ECEN 615
Methods of Electric Power Systems Analysis

Lecture 14: Contingency Analysis, Sensitivity Methods

Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

Announcements

- Homework 3 is due today
- Read Chapter 7 (the term reliability is now used instead of security)
- Midterm exam is Oct 18 in class
  - Off campus students should work with Iyke to get their exam proctoring setup
  - Closed book, closed notes, but calculators and one 8.5 by 11 inch note sheet allowed
  - Exam covers up to the end of today's lecture
  - Book material is intended to be supplementary; nothing from the book not covered in class or homework will be on the exam

Contingency Analysis

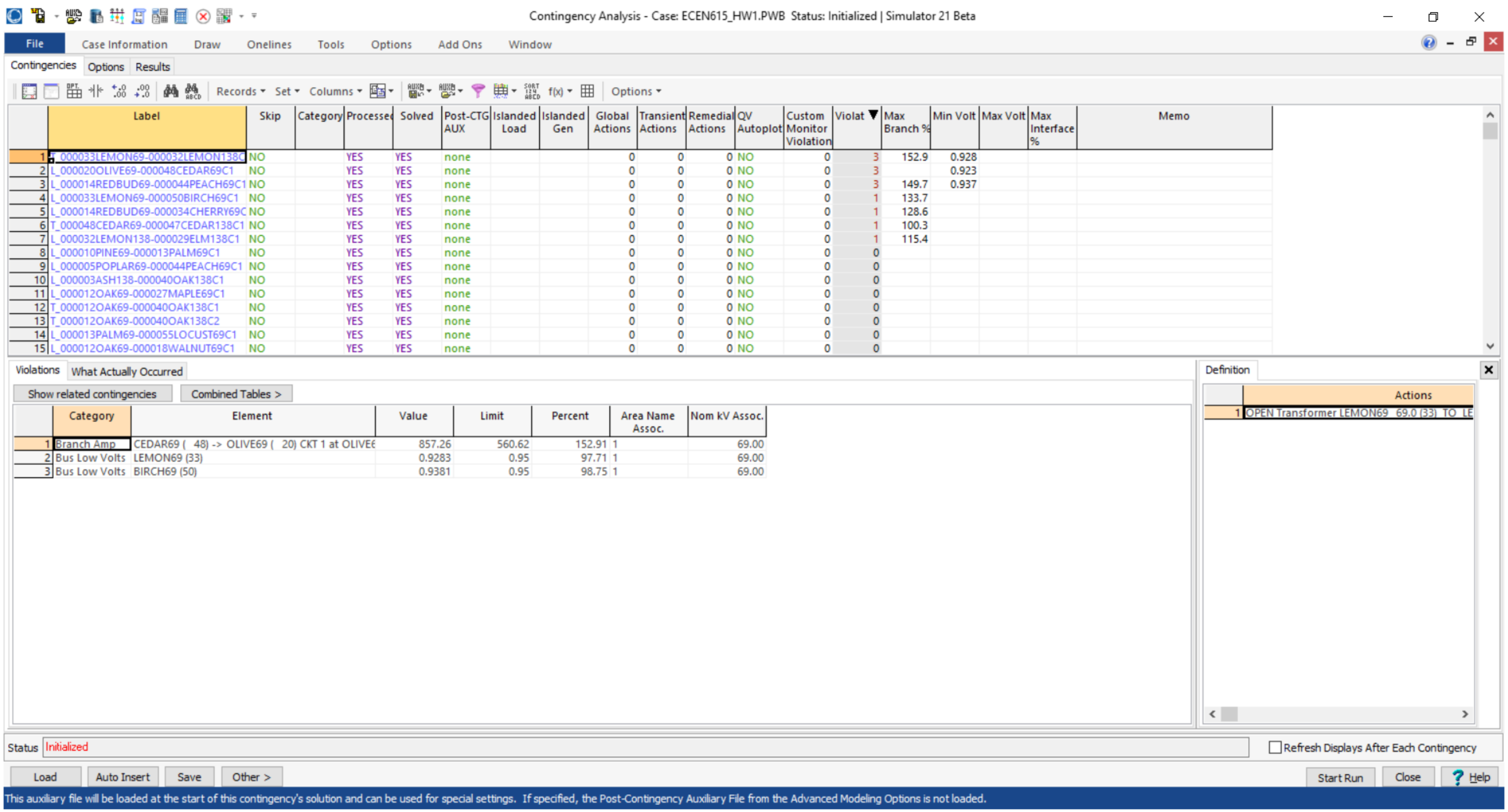
- Contingency analysis is the process of checking the impact of statistically likely contingencies
  - Example contingencies include the loss of a generator, the loss of a transmission line or the loss of all transmission lines in a common corridor
  - Statistically likely contingencies can be quite involved, and might include automatic or operator actions, such as switching load
- Reliable power system operation requires that the system be able to operate with no unacceptable violations even when these contingencies occur
  - N-1 reliable operation considers the loss of any single element

Contingency Analysis

- Of course this process can be automated with the usual approach of first defining a contingency set, and then sequentially applying the contingencies and checking for violations
  - This process can naturally be done in parallel
  - Contingency sets can get quite large, especially if one considers N-2 (outages of two elements) or N-1-1 (initial outage, followed by adjustment, then second outage
- The assumption is usually most contingencies will not cause problems, so screening methods can be used to quickly eliminate many contingencies
  - We'll cover these later (Gabe Ejebe is NAE class of 2018)

Contingency Analysis in PowerWorld

- Automated using the Contingency Analysis tool

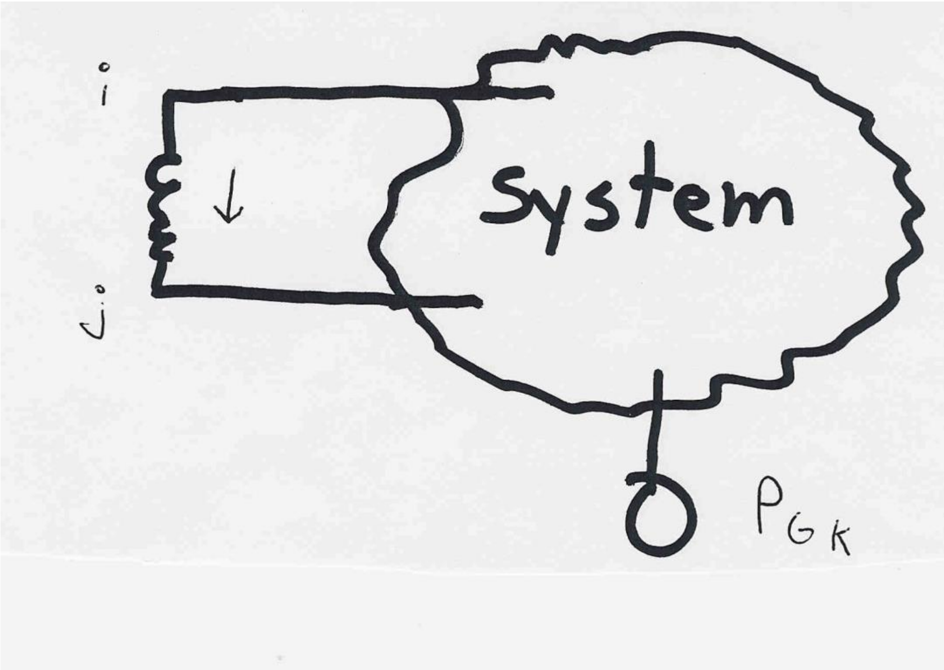


Power System Control and Sensitivities

- A major issue with power system operation is the limited capacity of the transmission system
  - lines/transformers have limits (usually thermal)
  - no direct way of controlling flow down a transmission line (e.g., there are no valves to close to limit flow)
  - open transmission system access associated with industry restructuring is stressing the system in new ways
- We need to indirectly control transmission line flow by changing the generator outputs
- Similar control issues with voltage

Indirect Transmission Line Control

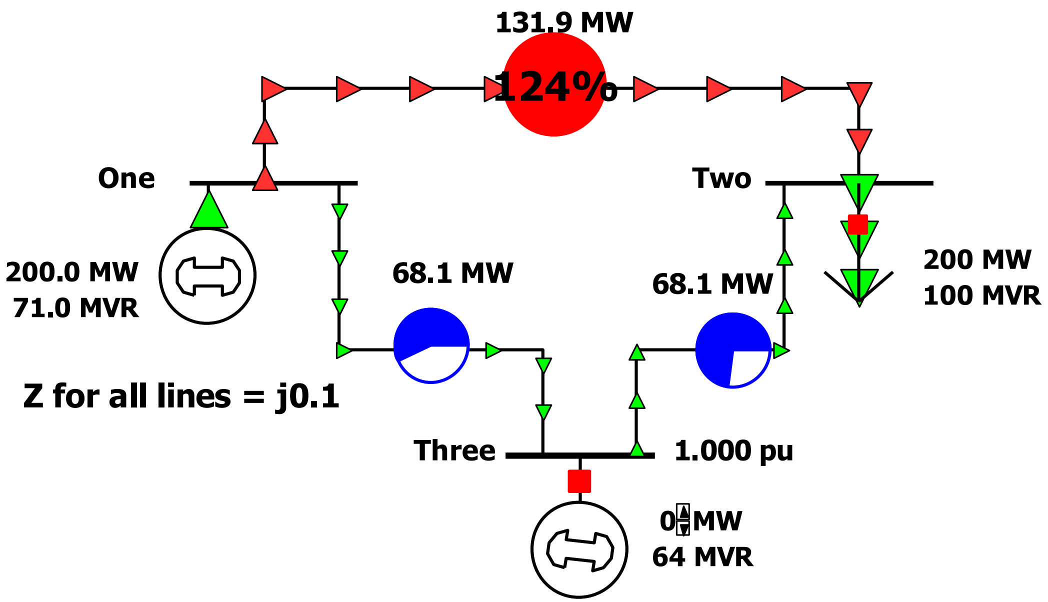
- What we would like to determine is how a change in generation at bus k affects the power flow on a line from bus i to bus j.



