ECEN 615 Methods of Electric Power Systems Analysis

Lecture 20: SE, EMSs, Equivalents

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Announcements

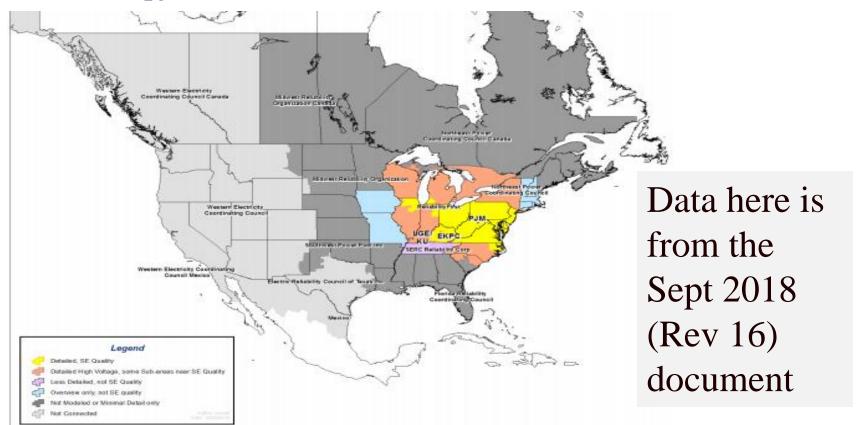


- Homework 5 is due on Tuesday Nov 13
- Final exam is Wednesday Dec 12, 1 to 3pm (the syllabus had indicated Wednesday Dec 13)

Example SE Application: PJM and MISO



- PJM provides information about their EMS model in
 - www.pjm.com/-/media/documents/manuals/m03a.ashx



Example SE Application: PJM and MISO



- PJM measurements are required for 69 kV and up
- PJM SE is triggered to execute every minute
- PJM SE solves well over 98% of the time
- Below reference provides info on MISO SE from March 2015
 - 54,433 buses
 - 54,415 network branches
 - 6332 generating units
 - 228,673 circuit breakers
 - 289,491 mapped points

QR Factorization



- Used in in SE since it handles ill-conditioned m by n matrices (with m >= n)
- Can be used with sparse matrices
- As before we will first split the \mathbf{R}^{-1} matrix $\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} = \mathbf{H}^T \mathbf{R}^{-\frac{1}{2}} \mathbf{R}^{-\frac{1}{2}} \mathbf{H} = \mathbf{H}^T \mathbf{H}^T$
- QR factorization represents the m by n \mathbf{H}' matrix as $\mathbf{H}' = \mathbf{Q} \mathbf{U}$

with \mathbf{Q} an m by m orthonormal matrix and \mathbf{U} an upper triangular matrix (most books use $\mathbf{Q} \mathbf{R}$ but we use \mathbf{U} to avoid confusion with the previous \mathbf{R})

QR Factorization



- We then have $\mathbf{H'}^T\mathbf{H'} = \mathbf{U}^T\mathbf{Q}^T\mathbf{Q}\mathbf{U}$
- But since \mathbf{Q} is an orthonormal matrix, $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$
- Hence we have $\mathbf{H}'^T \mathbf{H}' = \mathbf{U}^T \mathbf{U}$

Originally
$$\Delta \mathbf{x} = \left[\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} \right]^{-1} \mathbf{H}^T \mathbf{R}^{-1} \left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x}) \right]$$

With
$$\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} = \mathbf{H'}^T \mathbf{H'} = \mathbf{H'}^T \mathbf{H'} = \mathbf{U}^T \mathbf{U}$$

Let
$$\mathbf{z}' = \mathbf{Q}^T \mathbf{R}^{-\frac{1}{2}} \left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x}) \right]$$

Q is an m by m matrix

$$\Delta \mathbf{x} = \left[\mathbf{U}^T \mathbf{U} \right]^{-1} \mathbf{H}^T \mathbf{R}^{-\frac{1}{2}} \mathbf{R}^{-\frac{1}{2}} \left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x}) \right] = \left[\mathbf{U}^T \mathbf{U} \right]^{-1} \mathbf{U}^T \mathbf{Q}^T \mathbf{R}^{-\frac{1}{2}} \left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x}) \right]$$

$$\mathbf{U}^{T}\mathbf{U}\Delta\mathbf{x} = \mathbf{U}^{T}\mathbf{z}' \rightarrow \Delta\mathbf{x} = \mathbf{U}^{-1}\mathbf{z}'$$

QR Factorization



- Next issue we discuss the QR factorization algorithm
- When factored the **U** matrix (i.e., what most call the **R** matrix) will be an m by n upper triangular matrix
- Several methods are available including the Householder method and the Givens Method
- Givens is preferred when dealing with sparse matrices
- All good reference is Gene H. Golub and Charles F. Van Loan, "Matrix Computations," second edition, Johns Hopkins University Press, 1989.

