

ECEN 615

Methods of Electric Power Systems Analysis

Lecture 26: Optimal Power Flow, Security Constrained OPF, Optimization

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Announcements



- Homework 6 is due now
- Course evaluations are now available. Goto pica.tamu.edu
 - Please do the evaluation!!
- Final exam is Wednesday Dec 12, 1 to 3pm
 - Closed book, closed notes. Two 8.5 by 11 inch notesheets allowed; calculators allowed

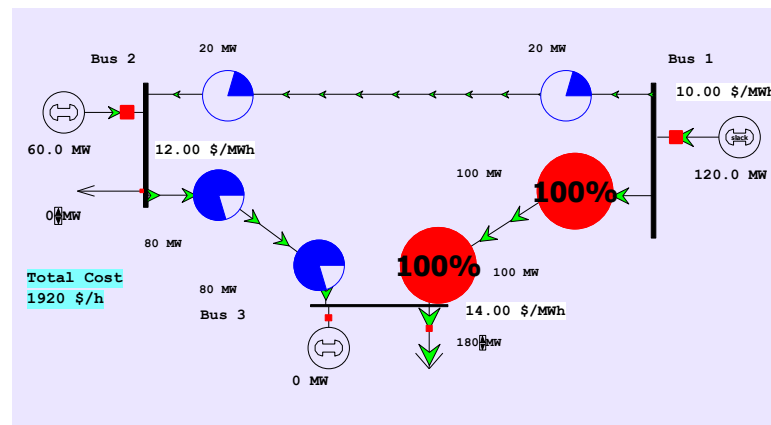
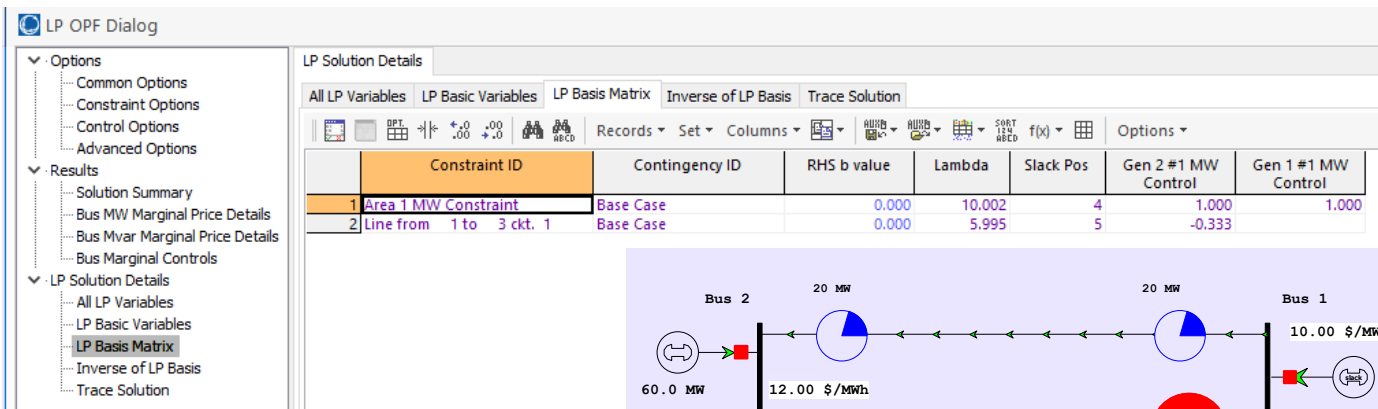
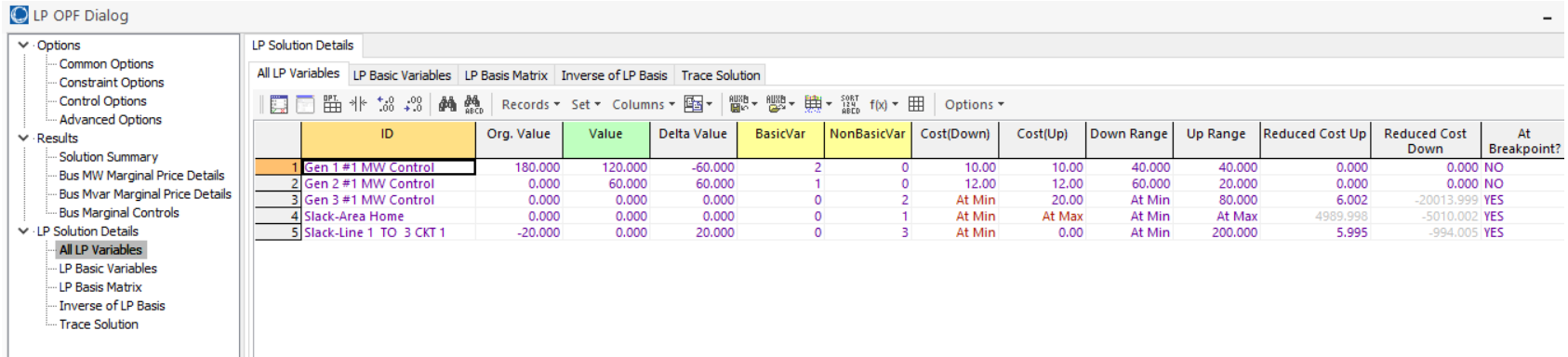
LP OPF Introductory Example, cont



- On use **Options, Constraint Options** to enable the enforcement of the Line/Transformer MVA limits

