ECEN 615
Methods of Electric Power Systems Analysis

Lecture 7: Advanced Power Flow

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Announcements

- Read Chapter 6
- Homework 2 is due on Sept 27

Tribute to Ti Xu, 1998-2018

Decoupled Power Flow

- Rather than not updating the Jacobian, the decoupled power flow takes advantage of characteristics of the power grid in order to decouple the real and reactive power balance equations
  - There is a strong coupling between real power and voltage angle, and reactive power and voltage magnitude
  - There is a much weaker coupling between real power and voltage angle, and reactive power and voltage angle
- Key reference is B. Stott, “Decoupled Newton Load Flow,” *IEEE Trans. Power. App and Syst.*, Sept/Oct. 1972, pp. 1955-1959

Decoupled Power Flow Formulation

General form of the power flow problem

$$-\begin{bmatrix} \frac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} & \frac{\partial \mathbf{P}}{\partial |\mathbf{V}|}^{(v)} \\ \frac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}^{(v)} & \frac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}^{(v)} \\ \Delta |\mathbf{V}|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \\ \Delta \mathbf{Q}(\mathbf{x}^{(v)}) \end{bmatrix} = \mathbf{f}(\mathbf{x}^{(v)})$$

where

$$\Delta \mathbf{P}(\mathbf{x}^{(v)}) = \begin{bmatrix} P_2(\mathbf{x}^{(v)}) + P_{D2} - P_{G2} \\ \vdots \\ P_n(\mathbf{x}^{(v)}) + P_{Dn} - P_{Gn} \end{bmatrix}$$

Decoupling Approximation

Usually the off-diagonal matrices, $\dfrac{\partial \mathbf{P}}{\partial |\mathbf{V}|}^{(v)}$ and $\dfrac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}^{(v)}$

are small. Therefore we approximate them as zero:

$$-\begin{bmatrix} \dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} & \mathbf{0} \\ \mathbf{0} & \dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}^{(v)} \\ \Delta |\mathbf{V}|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \\ \Delta \mathbf{Q}(\mathbf{x}^{(v)}) \end{bmatrix} = \mathbf{f}(\mathbf{x}^{(v)})$$

Then the problem can be decoupled

$$\Delta \boldsymbol{\theta}^{(v)} = -\left[\dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)}\right]^{-1} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \quad \Delta |\mathbf{V}|^{(v)} = -\left[\dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)}\right]^{-1} \Delta \mathbf{Q}(\mathbf{x}^{(v)})$$

Off-diagonal Jacobian Terms

Justification for Jacobian approximations:

1. Usually r $\ll$ x, therefore $|G_{ij}| \ll |B_{ij}|$
2. Usually θ_{ij} is small so $\sin\theta_{ij} \approx 0$

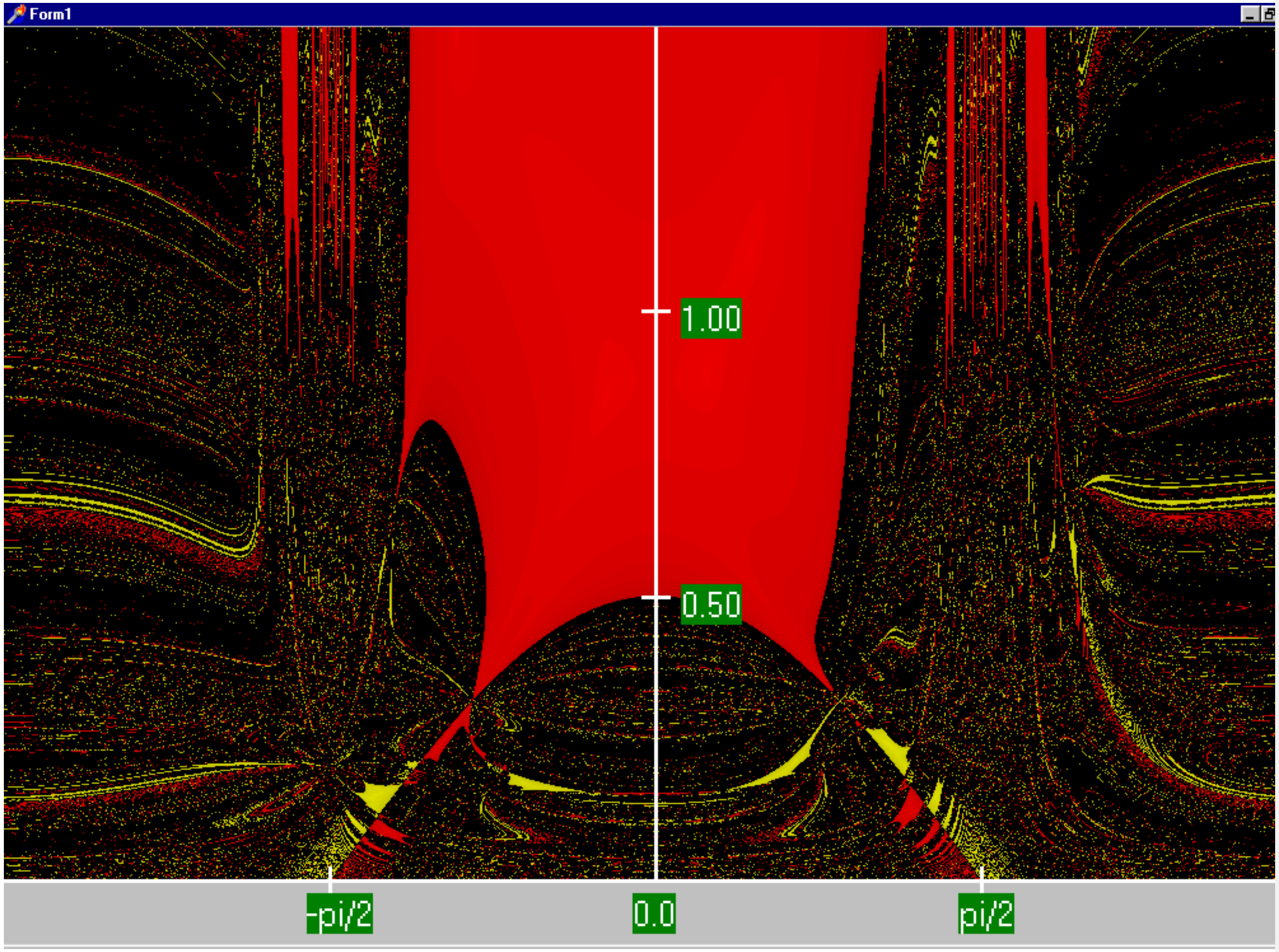
Therefore

$$\frac{\partial \mathbf{P}_i}{\partial |\mathbf{V}_j|} = |V_i|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \approx 0$$

$$\frac{\partial \mathbf{Q}_i}{\partial \boldsymbol{\theta}_j} = -|V_i||V_j|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \approx 0$$

By assuming ½ the elements are zero, we only have to do ½ the computations

Decoupled N-R Region of Convergence



The high solution ROC is actually larger than with the standard NPF. Obviously this is not a good a way to get the low solution

Fast Decoupled Power Flow

- By continuing with our Jacobian approximations we can actually obtain a reasonable approximation that is independent of the voltage magnitudes/angles.
- This means the Jacobian need only be built/inverted once per power flow solution
- This approach is known as the fast decoupled power flow (FDPF)

Fast Decoupled Power Flow, cont.

- FDPF uses the same mismatch equations as standard power flow (just scaled) so it should have same solution
- The FDPF is widely used, though usually only when we only need an approximate solution
- Key fast decoupled power flow reference is B. Stott, O. Alsac, “Fast Decoupled Load Flow,” *IEEE Trans. Power App. and Syst.*, May 1974, pp. 859-869
  - **Ongun Alsaç is NAE Class of 2018 (with Prof. Singh!)**
- Modified versions also exist, such as D. Jajicic and A. Bose, “A Modification to the Fast Decoupled Power Flow for Networks with High R/X Ratios, “*IEEE Transactions on Power Sys.*, May 1988, pp. 743-746

FDPF Approximations

The FDPF makes the following approximations:

1. $\left|G_{ij}\right| = 0$
2. $\left|V_i\right| = 1$
3. $\sin\theta_{ij} = 0 \qquad \cos\theta_{ij} = 1$

To see the impact on the real power equations recall

$$P_i = \sum_{k=1}^{n} V_i V_k (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

Which can also be written as

$$\frac{P_i}{V_i} = \sum_{k=1}^{n} V_k (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = \frac{P_{Gi} - P_{Di}}{V_i}$$

FDPF Approximations

- With the approximations for the diagonal term we get

$$\frac{\partial \mathrm{P_i}}{\partial \theta_i} \approx \sum_{\substack{k=1 \\ k\neq i}}^{n} B_{ik} = -B_{ii}$$

The for the off-diagonal terms (k $\neq$ i) with **G=0** and **V=1**

$$\frac{\partial P_i}{\partial \theta_k} = -B_{ik} \cos\theta_{ik} \approx -B_{ik}$$

- Hence the Jacobian for the real equations can be approximated as –**B**

FPDF Approximations

- For the reactive power equations we also scale by V_i

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

$$\frac{\mathrm{Q_i}}{V_i} = \sum_{k=1}^{n} V_k(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = \frac{Q_{Gi} - Q_{Di}}{V_i}$$

- For the Jacobian off-diagonals we get

$$\frac{\partial Q_i}{\partial V_k} = -B_{ik}\cos\theta_{ik} \approx -B_{ik}$$

FDPF Approximations

- And for the reactive power Jacobian diagonal we get

$$\frac{\partial Q_{\mathrm{i}}}{\partial V_i} \approx -2B_{ii} - \sum_{\substack{k=1 \\ k\neq i}}^{n} B_{ik} = -B_{ii}$$

- As derived the real and reactive equations have a constant Jacobian equal to –**B**
  - Usually modifications are made to omit from the real power matrix elements that affect reactive flow (like shunts) and from the reactive power matrix elements that affect real power flow, like phase shifters
  - We'll call the real power matrix **B**' and the reactive **B**"

