# ECEN 615
# Methods of Electric Power Systems Analysis

## Lecture 6: Power Flow

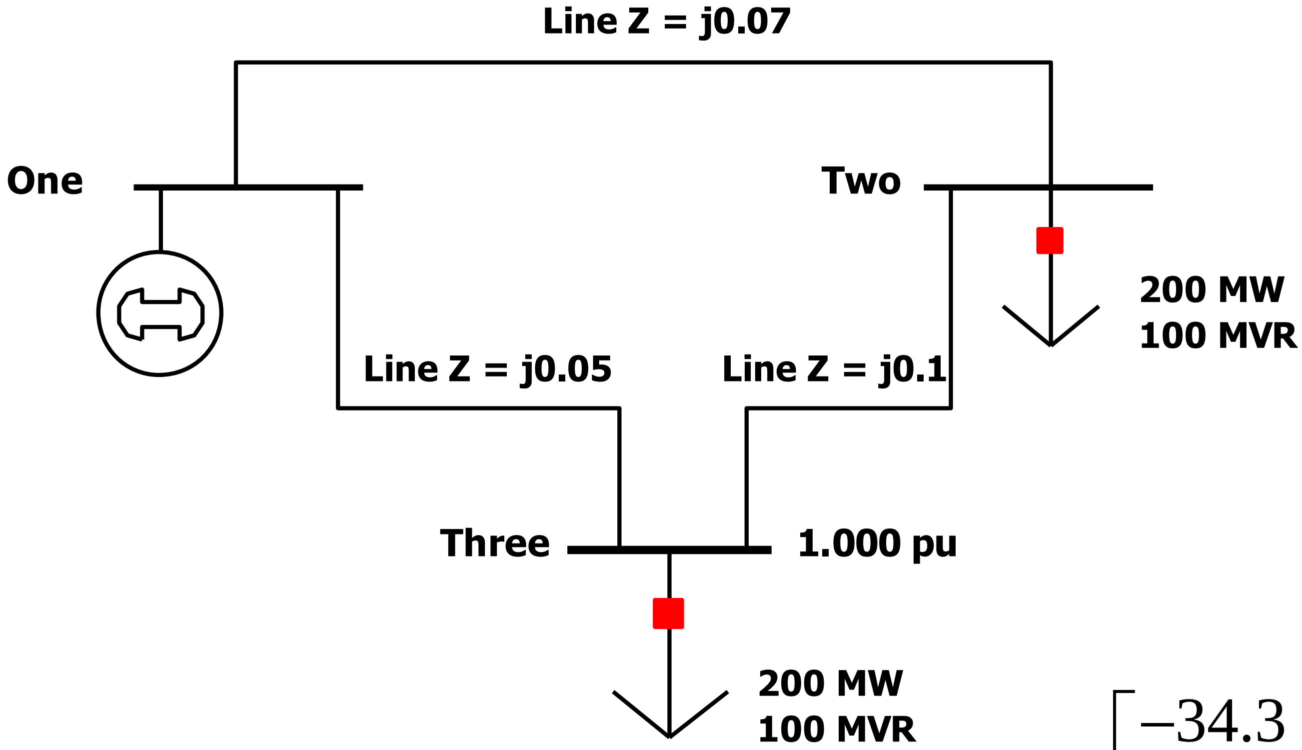
Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

# Announcements

- RSVP to Alex at zandra23@ece.tamu.edu for the TAMU ECE Energy and Power Group (EPG) picnic. It starts at 5pm on September 27, 2019
- Read Chapter 6 from the book
  - They formulate the power flow using the polar form for the $Y_{bus}$ elements
- Homework 1 is due today
- Homework 2 is due on Thursday September 26

# FDPF Three Bus Example

Use the FDPF to solve the following three bus system




$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix}$$

# FDPF Three Bus Example, cont'd

$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix} \rightarrow \mathbf{B} = \begin{bmatrix} -24.3 & 10 \\ 10 & -30 \end{bmatrix}$$

$$\mathbf{B}^{-1} = \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix}$$

Iteratively solve, starting with an initial voltage guess

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix}$$

# FDPF Three Bus Example, cont'd

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9364 \\ 0.9455 \end{bmatrix}$$

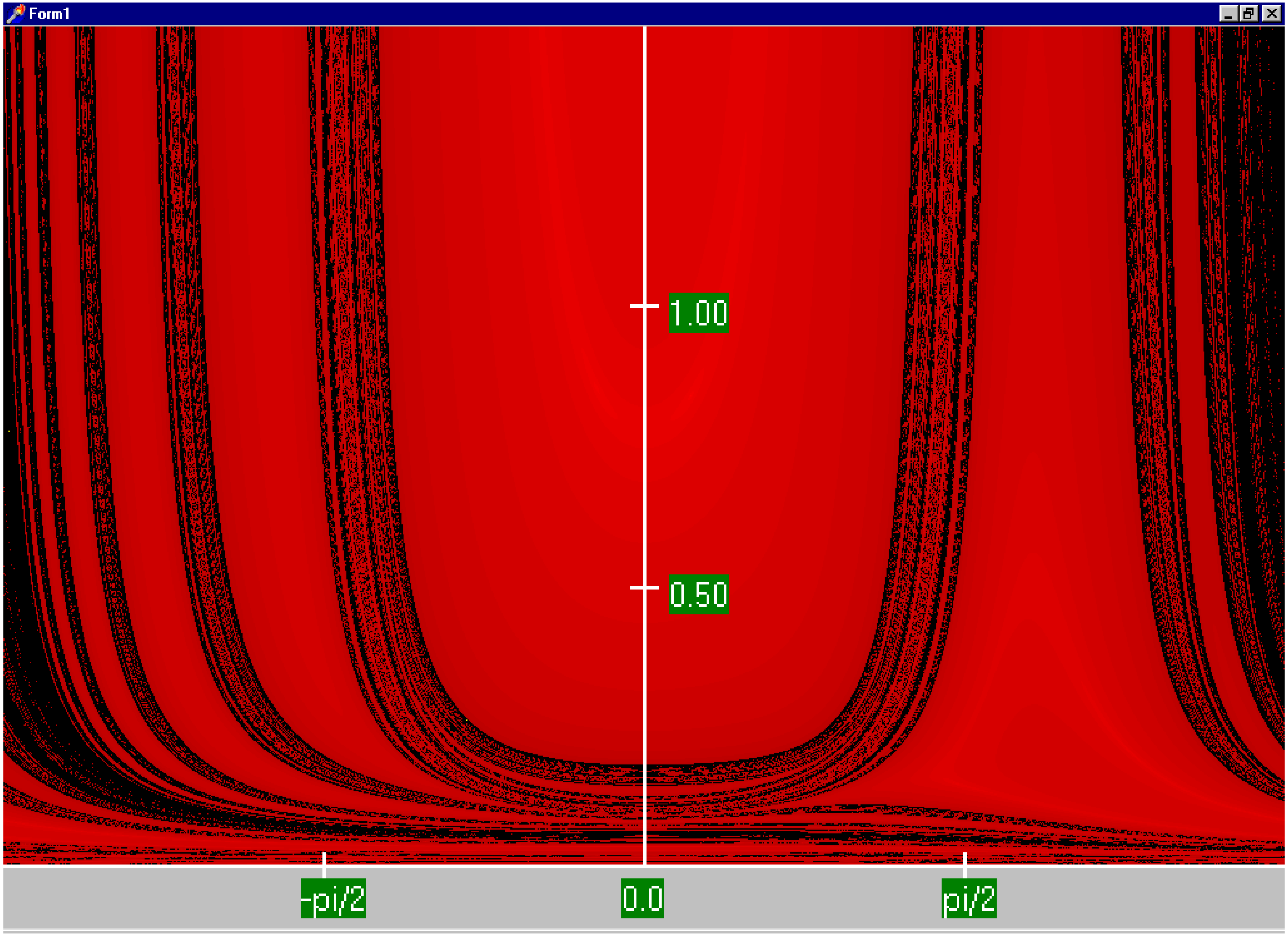
$$\frac{\Delta \mathrm{P_i}(\mathbf{x}\ )}{|\mathrm{V_i}|} = \sum_{k=1}^{n} |V_k| (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) + \frac{P_{Di} - P_{Gi}}{|\mathrm{V_i}|}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(2)} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 0.151 \\ 0.107 \end{bmatrix} = \begin{bmatrix} -0.1361 \\ -0.1156 \end{bmatrix}$$

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(2)} = \begin{bmatrix} 0.924 \\ 0.936 \end{bmatrix}$$

Actual solution: $\boldsymbol{\theta} = \begin{bmatrix} -0.1384 \\ -0.1171 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} 0.9224 \\ 0.9338 \end{bmatrix}$

# FDPF Region of Convergence



# FDPF Cautions

- The FDPF works well as long as the previous approximations hold for the entire system
- With the movement towards modeling larger systems, with more of the lower voltage portions of the system represented (for which r/x ratios are higher) it is quite common for the FDPF to get stuck because small portions of the system are ill-behaved
- The FDPF is commonly used to provide an initial guess of the solution or for contingency analysis

# DC Power Flow

- The “DC” power flow makes the most severe approximations:
  - completely ignore reactive power, assume all the voltages are always 1.0 per unit, ignore line conductance
- This makes the power flow a linear set of equations, which can be solved directly

$$\boldsymbol{\theta} = -\mathbf{B}^{-1}\,\mathbf{P}$$

**P** sign convention is generation is positive

- The term dc power flow actually dates from the time of the old network analyzers (going back into the 1930’s)
- Not to be confused with the inclusion of HVDC lines in the standard NPF

# DC Power Flow References

- I don’t think a classic dc power flow paper exists; a nice formulation is given in our book *Power Generation and Control* book by Wood, Wollenberg and Sheble
- The August 2009 paper in IEEE Transactions on Power Systems, “DC Power Flow Revisited” (by Stott, Jardim and Alsac) provides good coverage
- T. J. Overbye, X. Cheng, and Y. Sun, “A comparison of the AC and DC power flow models for LMP Calculations,” in *Proc. 37th Hawaii Int. Conf. System Sciences*, 2004, compares the accuracy of the approach

# DC Power Flow Example

**EXAMPLE 6.17**

Determine the dc power flow solution for the five bus from Example 6.9.

**SOLUTION** With bus 1 as the system slack, the **B** matrix and **P** vector for this system are

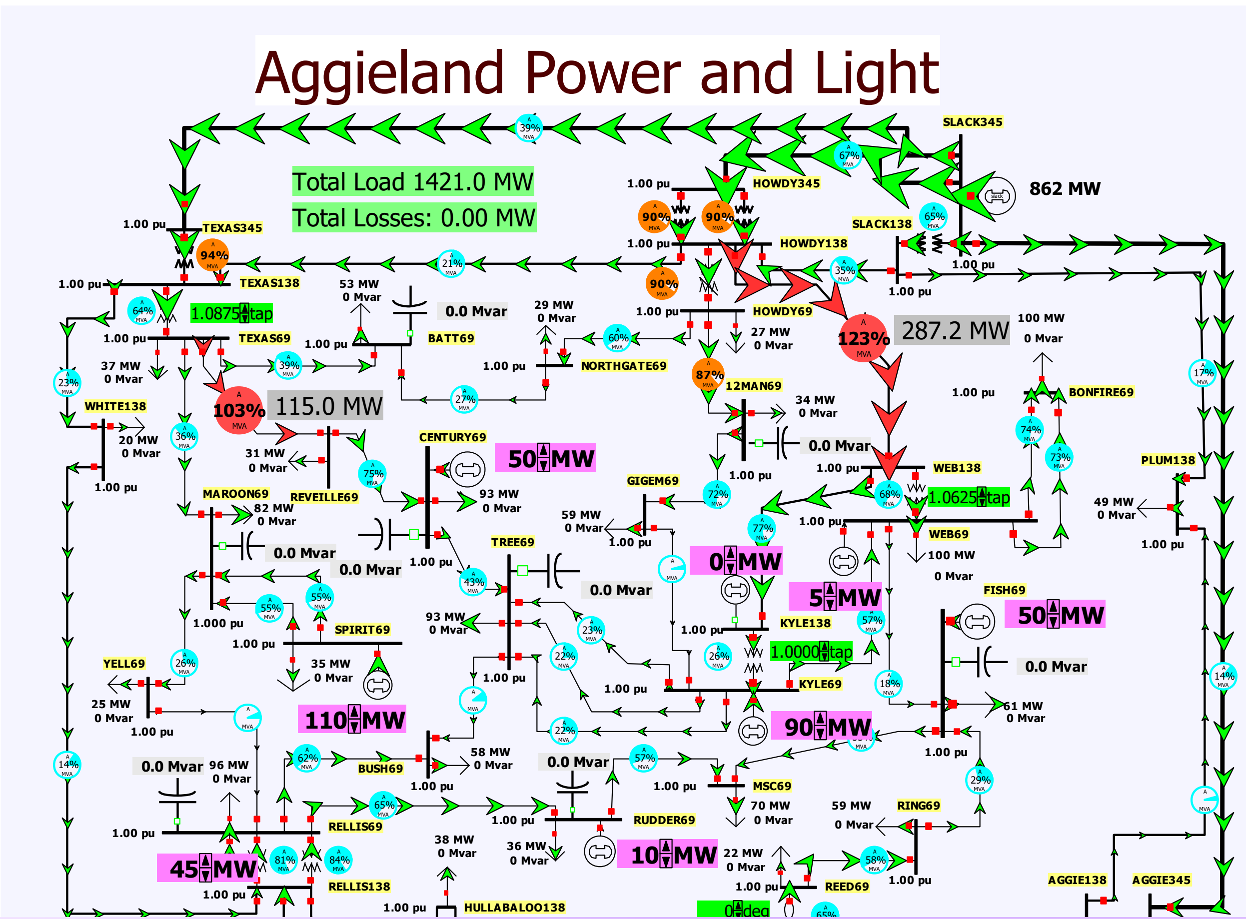
$$\mathbf{B} = \begin{bmatrix} -30 & 0 & 10 & 20 \\ 0 & -100 & 100 & 0 \\ 10 & 100 & -150 & 40 \\ 20 & 0 & 40 & -110 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} -8.0 \\ 4.4 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{\delta} = -\mathbf{B}^{-1}\mathbf{P} = \begin{bmatrix} -0.3263 \\ 0.0091 \\ -0.0349 \\ -0.0720 \end{bmatrix} \text{radians} = \begin{bmatrix} -18.70 \\ 0.5214 \\ -2.000 \\ -4.125 \end{bmatrix} \text{degrees}$$

Example from Power System Analysis and Design, by Glover, Overbye, Sarma, 6th Edition

# DC Power Flow in PowerWorld

- PowerWorld allows for easy switching between the dc and ac power flows (case **Aggieland37**)




To use the dc approach in PowerWorld select **Tools, Solve, DC Power Flow**

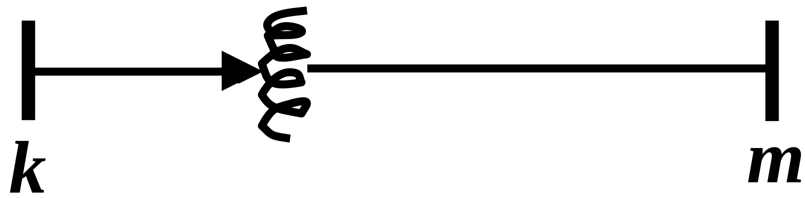
Notice there are no losses

# Modeling Transformers with Off-Nominal Taps and Phase Shifts

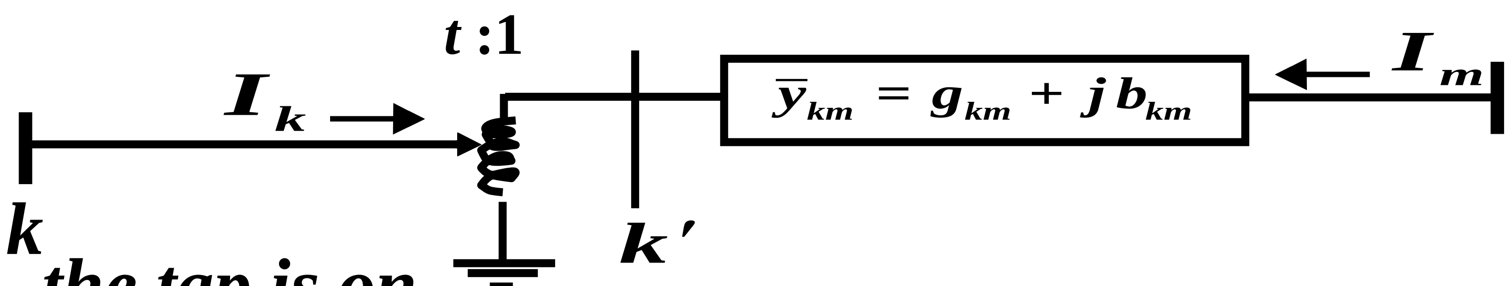
- If transformers have a turns ratio that matches the ratio of the per unit voltages than transformers are modeled in a manner similar to transmission lines.
- However it is common for transformers to have a variable tap ratio; this is known as an "off-nominal" tap ratio
  - The off-nominal tap is t, initially we'll consider it a real number
  - We'll cover phase shifters shortly in which t is complex

# Transformer Representation

- The one–line diagram of a branch with a variable tap transformer




- The network representation of a branch with off–nominal turns ratio transformer is




***the tap is on***

***the side of bus k***

# Transformer Nodal Equations

- From the network representation

$$\bar{I}_m = \bar{I}_{k'} = \bar{y}_{km}\left(\bar{E}_m - \bar{E}_{k'}\right) = \bar{y}_{km}\left(\bar{E}_m - \frac{\bar{E}_k}{t}\right)$$

$$= \left(\bar{y}_{km}\right)\bar{E}_m + \left(-\frac{\bar{y}_{km}}{t}\right)\bar{E}_k$$

- Also

$$\bar{I}_k = -\frac{1}{t}\bar{I}_{k'} = \left(-\frac{\bar{y}_{km}}{t}\right)\bar{E}_m + \left(\frac{\bar{y}_{km}}{t^2}\right)\bar{E}_k$$

# Transformer Nodal Equations

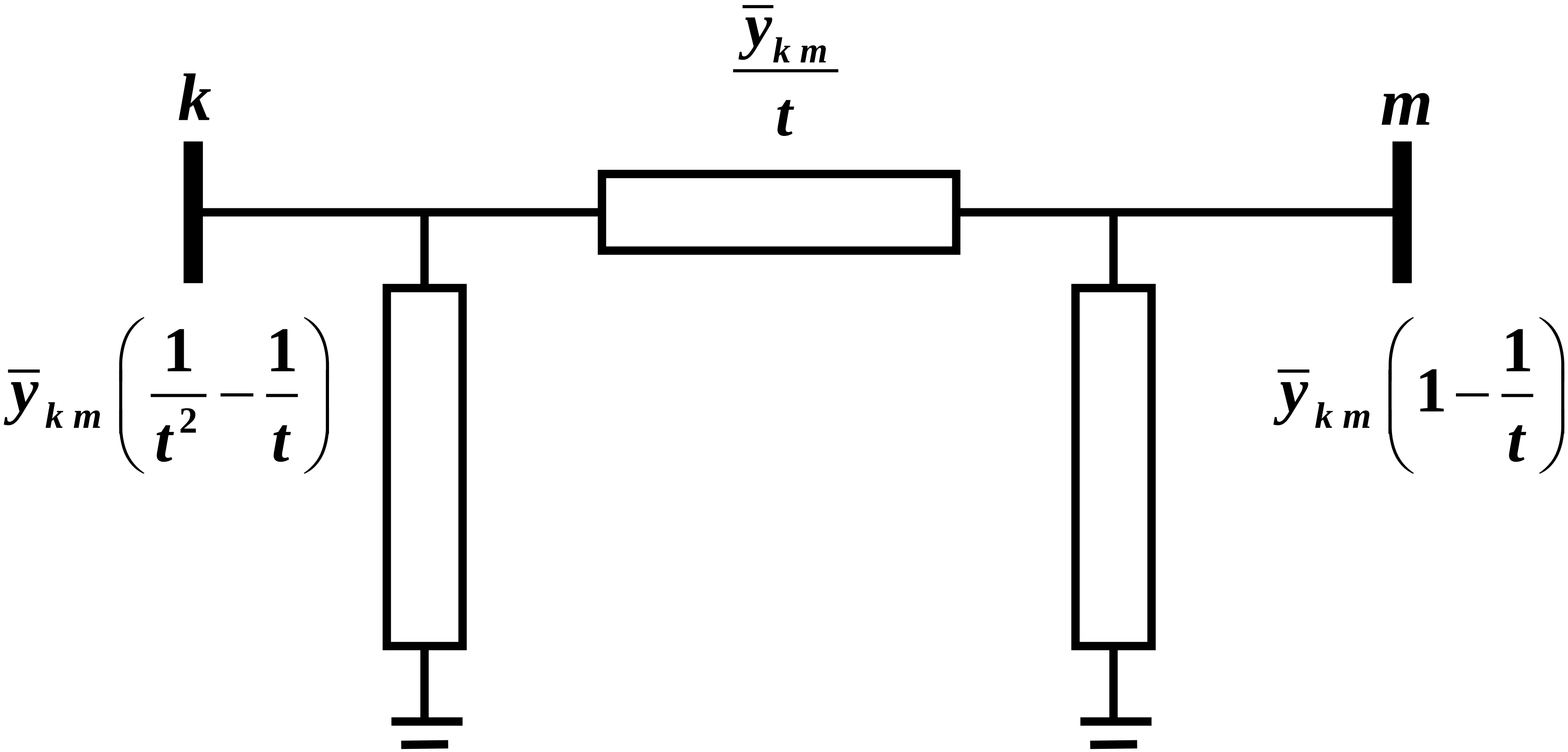
- We may rewrite these two equations as

$$\begin{bmatrix} \bar{I}_k \\ \bar{I}_m \end{bmatrix} = \begin{bmatrix} \dfrac{\bar{y}_{km}}{t^2} & -\dfrac{\bar{y}_{km}}{t} \\ -\dfrac{\bar{y}_{km}}{t} & \bar{y}_{km} \end{bmatrix} \begin{bmatrix} \bar{E}_k \\ \bar{E}_m \end{bmatrix}$$

**Y**$_{\text{bus}}$ is still symmetric here (though this will change with phase shifters)

This approach was first presented in F.L. Alvarado, "Formation of Y-Node using the Primitive Y-Node Concept," IEEE Trans. Power App. and Syst., December 1982

# The π-Equivalent Circuit for a Transformer Branch



# Variable Tap Voltage Control

- A transformer with a variable tap, i.e., the variable t is not constant, may be used to control the voltage at either the bus on the side of the tap or at the bus on the side away from the tap
- This constitutes an example of single criterion control since we adjust a single control variable (i.e., the transformer tap t) to achieve a specified criterion: the maintenance of a constant voltage at a designated bus
- Names for this type of control are on-load tap changer (LTC) transformer or tap changing under load (TCUL)
- Usually on low side; there may also be taps on high side that can be adjusted when it is de-energized

# Variable Tap Voltage Control

- An LTC is a discrete control, often with 32 incremental steps of 0.625% each, giving an automatic range of ± 10%
- It follows from the $\pi$–equivalent model for the transformer that the transfer admittance between the buses of the transformer branch and the contribution to the self admittance at the bus away from the tap explicitly depend on $t$
- However, the tap changes in discrete steps; there is also a built in time delay in how fast they respond
- Voltage regulators are devices with a unity nominal ratio, and then a similar tap range

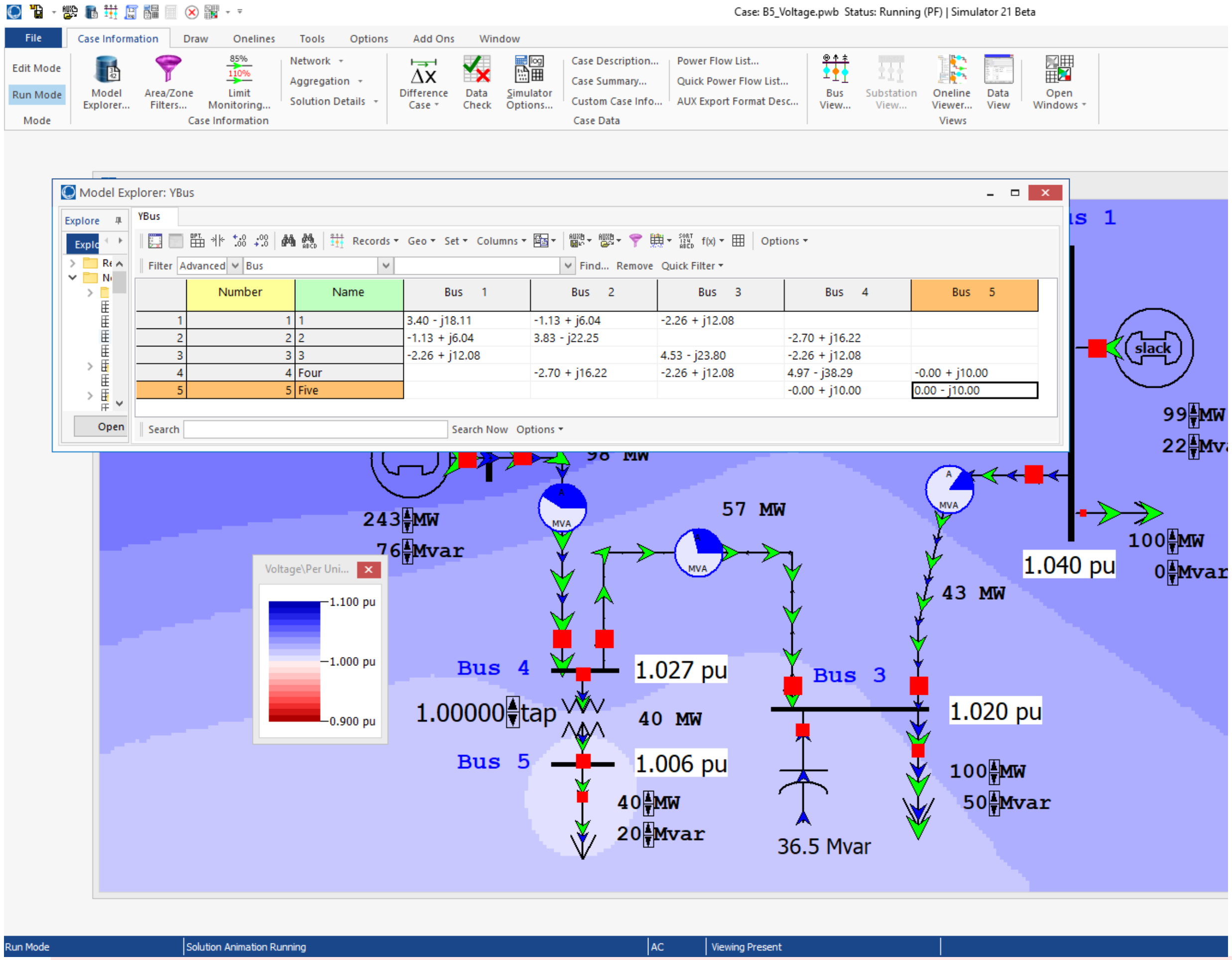
# Ameren Champaign (IL) Test Facility Voltage Regulators


These are connected on the low side of a 69/12.4 kV transformer; each phase can be regulated separately

# Variable Tap Voltage Control in the Power Flow

- LTCs (or voltage regulators) can be directly included in the power flow equations by modifying the $\mathbf{Y}_{\mathbf{bus}}$ entries; that is by scaling the terms by 1, 1/t or $1/t^2$ as appropriate
- If t is fixed then there is no change in the number of equations
- If t is variable, such as to enforce a voltage equality, then it can be included either by adding an additional equation and variable (t) directly, or by doing an "outer loop" calculation in which t is varied outside of the NR solution
  - The outer loop is used in PowerWorld because of limit issues

# Five Bus PowerWorld Example




With an impedance of j0.1 pu between buses 4 and 5, the y node primitive with t=1.0 is

$$\begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

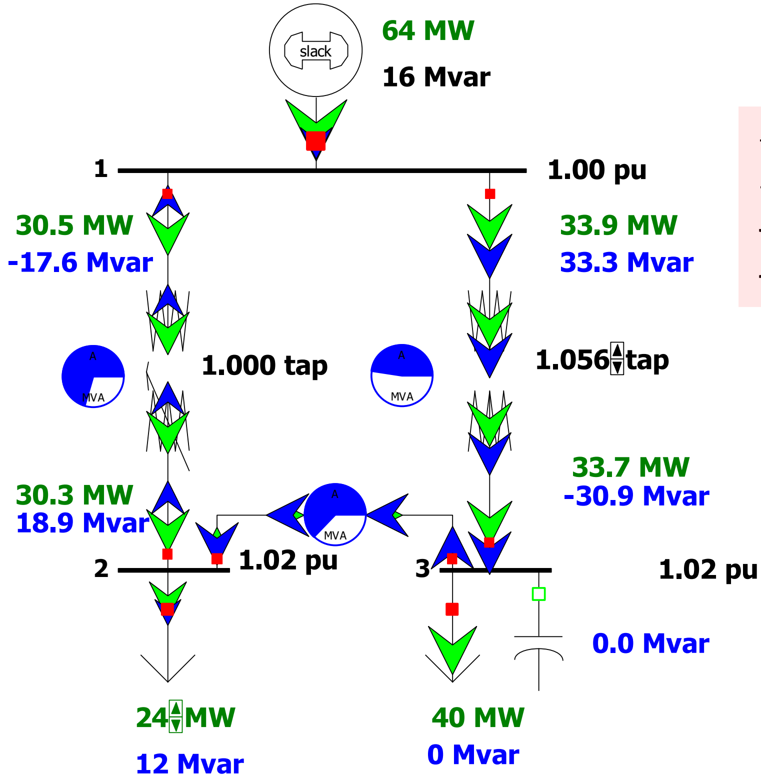
If t=1.1 then it is

$$\begin{bmatrix} -j10 & j9.09 \\ j9.09 & -j8.26 \end{bmatrix}$$

PowerWorld Case: **B5_Voltage**

# Circulating Reactive Power

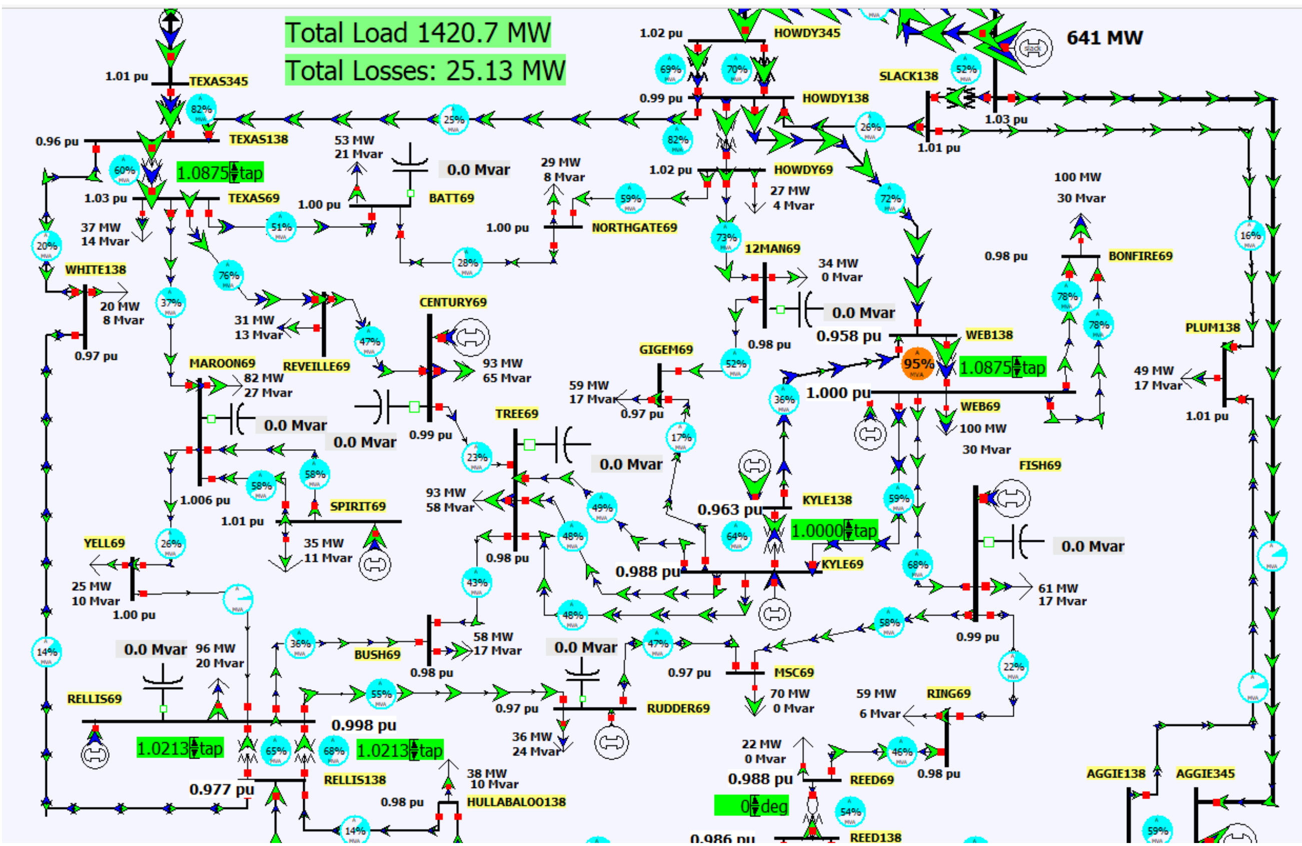
- Unbalanced transformer taps can cause large amounts of reactive power to circulating, increasing power system losses and overloading transformers




PowerWorld Case:
**Bus3CirculatingVars**

# LTC Tap Coordination

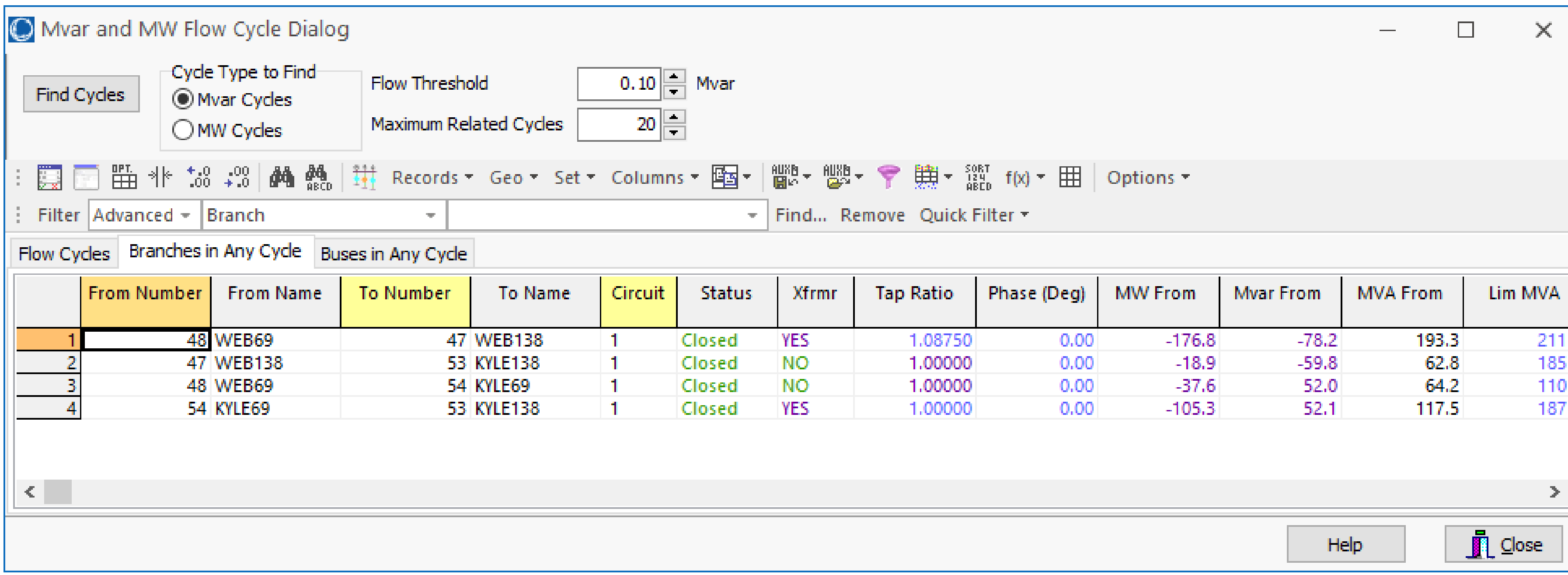
- Changing tap ratios can affect the voltages and var flow at nearby buses; hence coordinated control is needed




PowerWorld Case: **Aggieland37_LTC**

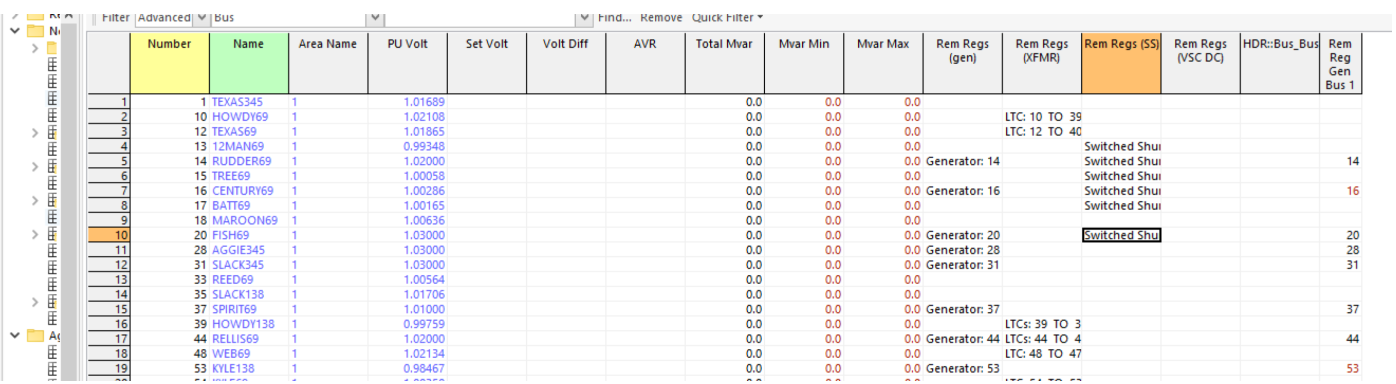
# Auto Detection of Circulating Reactive or Real Power

- Select **Tools, Connections, Find Circulating MW or Mvar Flows** to do an automatic determination of the circulating power in a case



# Coordinated Reactive Control

- A number of different devices may be doing automatic reactive power control. They must be considered in some control priority
  - One example would be 1) generator reactive power, 2) switched shunts, 3) LTCs
- You can see the active controls in PowerWorld with **Case Information, Solution Details, Remotely Regulated Buses**




| | Number | Name | Area Name | PU Volt | Set Volt | Volt Diff | AVR | Total Mvar | