ECEN 615
Methods of Electric Power Systems Analysis

Lecture 16: Sensitivity Analysis

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Announcements

- Exam average was 85.7, with a high of 100
- Read Chapter 7 (the term reliability is now used instead of security)
- Homework 4 is assigned today, due on Thursday Nov 1

Linearized Sensitivity Analysis

- By using the approximations from the fast decoupled power flow we can get sensitivity values that are independent of the current state. That is, by using the **B**' and **B**'' matrices
- For line flow we can approximate

$$h_\ell(\underline{s}) = g_\ell\left[\left(V^i\right)^2 - V^i V^j cos(\theta^i - \theta^j)\right] - b_\ell V^i V^j sin(\theta^i - \theta^j), \ \ell = \left(i, j\right)$$

By using the FDPF appxomations

$$h_\ell(\underline{s}) \approx -b_\ell(\theta^i - \theta^j) = \frac{(\theta^i - \theta^j)}{X_\ell}, \ \ell = \left(i, j\right)$$

Linearized Sensitivity Analysis

- Also, for each line ℓ

$$\frac{\partial \boldsymbol{h}_\ell}{\partial \boldsymbol{\theta}} \approx -\boldsymbol{b}_\ell \mathbf{a}_\ell \qquad\qquad \frac{\partial \boldsymbol{h}_\ell}{\partial \mathbf{V}} \approx \mathbf{0}$$

and so,

$$\frac{\partial \mathbf{h}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}} \\ \hline \dfrac{\partial \mathbf{h}}{\partial \mathbf{V}} \end{bmatrix} = -\left[\begin{array}{c|c|c} \begin{matrix} \boldsymbol{b}_{\ell_1}\mathbf{a}_1 & \cdots & \boldsymbol{b}_{\ell_L}\mathbf{a}_L \end{matrix} & \cdots & \\ \hline \mathbf{0} & & \mathbf{0} \end{array}\right] \approx \begin{bmatrix} \mathbf{A}^T \tilde{\mathbf{B}} \\ \mathbf{0} \\ \hline \end{bmatrix}$$

Sensitivity Analysis: Recall the Matrix Notation

- The series admittance of line ℓ is $g_\ell + jb_\ell$ and we define $$\tilde{\mathbf{B}} \triangleq -diag\left\{b_1, b_2, \cdots, b_L\right\}$$
- We define the L×N incidence matrix

$$\mathbf{A} \triangleq \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_L^T \end{bmatrix}$$

where the component j of $\mathbf{a}_i$ is nonzero whenever line ℓ_i is coincident with node j. Hence **A** is quite sparse, with at most two nonzeros per row

Linearized Active Power Flow Model

- Under these assumptions the change in the real power line flows are given as

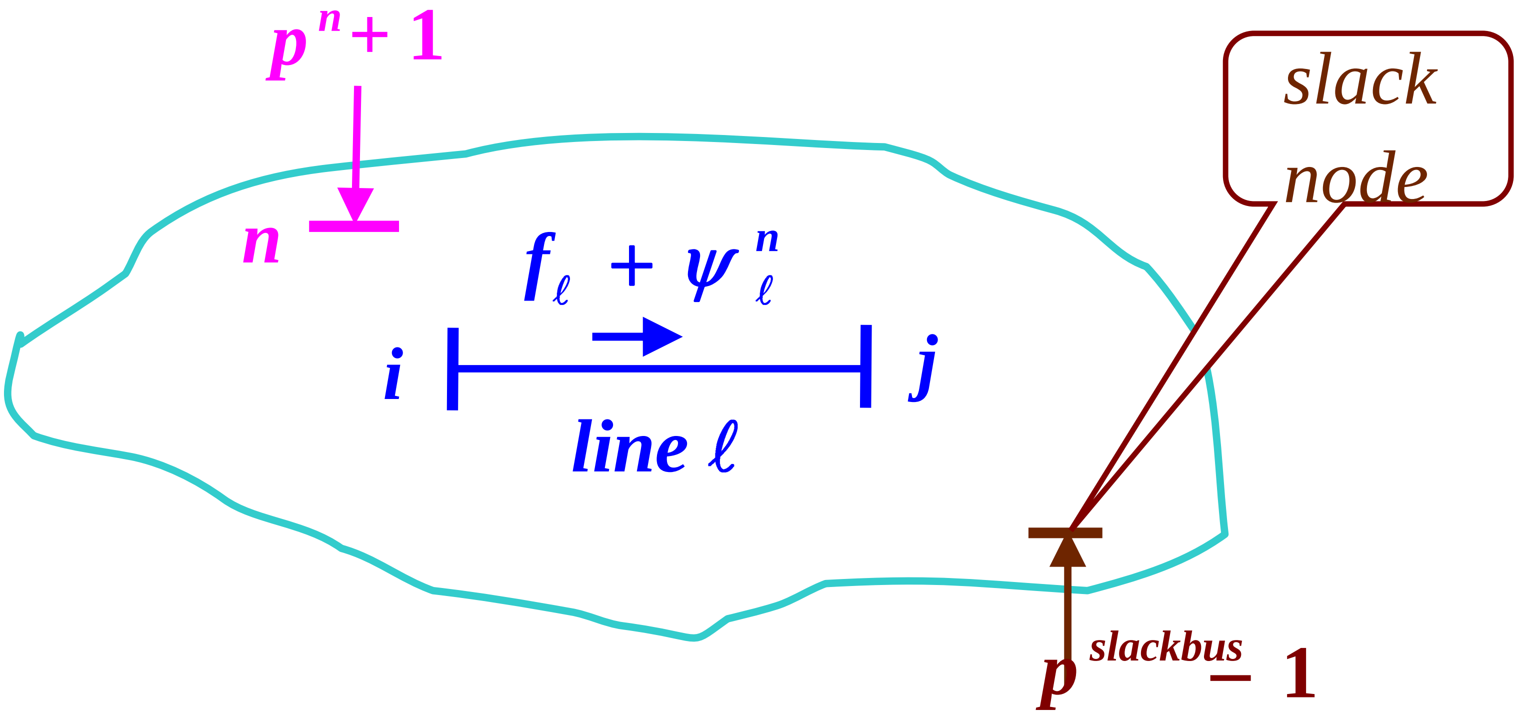
$$\Delta\mathbf{f} \approx \begin{bmatrix} \tilde{\mathbf{B}}\mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{B}' & \mathbf{0} \\ \mathbf{0} & \mathbf{B}'' \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta\mathbf{p} = \underbrace{\tilde{\mathbf{B}}\mathbf{A}\left[\mathbf{B}'\right]^{-1}\Delta\mathbf{p}} = \mathbf{\Psi}\Delta\mathbf{p}$$

- The constant matrix $\mathbf{\Psi} \triangleq \tilde{\mathbf{B}}\mathbf{A}\left[\mathbf{B}'\right]^{-1}$ is called the injection shift factor matrix (ISF)

Injection Shift Factors (ISFs)

- The element ψ_{ℓ}^{n} in row ℓ and column n of Ψ is called the injection shift factor (*ISF)* of line ℓ with respect to the injection at node *n*
  - Absorbed at the slack bus, so it is slack bus dependent
- Terms generation shift factor (GSF) and load shift factor (LSF) are also used (such as by NERC)
  - Same concept, just a variation in the sign whether it is a generator or a load
  - Sometimes the associated element is not a single line, but rather a combination of lines (an interface)
- Terms used in North America are defined in the NERC glossary (http://www.nerc.com/files/glossary_of_terms.pdf)

ISF Interpretation



ψ_ℓ^n is the fraction of the additional 1 *MW* injection at node *n* that goes though line ℓ

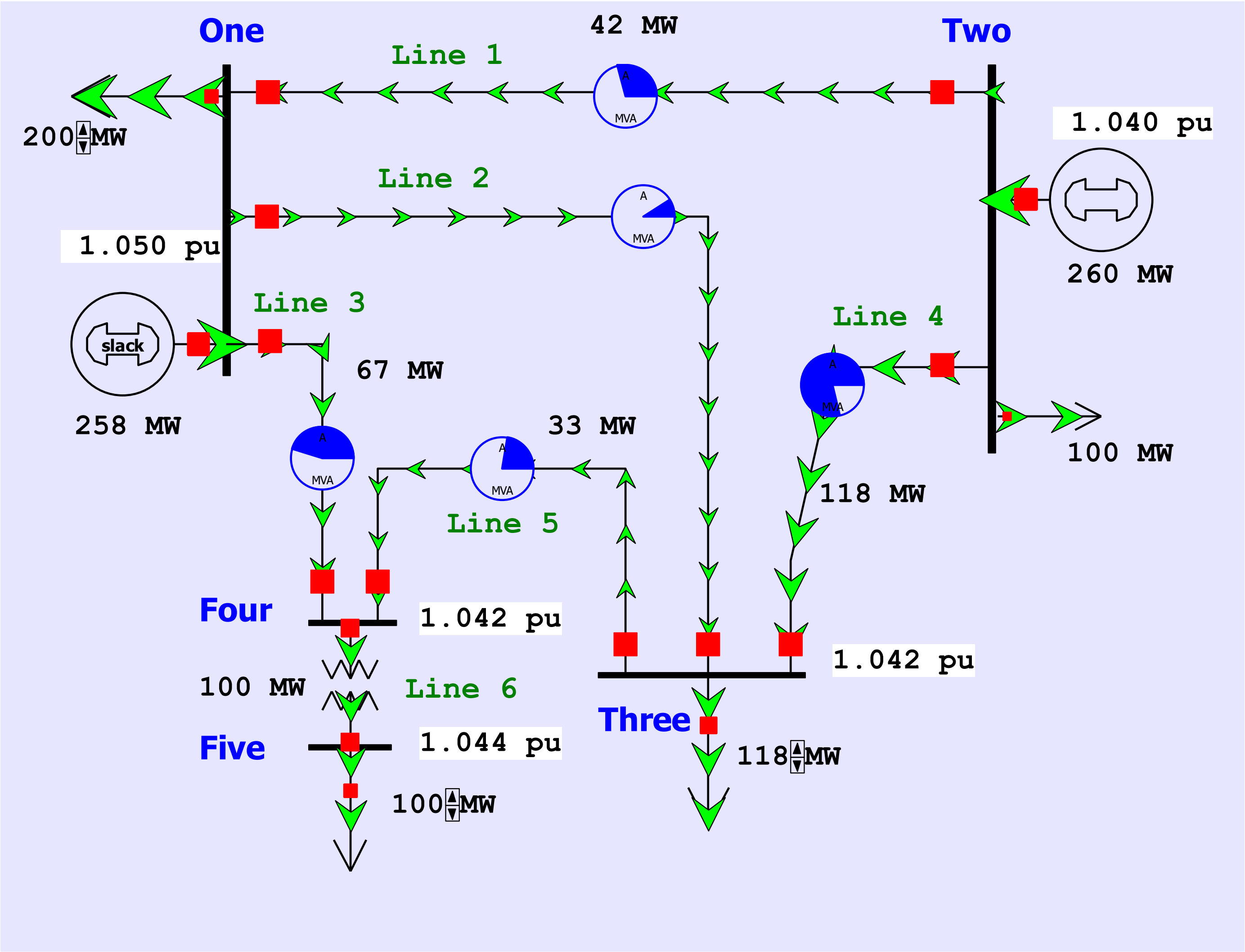
ISF Properties

- By definition, ψ_{ℓ}^{n} depends on the location of the slack bus
- By definition, $\psi_{\ell}^{slackbus} \equiv 0$ for $\forall \ell \in L$ since the injection and withdrawal buses are identical in this case and, consequently, no flow arises on any line ℓ
- The magnitude of ψ_{ℓ}^{n} is at most 1 since

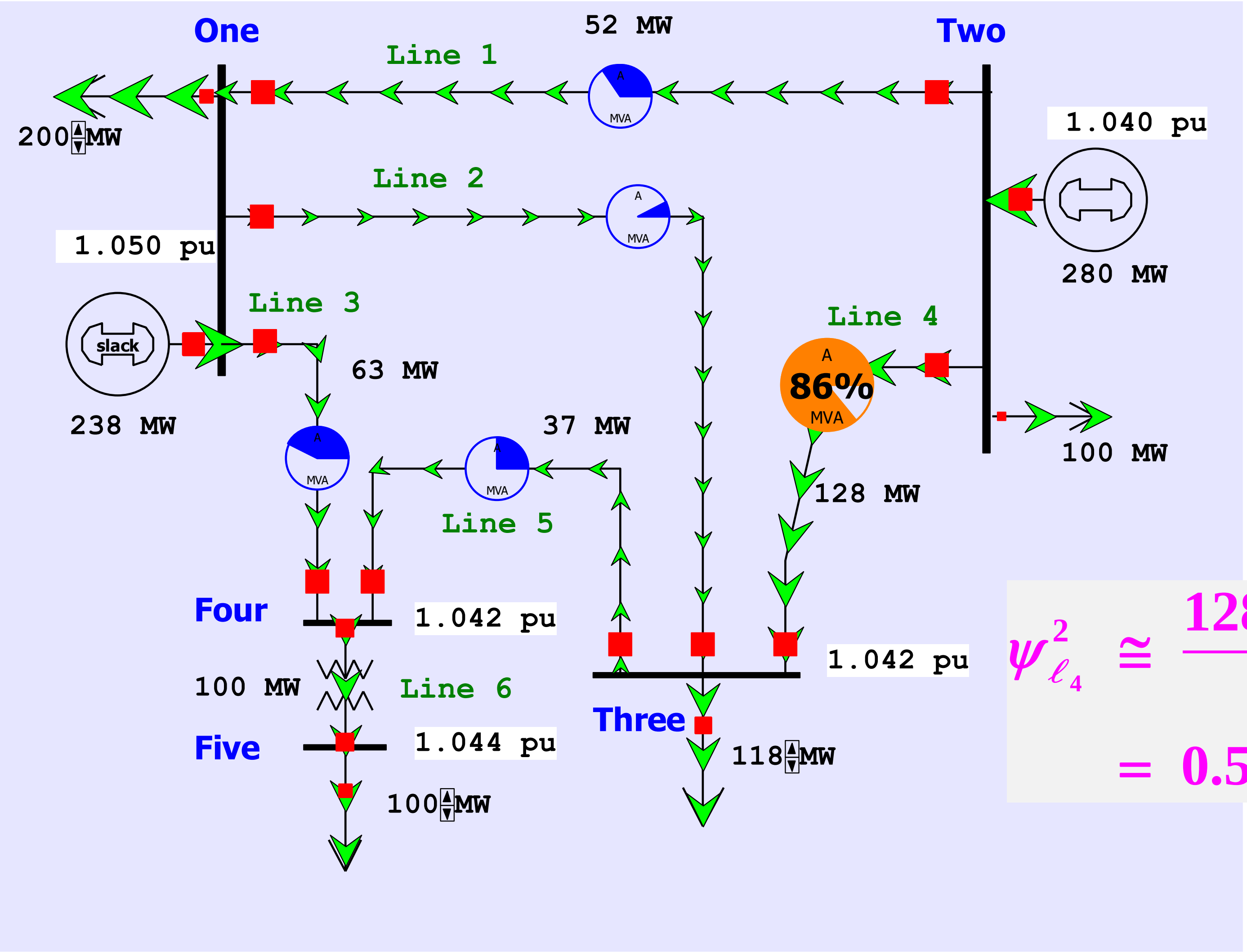
$$-1 \leq \psi_{\ell}^{n} \leq 1$$

Note, this is strictly true only for the linear (lossless) case. In the nonlinear case, it is possible that a transaction decreases losses. Hence a 1 MW injection could change a line flow by more than 1 MW.

Five Bus Example Reference



Five Bus ISF, Line 4, Bus 2 (to Slack)



$$\psi^2_{\ell_4} \cong \frac{128-118}{20} = 0.5$$

Five Bus Example

$$\tilde{\mathbf{B}} = -diag\{6.25,\quad 12.5,\quad 12.5,\quad 12.5,\quad 12.5,\quad 10\}$$

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

The row of **A** correspond to the lines and transformers, the columns correspond to the non-slack buses (buses 2 to 5); for each line there is a 1 at one end, a -1 at the other end (hence an assumed sign convention!). Here we put a 1 for the lower numbered bus, so positive flow is assumed from the lower numbered bus to the higher number

Five Bus Example

$$\mathbf{B}' = \mathbf{A}^T\tilde{\mathbf{B}}\mathbf{A} = \begin{bmatrix} -18.75 & 12.5 & 0 & 0 \\ 12.5 & -37.5 & 12.5 & 0 \\ 0 & 12.5 & -35 & 10 \\ 0 & 0 & 10 & -10 \end{bmatrix}$$

$$\underline{\Psi} = \underline{\tilde{B}}\,\underline{A}\left[\underline{B}'\right]^{-1} = \begin{bmatrix} -0.4545 & -0.1818 & -0.0909 & -0.0909 \\ -0.3636 & -0.5455 & -0.2727 & -0.2727 \\ -0.1818 & -0.2727 & -0.6364 & -0.6364 \\ 0.5455 & -0.1818 & -0.0909 & -0.0909 \\ 0.1818 & 0.2727 & -0.3636 & -0.3636 \\ 0 & 0 & 0 & -1.0000 \end{bmatrix}$$

With bus 1 as the slack, the buses (columns) go for 2 to 5

Five Bus Example Comments

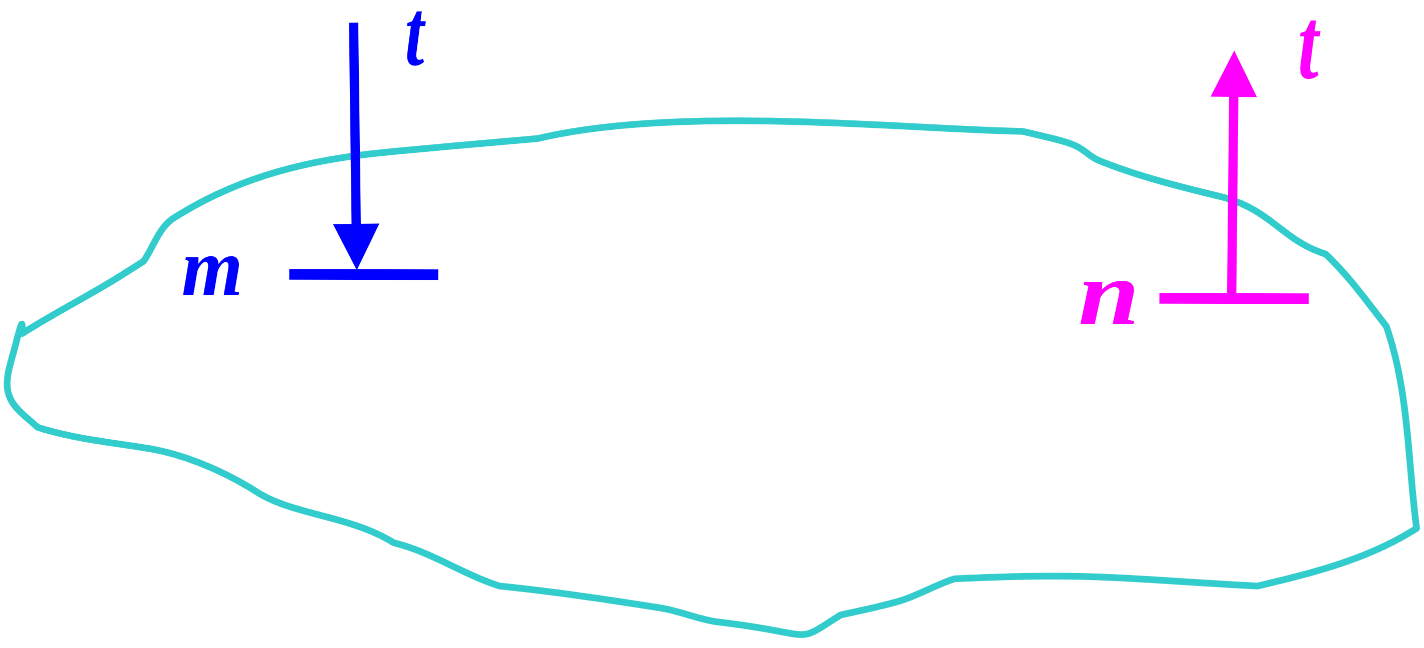
- At first glance the numerically determined value of (128-118)/20=0.5 does not match closely with the analytic value of 0.5455; however, in doing the subtraction we are losing numeric accuracy
  - Adding more digits helps (128.40 – 117.55)/20 = 0.5425
- The previous matrix derivation isn't intended for actual computation; Ψ is a full matrix so we would seldom compute all of its values
- Sparse vector methods can be used if we are only interested in the ISFs for certain lines and certain buses

Distribution Factors

- Various additional distribution factors may be defined
  - power transfer distribution factor (PTDF)
  - line outage distribution factor (LODF)
  - line addition distribution factor (LADF)
  - outage transfer distribution factor (OTDF)
- These factors may be derived from the ISFs making judicious use of the superposition principle

Definition: Basic Transaction

- A basic transaction involves the transfer of a specified amount of power t from an injection node m to a withdrawal node n



Definition: Basic Transaction

- We use the notation

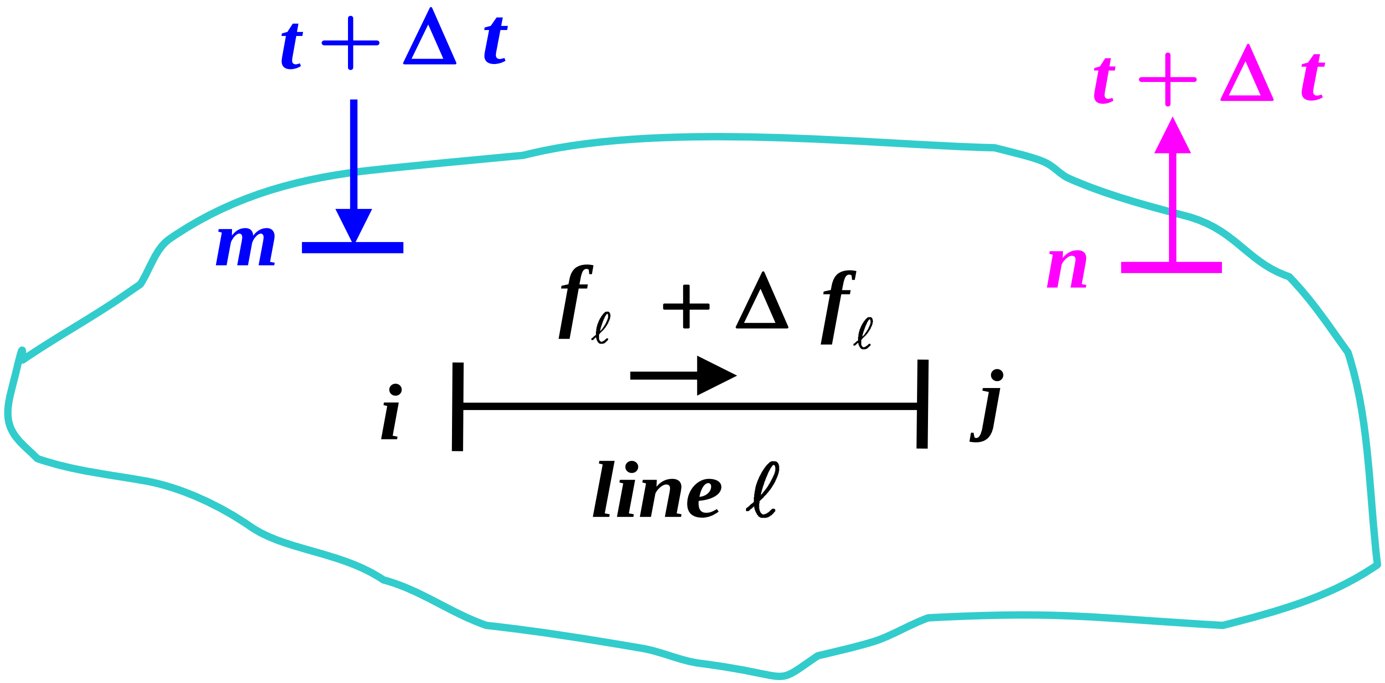
$$\boldsymbol{w} \triangleq \{\underbrace{\boldsymbol{m}}_{\textit{injection node}}, \underbrace{\boldsymbol{n}}_{\textit{withdrawal node}}, \underbrace{\boldsymbol{t}}_{\textit{quantity}}\}$$

to denote a basic transaction

Definition: PTDF

- NERC defines a PTDF as
  - "In the pre-contingency configuration of a system under study, a measure of the responsiveness or change in electrical loadings on transmission system Facilities due to a change in electric power transfer from one area to another, expressed in percent (up to 100%) of the change in power transfer"
  - Transaction dependent
- We'll use the notation $\varphi_{\ell}^{(w)}$ to indicate the PTDF on line ℓ with respect to basic transaction w
- In the lossless formulation presented here (and commonly used) it is slack bus independent

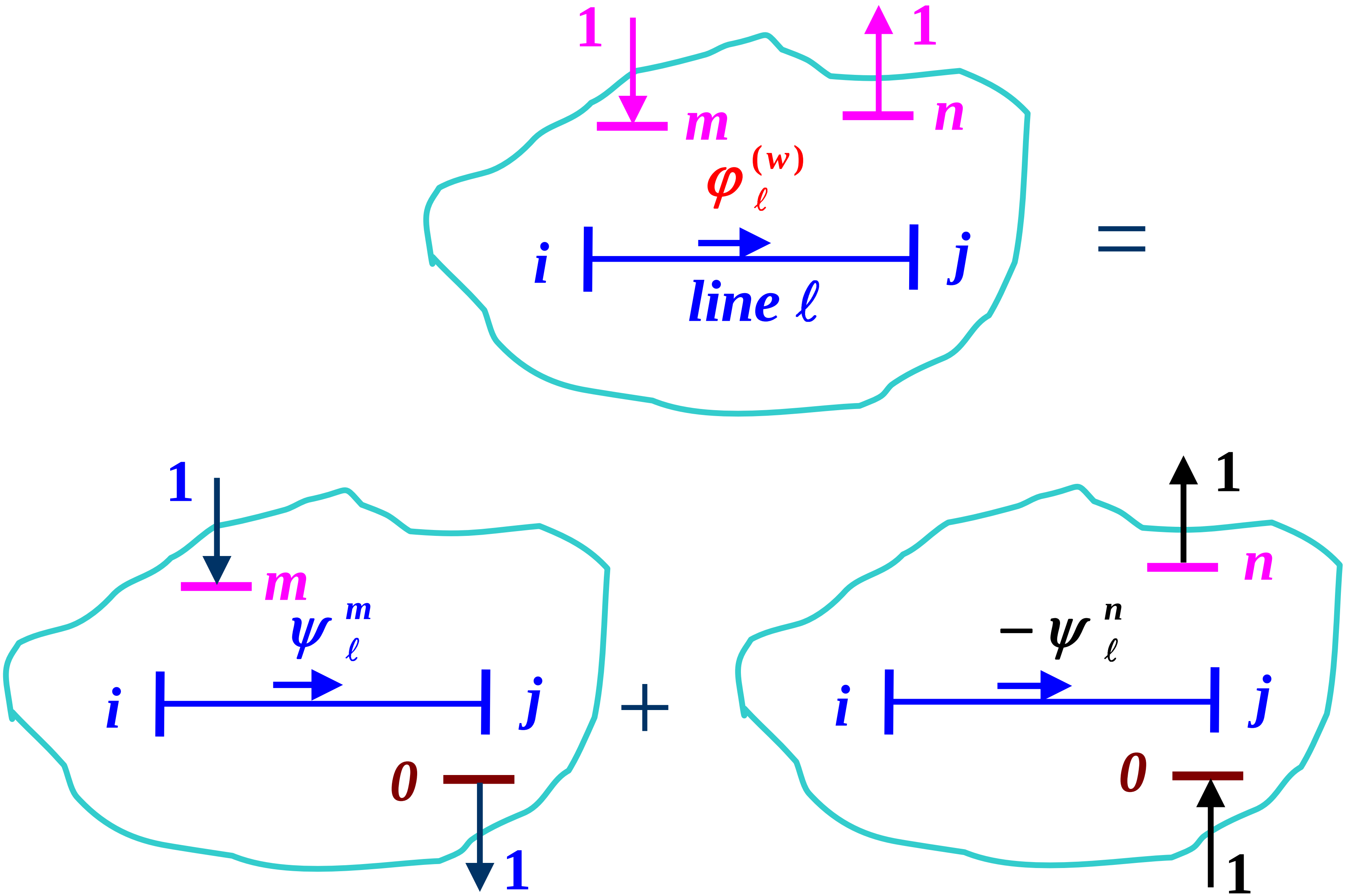
PTDFs



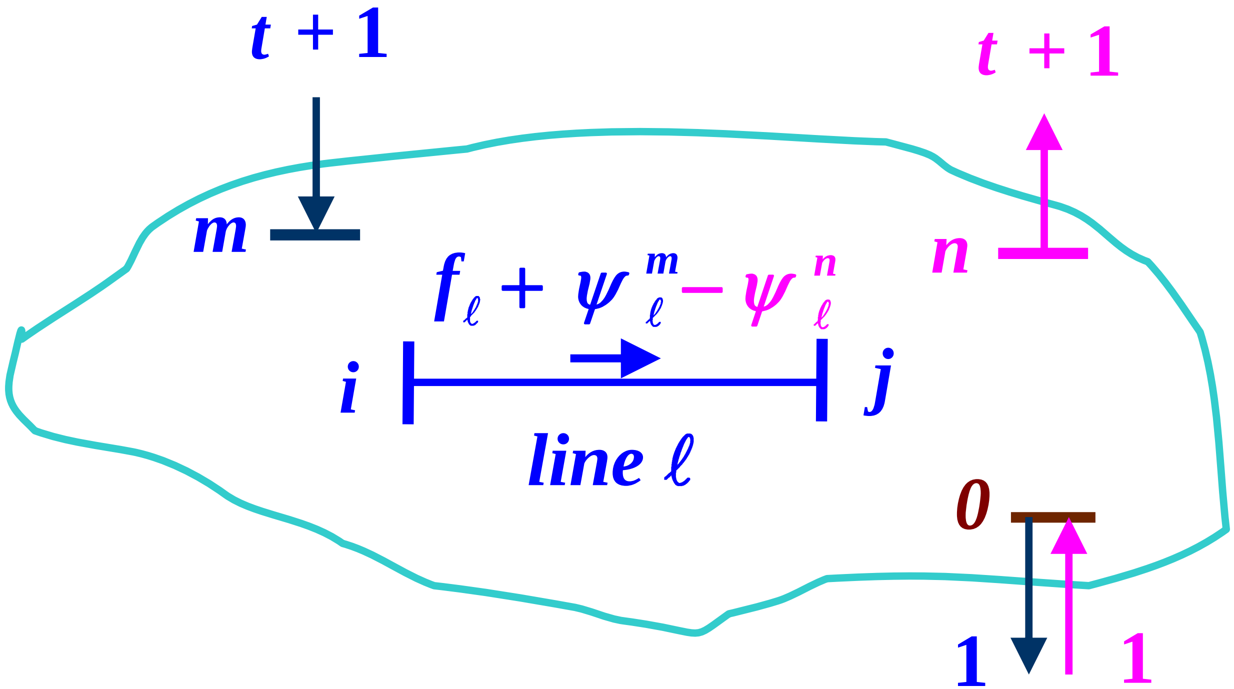
$$\varphi_\ell^{(w)} \triangleq \frac{\Delta f_\ell}{\Delta t}$$

Note, the PTDF is independent of the amount t; which is often expressed as a percent

PTDF Evaluation in Two Parts



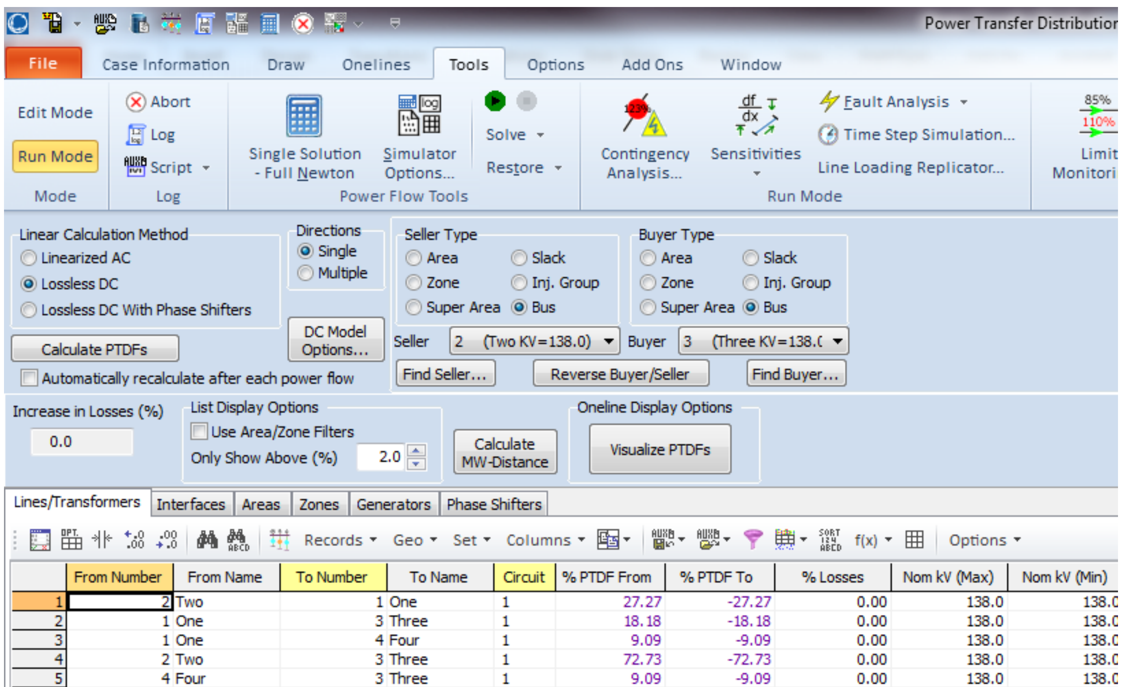
PTDF Evaluation



$$\boxed{\varphi_\ell^{(w)} = \psi_\ell^m - \psi_\ell^n}$$

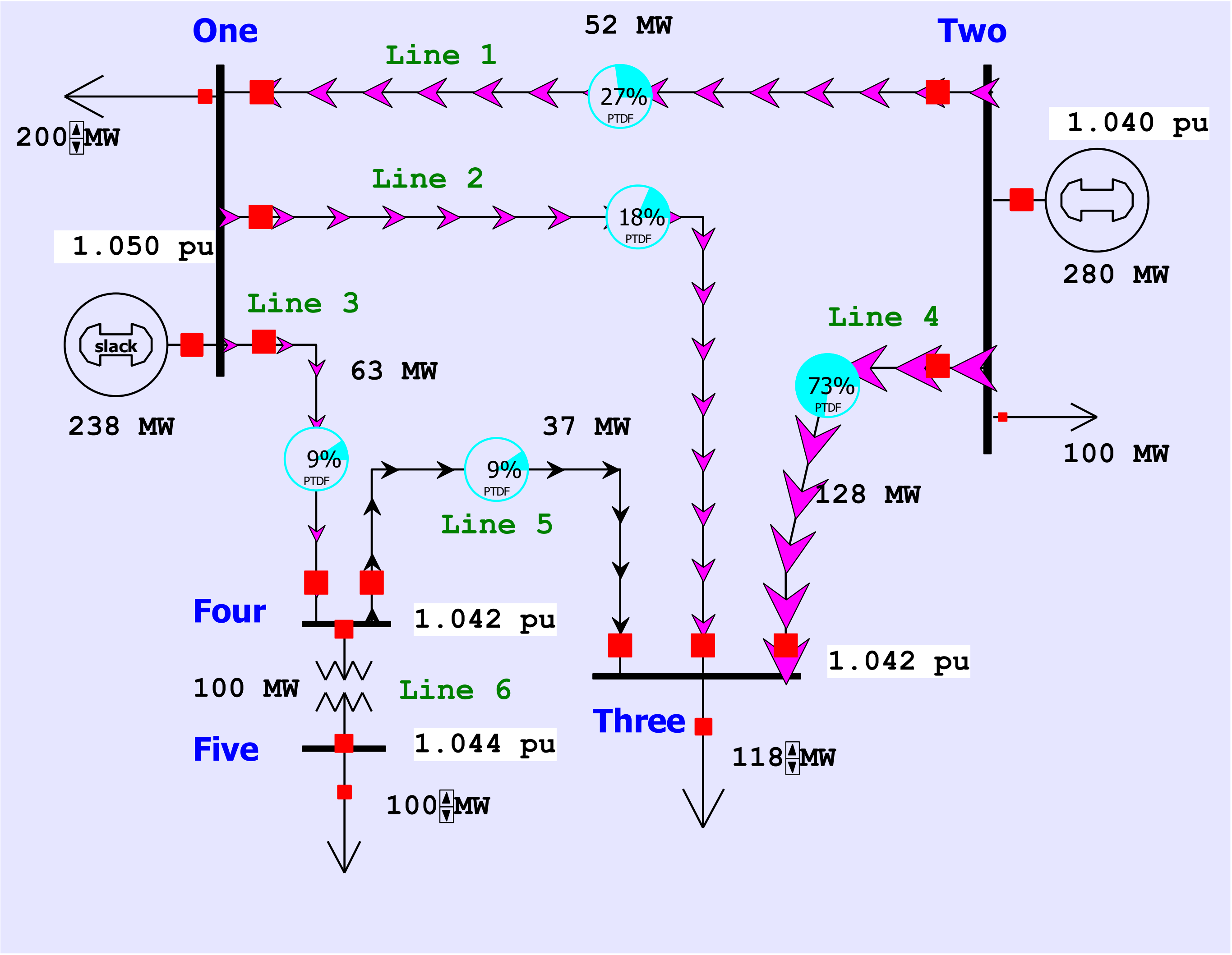
Calculating PTDFs in PowerWorld

- PowerWorld provides a number of options for calculating and visualizing PTDFs
  - Select Tools, Sensitivities, Power Transfer Distribution Factors (PTDFs)



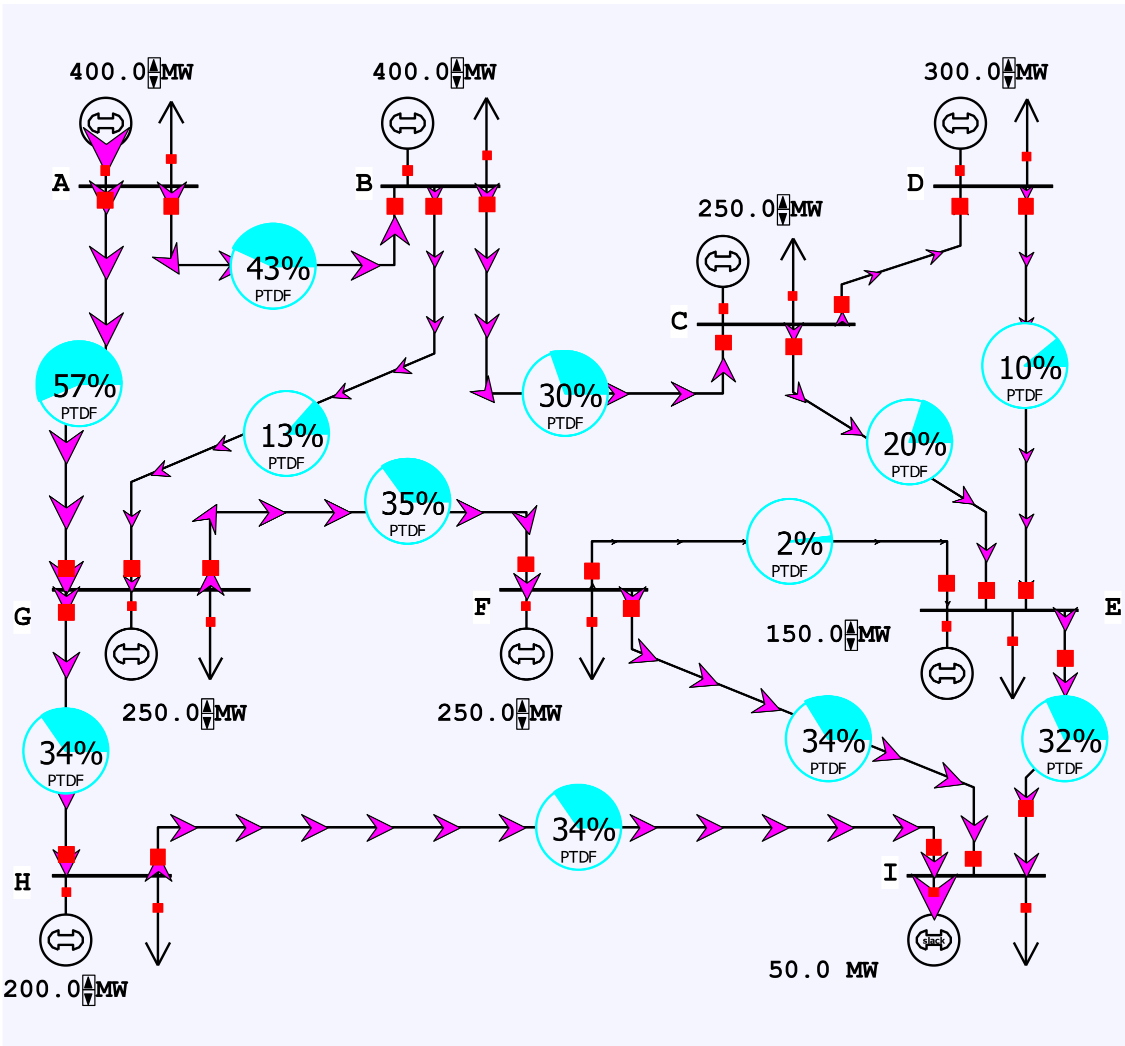
Results are shown for the five bus case for the Bus 2 to Bus 3 transaction

Five Bus PTDF Visualization



PowerWorld Case: B5_DistFact_PTDF

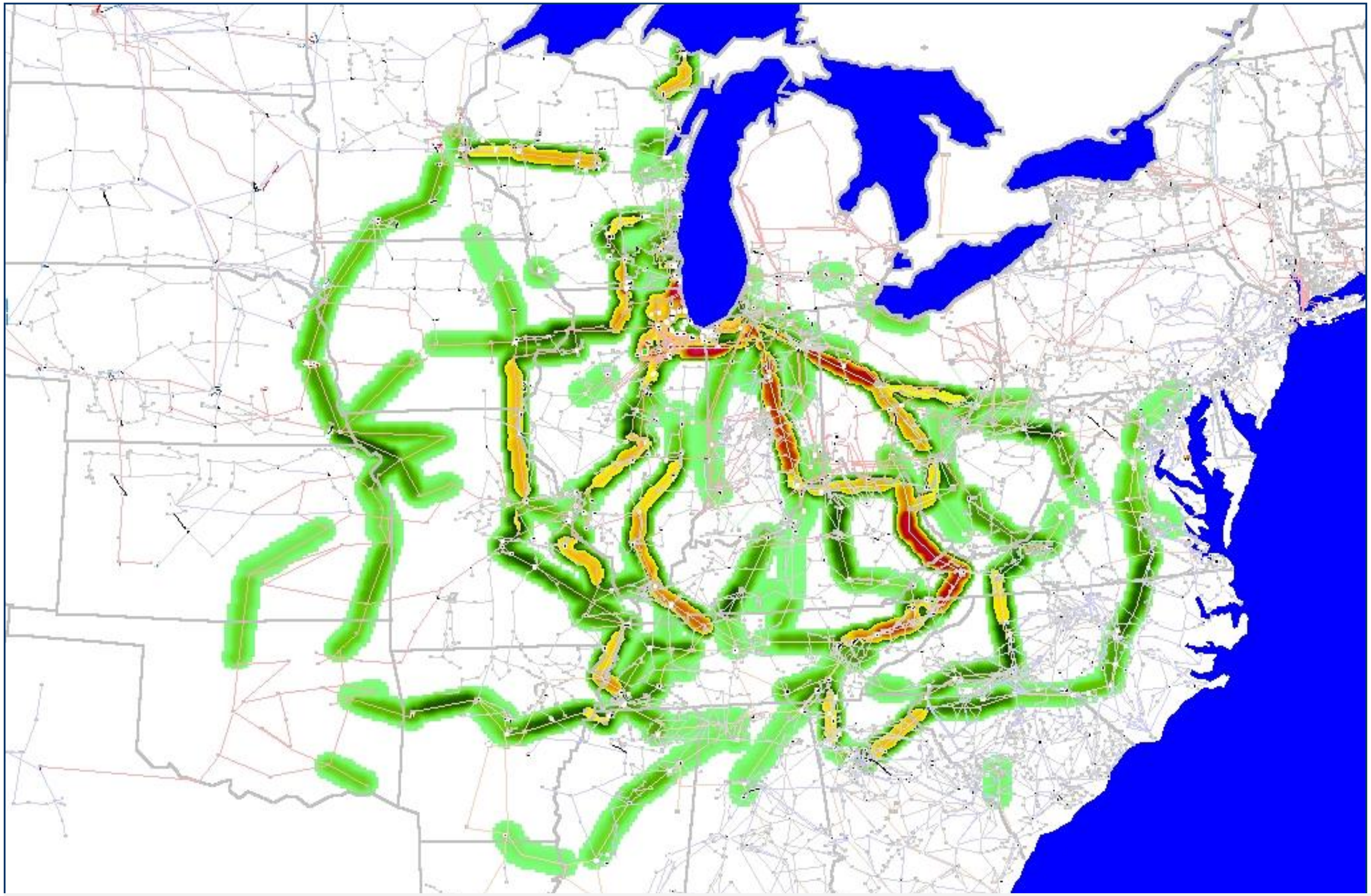
Nine Bus PTDF Example



Display shows the PTDFs for a basic transaction from Bus A to Bus I. Note that 100% of the transaction leaves Bus A and 100% arrives at Bus I

PowerWorld Case: B9_PTDF

Eastern Interconnect Example: Wisconsin Utility to TVA PTDFs



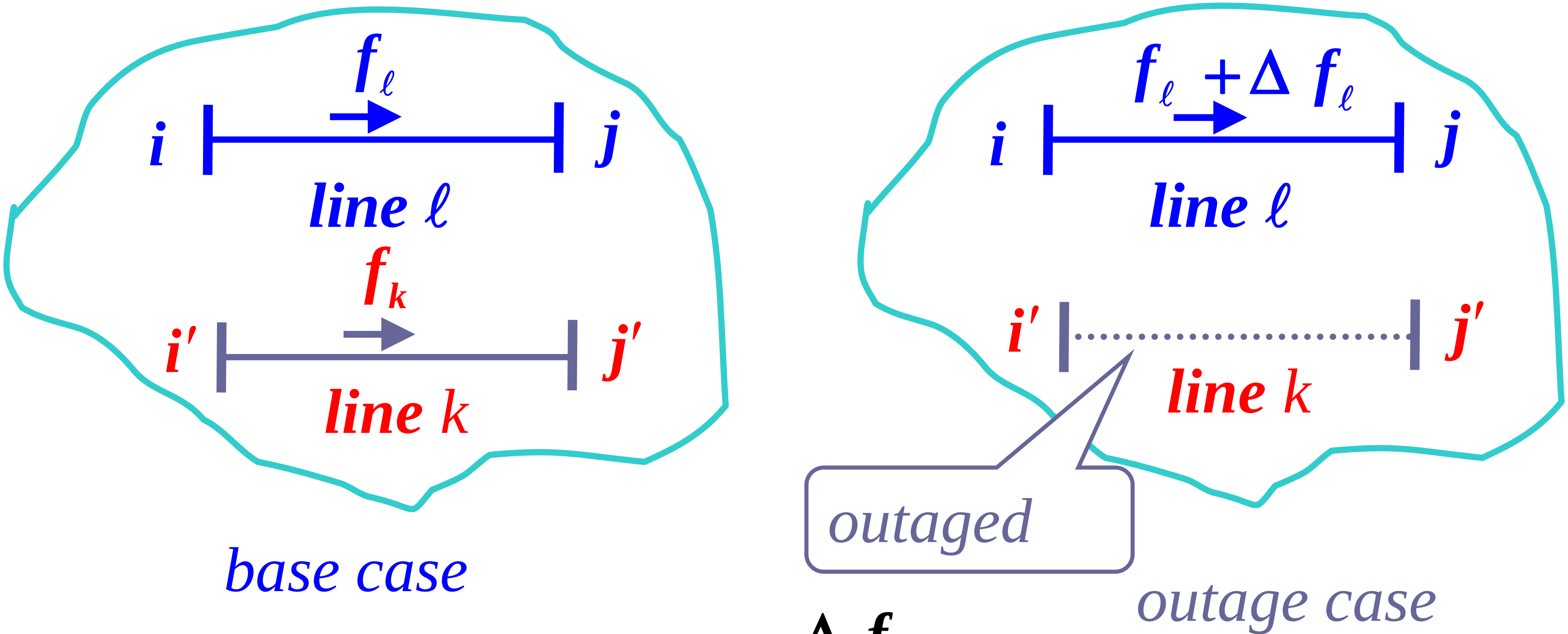
In this example multiple generators contribute for both the seller and the buyer

Contours show lines that would carry at least 2% of a power transfer from Wisconsin to TVA

Line Outage Distribution Factors (LODFs)

- Power system operation is practically always limited by contingencies, with line outages comprising a large number of the contingencies
- Desire is to determine the impact of a line outage (either a transmission line or a transformer) on other system real power flows without having to explicitly solve the power flow for the contingency
- These values are provided by the LODFs
- The LODF $\boldsymbol{d}_{\ell}^{k}$ is the portion of the pre-outage real power line flow on line k that is redistributed to line ℓ as a result of the outage of line k

LODFs

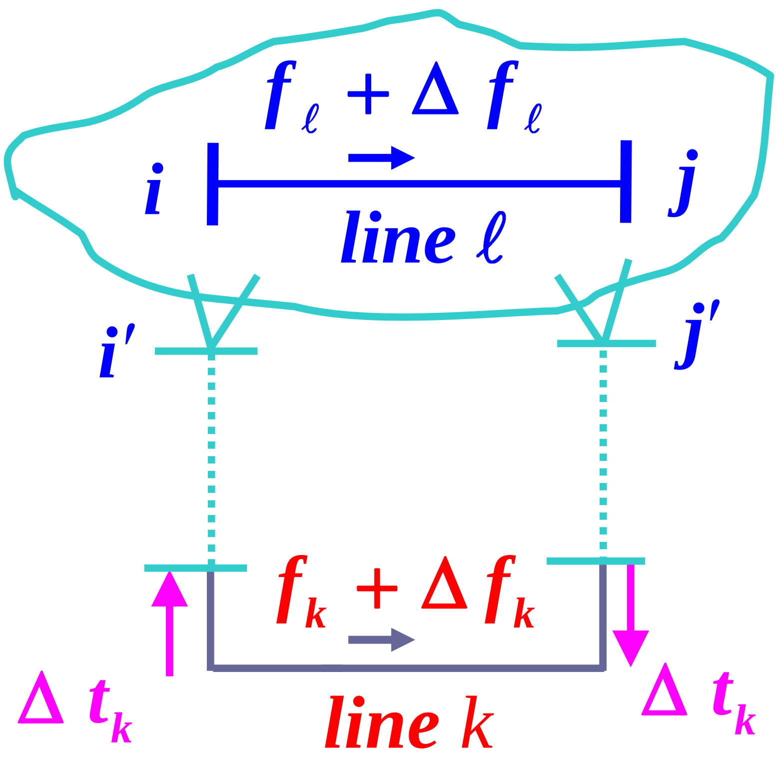


$$d_\ell^{k} = \frac{\Delta f_\ell}{f_k} = d_{\ell,k}$$

Best reference is Chapter 7 of the course book

LODF Evaluation

We simulate the impact of the outage of line k by adding the basic transaction $w_k = \{ i', j', \Delta t_k \}$



and selecting Δt_k in such a way that the flows on the dashed lines become exactly zero

In general this Δt_k is not equal to the original line flow

LODF Evaluation

- We select Δt_k to be such that

$$f_k + \Delta f_k - \Delta t_k = 0$$

where Δf_k is the active power flow change on the line k due to the transaction w_k

- The line k flow from w_k depends on its PTDF

$$\Delta f_k = \varphi_k^{(w_k)} \Delta t_k$$

it follows that $\Delta t_k = \dfrac{f_k}{1 - \varphi_k^{(w_k)}} = \dfrac{f_k}{1 - \left(\psi_k^{i'} - \psi_k^{j'}\right)}$

LODF Evaluation

- For the rest of the network, the impacts of the outage of line k are the same as the impacts of the additional basic transaction w_k

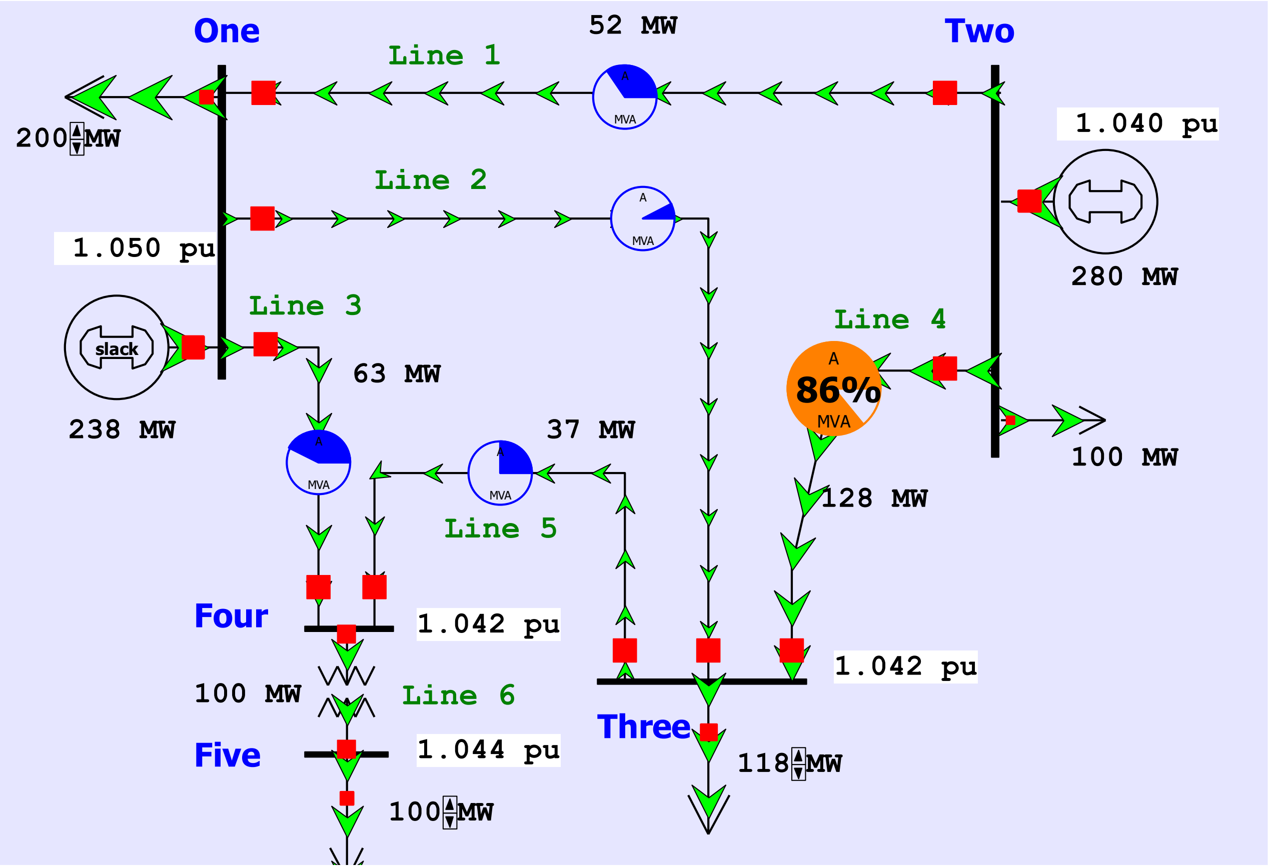
$$\Rightarrow \quad \Delta f_\ell = \varphi_\ell^{(w_k)} \Delta t_k = \frac{\varphi_\ell^{(w_k)}}{1 - \varphi_k^{(w_k)}} f_k$$

- Therefore, by definition the LODF is

$$d_\ell^k = \frac{\Delta f_\ell}{f_k} = \frac{\varphi_\ell^{(w_k)}}{1 - \varphi_k^{(w_k)}}$$

Five Bus Example

- Assume we wish to calculate the values for the outage of line 4 (between buses 2 and 3); this is line k



Say we wish to know the change in flow on the line 3 (Buses 3 to 4). PTDFs for a transaction from 2 to 3 are 0.7273 on line 4 and 0.0909 on line 3

Five Bus Example

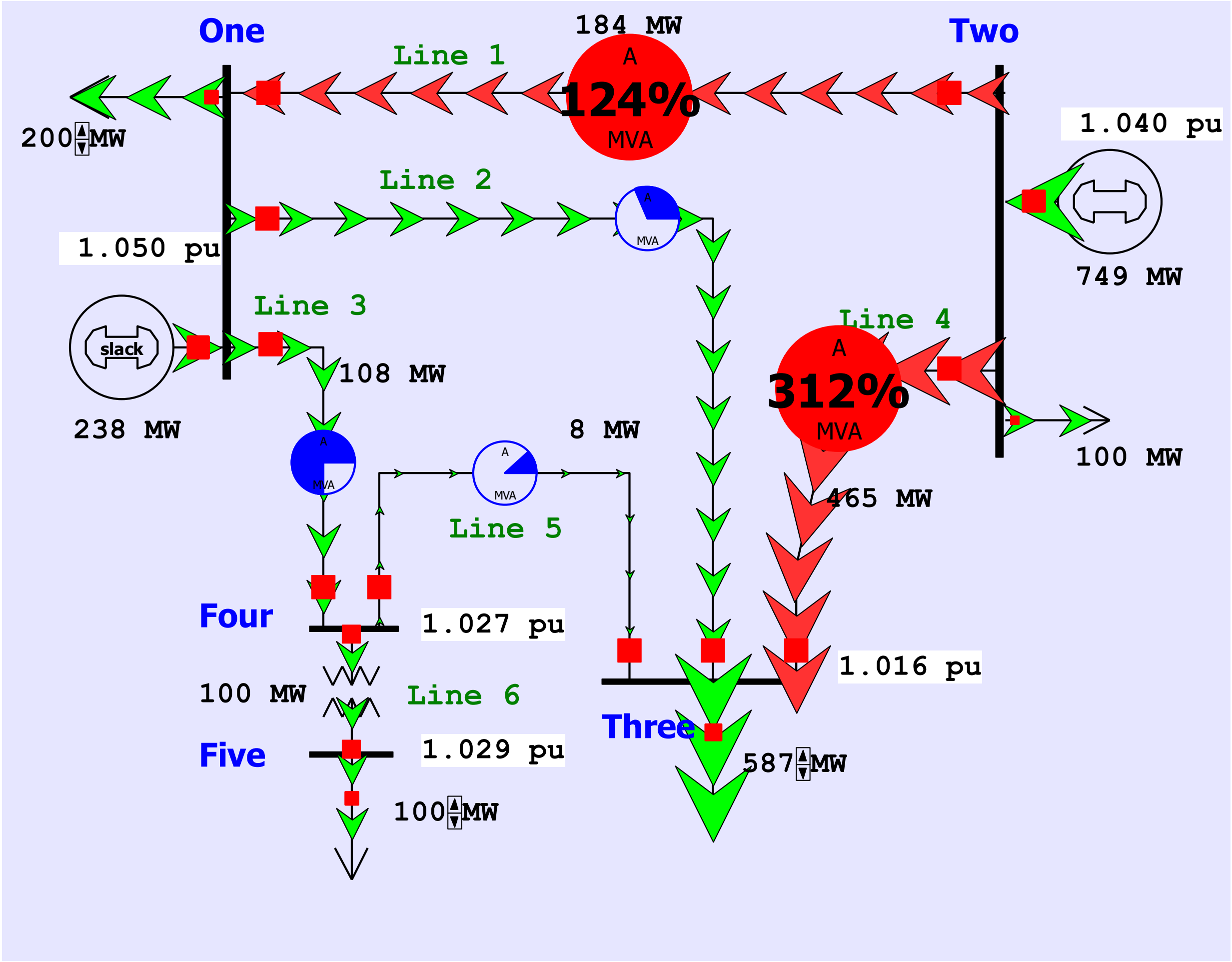
- Hence we get

$$\Delta t_k = \frac{f_k}{1-\varphi_k^{(w_k)}} = \frac{128}{1-0.7273} = 469.4$$

$$d_3^4 = \frac{\Delta f_3}{f_4} = \frac{\varphi_3^{(w_4)}}{1-\varphi_4^{(w_4)}} = \frac{0.0909}{1-0.7273} = 0.333$$

$$\Delta f_3 = \left(0.333\right) f_4 = 0.333 \times 128 = 42.66\,\mathrm{MW}$$

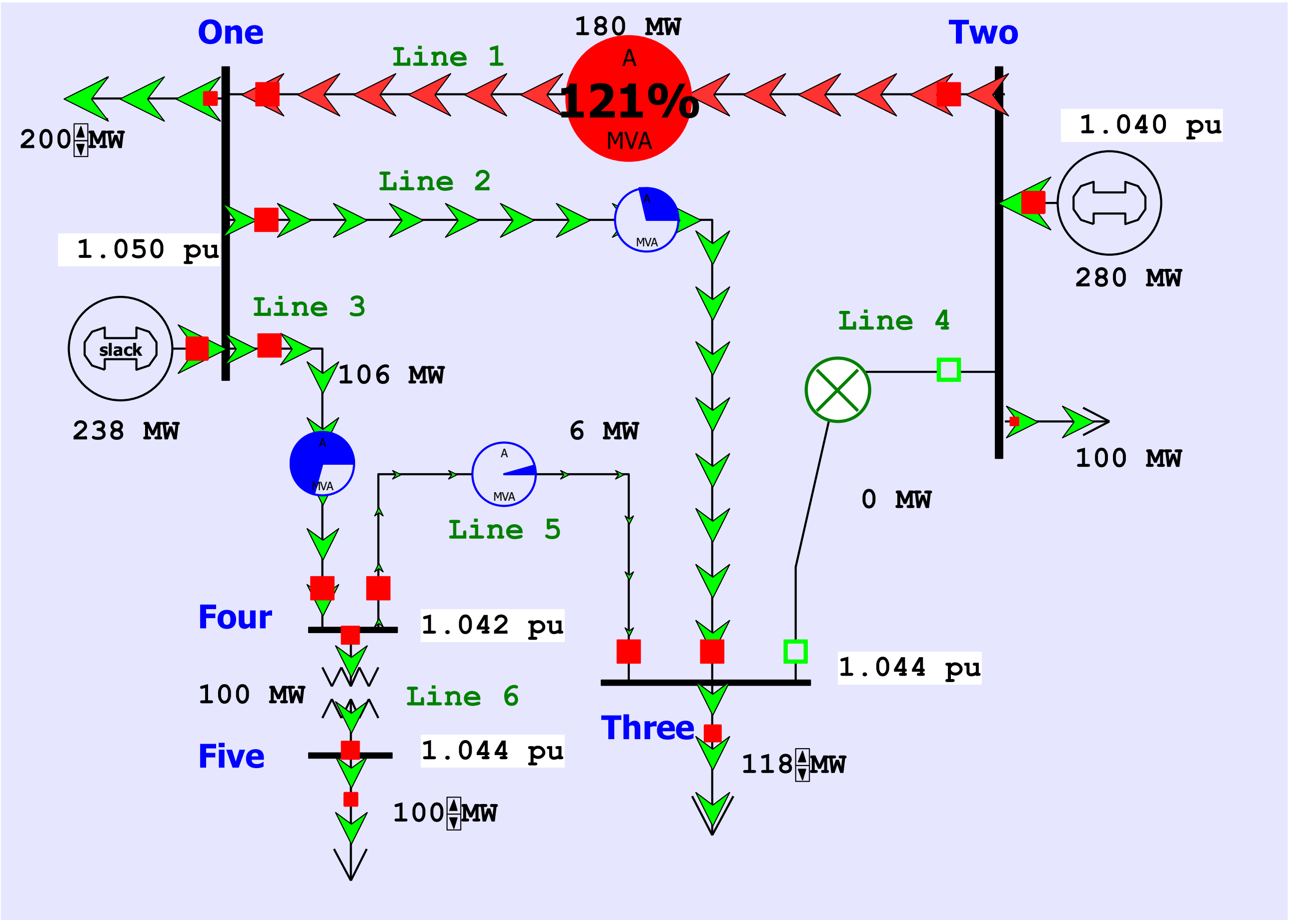
Five Bus Example Compensated



Here is the system with the compensation added to bus 2 and removed at bus 3; we are canceling the impact of the line 4 flow for the reset of the network.

Five Bus Example

- Below we see the network with the line actually outaged



The line 3 flow changed from 63 MW to 106 MW, an increase of 43 MW, matching the LODF value

Developing a Critical Eye

- In looking at the below formula you need to be thinking about what conditions will cause the formula to fail

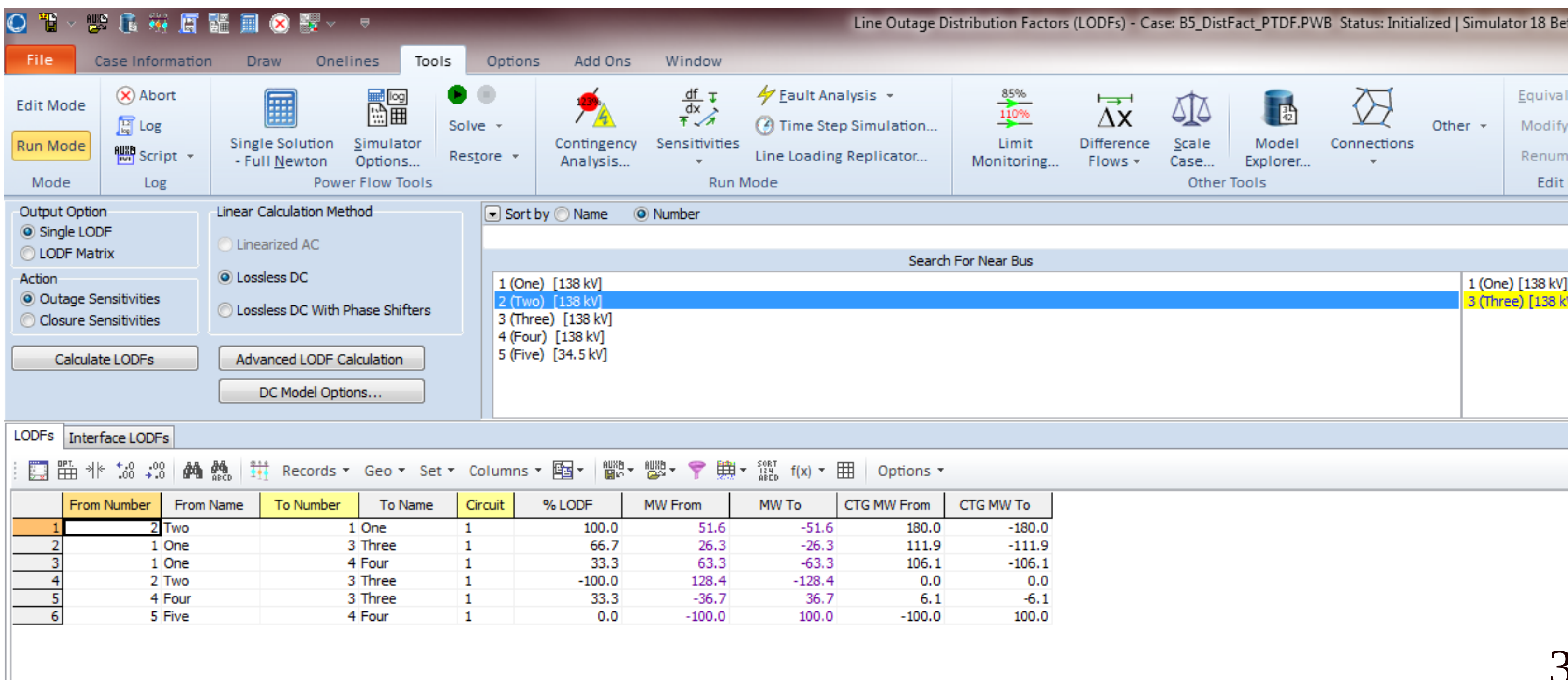
$$\Rightarrow \quad \Delta f_\ell = \varphi_\ell^{(w_k)} \Delta t_k = \frac{\varphi_\ell^{(w_k)}}{1 - \varphi_k^{(w_k)}} f_k$$

  Here the obvious situation is when the denominator is zero

- That corresponds to a situation in which the contingency causes system islanding
  - An example is line 6 (between buses 4 and 5)
  - Impact modeled by injections at the buses within each viable island

Calculating LODFs in PowerWorld

- Select Tools, Sensitivities, Line Outage Distribution Factors
  - Select the Line using dialogs on right, and click Calculate LODFS; below example shows values for line 4

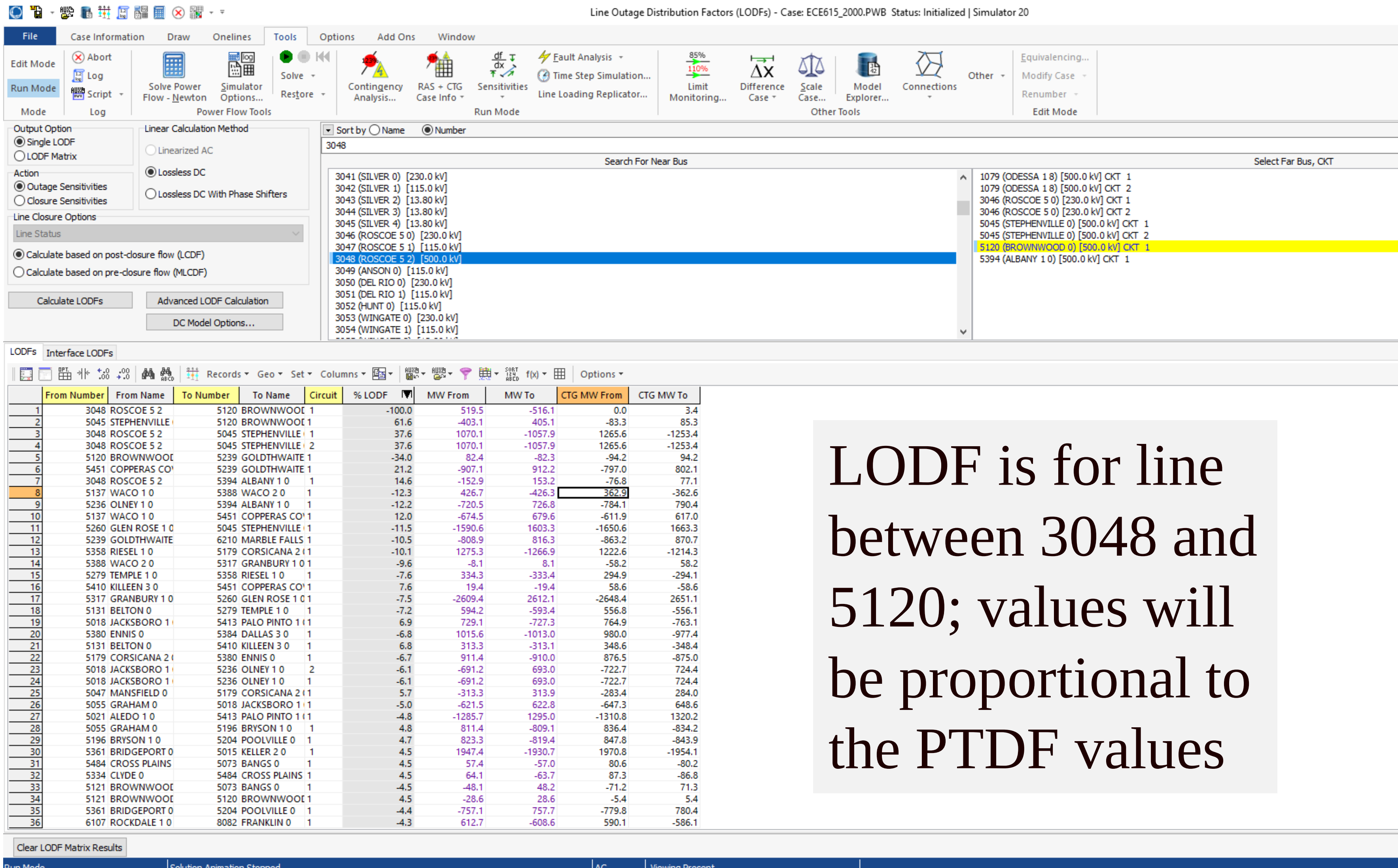


	From Number	From Name	To Number	To Name	Circuit	% LODF	MW From	MW To	CTG MW From	CTG MW To
1	2	Two	1	One	1	100.0	51.6	-51.6	180.0	-180.0
2	1	One	3	Three	1	66.7	26.3	-26.3	111.9	-111.9
3	1	One	4	Four	1	33.3	63.3	-63.3	106.1	-106.1
4	2	Two	3	Three	1	-100.0	128.4	-128.4	0.0	0.0
5	4	Four	3	Three	1	33.3	-36.7	36.7	6.1	-6.1
6	5	Five	4	Four	1	0.0	-100.0	100.0	-100.0	100.0

Blackout Case LODFs

- One of the issues associated with the 8/14/03 blackout was the LODF associated with the loss of the Hanna-Juniper 345 kV line (21350-22163) that was being used in a flow gate calculation was not correct because the Chamberlin-Harding 345 kV line outage was missed
  - With the Chamberlin-Harding line assumed in-service the value was 0.362
  - With this line assumed out-of-service (which indeed it was) the value increased to 0.464

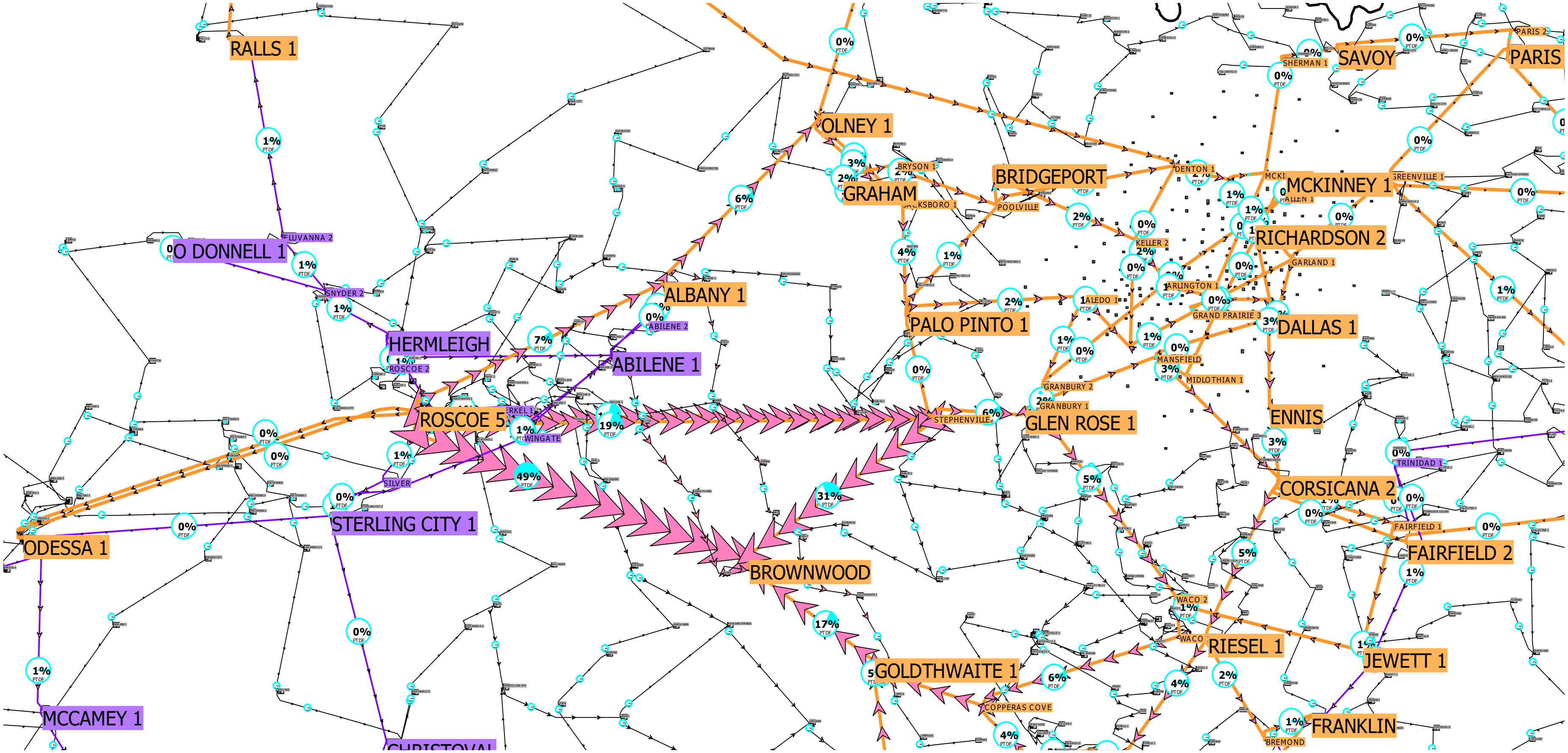
2000 Bus LODF Example



	From Number	From Name	To Number	To Name	Circuit	% LODF	MW From	MW To	CTG MW From	CTG MW To
1	3048	ROSCOE 5 2	5120	BROWNWOOI	1	-100.0	519.5	-516.1	0.0	3.4
2	5045	STEPHENVILLE	5120	BROWNWOOI	1	61.6	-403.1	405.1	-83.3	85.3
3	3048	ROSCOE 5 2	5045	STEPHENVILLE	1	37.6	1070.1	-1057.9	1265.6	-1253.4
4	3048	ROSCOE 5 2	5045	STEPHENVILLE	2	37.6	1070.1	-1057.9	1265.6	-1253.4
5	5120	BROWNWOOI	5239	GOLDTHWAITE	1	-34.0	82.4	-82.3	-94.2	94.2
6	5451	COPPERAS CO	5239	GOLDTHWAITE	1	21.2	-907.1	912.2	-797.0	802.1
7	3048	ROSCOE 5 2	5394	ALBANY 1 0	1	14.6	-152.9	153.2	-76.8	77.1
8	5137	WACO 1 0	5388	WACO 2 0	1	-12.3	426.7	-426.3	362.9	-362.6
9	5236	OLNEY 1 0	5394	ALBANY 1 0	1	-12.2	-720.5	726.8	-784.1	790.4
10	5137	WACO 1 0	5451	COPPERAS CO	1	12.0	-674.5	679.6	-611.9	617.0
11	5260	GLEN ROSE 1 0	5045	STEPHENVILLE	1	-11.5	-1590.6	1603.3	-1650.6	1663.3
12	5239	GOLDTHWAITE	6210	MARBLE FALLS	1	-10.5	-808.9	816.3	-863.2	870.7
13	5358	RIESEL 1 0	5179	CORSICANA 2	1	-10.1	1275.3	-1266.9	1222.6	-1214.3
14	5388	WACO 2 0	5317	GRANBURY 1 0	1	-9.6	-8.1	8.1	-58.2	58.2
15	5279	TEMPLE 1 0	5358	RIESEL 1 0	1	-7.6	334.3	-333.4	294.9	-294.1
16	5410	KILLEEN 3 0	5451	COPPERAS CO	1	7.6	19.4	-19.4	58.6	-58.6
17	5317	GRANBURY 1 0	5260	GLEN ROSE 1 0	1	-7.5	-2609.4	2612.1	-2648.4	2651.1
18	5131	BELTON 0	5279	TEMPLE 1 0	1	-7.2	594.2	-593.4	556.8	-556.1
19	5018	JACKSBORO 1	5413	PALO PINTO 1	1	6.9	729.1	-727.3	764.9	-763.1
20	5380	ENNIS 0	5384	DALLAS 3 0	1	-6.8	1015.6	-1013.0	980.0	-977.4
21	5131	BELTON 0	5410	KILLEEN 3 0	1	6.8	313.3	-313.1	348.6	-348.4
22	5179	CORSICANA 2	5380	ENNIS 0	1	-6.7	911.4	-910.0	876.5	-875.0
23	5018	JACKSBORO 1	5236	OLNEY 1 0	2	-6.1	-691.2	693.0	-722.7	724.4
24	5018	JACKSBORO 1	5236	OLNEY 1 0	1	-6.1	-691.2	693.0	-722.7	724.4
25	5047	MANSFIELD 0	5179	CORSICANA 2	1	5.7	-313.3	313.9	-283.4	284.0
26	5055	GRAHAM 0	5018	JACKSBORO 1	1	-5.0	-621.5	622.8	-647.3	648.6
27	5021	ALEDO 1 0	5413	PALO PINTO 1	1	-4.8	-1285.7	1295.0	-1310.8	1320.2
28	5055	GRAHAM 0	5196	BRYSON 1 0	1	4.8	811.4	-809.1	836.4	-834.2
29	5196	BRYSON 1 0	5204	POOLVILLE 0	1	4.7	823.3	-819.4	847.8	-843.9
30	5361	BRIDGEPORT 0	5015	KELLER 2 0	1	4.5	1947.4	-1930.7	1970.8	-1954.1
31	5484	CROSS PLAINS	5073	BANGS 0	1	4.5	57.4	-57.0	80.6	-80.2
32	5334	CLYDE 0	5484	CROSS PLAINS	1	4.5	64.1	-63.7	87.3	-86.8
33	5121	BROWNWOOI	5073	BANGS 0	1	-4.5	-48.1	48.2	-71.2	71.3
34	5121	BROWNWOOI	5120	BROWNWOOI	1	4.5	-28.6	28.6	-5.4	5.4
35	5361	BRIDGEPORT 0	5204	POOLVILLE 0	1	-4.4	-757.1	757.7	-779.8	780.4
36	6107	ROCKDALE 1 0	8082	FRANKLIN 0	1	-4.3	612.7	-608.6	590.1	-586.1

LODF is for line between 3048 and 5120; values will be proportional to the PTDF values

2000 Bus LODF Example



Multiple Line LODFs

- LODFs can also be used to represent multiple device contingencies, but it is usually more involved than just adding the effects of the single device LODFs
- Assume a simultaneous outage of lines k_1 and k_2
- Now setup two transactions, w_{k1} (with value Δt_{k1})and w_{k2} (with value Δt_{k2}) so

$$f_{k1} + \Delta f_{k1} + \Delta f_{k2} - \Delta t_{k1} = 0$$

$$f_{k2} + \Delta f_{k1} + \Delta f_{k2} - \Delta t_{k2} = 0$$

$$f_{k1} + \varphi_{k1}^{(w_{k1})} \Delta t_{k1} + \varphi_{k1}^{(w_{k2})} \Delta t_{k2} - \Delta t_{k1} = 0$$

$$f_{k2} + \varphi_{k2}^{(w_{k1})} \Delta t_{k1} + \varphi_{k2}^{(w_{k2})} \Delta t_{k1} - \Delta t_{k2} = 0$$

Multiple Line LODFs

- Hence we can calculate the simultaneous impact of multiple outages; details for the derivation are given in C.Davis, T.J. Overbye, "Linear Analysis of Multiple Outage Interaction," *Proc. 42nd HICSS*, 2009
- Equation for the change in flow on line ℓ for the outage of lines k_1 and k_2 is

$$\Delta f_\ell = \begin{bmatrix} d_\ell^{k1} & d_\ell^{k2} \end{bmatrix} \begin{bmatrix} 1 & -d_{k1}^{k2} \\ -d_{k2}^{k1} & 1 \end{bmatrix}^{-1} \begin{bmatrix} f_{k1} \\ f_{k2} \end{bmatrix}$$

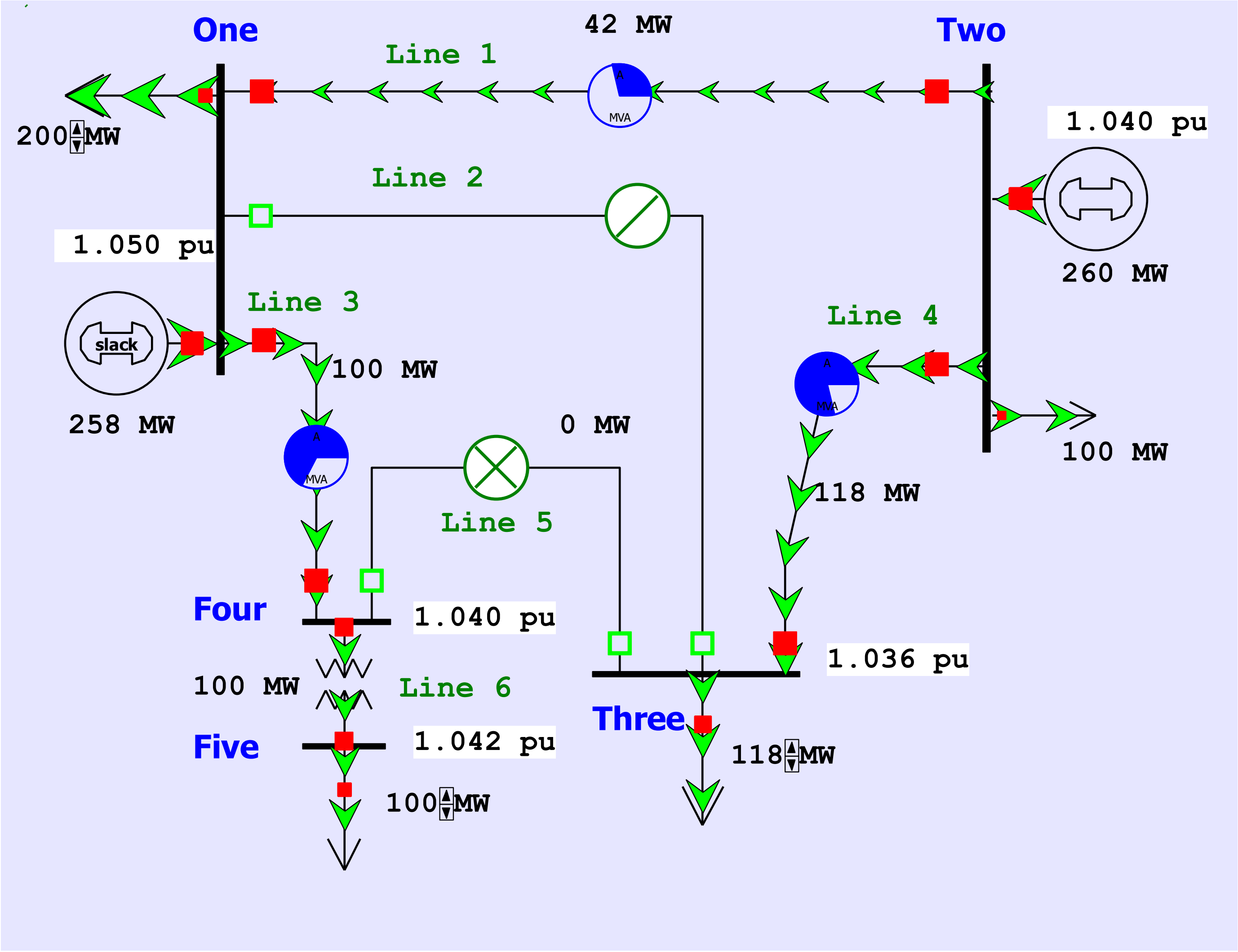
Multiple Line LODFs

- Example: Five bus case, outage of lines 2 and 5 to flow on line 4.

$$\Delta f_\ell = \begin{bmatrix} d_\ell^{k1} & d_\ell^{k2} \end{bmatrix} \begin{bmatrix} 1 & -d_{k1}^{k2} \\ -d_{k2}^{k1} & 1 \end{bmatrix}^{-1} \begin{bmatrix} f_{k1} \\ f_{k2} \end{bmatrix}$$

$$\Delta f_\ell = \begin{bmatrix} 0.4 & 0.25 \end{bmatrix} \begin{bmatrix} 1 & -0.75 \\ -0.6 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.336 \\ -0.331 \end{bmatrix} = 0.005$$

Multiple Line LODFs



Flow goes from 117.5 to 118.0