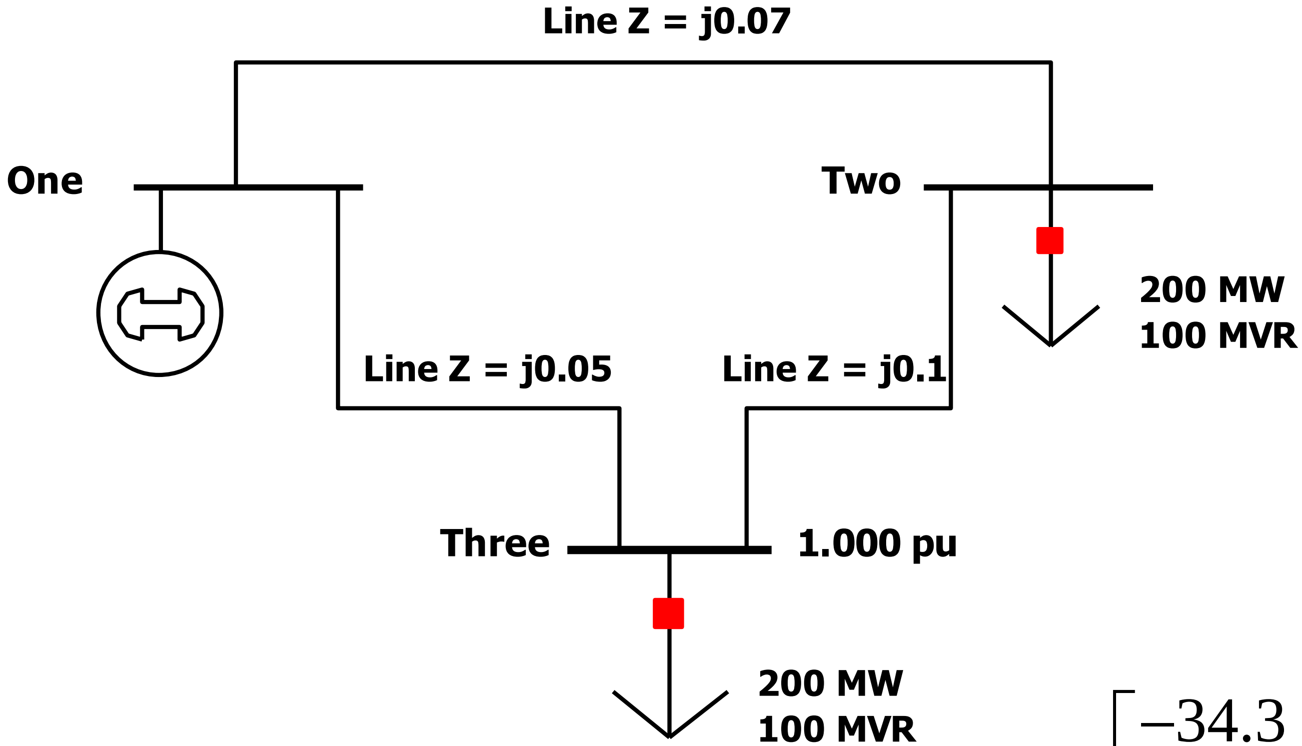
FDPF Approximations

- It is also common to flip the sign on the mismatch equation, by changing it from (summation – injection) to (injection – summation)
  - Other modifications on the **B** matrix have been presented in the literature (such as in the Bose paper)
- Hence we have

$$\Delta\boldsymbol{\theta}^{(v)} = \mathbf{B}'^{-1}\frac{\Delta\mathbf{P}(\mathbf{x}^{(v)})}{\mathbf{V}^{(v)}} \qquad \Delta\left|\mathbf{V}\right|^{(v)} = \mathbf{B}''^{-1}\frac{\Delta\mathbf{Q}(\mathbf{x}^{(v)})}{\mathbf{V}^{(v)}}$$

FDPF Three Bus Example

Use the FDPF to solve the following three bus system



$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix}$$

FDPF Three Bus Example, cont'd

$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix} \rightarrow \mathbf{B} = \begin{bmatrix} -24.3 & 10 \\ 10 & -30 \end{bmatrix}$$

$$\mathbf{B}^{-1} = \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix}$$

Iteratively solve, starting with an initial voltage guess

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix}$$

FDPF Three Bus Example, cont'd

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9364 \\ 0.9455 \end{bmatrix}$$

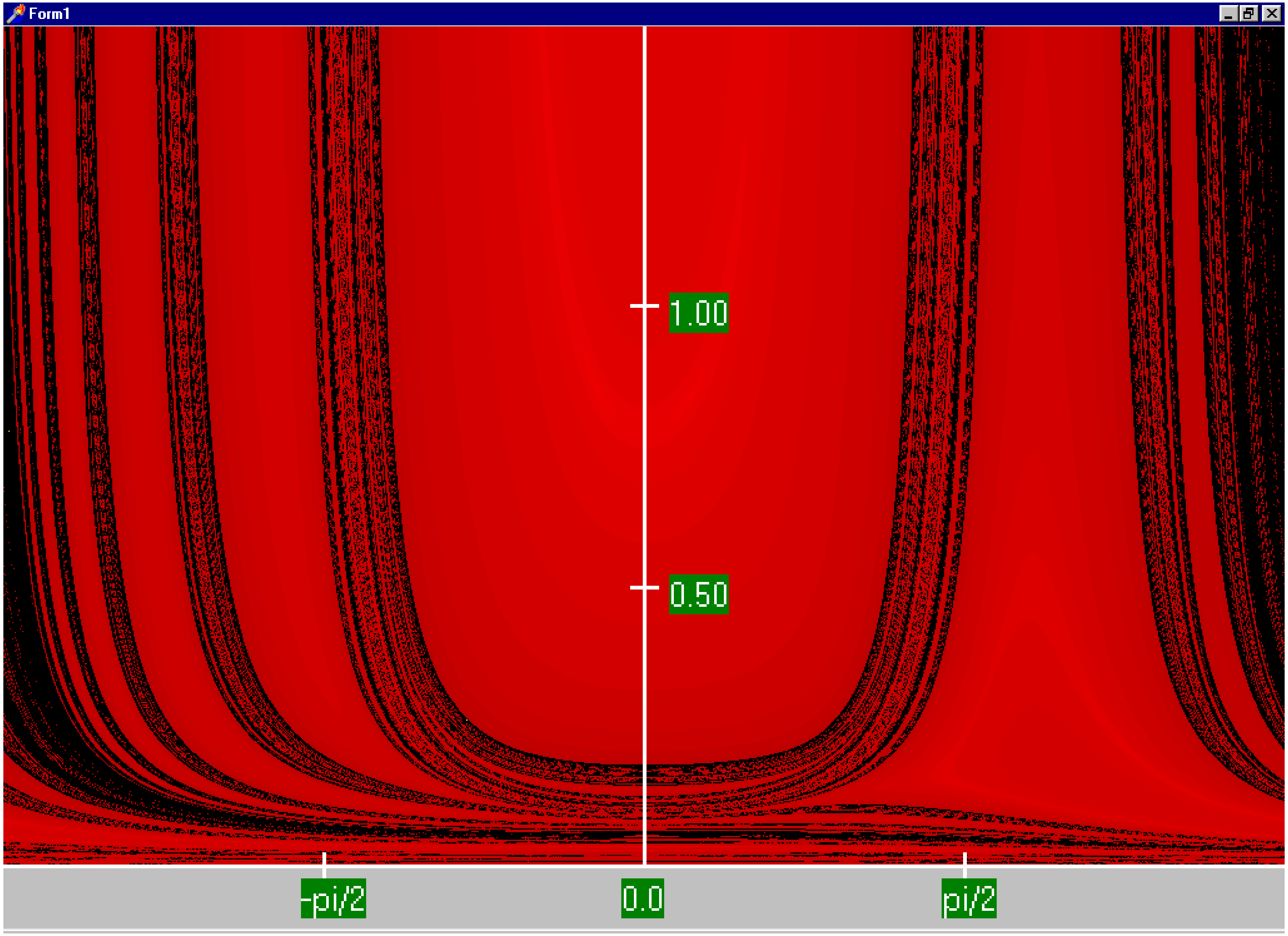
$$\frac{\Delta \mathrm{P_i}(\mathbf{x}\ )}{|\mathrm{V_i}|} = \sum_{k=1}^{n} |V_k| (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) + \frac{P_{Di} - P_{Gi}}{|\mathrm{V_i}|}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(2)} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 0.151 \\ 0.107 \end{bmatrix} = \begin{bmatrix} -0.1361 \\ -0.1156 \end{bmatrix}$$

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(2)} = \begin{bmatrix} 0.924 \\ 0.936 \end{bmatrix}$$

Actual solution: $\boldsymbol{\theta} = \begin{bmatrix} -0.1384 \\ -0.1171 \end{bmatrix}$ $\quad \mathbf{V} = \begin{bmatrix} 0.9224 \\ 0.9338 \end{bmatrix}$

FDPF Region of Convergence



FDPF Cautions

- The FDPF works well as long as the previous approximations hold for the entire system
- With the movement towards modeling larger systems, with more of the lower voltage portions of the system represented (for which r/x ratios are higher) it is quite common for the FDPF to get stuck because small portions of the system are ill-behaved
- The FDPF is commonly used to provide an initial guess of the solution or for contingency analysis

DC Power Flow

- The "DC" power flow makes the most severe approximations:
  - completely ignore reactive power, assume all the voltages are always 1.0 per unit, ignore line conductance
- This makes the power flow a linear set of equations, which can be solved directly

$$\boldsymbol{\theta} = -\mathbf{B}^{-1}\,\mathbf{P}$$

P sign convention is generation is positive

- The term dc power flow actually dates from the time of the old network analyzers (going back into the 1930's)
- Not to be confused with the inclusion of HVDC lines in the standard NPF

DC Power Flow References

- I don't think a classic dc power flow paper exists; a nice formulation is given in our book *Power Generation and Control* book by Wood and Wollenberg
- The August 2009 paper in IEEE Transactions on Power Systems, "DC Power Flow Revisited" (by Stott, Jardim and Alsac) provides good coverage
- T. J. Overbye, X. Cheng, and Y. Sun, "A comparison of the AC and DC power flow models for LMP Calculations," in *Proc. 37th Hawaii Int. Conf. System Sciences*, 2004, compares the accuracy of the approach

DC Power Flow Example

EXAMPLE 6.17

Determine the dc power flow solution for the five bus from Example 6.9.

SOLUTION With bus 1 as the system slack, the **B** matrix and **P** vector for this system are

