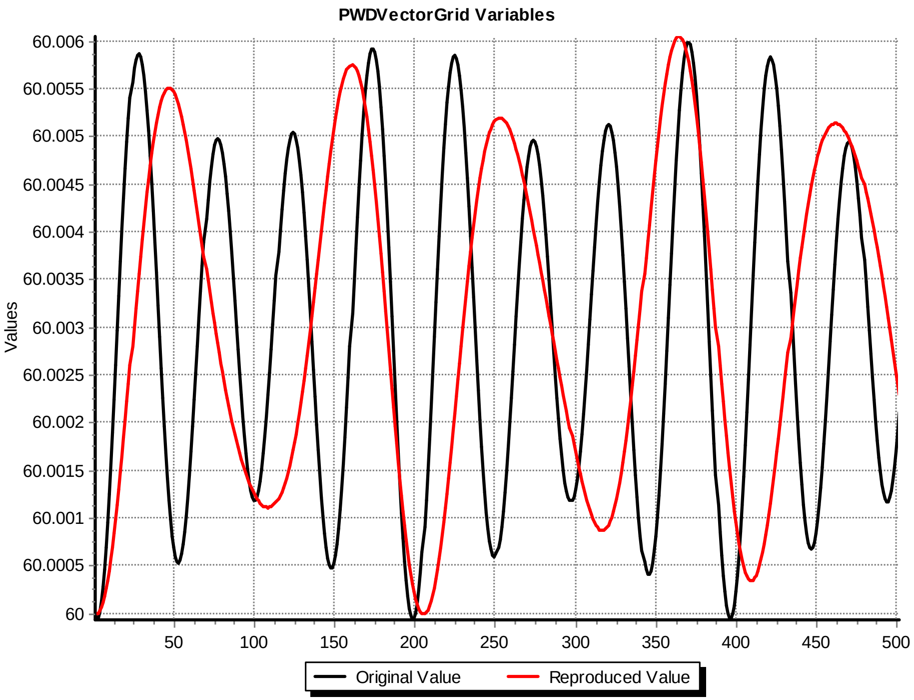
Example using PowerWorld Modal Analysis Dialog

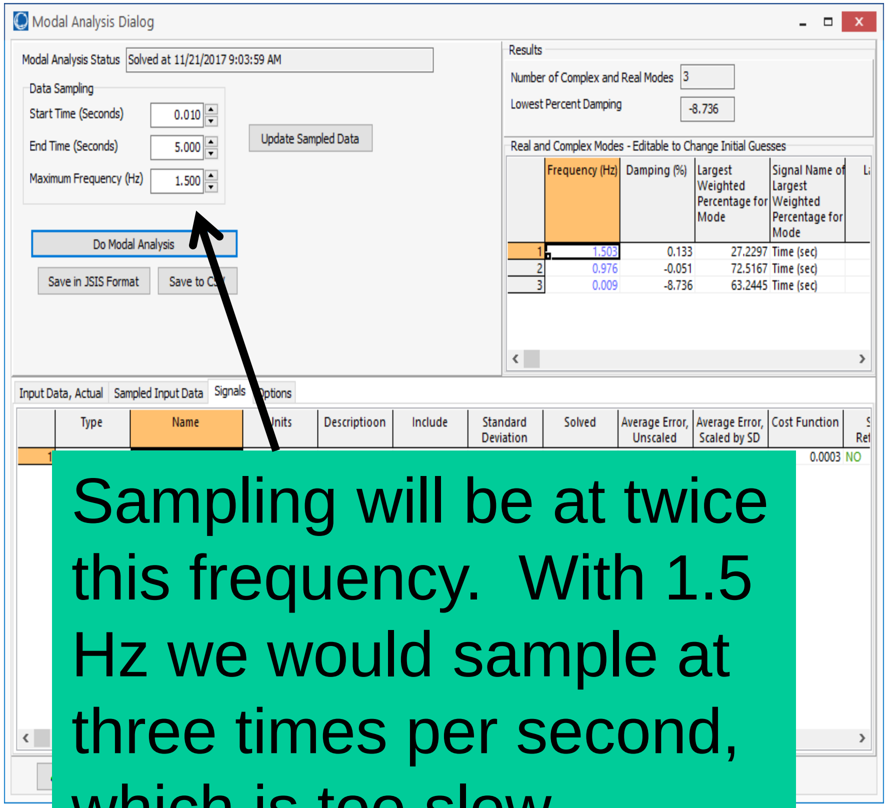


Details of solution process are available by selecting “Show Solution Details.”