The assumption is that the change in generation is absorbed by the slack bus

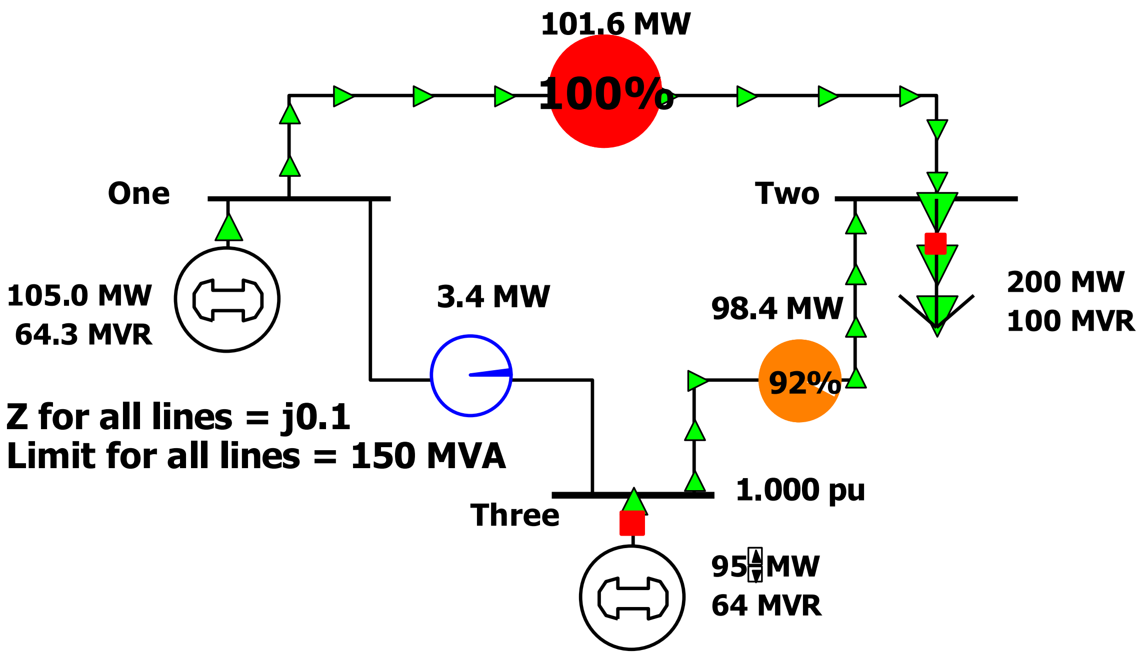
Power Flow Simulation - Before

- One way to determine the impact of a generator change is to compare a before/after power flow.
- For example below is a three bus case with an overload



Power Flow Simulation - After

- Increasing the generation at bus 3 by 95 MW (and hence decreasing it at bus 1 by a corresponding amount), results in a 30.3 MW drop in the MW flow on the line from bus 1 to 2, and a 64.7 MW drop on the flow from 1 to 3.



Expressed as a percent, 30.3/95 =32% and 64.7/95=68%

Analytic Calculation of Sensitivities

- Calculating control sensitivities by repeat power flow solutions is tedious and would require many power flow solutions. An alternative approach is to analytically calculate these values

The power flow from bus i to bus j is

$$P_{ij} \approx \frac{|V_i||V_j|}{X_{ij}} \sin(\theta_i - \theta_j) \approx \frac{\theta_i - \theta_j}{X_{ij}}$$

So $\Delta P_{ij} \approx \dfrac{\Delta\theta_i - \Delta\theta_j}{X_{ij}}$ We just need to get $\dfrac{\Delta\theta_{ij}}{\Delta P_{Gk}}$

Analytic Sensitivities

From the fast decoupled power flow we know

$$\Delta\boldsymbol{\theta} = \mathbf{B}^{-1}\Delta\mathbf{P}(\mathbf{x})$$

So to get the change in $\Delta\boldsymbol{\theta}$ due to a change of generation at bus k, just set $\Delta\mathbf{P}(\mathbf{x})$ equal to all zeros except a minus one at position k.

$$\Delta\mathbf{P} = \begin{bmatrix} 0 \\ \vdots \\ -1 \\ 0 \\ \vdots \end{bmatrix} \begin{matrix} \\ \\ \leftarrow \text{ Bus k} \\ \\ \\ \end{matrix}$$

Three Bus Sensitivity Example

For a three bus, three line case with $Z_{\text{line}} = j0.1$

$$\mathbf{Y}_{\text{bus}} = j\begin{bmatrix} -20 & 10 & 10 \\ 10 & -20 & 10 \\ 10 & 10 & -20 \end{bmatrix} \rightarrow \mathbf{B} = \begin{bmatrix} -20 & 10 \\ 10 & -20 \end{bmatrix}$$

Hence for a change of generation at bus 3

$$\begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \end{bmatrix} = \begin{bmatrix} -20 & 10 \\ 10 & -20 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.0333 \\ 0.0667 \end{bmatrix}$$

Then $\Delta P_{3 \text{ to } 1} = \dfrac{0.0667 - 0}{0.1} = 0.667 \text{ pu}$

$\Delta P_{3 \text{ to } 2} = 0.333 \text{ pu}$ $\quad$ $\Delta P_{2 \text{ to } 1} = 0.333 \text{ pu}$

More General Sensitivity Analysis: Notation

- We consider a system with *n* buses and L lines given by the set given by the set $L \triangleq \{\ell_1, \ell_2, \cdots, \ell_L\}$
  - Some authors designate the slack as bus zero; an alternative approach, that is easier to implement in cases with multiple islands and hence slacks, is to allow any bus to be the slack, and just set its associated equations to trivial equations just stating that the slack bus voltage is constant
- We may denote the k^th transmission line or transformer in the system, ℓ_k , as

$$\ell_k \triangleq (i_k, j_k),$$

from node

to node

Notation, cont.