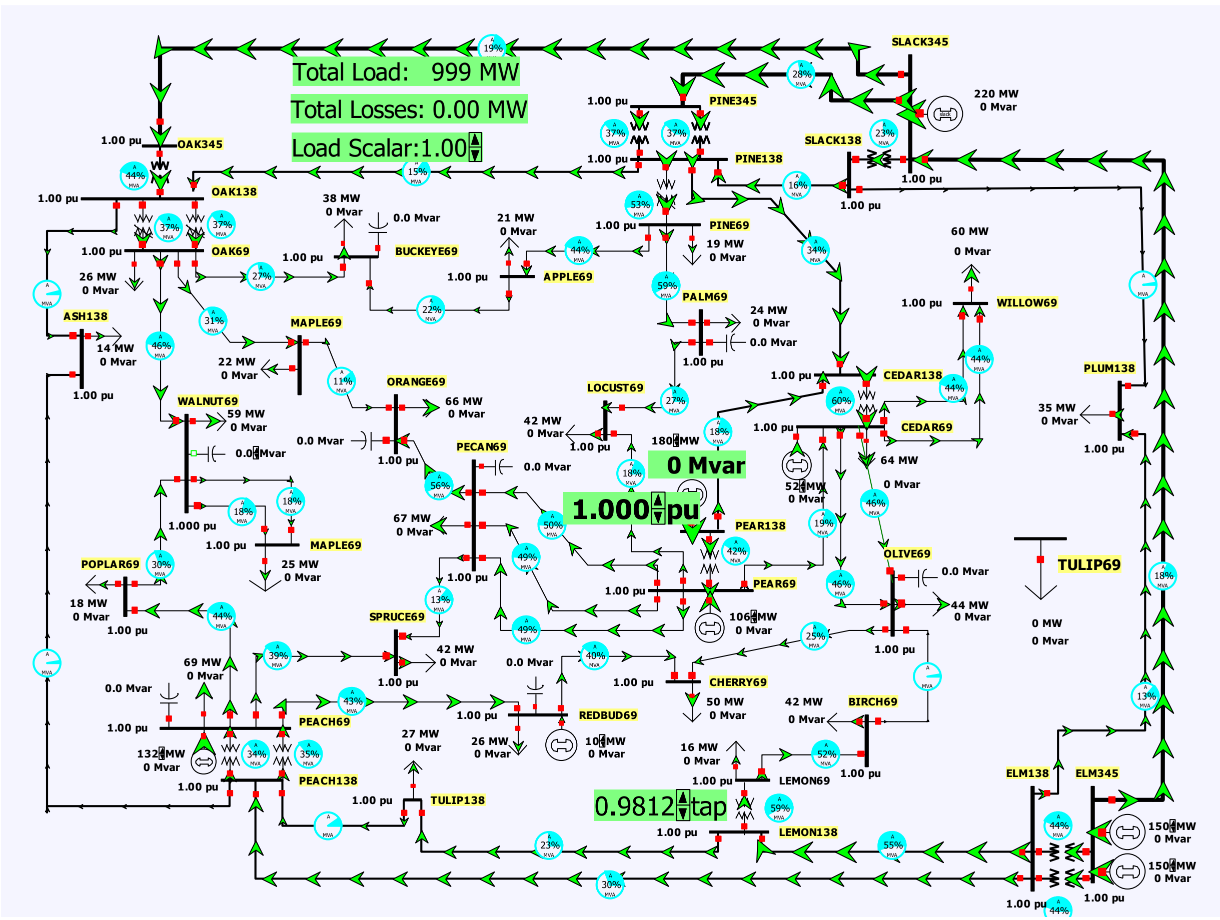
$$\mathbf{B} = \begin{bmatrix} -30 & 0 & 10 & 20 \\ 0 & -100 & 100 & 0 \\ 10 & 100 & -150 & 40 \\ 20 & 0 & 40 & -110 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} -8.0 \\ 4.4 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{\delta} = -\mathbf{B}^{-1}\mathbf{P} = \begin{bmatrix} -0.3263 \\ 0.0091 \\ -0.0349 \\ -0.0720 \end{bmatrix} \text{radians} = \begin{bmatrix} -18.70 \\ 0.5214 \\ -2.000 \\ -4.125 \end{bmatrix} \text{degrees}$$

Example from Power System Analysis and Design, by Glover, Overbye, Sarma

DC Power Flow in PowerWorld

- PowerWorld allows for easy switching between the dc and ac power flows



To use the dc approach in PowerWorld select Tools, Solve, DC Power Flow

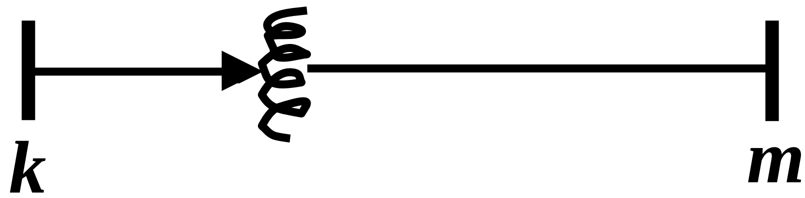
Notice there are no losses

Modeling Transformers with Off-Nominal Taps and Phase Shifts

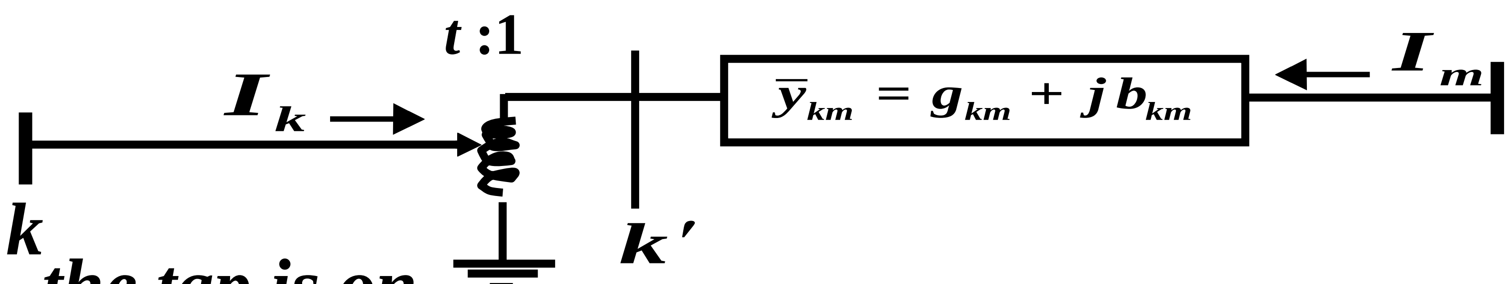
- If transformers have a turns ratio that matches the ratio of the per unit voltages than transformers are modeled in a manner similar to transmission lines.
- However it is common for transformers to have a variable tap ratio; this is known as an “off-nominal” tap ratio
  - The off-nominal tap is t, initially we’ll consider it a real number
  - We’ll cover phase shifters shortly in which t is complex

Transformer Representation

- The one–line diagram of a branch with a variable tap transformer



- The network representation of a branch with off–nominal turns ratio transformer is



the tap is on

the side of bus k

Transformer Nodal Equations

- From the network representation

$$\bar{I}_m = \bar{I}_{k'} = \bar{y}_{km}\left(\bar{E}_m - \bar{E}_{k'}\right) = \bar{y}_{km}\left(\bar{E}_m - \frac{\bar{E}_k}{t}\right)$$

$$= \left(\bar{y}_{km}\right)\bar{E}_m + \left(-\frac{\bar{y}_{km}}{t}\right)\bar{E}_k$$