The screenshot displays the 'LP OPF Dialog' software interface. On the left is a tree view with categories: Options (sub-items: Common Options, Constraint Options, Control Options, Advanced Options), Results (sub-items: Solution Summary, Bus MW Marginal Price Details, Bus Mvar Marginal Price Details, Bus Marginal Controls), and LP Solution Details (sub-items: All LP Variables, LP Basic Variables, LP Basis Matrix, Inverse of LP Basis, Trace Solution). The 'Constraint Options' tab is selected in the main window. The 'Line/Transformer Constraints' section includes: Disable Line/Transformer MVA Limit Enforcement; Percent Correction Tolerance (2.0); MVA Auto Release Percentage (75.0); Maximum Violation Cost (\$/MWhr) (1000.0); and Enforce Line/Transformer MW Flow Limits (not MVA). The 'Interface Constraints' section includes: Disable Interface MW Limit Enforcement; Percent Correction Tolerance (2.0); MW Auto Release Percentage (75.0); Maximum Violation Cost (\$/MWhr) (1000.0). The 'Phase Shifting Transformer Regulation Limits' section includes: Disable Phase Shifter Regulation Limit Enforcement; In Range Cost (\$/MWhr) (0.10); Maximum Violation Cost (\$/MWhr) (1000.0). The 'Bus Constraints' section includes: Disable Bus Angle Enforcement; Maximum Violation Cost (\$/deg-h) (1000.0). The 'D-FACTS Constraints' section includes: Enforce Limits on Number of D-FACTS Devices in OPF; Maximum Number of D-FACTS Devices (1000); Maximum Violation Cost (\$/num-h) (1000.0). A callout box states: 'If you want to change enforcement percentages, modify the Limit Monitoring Settings' with a 'Limit Monitoring Settings ...' button.

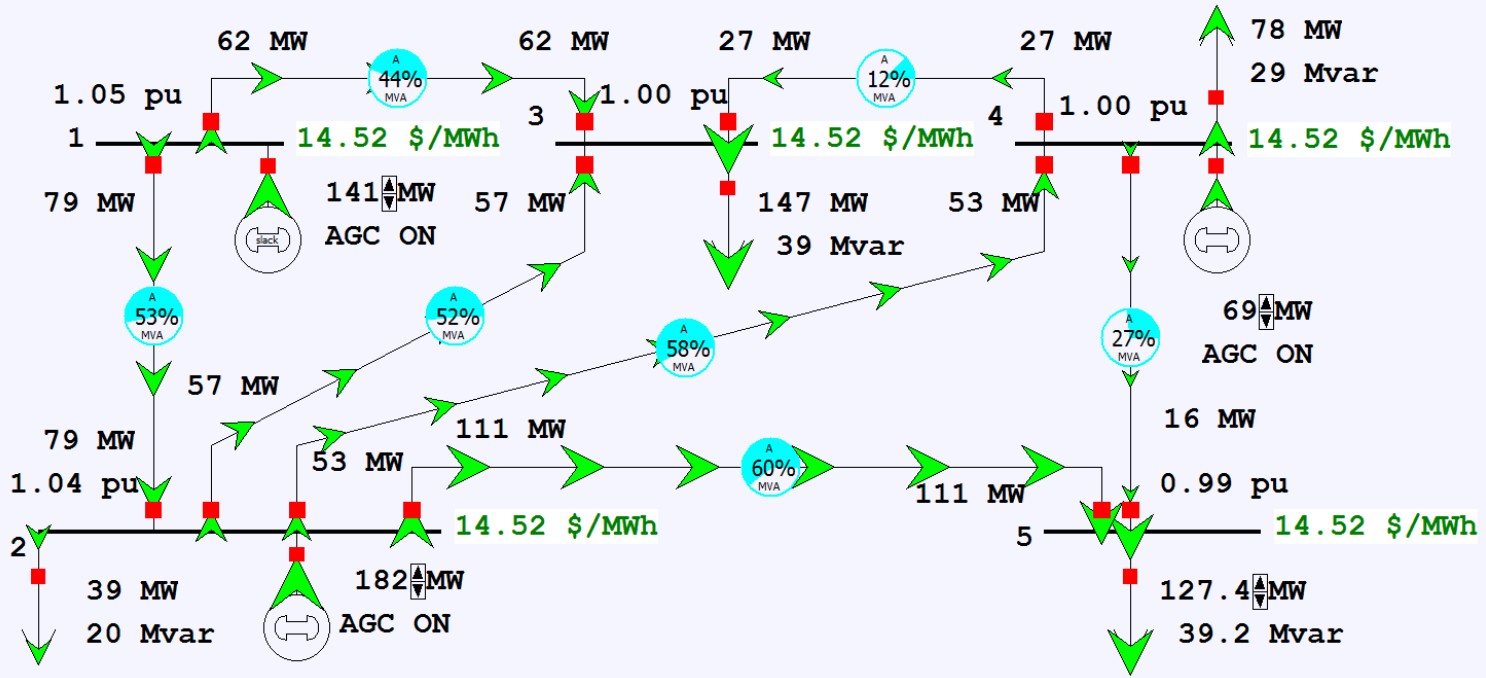
LP OPF Introductory Example, cont



Example 6_23 Optimal Power Flow



Example6_23 - Case: Example6_23.PWB Status: Initialized | Simulator 20 Beta



Total Hourly Cost: 5724.32 \$/h Load Scalar: 1.00

Total Area Load: 392.0 MW

Marginal Cost (\$/MWh): 14.52 \$/MWh

In the example the load is gradually increased

Locational Marginal Costs (LMPs)