- We'll denote the real power flowing on ℓ_k from bus i to bus j as f_k
- The vector of real power flows on the L lines is:

$$\mathbf{f} \triangleq [\, f_{\ell_1}, f_{\ell_2}, \cdots, f_{\ell_L} \,]^T$$

  which we simplify to $\mathbf{f} = [\, f_1, f_2, \cdots, f_L \,]^T$
- The bus real and reactive power injection vectors are

$$\mathbf{p} \triangleq [\, p^1, p^2, \cdots, p^N \,]^T$$

$$\mathbf{q} \triangleq [\, q^1, q^2, \cdots, q^N \,]^T$$

Notation, cont.

- The series admittance of line ℓ is $g_\ell + jb_\ell$ and we define $$\tilde{\mathbf{B}} \triangleq -diag\left\{b_1, b_2, \cdots, b_L\right\}$$
- We define the L×N incidence matrix $$\mathbf{A} \triangleq \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_L^T \end{bmatrix}$$

where the component j of $\mathbf{a}_i$ is nonzero whenever line ℓ_i is coincident with node j. Hence **A** is quite sparse, with two nonzeros per row

Analysis Example: Available Transfer Capability

- The power system available transfer capability or ATC is defined as the maximum additional MW that can be transferred between two specific areas, while meeting all the specified pre- and post-contingency system conditions
- ATC impacts measurably the market outcomes and system reliability and, therefore, the ATC values impact the system and market behavior
- A useful reference on ATC is *Available Transfer Capability Definitions and Determination* from NERC, June 1996 (available freely online)

ATC and Its Key Components

- Total transfer capability (TTC)
  - Amount of real power that can be transmitted across an interconnected transmission network in a reliable manner, including considering contingencies
- Transmission reliability margin (TRM)
  - Amount of TTC needed to deal with uncertainties in system conditions; typically expressed as a percent of TTC
- Capacity benefit margin (CBM)
  - Amount of TTC needed by load serving entities to ensure access to generation; typically expressed as a percent of TTC

ATC and Its Key Components

- Uncommitted transfer capability (UTC)

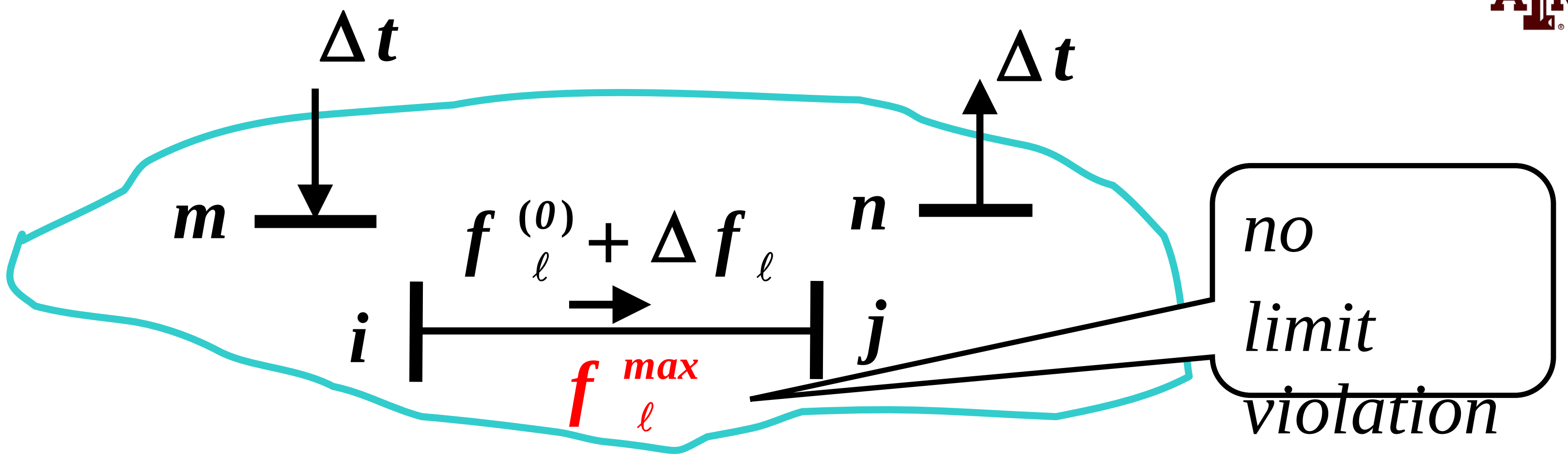
  $$UTC \triangleq TTC - \text{ existing transmission commitment}$$

- Formal definition of ATC is

  $$ATC \triangleq UTC - CBM - TRM$$

- We focus on determining $U_{m,n}$, the UTC from node m to node n
- $U_{m,n}$ is defined as the maximum additional MW that can be transferred from node m to node n without violating any limit in either the base case or in any post-contingency conditions

UTC (or TTC) Evaluation



$$U_{m,n} = max \quad \Delta t$$

s.t.

$$f_\ell^{(j)} + \Delta f_\ell \leq f_\ell^{max} \quad \forall \ell \in L$$

for the base case j = 0 and each contingency case

$j = 1,2 \ldots , J$

Conceptual Solution Algorithm

1. Solve the initial power flow, corresponding to the initial system dispatch (i.e., existing commitments); set the change in transfer $\Delta t^{(0)} = 0$, k=0; set step size δ; j is used to indicate either the base case (j=0) or a contingency, j= 1,2,3…J
2. Compute $\Delta t^{(k+1)} = \Delta t^{(k)} + \delta$
3. Solve the power flow for the new $\Delta t^{(k+1)}$
4. Check for limit violations: if violation is found set $U^{j}_{m,n} = \Delta t^{(k)}$ and stop; else set k=k+1, and goto 2