- Also

$$\bar{I}_k = -\frac{1}{t}\bar{I}_{k'} = \left(-\frac{\bar{y}_{km}}{t}\right)\bar{E}_m + \left(\frac{\bar{y}_{km}}{t^2}\right)\bar{E}_k$$

Transformer Nodal Equations

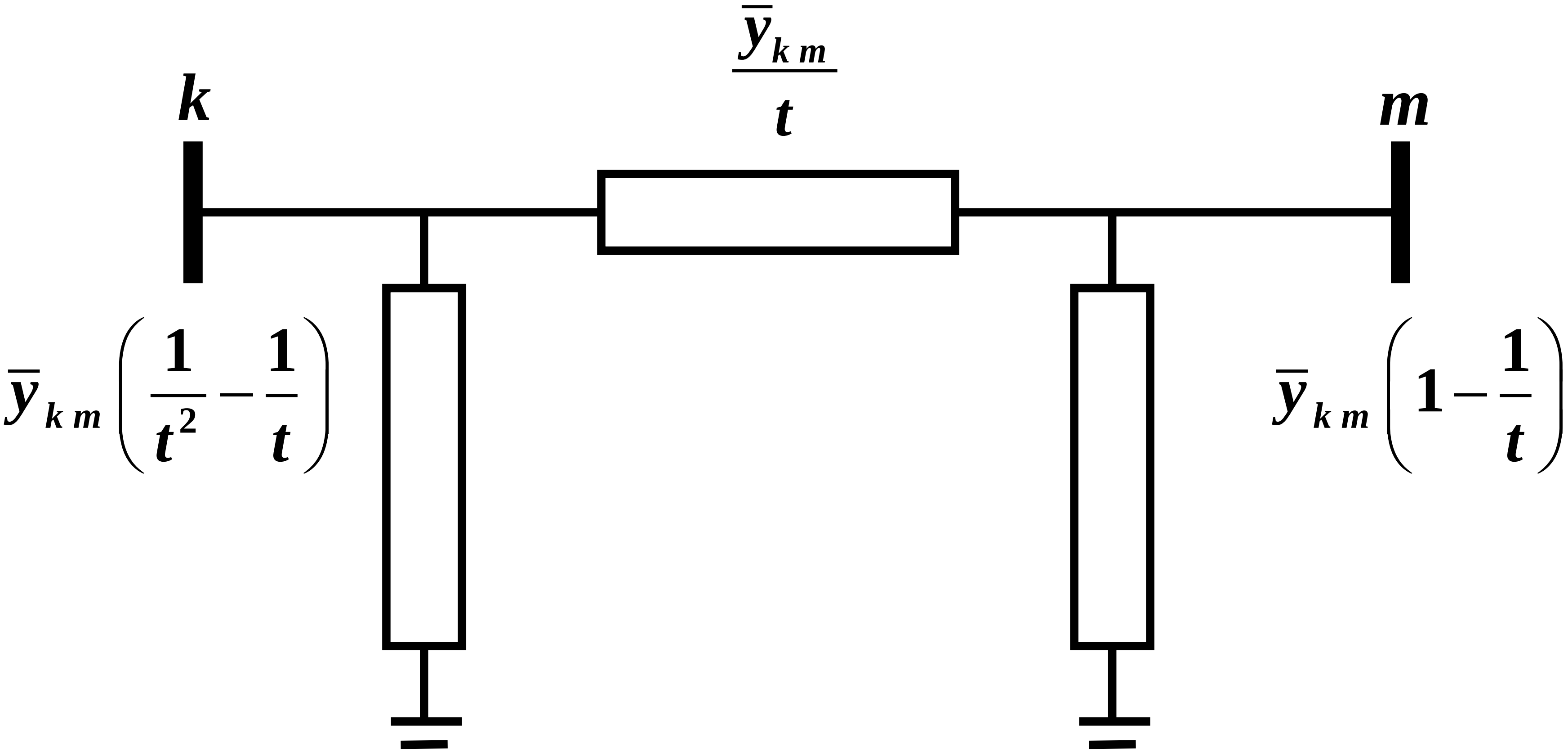
- We may rewrite these two equations as

$$\begin{bmatrix} \bar{I}_k \\ \bar{I}_m \end{bmatrix} = \begin{bmatrix} \dfrac{\bar{y}_{km}}{t^2} & -\dfrac{\bar{y}_{km}}{t} \\ -\dfrac{\bar{y}_{km}}{t} & \bar{y}_{km} \end{bmatrix} \begin{bmatrix} \bar{E}_k \\ \bar{E}_m \end{bmatrix}$$

$\mathbf{Y}_{\text{bus}}$ is still symmetric here (though this will change with phase shifters)

This approach was first presented in F.L. Alvarado, "Formation of Y-Node using the Primitive Y-Node Concept," IEEE Trans. Power App. and Syst., December 1982

The π-Equivalent Circuit for a Transformer Branch



Variable Tap Voltage Control

- A transformer with a variable tap, i.e., the variable t is not constant, may be used to control the voltage at either the bus on the side of the tap or at the bus on the side away from the tap
- This constitutes an example of single criterion control since we adjust a single control variable (i.e., the transformer tap t) to achieve a specified criterion: the maintenance of a constant voltage at a designated bus
- Names for this type of control are on-load tap changer (LTC) transformer or tap changing under load (TCUL)
- Usually on low side; there may also be taps on high side that can be adjusted when it is de-energized