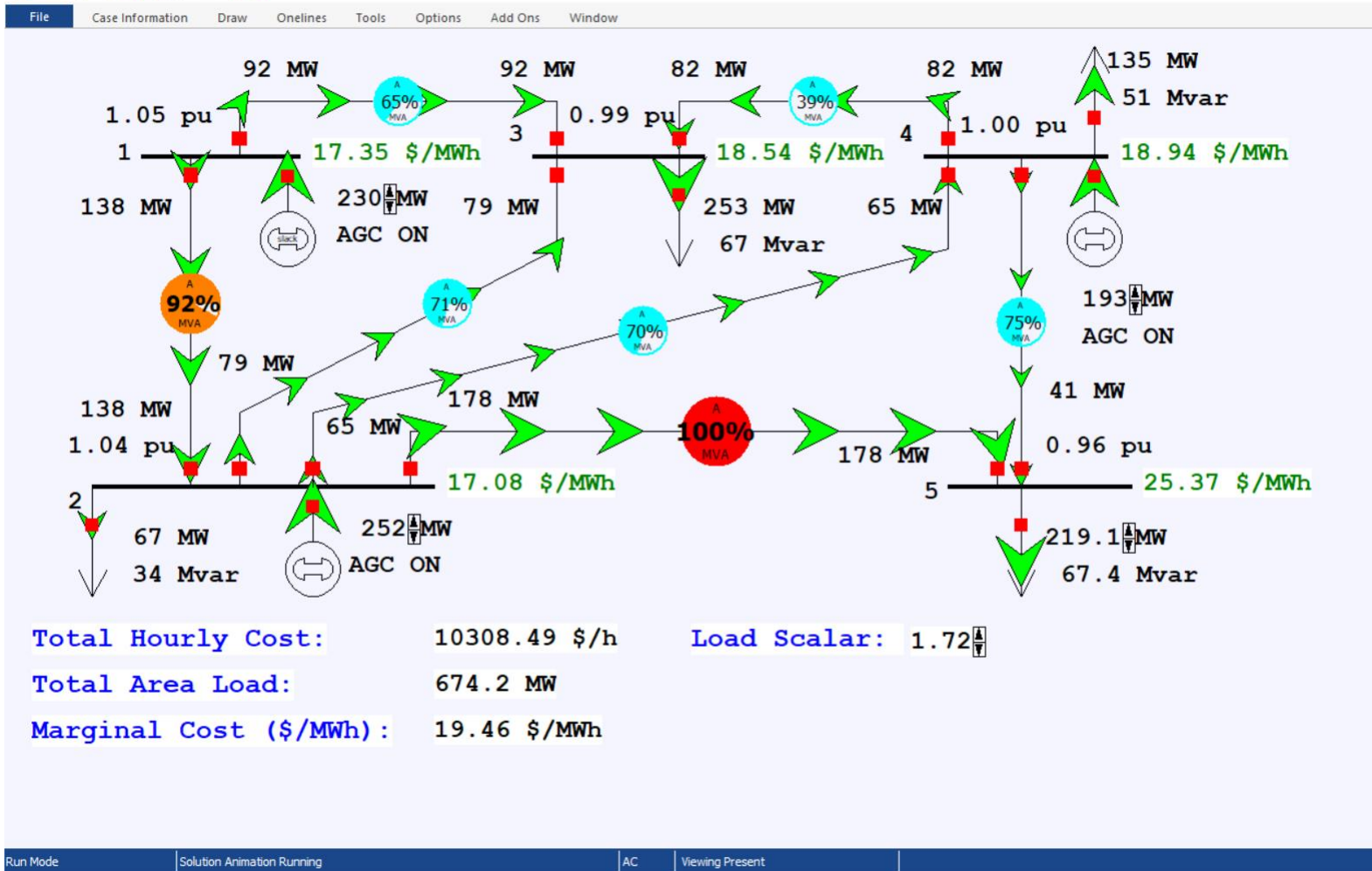


- In an OPF solution, the bus LMPs tell the marginal cost of supplying electricity to that bus
- The term “congestion” is used to indicate when there are elements (such as transmission lines or transformers) that are at their limits; that is, the constraint is binding
- Without losses and without congestion, all the LMPs would be the same
- Congestion or losses causes unequal LMPs
- LMPs are often shown using color contours; a challenge is to select the right color range!

Example 6_23 Optimal Power Flow with Load Scale = 1.72



Example6_23 - Case: Example6_23.pwb Status: Initialized | Simulator 20



Example 6_23 Optimal Power Flow with Load Scale = 1.72



- LP Sensitivity Matrix (A Matrix)

LP OPF Dialog - Case: Example6_23.pwb Status: Paused | Simulator 20

File Case Information Draw Onelines Tools Options Add Ons Window

LP OPF Dialog

Options
Common Options
Constraint Options
Control Options
Advanced Options

Results
Solution Summary
Bus MW Marginal Price Details
Bus Mvar Marginal Price Details
Bus Marginal Controls

LP Solution Details
All LP Variables LP Basic Variables LP Basis Matrix Inverse of LP Basis Trace Solution

	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 1 #1 MW Control	Gen 2 #1 MW Control	Gen 4 #1 MW Control	Slack-Area Top	Slack-Line 2 TO 5 CKT 1
1	Area 1 MW Constraint	Base Case	0.000	17.352	4	1.000	1.000	1.000	1.000	
2	Line from 2 to 5 ckt. 1	Base Case	0.000	10.541	5		0.026	-0.151		1.000

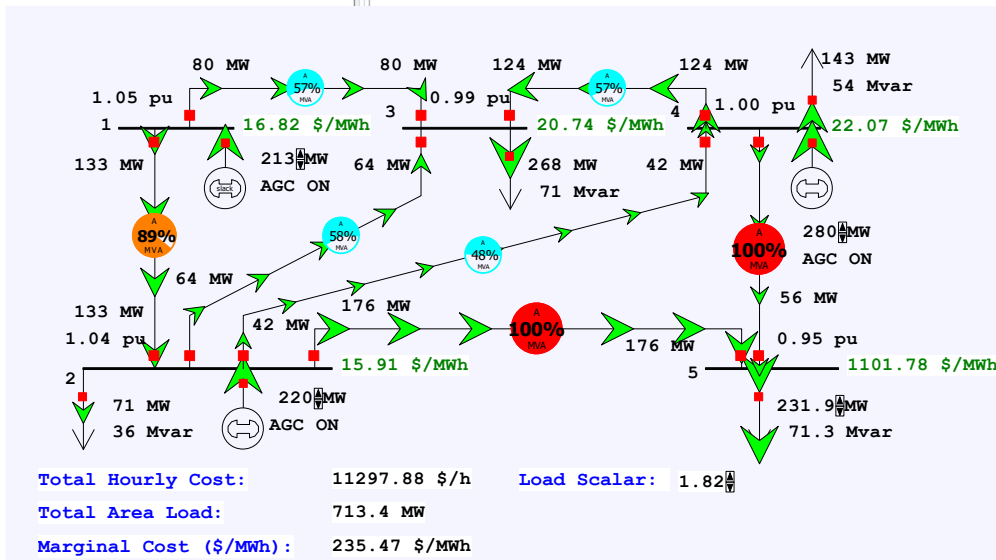
The first row is the power balance constraint, while the second row is the line flow constraint. The matrix only has the line flows that are being enforced.

Example 6_23 Optimal Power Flow with Load Scale = 1.82



- This situation is infeasible, at least with available controls. There is a solution because the OPF is allowing one of the constraints to violate (at high cost)

				Control	Control	Control	CKT 1	
1	Area 1 MW Constraint	Base Case	0.000	16.824	4	1.000	1.000	1.000
2	Line from 2 to 5 ckt. 1	Base Case	0.000	993.664	5	0.026	-0.146	1.000
3	Line from 4 to 5 ckt. 1	Base Case	-0.002	1000.000	6	-0.024	0.140	



Generator Cost Curve Modeling



- LP algorithms require linear cost curves, with piecewise linear curves used to approximate a nonlinear cost function
- Two common ways of entering cost information are
 - Quadratic function
 - Piecewise linear curve
- The PowerWorld OPF supports both types

Security Constrained OPF



- Security constrained optimal power flow (SCOPF) is similar to OPF except it also includes contingency constraints
 - Again the goal is to minimize some objective function, usually the current system cost, subject to a variety of equality and inequality constraints
 - This adds significantly more computation, but is required to simulate how the system is actually operated (with N-1 reliability)
- A common solution is to alternate between solving a power flow and contingency analysis, and an LP

Security Constrained OPF, cont.

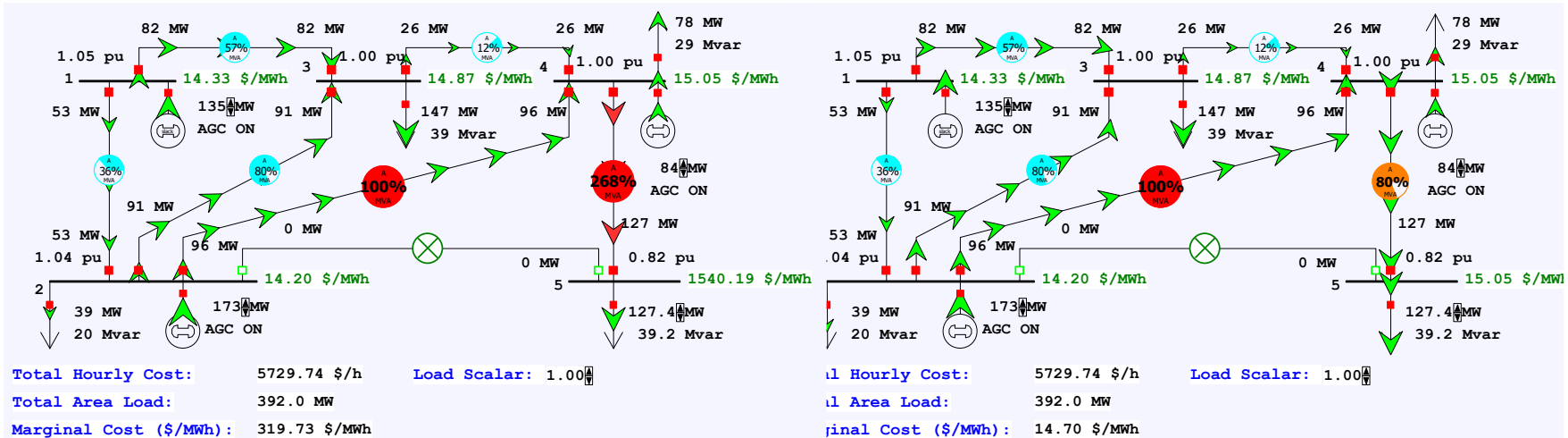


- With the inclusion of contingencies, there needs to be a distinction between what control actions must be done pre-contingent, and which ones can be done post-contingent
 - The advantage of post-contingent control actions is they would only need to be done in the unlikely event the contingency actually occurs
- Pre-contingent control actions are usually done for line overloads, while post-contingent control actions are done for most reactive power control and generator outage re-dispatch

SCOPF Example



- We'll again consider Example 6_23, except now it has been enhanced to include contingencies and we've also greatly increased the capacity on the line between buses 4 and 5



Original with line 4-5 limit of 60 MW with 2-5 out

Modified with line 4-5 limit of 200 MVA with 2-5 out

PowerWorld SCOPF Application



Just click the button to solve

The screenshot shows the PowerWorld SCOPF application interface. The title bar reads "Security Constrained Optimal Power Flow Form - Case: Example6_2?". The menu bar includes "File", "Case Information", "New", "Onlines", "Tools", "Options", "Add Ons", and "Window". A toolbar contains buttons for "Close", "Help", "Save As Aux", and "Load Aux". The "Run Full Security Constrained OPF" button is highlighted with a blue arrow pointing to it from the text "Just click the button to solve". Below the toolbar, the "SCOPF Status" is "SCOPF Solved Correctly".

The "Options" panel is open, showing "SCOPF Specific Options". The "Maximum Number of Outer Loop Iterations" is set to 1, with a blue arrow pointing to the input field. Other options include "Consider Binding Contingent Violations from Last SCOPF Solution" (checked), "Initialize SCOPF with Previously Binding Constraints" (checked), and "Set Solution as Contingency Analysis Reference Case" (checked). The "Maximum Number of Contingency Violations Allow Per Element" is set to 12. The "Basecase Solution Method" is "Solve base case using the power flow". The "Handling of Contingent Violations Due to Radial Load" is "Flag violations but do not include them in SCOPF". The "DC SCOPF Options" are "None (used and disgarded)".

The "SCOPF Results Summary" panel shows the following data:

SCOPF Results Summary	Value
Number of Outer Loop Iterations	1
Number of Contingent Violations	0
SCOPF Start Time	
SCOPF End Time	
Total Solution Time (Seconds)	0.136
Total LP Iterations	24
Final Cost Function (\$/Hr)	6301.94

The "Contingency Analysis Input" panel shows "Number of Active Contingencies: 7" and a "View Contingency Analysis Form" button.