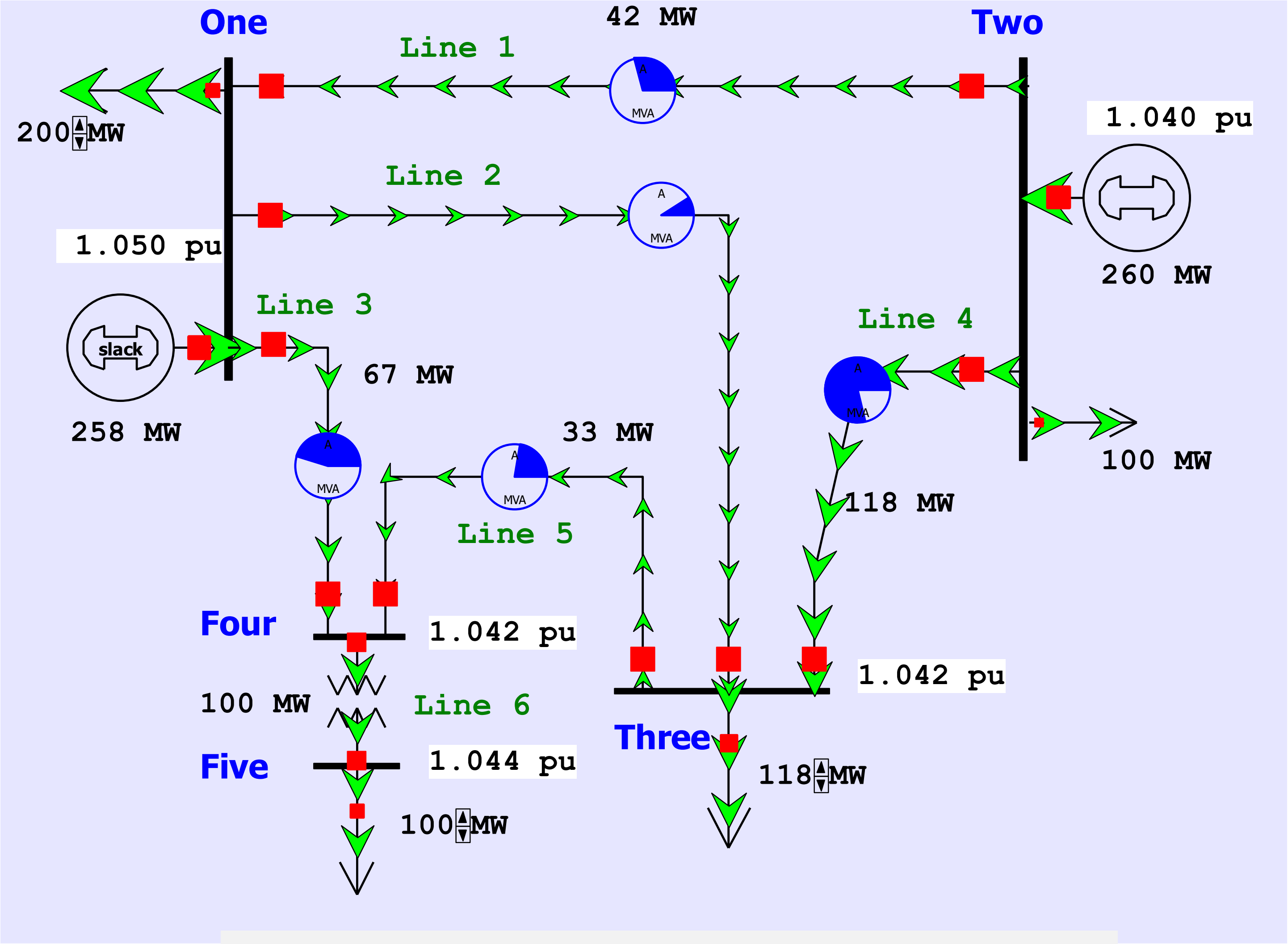
Conceptual Solution Algorithm, cont.

- This algorithm is applied for the base case (j=0) and each specified contingency case, j=1,2,..J
- The final UTC, $U_{m,n}$ is then determined by

$$U_{m,n} = \min_{0 \le j \le J} \left\{ U_{m,n}^{(j)} \right\}$$

- This algorithm can be easily performed on parallel processors since each contingency evaluation is independent of the other

Five Bus Example: Reference



PowerWorld Case: B5_DistFact

Five Bus Example: Reference

ℓ	i	j	g_ℓ	b_ℓ	f_ℓ^{max} (MW)
ℓ_1	1	2	0	6.25	150
ℓ_2	1	3	0	12.5	400
ℓ_3	1	4	0	12.5	150
ℓ_4	2	3	0	12.5	150
ℓ_5	3	4	0	12.5	150
ℓ_6	4	5	0	10	1,000

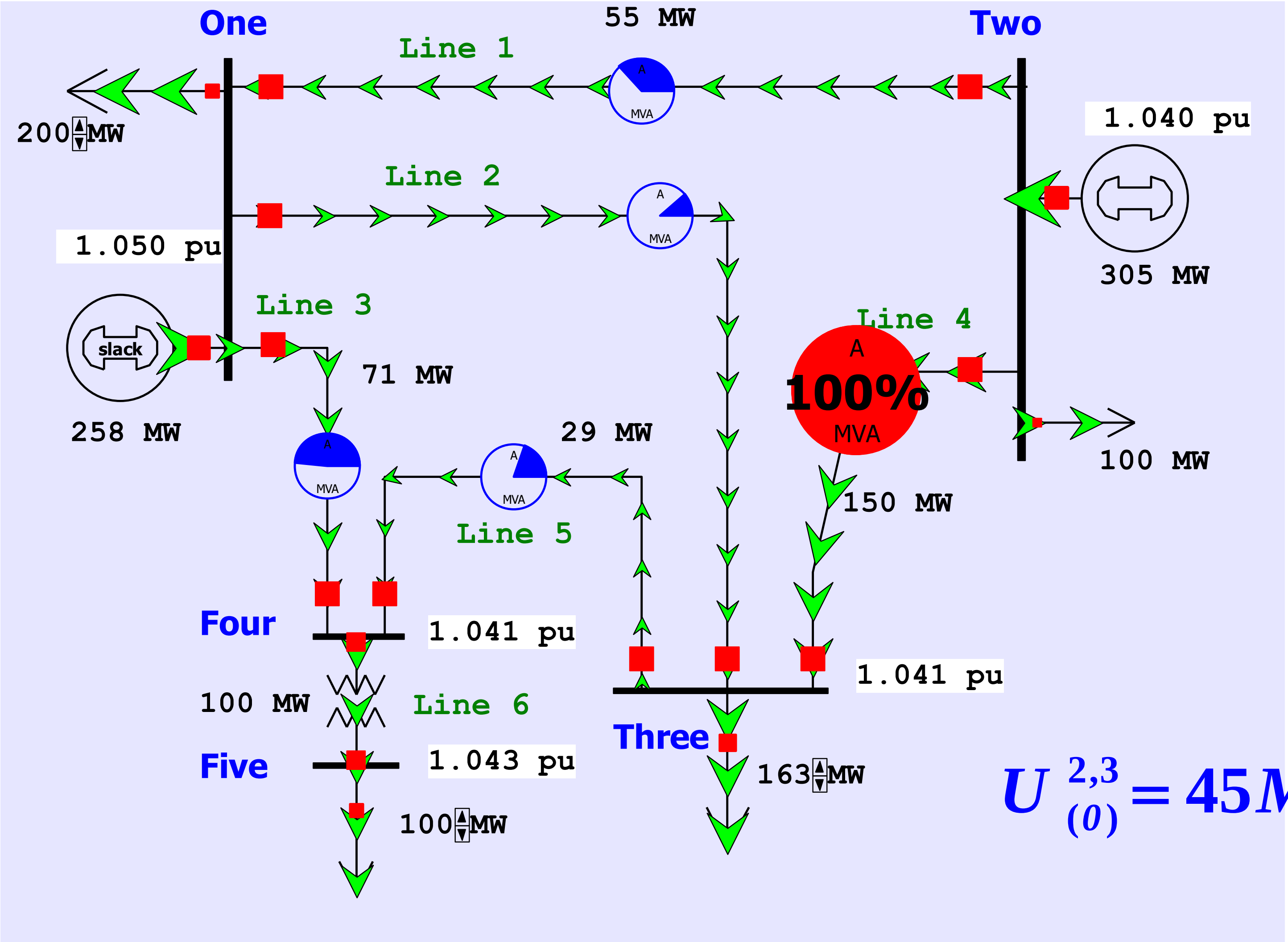
5 - BUS SYSTEM EXAMPLE

- We evaluate $U_{2,3}$ using the previous procedure
  - Gradually increase generation at Bus 2 and load at Bus 3
- We consider the base case and the single contingency with line 2 outaged (between 1 and 3): $J = 1$
- Simulation results show for the base case that