Variable Tap Voltage Control

- An LTC is a discrete control, often with 32 incremental steps of 0.625% each, giving an automatic range of ± 10%
- It follows from the π–equivalent model for the transformer that the transfer admittance between the buses of the transformer branch and the contribution to the self admittance at the bus away from the tap explicitly depend on t
- However, the tap changes in discrete steps; there is also a built in time delay in how fast they respond
- Voltage regulators are devices with a unity nominal ratio, and then a similar tap range

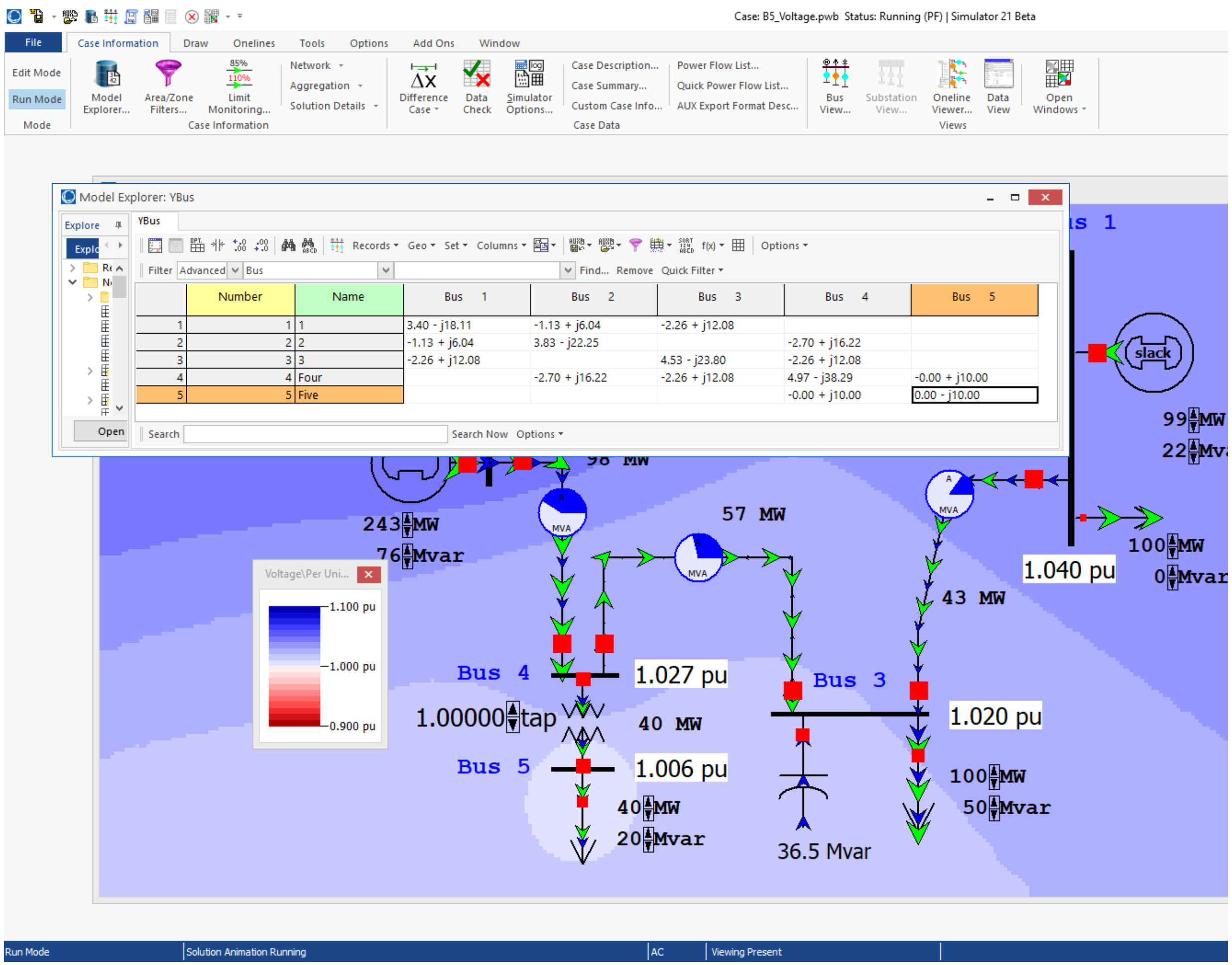
Ameren Champaign (IL) Test Facility Voltage Regulators

These are connected on the low side of a 69/12.4 kV transformer; each phase can be regulated separately

Variable Tap Voltage Control

- LTCs (or voltage regulators) can be directly included in the power flow equations by modifying the $\mathbf{Y_{bus}}$ entries; that is by scaling the terms by 1, 1/t or $1/t^2$ as appropriate
- If t is fixed then there is no change in the number of equations
- If t is variable, such as to enforce a voltage equality, then it can be included either by adding an additional equation and variable (t) directly, or by doing an "outer loop" calculation in which t is varied outside of the NR solution
  - The outer loop is used in PowerWorld because of limit issues

Five Bus PowerWorld Example



PowerWorld Case: B5_Voltage

With an impedance of j0.1 pu between buses 4 and 5, the y node primitive with t=1.0 is