VPM: Example 2

- Results are quite poor if sampling is reduced to 1.5 Hz

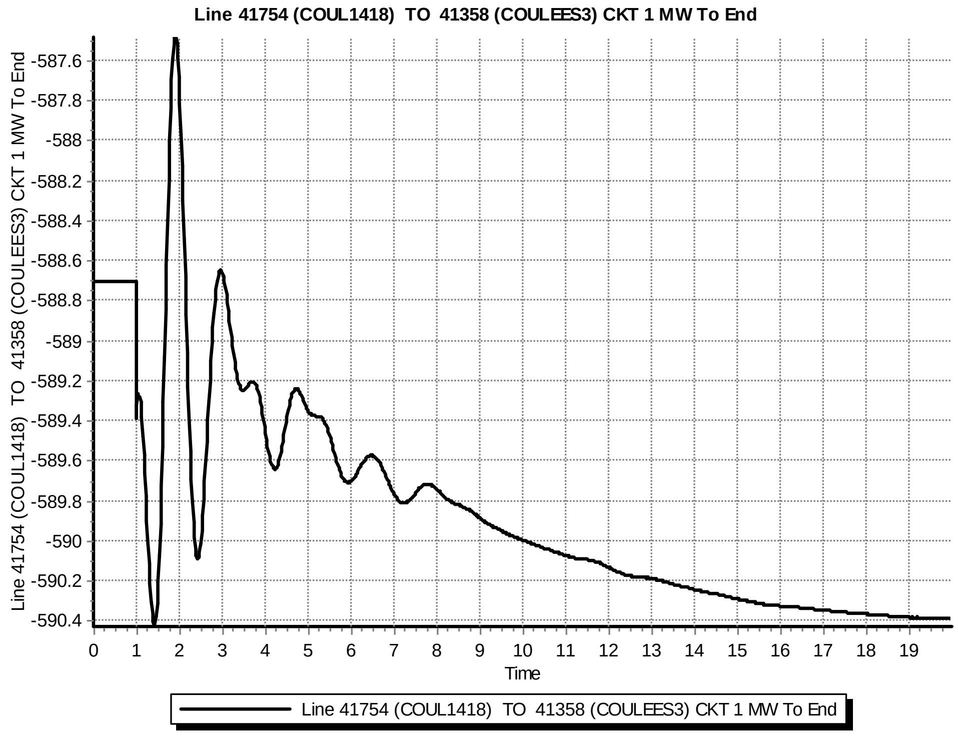




Sampling will be at twice this frequency. With 1.5 Hz we would sample at three times per second, which is too slow.

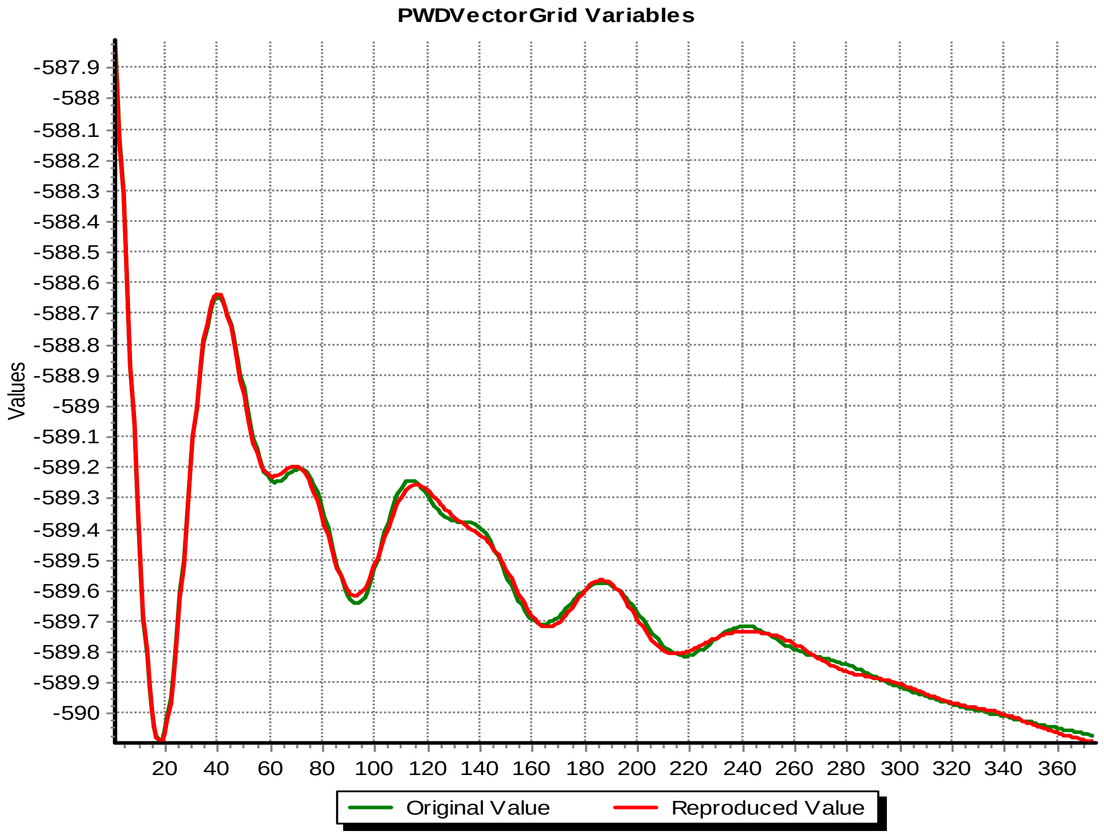
Moving Forward with VPM

- Not all signals exhibit such straightforward oscillations, since there can be other slower dynamics superimposed
- How can this method be extended to handle these situations?



Moving Forward with VPM

- Here are the results



This is made up of five signals with frequencies from 0.123 to 1.1 Hz, with the highest magnitude at 0.634 Hz