$$\boldsymbol{U_{2,3}^{(0)} = 45\ MW}$$

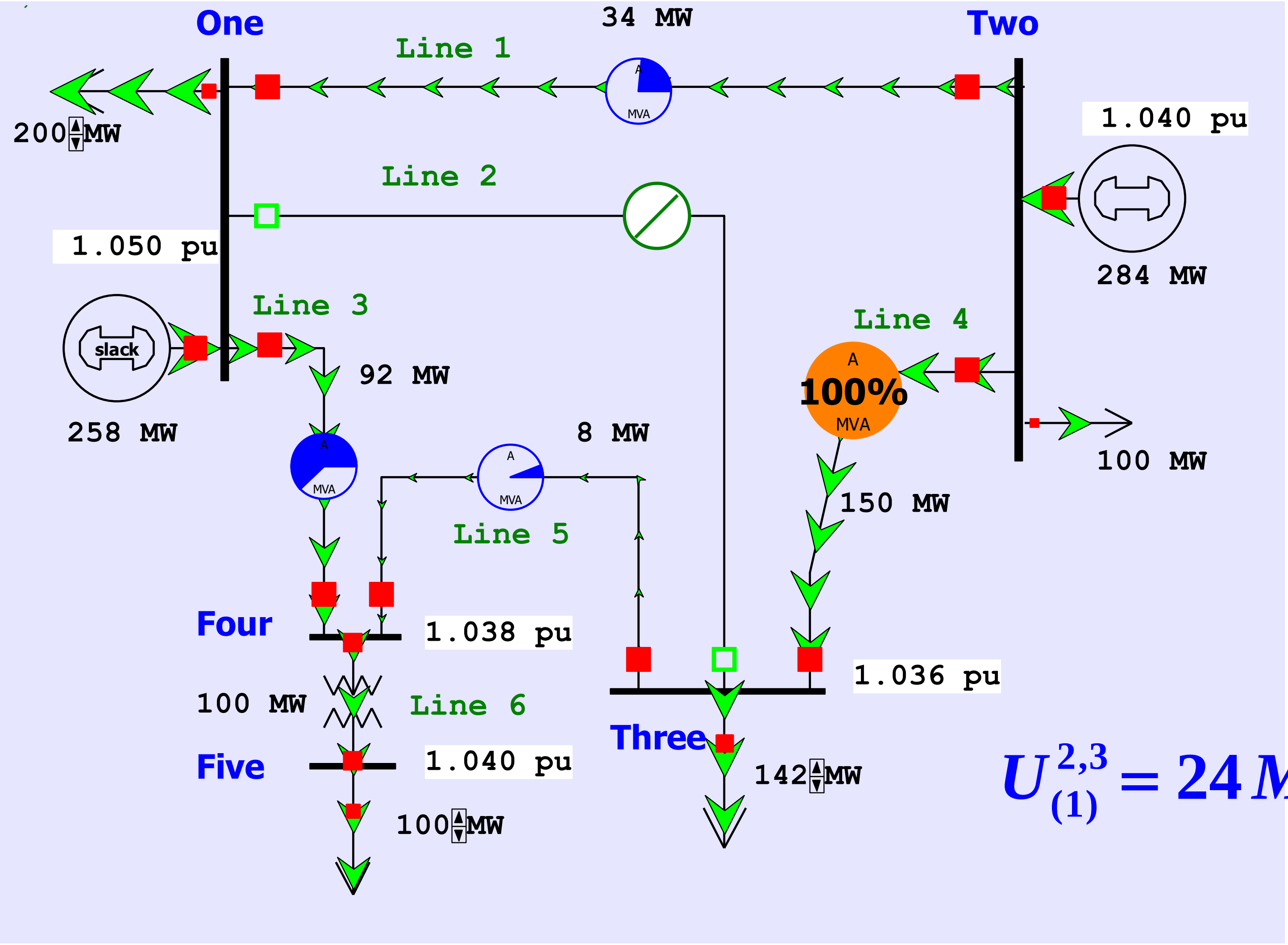
- And for the contingency that $\boldsymbol{U_{2,3}^{(1)} = 24\ MW}$
- Hence $\boldsymbol{U_{2,3} = min\left\{U_{2,3}^{(0)}, U_{2,3}^{(1)}\right\} = 24\ MW}$

Five Bus: Maximum Base Case Transfer



$$U_{(0)}^{2,3} = 45\,MW$$

Five Bus: Maximum Contingency Transfer



$$U_{(1)}^{2,3} = 24\,MW$$

Computational Considerations

- Obviously such a brute force approach can run into computational issues with large systems
- Consider the following situation:
  - 10 iterations for each case
  - 6,000 contingencies
  - 2 seconds to solve each power flow
- It will take over 33 hours to compute a single UTC for the specified transfer direction from m to n.
- Consequently, there is an acute need to develop fast tools that can provide satisfactory estimates

Problem Formulation

- Denote the system state by

$$\mathbf{x} \triangleq \begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{V} \end{bmatrix} \qquad \begin{aligned} \boldsymbol{\theta} &\triangleq [\theta^1, \theta^2, \cdots, \theta^N]^T \\ \mathbf{V} &\triangleq [V^1, V^2, \cdots, V^N]^T \end{aligned}$$

- Denote the conditions corresponding to the existing commitment/dispatch by $\mathbf{s}^{(0)}$, $\mathbf{p}^{(0)}$ and $\mathbf{f}^{(0)}$ so that

$$\begin{cases} \mathbf{g}(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}) = \mathbf{0} & \text{the power flow equations} \\ \mathbf{f}^{(0)} = \mathbf{h}(\mathbf{x}^{(0)}) & \text{line real power flow vector} \end{cases}$$

Problem Formulation

$$\mathbf{g}(\mathbf{x},\mathbf{p}) = \begin{bmatrix} \mathbf{g}^P(\mathbf{x},\mathbf{p}) \\ \mathbf{g}^Q(\mathbf{x},\mathbf{p}) \end{bmatrix}$$

g includes the real and reactive power balance equations

$$g_k^P(\underline{s},\underline{p}) = V^k \sum_{m=1}^{N} \left(V^m \left[G_{km} cos\left(\theta^k - \theta^m\right) + B_{km} sin\left(\theta^k - \theta^m\right) \right] \right) - p^k$$

$$g_k^Q(\underline{s},\underline{p}) = V^m \sum_{m=1}^{N} \left(V^m \left[G_{km} sin\left(\theta^k - \theta^m\right) - B_{km} cos\left(\theta^k - \theta^m\right) \right] \right) - q^k$$

$$h_\ell(\underline{s}) = g_\ell \left[\left(V^i\right)^2 - V^i V^j cos(\theta^i - \theta^j) \right] - b_\ell V^i V^j sin(\theta^i - \theta^j), \ \ell = \left(i, j\right)$$

Problem Formulation

- For a small change, $\Delta\mathbf{p}$, that moves the injection from $\mathbf{p}^{(0)}$ to $\mathbf{p}^{(0)} + \Delta\mathbf{p}$, we have a corresponding change in the state $\Delta\mathbf{x}$ with

$$\mathbf{g}\,(\mathbf{x}^{(0)} + \Delta\mathbf{x}, p^{(0)} + \Delta\mathbf{p}) = \mathbf{0}$$