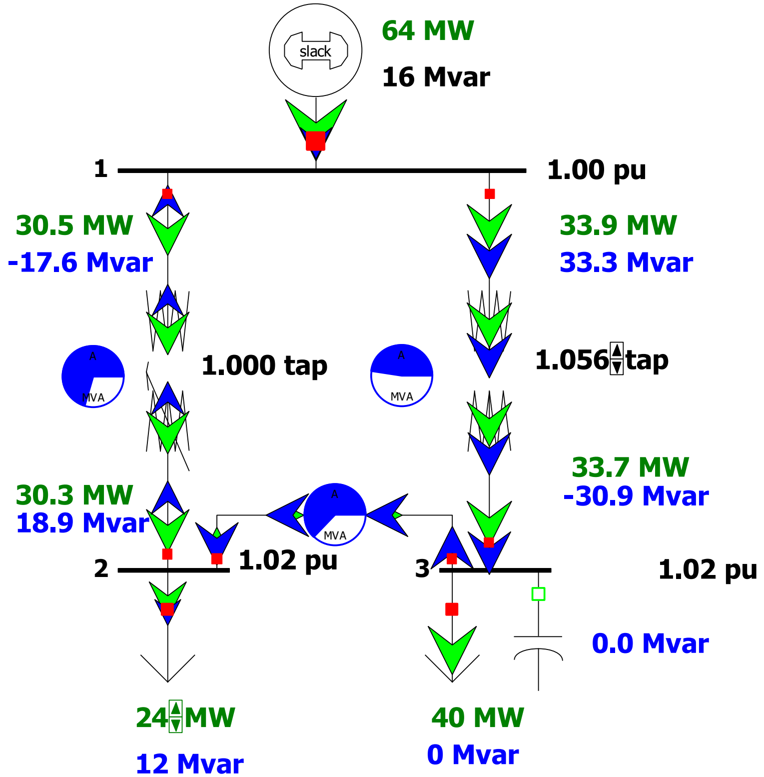
$$\begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

If t=1.1 then it is

$$\begin{bmatrix} -j10 & j9.09 \\ j9.09 & -j8.26 \end{bmatrix}$$

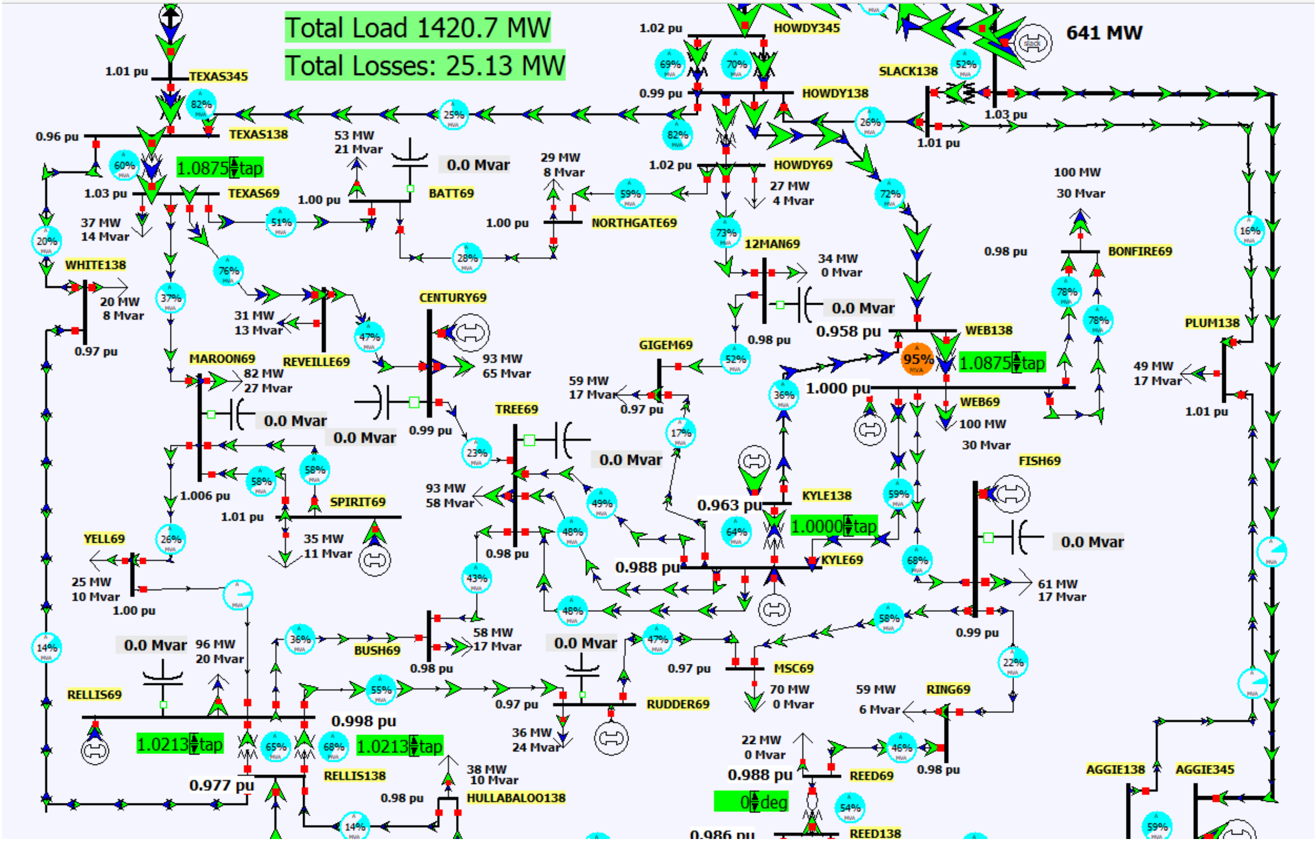
Circulating Reactive Power

- Unbalanced transformer taps can cause large amounts of reactive power to circulating, increasing power system losses and overloading transformers



LTC Tap Coordination

- Changing tap ratios can affect the voltages and var flow at nearby buses; hence coordinated control is needed



PowerWorld Case: Aggieland37_LTC