Givens Algorithm for Factoring a Matrix A



- The Givens algorithm works by pre-multiplying the initial matrix, \mathbf{A} , by a series of matrices and their transposes, starting with $\mathbf{G}_1\mathbf{G}_1^T$
 - If **A** is m by n, then each **G** is an m by m matrix
- Algorithm proceeds column by column, sequentially zeroing out elements in the lower triangle of **A**, starting at the bottom of each column

$$\mathbf{G}_1 \dots \mathbf{G}_p \mathbf{G}_p^T \dots \mathbf{G}_1^T \mathbf{A} = \mathbf{Q} \mathbf{U}$$

$$\mathbf{G}_1 \dots \mathbf{G}_p = \mathbf{Q}$$

$$\mathbf{G}_{p}^{T}...\mathbf{G}_{1}^{T}\mathbf{A}=\mathbf{U}$$

If A is sparse, then we can take advantage of sparsity going up the column

Givens Algorithm



• To zero out element A[i,j], with i > j we first solve with a=A[k,j], b=A[i,j]

$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix}^T \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

A numerically safe algorithm is
 If b=0 then c=1, s=0 // i.e, no rotation is needed

Else If
$$|b| > |a|$$
 then $\tau = -a/b$; $s = 1/\sqrt{1 + \tau^2}$; $c = s\tau$

Else
$$\tau = -b/a$$
; $c = 1/\sqrt{1+\tau^2}$; $s = c\tau$

Givens G Matrix



• The orthogonal $G(i,k,\theta)$ matrix is then

$$\mathbf{G}(i,\mathbf{k},\theta) = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & c & \cdots & s & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -s & \cdots & c & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

• Premultiplication by $G(i,k,\theta)^T$ is a rotation by θ radians in the (i,k) coordinate plane

Small Givens Example



$$\mathbf{A} = \begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 0 & 5 \\ 2 & 1 \end{bmatrix}$$

• Let $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 0 & 5 \\ 2 & 1 \end{bmatrix}$ First start in column j=1; we will zero out A[4,1] with i=4, k=2

• First we zero out A[4,1], a=1, b=2 giving s=0.8944, c = -0.4472

$$\mathbf{G}_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.4472 & 0 & 0.8944 \\ 0 & 0 & 1 & 0 \\ 0 & -0.8944 & 0 & -0.4472 \end{bmatrix} \quad \mathbf{G}_{1}^{T} \mathbf{A} = \begin{bmatrix} 4 & 2 \\ -2.236 & -0.8944 \\ 0 & 5 \\ 0 & -0.4472 \end{bmatrix}$$

$$\mathbf{G}_{1}^{T}\mathbf{A} = \begin{vmatrix} 4 & 2 \\ -2.236 & -0.8944 \\ 0 & 5 \\ 0 & -0.4472 \end{vmatrix}$$

Small Givens Example



• Next zero out A[2,1] with a=4, b=-2.236, giving c= -0.8729, s=0.4880

$$\mathbf{G}_{2} = \begin{bmatrix} 0.873 & 0.488 & 0 & 0 \\ -0.488 & 0.873 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{G}_{2}^{T} \mathbf{G}_{1}^{T} \mathbf{A} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5 \\ 0 & -0.447 \end{bmatrix}$$

• Next zero out A[4,2] with a=5, b=-0.447, c=0.996,

$$\mathbf{S} = \mathbf{O.089} \qquad \mathbf{G}_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.996 & 0.089 \\ 0 & 0 & -0.089 & 0.996 \end{bmatrix} \quad \mathbf{G}_{3}^{T} \mathbf{G}_{2}^{T} \mathbf{A} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5.020 \\ 0 & 0 \end{bmatrix}$$

Small Givens Example



• Next zero out A[3,2] with a=0.195, b=5.02, c=0.996, s=0.089

$$\mathbf{G}_{4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.039 & 0.999 & 0 \\ 0 & -0.999 & -0.039 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{G}_{4}^{T} \mathbf{G}_{3}^{T} \mathbf{G}_{2}^{T} \mathbf{A} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & -5.023 \\ 0 & 0 \end{bmatrix}$$

Also we have

$$\mathbf{Q} = \mathbf{G}_1 \mathbf{G}_2 \mathbf{G}_3 \mathbf{G}_4 = \begin{bmatrix} 0.872 & -0.019 & 0.487 & 0 \\ 0.218 & 0.094 & -0.387 & 0.891 \\ 0 & -0.995 & -0.039 & 0.089 \\ 0.436 & -0.009 & -0.782 & -0.445 \end{bmatrix}$$

Start of Givens for SE Example



Starting with the **H** matrix we get

atrix we get
$$\mathbf{H'} = \mathbf{R}^{-\frac{1}{2}}\mathbf{H} = 100 \times \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ -10 & 0 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
we have