The "Contingency Analysis Results" panel shows the following text:

```
Solving contingency L_000003Three-000004FourC1
Applied:
  OPEN Line Three_138.0 (3) TO Four_138.0 (4) CKT 1 | | CHECK | | Oper
Contingency L_000003Three-000004FourC1 successfully solved.
Solving contingency L_000004Four-000005FiveC1
Applied:
  OPEN Line Four_138.0 (4) TO Five_138.0 (5) CKT 1 | | CHECK | | Oper
Contingency L_000004Four-000005FiveC1 successfully solved.
Contingency Analysis finished at November 01, 2017 07:55:50
```

Number of times to redo contingency analysis

LP OPF and SCOPF Issues



- The LP approach is widely used for the OPF and SCOPF, particularly when implementing a dc power flow approach
- A key issue is determining the number of binding constraints to enforce in the LP tableau
 - Enforcing too many is time-consuming, enforcing too few results in excessive iterations
- The LP approach is limited by the degree of linearity in the power system
 - Real power constraints are fairly linear, reactive power constraints much less so

OPF Solution by Newton's Method



- An alternative to using the LP approach is to use Newton's method, in which all the equations are solved simultaneously
- Key paper in area is
 - D.I. Sun, B. Ashley, B. Brewer, B.A. Hughes, and W.F. Tinney, "Optimal Power Flow by Newton Approach", *IEEE Trans. Power App and Syst.*, October 1984

- Problem is

Minimize $f(\mathbf{x})$

s.t. $\mathbf{g}(\mathbf{x}) = \mathbf{0}$

$\mathbf{h}(\mathbf{x}) \leq \mathbf{0}$

For simplicity \mathbf{x} represents all the variables and we can use \mathbf{h} to impose limits on individual variables

OPF Solution by Newton's Method



- During the solution the inequality constraints are either binding ($=0$) or nonbinding (<0)
 - The nonbinding constraints do not impact the final solution
- We'll modify the problem to split the \mathbf{h} vector into the binding constraints, \mathbf{h}_1 and the nonbinding constraints, \mathbf{h}_2

Minimize $f(\mathbf{x})$

s.t. $\mathbf{g}(\mathbf{x}) = \mathbf{0}$

$\mathbf{h}_1(\mathbf{x}) = \mathbf{0}$

$\mathbf{h}_2(\mathbf{x}) < \mathbf{0}$

OPF Solution by Newton's Method



- To solve first define the Lagrangian

$$L(\mathbf{x}, \lambda_1, \lambda_2) = f(\mathbf{x}) + \boldsymbol{\mu}^T \mathbf{g}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{h}_1(\mathbf{x})$$

$$\text{Let } \mathbf{z} = [\mathbf{x} \quad \boldsymbol{\mu} \quad \boldsymbol{\lambda}]$$

- A necessary condition for a minimum is that the gradient is zero

$$\nabla L(\mathbf{z}) = \mathbf{0} = \begin{bmatrix} \frac{\partial L(\mathbf{z})}{\partial z_1} \\ \frac{\partial L(\mathbf{z})}{\partial z_2} \\ \vdots \end{bmatrix}$$

Both $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are Lagrange Multipliers

OPF Solution by Newton's Method



- Solve using Newton's method. To do this we need to define the Hessian matrix

$$\nabla^2 L(\mathbf{z}) = \mathbf{H}(\mathbf{z}) = \begin{bmatrix} \frac{\partial^2 L(\mathbf{z})}{\partial z_i \partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 L(\mathbf{z})}{\partial x_i \partial x_j} & \frac{\partial^2 L(\mathbf{z})}{\partial x_i \partial \mu_j} & \frac{\partial^2 L(\mathbf{z})}{\partial x_i \partial \lambda_j} \\ \frac{\partial^2 L(\mathbf{z})}{\partial \mu_i \partial x_j} & \mathbf{0} & \mathbf{0} \\ \frac{\partial^2 L(\mathbf{z})}{\partial \lambda \partial x_{ji}} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

- Because this is a second order method, as opposed to a first order linearization, it can better handle system nonlinearities

OPF Solution by Newton's Method



- Solution is then via the standard Newton's method.

That is

Set iteration counter $k=0$, set k_{\max}

Set convergence tolerance ε

Guess $\mathbf{z}^{(k)}$

While $(\|\nabla L(\mathbf{z})\| \geq \varepsilon)$ and $(k < k_{\max})$

$$\mathbf{z}^{(k+1)} = \mathbf{z}^{(k)} - [\mathbf{H}(\mathbf{z})]^{-1} \nabla L(\mathbf{z})$$

$$k = k + 1$$

End While

No iteration is needed for a quadratic function with linear constraints

Example



- Solve

Minimize $x_1^2 + x_2^2$ such that $x_1 + x_2 - 2 \geq 0$

Solve initially assuming the constraint is binding

$$L(\mathbf{x}, \lambda) = x_1^2 + x_2^2 + \lambda(3x_1 + x_2 - 2)$$

$$\nabla L(\mathbf{x}, \lambda) = \begin{bmatrix} \frac{\partial L}{\partial x_1} \\ \frac{\partial L}{\partial x_2} \\ \frac{\partial L}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} 2x_1 + 3\lambda \\ 2x_2 + \lambda \\ 3x_1 + x_2 - 2 \end{bmatrix}$$

No iteration is needed so any “guess” is fine.
Pick (1,1,0)

$$\nabla^2 L(\mathbf{x}, \lambda) = \mathbf{H}(\mathbf{x}, \lambda) = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.2 \\ 0.4 \end{bmatrix}$$

Because λ is positive the constraint is binding

Newton OPF Comments



- The Newton OPF has the advantage of being better able to handle system nonlinearities
- There is still the issue of having to deal with determining which constraints are binding
- The Newton OPF needs to implement second order derivatives plus all the complexities of the power flow solution
 - The power flow starts off simple, but can rapidly get complex when dealing with actual systems
- There is still the issue of handling integer variables

Mixed-Integer Programming



- A mixed-integer program (MIP) is an optimization problem of the form

Minimize $\mathbf{c}\mathbf{x}$

s.t. $\mathbf{A}\mathbf{x} = \mathbf{b}$

$\mathbf{x} \geq \mathbf{0}$

where \mathbf{x} = n-dimensional column vector

\mathbf{c} = n-dimensional row vector

\mathbf{b} = m-dimensional column vector

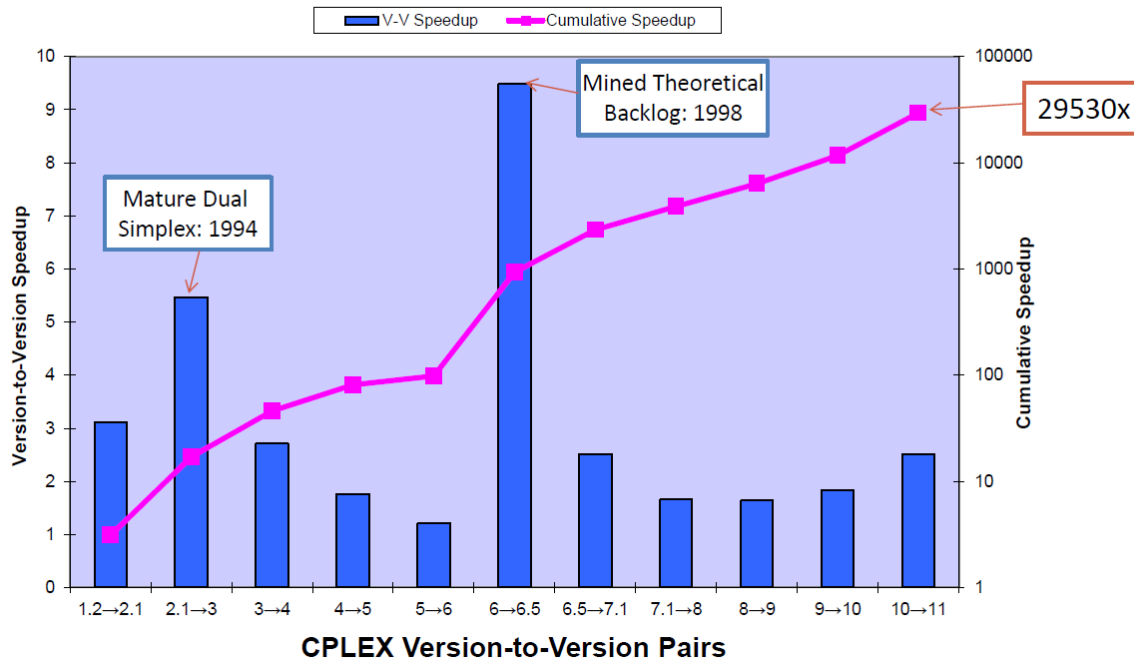
\mathbf{A} = $m \times n$ matrix

some or all x_j integer

Mixed-Integer Programming



- The advances in the algorithms have been substantial
Speedups 1991-2008



Speedups from 2009 to 2015 were about a factor of 30!

Notes are partially based on a presentation at Feb 2015 US National Academies Analytic Foundations of the Next Generation Grid by Robert Bixby from Gurobi Optimization titled “Advances in Mixed-Integer Programming and the Impact on Managing Electrical Power Grids”

Mixed-Integer Programming



- Suppose you were given the following choices?
 - Solve a MIP with today's solution technology on a 1991 machine
 - Solve a MIP with a 1991 solution on a machine from today?
- The answer is to choose option 1, by a factor of approximately 300
- This leads to the current debate of whether the OPF (and SCOPF) should be solved using generic solvers or more customized code (which could also have quite good solvers!)

More General Solvers Overview



- OPF is currently an area of active research
- Many formulations and solution methods exist...
 - As do many *tools* for highly complex, large-scale computing!
- While many options exist, some may work better for certain problems or with certain programs you already use
- Consider experimenting with a new language/solver!

Gurobi and CPLEX



- Gurobi and CPLEX are two well-known commercial optimization solvers/packages for **linear programming (LP)**, quadratic programming (QP), quadratically constrained programming (QCP), and the mixed integer (MI) counterparts of LP/QP/QCP
- Gurobi and CPLEX are accessible through object-oriented interfaces (C++, Java, Python, C), matrix-oriented interfaces (MATLAB) and other modeling languages (AMPL, GAMS)

Solver Comparison



Algorithm Type ----- <i>Solver</i>	LP/MILP linear/mixed integer linear program	QP/MIQP quadratic/mixed integer quadratic program	SOCP second order cone program	SDP semidefinite program
<i>CPLEX*</i>	X	X	X	
<i>GLPK</i>	X			
<i>Gurobi*</i>	X	X	X	
<i>IPOPT</i>		X		
<i>Mosek*</i>	X	X	X	X
<i>SDPT3/SeDuMi</i>			X	X

Linear programming can be solved by quadratic programming, which can be solved by second-order cone programming, which can be solved by semidefinite programming.

Modeling Tools



	AMPL	CVX (Matlab)	GAMS	Pyomo (Python)	YALMIP (Matlab)
<i>CPLEX</i>	x		x	x	x
<i>GLPK</i>				x	x
<i>Gurobi</i>	x	x	x	x	x
<i>IPOPT</i>	x			x	x
<i>Mosek</i>	x	x	x		x
<i>SDPT3/SeDu</i> <i>Mi</i>		x	x		x

Introduction to AMPL



- AMPL (A Mathematical Programming Language) is a modeling language that enables the compact and logical representation of optimization models
- Visit <http://www.ampl.com> to download and start using a student version.
 - To actually solve problems with AMPL, you'll also need a solver!
 - CPLEX is a good one to start with (already included in Windows download)
- There is an AMPL book and many examples also available for download at <http://www.ampl.com>

Introduction to AMPL



- The **model file** (.mod) declares the *data parameters*, the *variables*, the *objective function* and the *constraints*
- The *data* is provided in the **data file** (.dat)
 - You don't need to change the model for every small change in the data!
- Every declaration ends in a semicolon ;
- Comments begin with a pound sign #
- Parameters in the data file must first be declared in the model file

An AMPL Example



- Example 1 (lumber mill problem) from Lecture 24

$$\begin{aligned} &\text{Maximize} && 100x_1 + 120x_2 \\ &\text{s.t.} && 2x_1 + 2x_2 \leq 8 \\ &&& 3x_1 + 5x_2 \leq 15 \\ &&& x_1, x_2 \geq 0 \end{aligned}$$

Console

```
AMPL
ampl: option solver gurobi;
ampl: model example.mod
ampl: solve;
Gurobi 8.1.0: optimal solution; objective 430
2 simplex iterations
ampl: display x1,x2;
x1 = 2.5
x2 = 1.5
ampl: |
```

example.mod

```
# Problem 1

var x1 >= 0;
var x2 >= 0;

maximize profit: 100*x1 +120*x2;
subject to saw: 2*x1+2*x2 <= 8;
subject to plane: 3*x1+5*x2 <= 15;
```


An (Improved) AMPL Example



- Example 1 (lumber mill problem) from Lecture 24

```
example_improved.mod example_improved.dat
# Problem 1, improved

set B; # Boards (products)
set R; # Resources

param c {i in B};
param b {j in R};
param A {j in R, i in B};

var x{i in B} >= 0;

maximize profit: sum {i in B} c[i] * x[i];
subject to R_constr {j in R}: sum {i in B} A[j,i] * x[i] <= b[j];
```

```
example_improved.mod example_improved.dat
set B := construction finish; # grade boards
set R := saw plane; # resources

param c :=
construction 100
finish 120;

param b :=
saw 8
plane 15;

param A: construction finish :=
saw 2 2
plane 3 5;
```

AMPL

```
ampl: option solver gurobi;
ampl: model example_improved.mod
ampl: data example_improved.dat
ampl: solve;
Gurobi 8.1.0: optimal solution; objective 430
2 simplex iterations
ampl: display x, profit;
x [*] :=
construction 2.5
finish 1.5
;

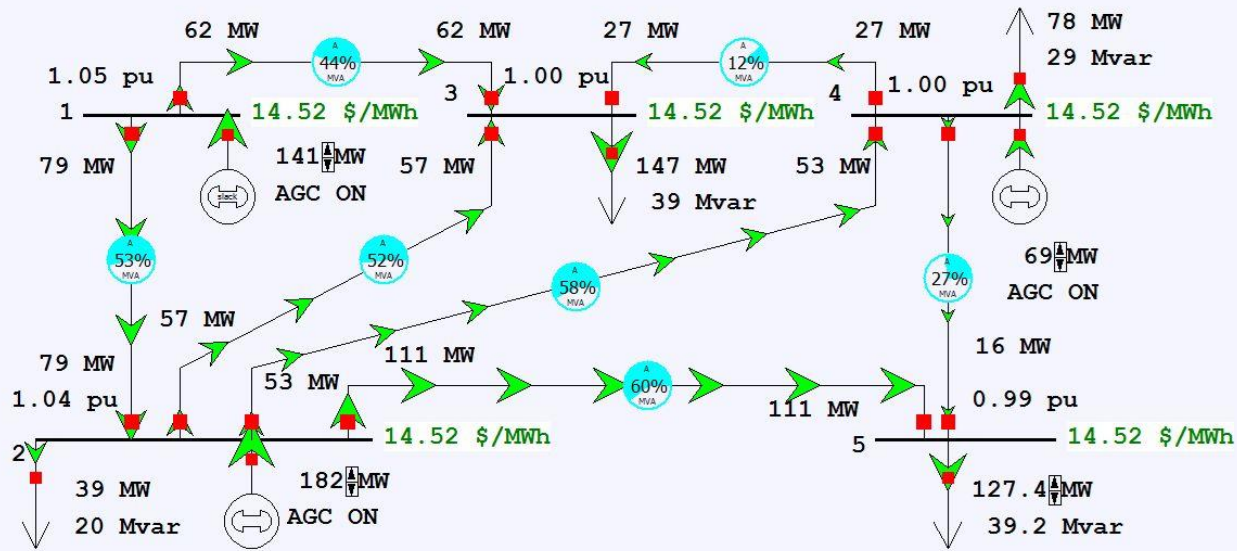
profit = 430
```

Can change the values in the .dat file, change solver, etc. with just a few quick clicks!

DCOPF in AMPL



- Example 6_23



Total Hourly Cost: 5724.32 \$/h Load Scalar: 1.00
 Total Area Load: 392.0 MW

DCOPF in AMPL: Parameters



- LOTS of parameters (not all are used in this example, but the framework is there for more involved examples)

```
case4.mod ✕
set BUS;
set BR within {1..4000} cross BUS cross BUS;
set GEN;

## Parameters ##
#param bus_number {BUS};
param bus_type {BUS};
param bus_p_load {BUS};
param bus_q_load {BUS};
param bus_g_shunt {BUS};
param bus_b_shunt {BUS};
param bus_area {BUS};
param bus_voltage {BUS};
param bus_angle0 {BUS};
param base_volt {BUS};
param loss_zone {BUS};
param vmax {BUS};
param vmin {BUS};
param lam_p {BUS};
param lam_q {BUS};
param mu_vmax {BUS};
param mu_vmin {BUS};

param f_bus {BR};
param t_bus {BR};
param br_type {BR};
param br_r {BR};
param br_x {BR};
param br_b {BR};
param rate_a {BR};
param rate_b {BR};
param rate_c {BR};
param tap {BR};
param shift {BR};
param br_status {BR};
param angmin {BR};
param angmax {BR};
param pf {BR};
param qf {BR};
param pt {BR};
param qt {BR};
param mu_sf {BR};
param mu_st {BR};
param mu_angmin {BR};
param mu_angmax {BR};

param gen_bus {GEN};
param pg0 {GEN};
param qg0 {GEN};
param qmax {GEN};
param qmin {GEN};
param vg {GEN};
param mbase {GEN};
param gen_status {GEN};
param pmax {GEN};
param pmin {GEN};
#param pc1 {GEN};
#param pc2 {GEN};
#param qc1min {GEN};
#param qc1max {GEN};
#param qc2min {GEN};
#param qc2max {GEN};
#param ramp_agc {GEN};
#param ramp_10 {GEN};
#param ramp_30 {GEN};
#param ramp_q {GEN};
#param apf {GEN};
#param mu_pmax {GEN};
#param mu_pmin {GEN};
#param mu_qmax {GEN};
#param mu_qmin {GEN};
```

DCOPF in AMPL: Parameters, cont.



```
param deg2rad := 3.14156/180;
param base_mva := 100;

# Make YBUS
set YBUS := setof{i in BUS} (i,i) union
            setof {(l,k,m) in BR} (k,m) union
            setof {(l,k,m) in BR} (m,k);

param B{(k,m) in YBUS} := if(k ==m) then (sum{(l,k,i) in BR} (0)
                                     +sum{(l,i,k) in BR} (0))
                          else if(k != m) then (sum{(l,k,m) in BR} 1/br_x[l,k,m]
                                               +sum{(l,m,k) in BR} 1/br_x[l,m,k]);
```

- Initialization

```
# data init
for{i in BUS} {
  let bus_angle[i] := bus_angle0[i]*deg2rad;
  let pg0[i] := pg0[i]/base_mva;
  let pmin[i] := pmin[i]/base_mva;
  let pmax[i] := pmax[i]/base_mva;
  let bus_p_load[i] := bus_p_load[i]/base_mva;
};

# fixing variables

fix {i in BUS : bus_type[i] == 3} bus_angle[i]; # slack angle fixed
```

DCOPF in AMPL: Model



- Objective: Polynomial

```
## Available Controls
var bus_angle {BUS};
var pflow      {BR}; # used for output

# generator MW outputs
var pg {k in BUS} = bus_p_load[k]
                + sum{(k,m) in YBUS} (B[k,m]*(bus_angle[k] - bus_angle[m]));

## Objective ##
minimize cost: 0.016*(pg[1]*base_mva)^2+10*pg[1]*base_mva+373.50 +
              0.018*(pg[2]*base_mva)^2+8*pg[2]*base_mva+403.60 +
              0.018*(pg[4]*base_mva)^2+12*pg[4]*base_mva+253.20;

## Equality Constraints
# bus real power balance
subject to pL {k in BUS}: pg[k] - bus_p_load[k]
                    - sum{(k,m) in YBUS} (B[k,m]*(bus_angle[k] - bus_angle[m])) = 0;

## Inequality Constraints
# generator MW limits
subject to pG {k in BUS}:
    pmin[k] <= pg[k] <= pmax[k];
```



DCOPF in AMPL: Data



- Bus, Branch, and Generator data saved in .txt files

```
data;
param: BUS: bus_type bus_p_load bus_q_load
        bus_g_shunt bus_b_shunt bus_area bus_voltage bus_angle0
        base_volt loss_zone vmax vmin lam_p lam_q mu_vmax mu_vmin :=
include 'C:\Users\Documents\AMPL\amplide.mswin64\amplide\busdat.bus.txt';

data;
param: BR: br_r br_x br_b
        rate_a rate_b rate_c tap shift br_status angmin angmax
        pf qf pt qt mu_sf mu_st mu_angmin mu_angmax :=
include 'C:\Users\Documents\AMPL\amplide.mswin64\amplide\brdat.br.txt';

data;
param: GEN: pg0 qg0 qmax qmin vg mbase gen_status
        pmax pmin #pc1 pc2 qc1min qc1max qc2min qc2max
        #ramp_agc ramp_10 ramp_30 ramp_q apf mu_pmax mu_pmin mu_qmax mu_qmin
:= include 'C:\Users\Documents\AMPL\amplide.mswin64\amplide\gendat.gen.txt';
```

	1	2	3	4	5	6	7	8	9	10
1	141.31	15.03	9900.00	-9900.00	1.0500	100.00	1	400.00	100.00	
2	181.50	70.93	9900.00	-9900.00	1.0400	100.00	1	500.00	150.00	
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
4	69.19	6.26	9900.00	-9900.00	1.0000	100.00	1	300.00	0.00	
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

DCOPF in AMPL: Solution



Gurobi 8.1.0: optimal solution; objective 5724.293529

7 barrier iterations

No basis.

bus_angle [*] :=

```
1  0.130103
2  0.082881
3 -0.0196945
4 -0.0114151
5 -0.0504711
;
```



Ang (deg)
7.454
4.748
-1.128
-0.654
-2.892

AMPL+Gurobi
DCOPF with
polynomial

MATPOWER Version 4.1, 14-Dec-2011 -- AC Optimal Power Flow
MATLAB Interior Point Solver -- MIPS, Version 1.0, 07-Feb-2011
Converged!

Converged in 0.47 seconds
Objective Function Value = 5724.29 \$/hr

Bus #	Voltage Mag (pu)	Ang (deg)
1	1.020	7.454*
2	1.021	4.846
3	1.004	-0.916
4	1.012	-0.448
5	0.983	-2.763

MATPOWER ACOPF
with polynomial objective
function

Angle (Deg)
7.45
4.97
-0.73
-0.26
-2.48

PowerWorld OPF
solution

Total Hourly Cost: 5724.32 \$/h

MATPOWER



- Uses data stored in MATLAB struct

```
>> runopf(Example6_23)
```

```
MATPOWER Version 4.1, 14-Dec-2011 -- AC Optimal Power Flow
```

```
MATLAB Interior Point Solver -- MIPS, Version 1.0, 07-Feb-2011
```

```
Converged!
```

```
Converged in 1.13 seconds
```

```
Objective Function Value = 5724.29 $/hr
```

```
=====
```

```
|      System Summary      |
```

```
=====
```

How many?		How much?	P (MW)	Q (MVar)
Buses	5	Total Gen Capacity	1200.0	-29700.0 to 29700.0
Generators	3	On-line Capacity	1200.0	-29700.0 to 29700.0
Committed Gens	3	Generation (actual)	392.0	93.0
Loads	4	Load	392.0	127.4
Fixed	4	Fixed	392.0	127.4
Dispatchable	0	Dispatchable	-0.0 of -0.0	-0.0
Shunts	0	Shunt (inj)	-0.0	0.0
Branches	7	Losses ($I^2 * Z$)	0.00	41.01
Transformers	0	Branch Charging (inj)	-	75.4
Inter-ties	0	Total Inter-tie Flow	0.0	0.0
Areas	1			

MATPOWER Solvers



- Section 6.5 of manual
 - Originally used MATLAB's Optimization Toolbox
 - Now can use MINOPF/TSPOPF packages, IPOPT solver (open-source), CPLEX/MOSEK/Gurobi (for DC OPF), KNITRO (for AC OPFs)
 - Default: own primal-dual interior point method implementation MIPS (MATPOWER Interior Point Solver) for AC and DC (QP solver)

<http://www.pserc.cornell.edu/matpower/>

Questions



- What changes would you make to the .mod file to do a full ACOPF?
- What sensitivity analysis could you do by only changing the .dat/.txt files?
- What might you consider when comparing solvers?
- **What tools best fit your needs?**