- We then apply a first order Taylor's series expansion

$$\mathbf{g}\left(\mathbf{x}^{(0)} + \Delta\mathbf{x}, \mathbf{p}^{(0)} + \Delta\mathbf{p}\right) = \mathbf{g}\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right) + \left.\frac{\partial\mathbf{g}}{\partial\mathbf{x}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta\mathbf{x}$$

$$+ \left.\frac{\partial\mathbf{g}}{\partial\mathbf{p}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta p + h.o.t.$$

Problem Formulation

- We consider this to be a “small signal” change, so we can neglect the higher order terms (h.o.t.) in the expansion
- Hence we should still be satisfying the power balance equations with this perturbation; so

$$\left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta \mathbf{x} + \left.\frac{\partial \mathbf{g}}{\partial \mathbf{p}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta \boldsymbol{p} \approx \mathbf{0}$$

Problem Formulation

- Also, from the power flow equations, we obtain

$$\frac{\partial \mathbf{g}}{\partial \mathbf{p}} = \begin{bmatrix} \dfrac{\partial \mathbf{g}^{P}}{\partial \mathbf{p}} \\ \hdashline \dfrac{\partial \mathbf{g}^{Q}}{\partial \mathbf{p}} \end{bmatrix} = \begin{bmatrix} -\mathbf{I} \\ \hdashline \mathbf{0} \end{bmatrix}$$

and then just the power flow Jacobian

$$\frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{g}^{P}}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{g}^{P}}{\partial \mathbf{V}} \\ \dfrac{\partial \mathbf{g}^{Q}}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{g}^{Q}}{\partial \mathbf{V}} \end{bmatrix} = \mathbf{J}(\mathbf{x},\mathbf{p})$$

Problem Formulation

- With the standard assumption that the power flow Jacobian is nonsingular, then

$$\Delta \mathbf{x} \approx \left[\mathbf{J}\left(\mathbf{x}^{(0)},\mathbf{p}^{(0)}\right)\right]^{-1}\begin{bmatrix}\mathbf{I}\\ \mathbf{0}\end{bmatrix}\Delta \mathbf{p}$$

- We can then compute the change in the line real power flow vector

$$\Delta \mathbf{f} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right]^{T}\Delta \mathbf{s} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right]^{T}\left[J\left(\mathbf{x}^{(0)},\mathbf{p}^{(0)}\right)\right]^{-1}\begin{bmatrix}\mathbf{I}\\ \mathbf{0}\end{bmatrix}\Delta \mathbf{p}$$

Sensitivity Comments

- Sensitivities can easily be calculated even for large systems
  - If $\Delta \mathbf{p}$ is sparse (just a few injections) then we can use a fast forward; if sensitivities on a subset of lines are desired we could also use a fast backward
- Sensitivities are dependent upon the operating point
  - They also include the impact of marginal losses
- Sensitivities could easily be expanded to include additional variables in $\mathbf{x}$ (such as phase shifter angle), or additional equations, such as reactive power flow

Sensitivity Comments, cont.

- Sensitivities are used in the optimal power flow; in that context a common application is to determine the sensitivities of an overloaded line to injections at all the buses
- In the below equation, how could we quickly get these values?

$$\Delta \mathbf{f} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right]^T \Delta f \approx \left[\frac{\partial \mathbf{h}}{\partial x}\right]^T \left[J\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right)\right]^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p}$$

  – A useful reference is O. Alsac, J. Bright, M. Prais, B. Stott, "Further Developments in LP-Based Optimal Power Flow," IEEE. Trans. on Power Systems, August 1990, pp. 697-711; especially see equation 3.

Sensitivity Example in PowerWorld

- Open case B5_DistFact and then Select **Tools, Sensitivities, Flow and Voltage Sensitivities**
  - Select **Single Meter, Multiple Transfers**, **Buses** page
  - Select the **Device Type (Line/XFMR)**, **Flow Type (MW)**, then select the line (from Bus 2 to Bus 3)
  - Click **Calculate Sensitivities**; this shows impact of a single injection going to the slack bus (Bus 1)
  - For our example of a transfer from 2 to 3 the value is the result we get for bus 2 (0.5440) minus the result for bus 3 (-0.1808) = 0.7248
  - With a flow of 118 MW, we would hit the 150 MW limit with (150-118)/0.7248 =44.1MW, close to the limit we found of 45MW

Sensitivity Example in PowerWorld

- If we change the conditions to the anticipated maximum loading (changing the load at 2 from 118 to 118+44=162 MW) and we re-evaluate the sensitivity we note it has changed little (from -0.7248 to -0.7241)
  - Hence a linear approximation (at least for this scenario) could be justified
- With what we know so far, to handle the contingency situation, we would have to simulate the contingency, and reevaluate the sensitivity values
  - We'll be developing a quicker (but more approximate) approach next

Linearized Sensitivity Analysis

- By using the approximations from the fast decoupled power flow we can get sensitivity values that are independent of the current state. That is, by using the **B**' and **B**'' matrices
- For line flow we can approximate

$$h_\ell(\underline{s}) = g_\ell\left[\left(V^i\right)^2 - V^i V^j \cos(\theta^i - \theta^j)\right] - b_\ell V^i V^j \sin(\theta^i - \theta^j),\ \ell = \left(i, j\right)$$

By using the FDPF appxomations

$$h_\ell(\underline{s}) \approx -b_\ell(\theta^i - \theta^j) = \frac{(\theta^i - \theta^j)}{X_\ell},\ \ell = \left(i, j\right)$$