To zero out $\mathbf{H}'[5,1]=1$ we have

$$\mathbf{G}_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.995 & 0.0995 & 0 \\ 0 & 0 & 0 & -0.0995 & 0.995 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Start of Givens for SE Example



Which gives

$$\mathbf{G}_{1}^{T}\mathbf{H'} = 100 \times \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ 10.049 & 0 & -9.95 \\ 0 & 0 & 0.995 \\ 0 & 0 & 1 \end{bmatrix}$$

• The next rotation would be to zero out element [4,1], continuing until all the elements in the lower triangle have been reduced

Givens Comments



- For a full matrix, Givens is O(mn²) since each element in the lower triangle needs to be zeroed O(nm), and each operation is O(n)
- Computation can be drastically reduced for a sparse matrix since we only need to zero out the elements that are initially non-zero, and any that become non-zero (i.e., the fills)
 - Also, for each multiply we only need to deal with the nonzeros in the impacted row
- Givens rotation is commonly used to solve the SE

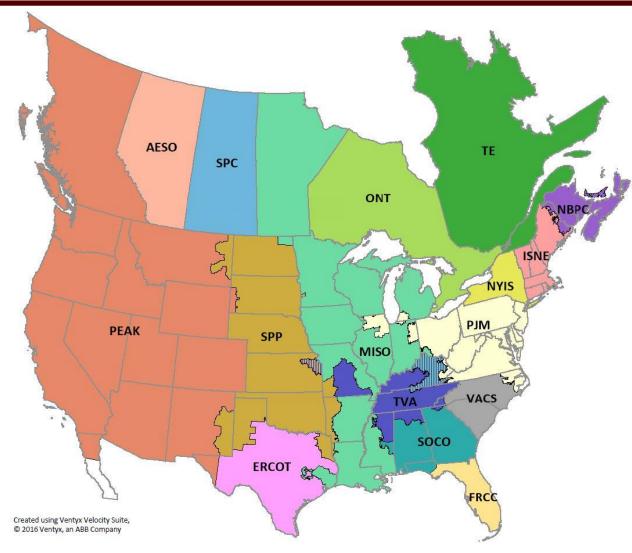
Energy Management Systems (EMSs)



- EMSs are now used to control most large scale electric grids
- EMSs developed in the 1970's and 1980's out of SCADA systems
 - An EMS usually includes a SCADA system; sometimes called a SCADA/EMS
- Having a SE is almost the definition of an EMS. The SE then feeds data to the more advanced functions
- EMSs have evolved as the industry as evolved as the industry has evolved, with functionality customized for the application (e.g., a reliability coordinator or a vertically integrated utility)

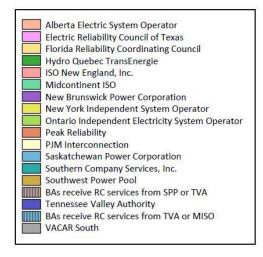
NERC Reliability Coordinators





NERC Reliability Coordinators

As of June 1, 2015



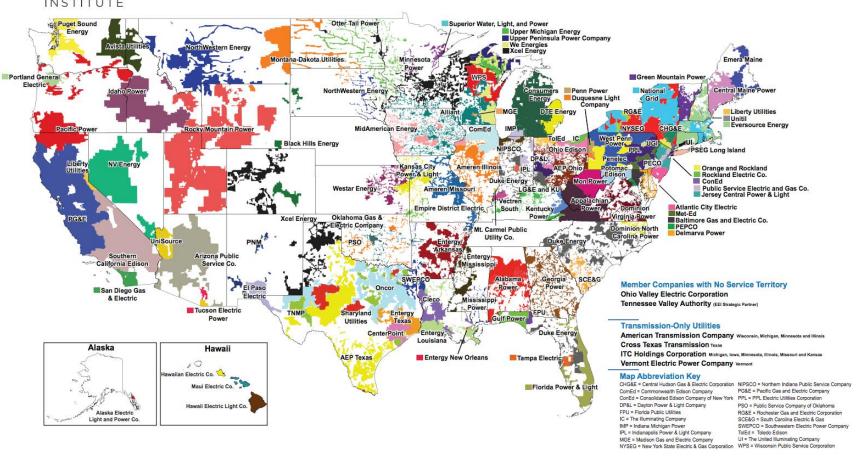
Source: www.nerc.com/pa/rrm/TLR/Pages/Reliability-Coordinators.aspx

EEI Member Companies



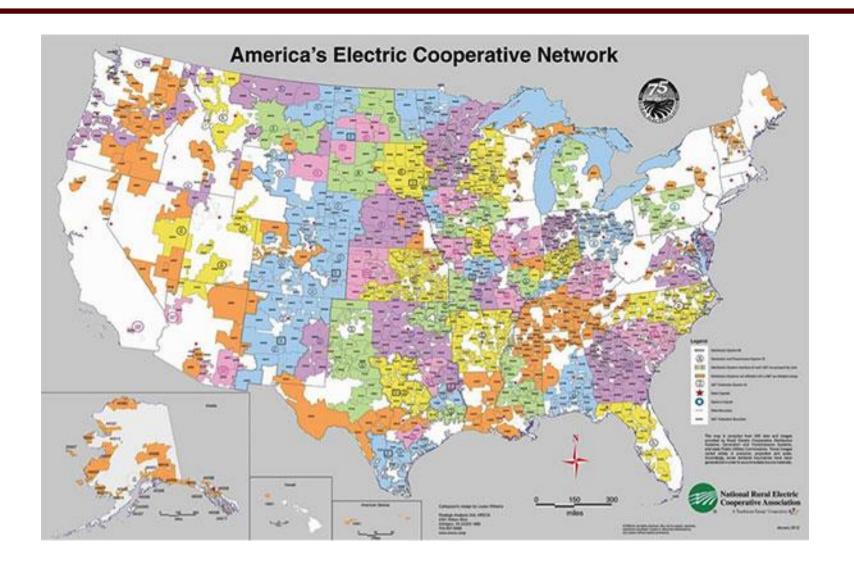


EEI U.S. Member Company Service Territories



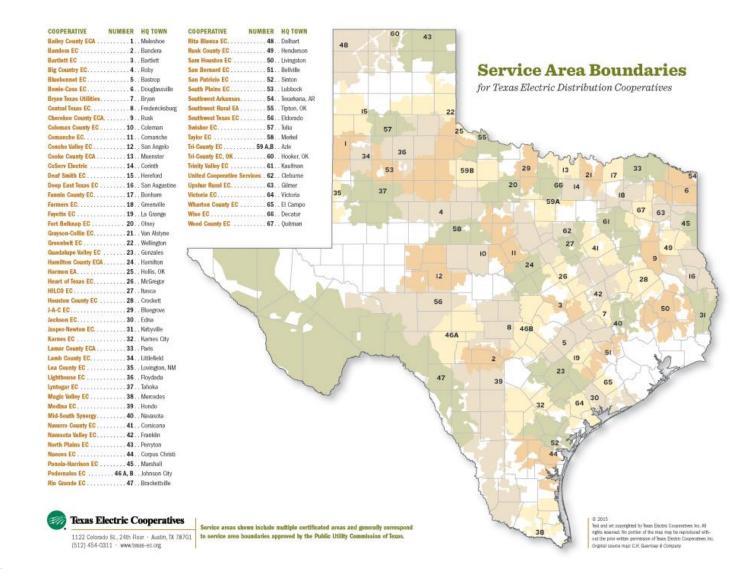
Electric Coops





Texas Electric Coops





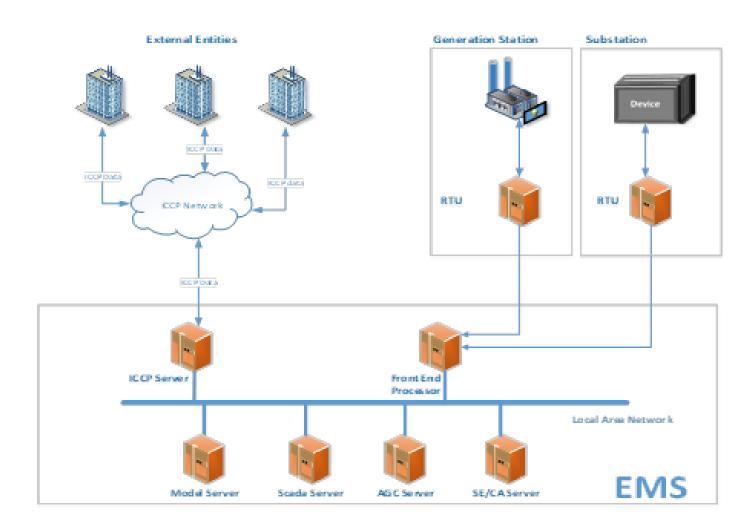
ERCOT Control Center with EMS





ERCOT EMS

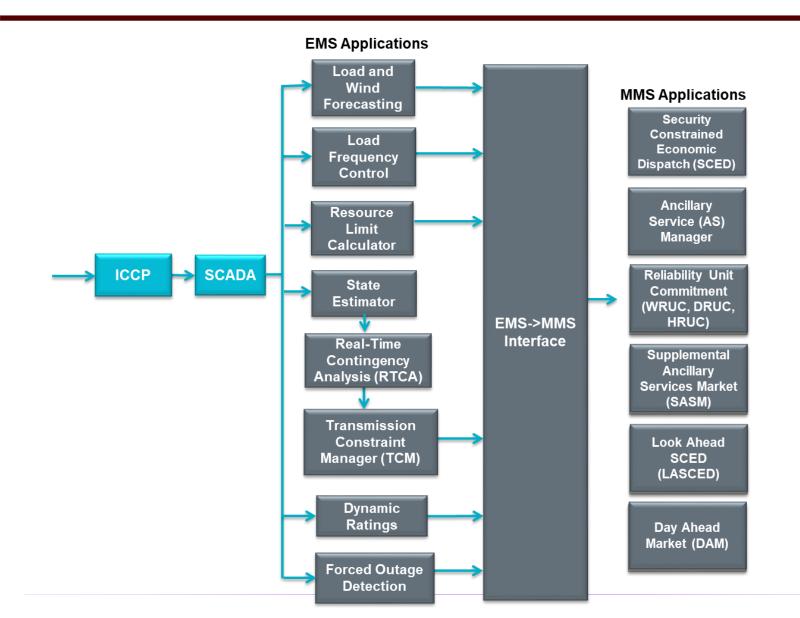




Slide source: ERCOT, D. Penney, J. Mandavilli, M. Henry, "Loss of SCADA, EMS or LCC"

ERCOT EMS





Power System Equivalents



- No electric grid model is ever going to completely represent a real electric grid
 - "All models are wrong but some models are useful"
- A key modeling consideration is how much of the electric grid to represent
 - For large-scale systems the distribution system is usually equivalence at some point; this has few system level ramifications if it is radial; if it is networked then there are potential issues
 - At the transmission level either the full interconnect is represented or it is equivalence
 - In an SE model in large grids (like the Eastern Interconnect) it is always an electrical equivalent

Kron Reduction, Ward Equivalents



- For decades power system network models have been equivalenced using the approach originally presented by J.B. Ward in 1949 AIEE paper "Equivalent Circuits for Power-Flow Studies"
 - Paper's single reference is to 1939 book by Gabriel Kron, so this is also known as Kron's reduction or Ward equivalent
- System buses are partitioned into a study system (s) to be retained and an external system (e) to be eliminated; buses in study system that connect to the external are known as boundary buses

Ward Equivalents



Ward approach is based on the below relationship

$$\begin{bmatrix} I_s \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ss} & Y_{se} \\ Y_{es} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_s \\ V_e \end{bmatrix}$$

$$(I_s - Y_{se} Y_{ee}^{-1} I_e) = (Y_{ss} - Y_{se} Y_{ee}^{-1} Y_{es}) V_s$$

- No impact on study, non-boundary buses
- Equivalent is created by doing a partial factorization of the \mathbf{Y}_{bus}
 - Computationally efficient

Other Types of Equivalents



- There are many different methods available for creating power system equivalents
 - A classic paper is by S. Deckmann, et. al., "Studies on Power System Load Flow Equivalencing," *IEEE Transactions Power App. And Syst.*, Nov/Dec 1980
 - Companion paper covers numerical testing of equivalents
- The major equivalencing types are
 - Ward-Type Equivalence: this is what we'll be covering, with the major differences associated with how the generator buses and equivalent loads are represented
 - REI Equivalents: All boundary buses connect to one "REI" bus
 - Linearized Methods: Linearize about an operating point
 - Others: PTDF-based, backbone type

Equivalent System Properties



- An equivalent is usually created from a larger model
 - In the Eastern Interconnect there are full grid models that are used for wide-area planning, these are equivalenced for realtime usage or more specialized studies
- The equivalent is usually smaller and less detailed
 - Solves quicker
 - Requires less storage
 - Requires less up-to-date data
- Equivalences contain fictitious elements
 - This can make modeling/updating more difficult
- The equivalent only approximates the behavior of the original

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Study vs External System

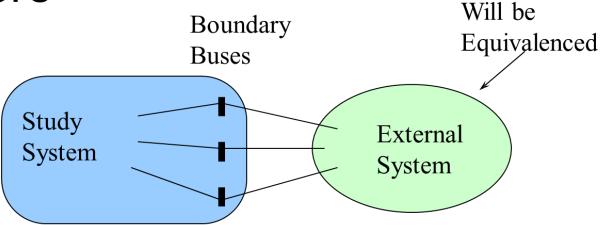


- The key decision in creating an equivalent is to divide the system into a study portion that is represented in detail, and an external portion that is represented by the equivalent
- The two systems are joined at boundary buses, which are part of the study subsystem
- How this is done is application specific; for example:
 - for real-time use it does not make sense to retain significant portions of the grid for which there is no real-time information
 - for contingency analysis the impact of the contingency is localized
 - for planning the new system additions have localized impacts 30

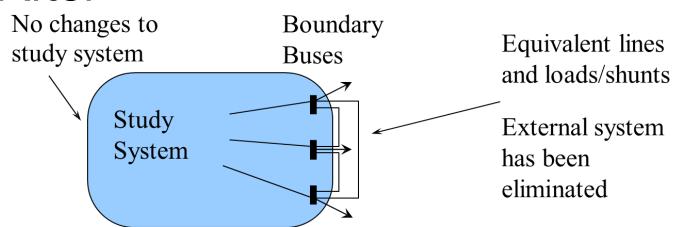
Ward Type Equivalencing







After



Ward Type Equivalencing Considerations



- The Ward equivalent is calculated by doing a partial factorization of the \mathbf{Y}_{bus}
 - The equivalent buses are numbered after the study buses
 - As the equivalent buses are eliminated their first neighbors are joined together
 - At the end, many of the boundary buses are connected
 - This can GREATLY decrease the sparsity of the system
 - Buses with different voltages can be directly connected

$$\begin{bmatrix} I_s \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ss} & Y_{se} \\ Y_{es} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_s \\ V_e \end{bmatrix}$$

$$(I_s - Y_{se}Y_{ee}^{-1}I_e) = (Y_{ss} - Y_{se}Y_{ee}^{-1}Y_{es})V_s$$

Ward Type Equivalencing Considerations

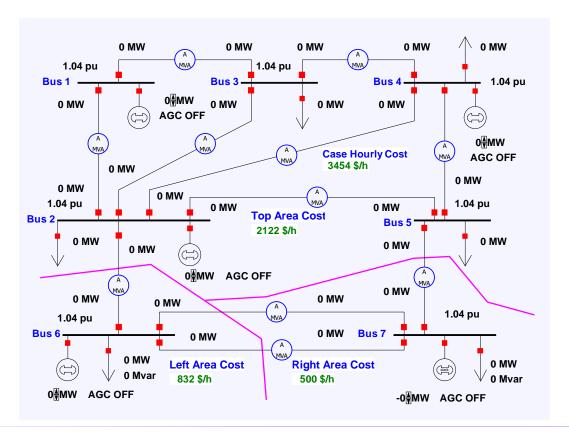


- At the end of the Ward process often many of the new equivalent lines have high impedances
 - Often there is an impedance threshold, and lines with impedances above this value are eliminated
- The equivalent lines may have unusual values, including negative resistances
- Load and generation is represented as equivalent current injections or shunts; sometimes these values are converted back to constant power
- Consideration needs to be given to loss of reactive support
- The equivalent embeds the present load and gen values3

B7Flat_Eqv Example



• In this example the B7Flat_Eqv case is reduced, eliminating buses 1, 3 and 4. The study system is then 2, 5, 6, 7, with buses 2 and 5 the boundary buses



For ease of comparison system is modeled unloaded

B7Flat_Eqv Example



Original Y_{bus}

$$\mathbf{Y}_{bus} = j \begin{bmatrix} -20.83 & 16.67 & 4.17 & 0 & 0 & 0 & 0 \\ 16.67 & -52.78 & 5.56 & 5.56 & 8.33 & 16.67 & 0 \\ 4.17 & 5.56 & -43.1 & 33.3 & 0 & 0 & 0 \\ 0 & 5.56 & 33.3 & -43.1 & 4.17 & 0 & 0 \\ 0 & 8.33 & 0 & 4.17 & -29.17 & 0 & 16.67 \\ 0 & 16.67 & 0 & 0 & 0 & -25 & 8.33 \\ 0 & 0 & 0 & 0 & 16.67 & 8.33 & -25 \end{bmatrix}$$

$$\mathbf{Y}_{ee} = j \begin{bmatrix} -20.833 & 4.167 & 0 \\ 4.167 & -43.056 & 33.333 \\ 0 & 33.333 & -43.056 \end{bmatrix}$$

B7Flat_Eqv Example



$$\mathbf{Y}_{es} = j \begin{bmatrix} 16.667 & 0 & 0 & 0 \\ 5.556 & 0 & 0 & 0 \\ 5.556 & 4.167 & 0 & 0 \end{bmatrix} \quad \mathbf{Y}_{se} = j \begin{bmatrix} 16.667 & 5.556 & 5.556 \\ 0 & 0 & 4.167 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{Y}_{ss} = j \begin{bmatrix} -52.778 & 8.333 & 16.667 & 0 \\ 8.333 & -29.167 & 0 & 16.667 \\ 16.667 & 0 & -25.0 & 8.333 \\ 0 & 16.667 & 8.333 & -25.0 \end{bmatrix}$$

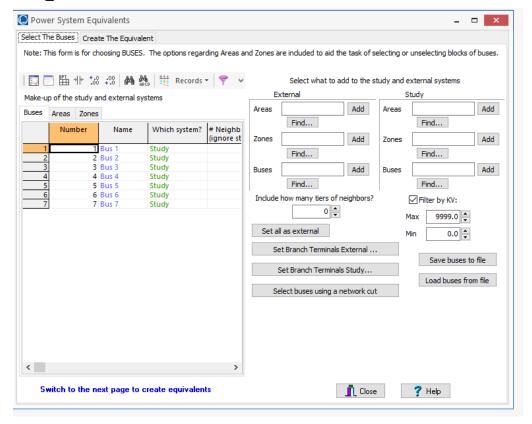
Note $Y_{es} = Y_{se}'$ if no phase shifters

$$(\mathbf{Y}_{ss} - \mathbf{Y}_{se} \mathbf{Y}_{ee}^{-1} \mathbf{Y}_{es}) = j \begin{bmatrix} -28.128 & 11.463 & 16.667 & 0\\ 11.463 & -28.130 & 0 & 16.667\\ 16.667 & 0 & -25.0 & 8.333\\ 0 & 16.667 & 8.333 & -25.0 \end{bmatrix}$$

Equivalencing in PowerWorld



 Open a case and solve it; then select Edit Mode, Tools, Equivalencing; this displays the Power System Equivalents Form

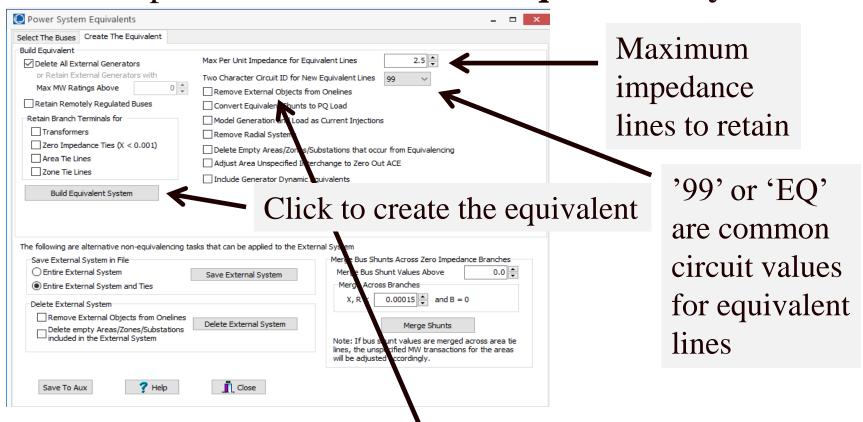


Next step is then to divide the buses into the study system and the external system; buses can be loaded from a text file as well

Equivalencing in PowerWorld



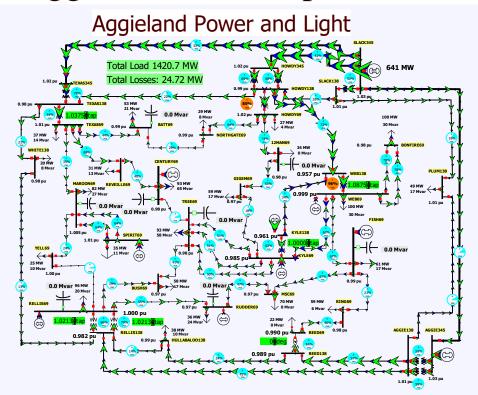
• Then go to the Create The Equivalent page, select the desired options and select Build Equivalent System





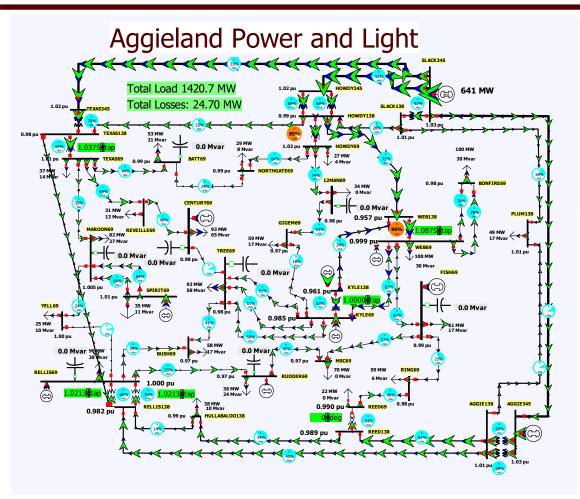
Example shows the creation of an equivalent for

Aggieland37 example



First example is simple, just removing WHITE138 (bus 3); note TEXAS138 is now directly joined to RELLIS138..





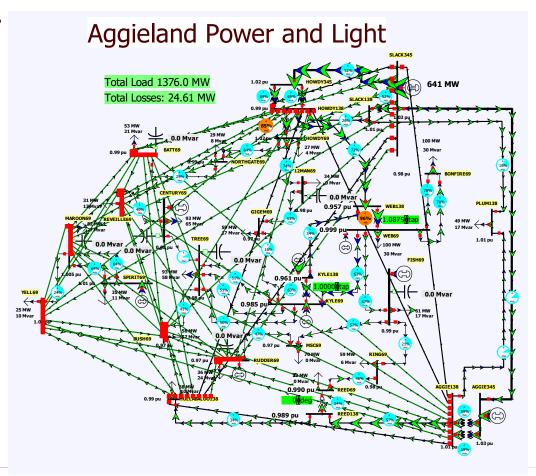
Only bus 3 was removed; the new equivalent line was auto-inserted.

Don't save the equivalent with the same name as the original, unless you want to lose the original



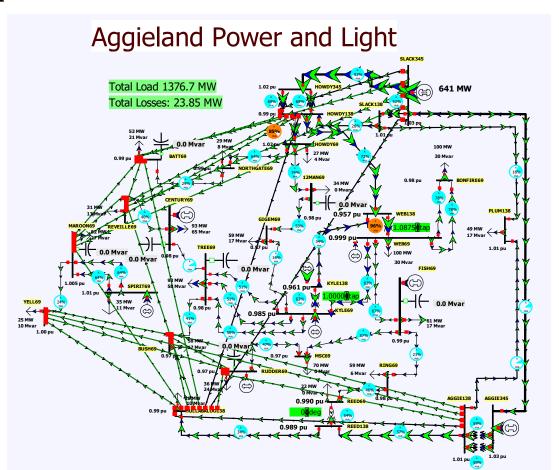
Now remove buses at WHITE138 and TEXAS and RELLIS (1, 3, 12, 40, 41, 44); set **Max Per Unit**

Impedance for **Equivalent** Lines to 99 (per unit) to retain all lines. Again to an autoinsert to show the equivalent lines.





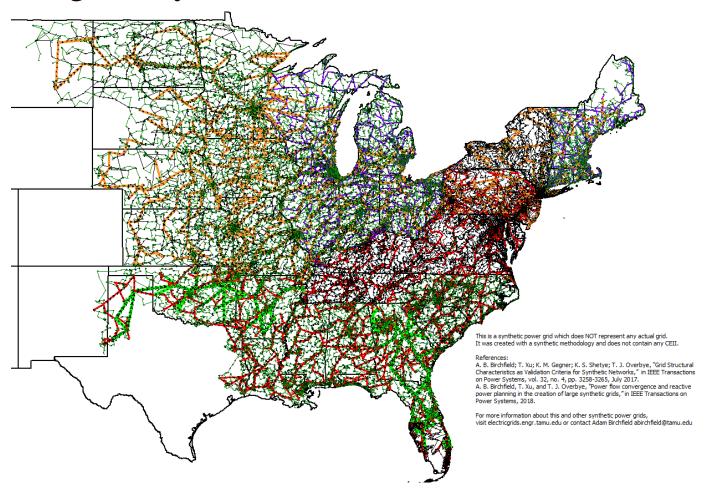
• Now set the Max Per Unit Impedance for Equivalent Lines to 2.5.



Large System Example: 70K Case



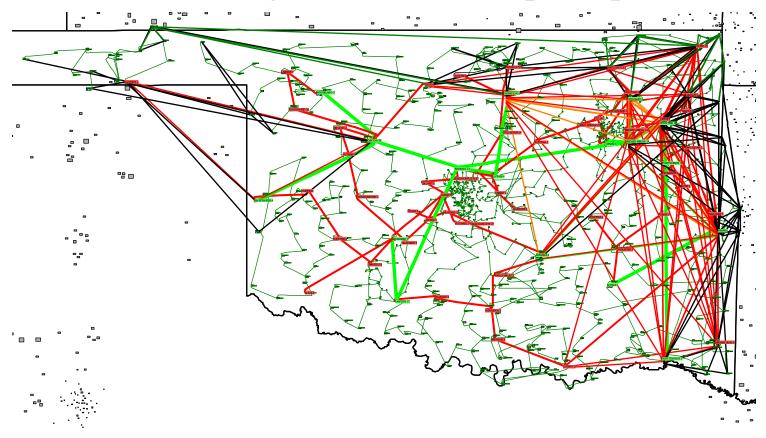
• Original System has 70,000 buses and 71,343 lines



Large System Example: 70K Case



 Just retain the Oklahoma Area; now 1591 buses and 1745 lines (deleting ones above 2.5 pu impedance)



Grid Equivalent Examples



- A 2016 EI case had about 350 lines with a circuit ID of '99' and about 60 with 'EQ' (out of a total of 102,000)
 - Both WECC and the EI use '99' or 'EQ' circuit IDs to indicate equivalent lines
 - One would expect few equivalent lines in interconnect wide models
- A ten year old EI case had about 1633 lines with a circuit ID of '99' and 400 with 'EQ' (out of a total of 65673)
- A ten year old case with about 5000 buses and 5000 lines had 600 equivalent lines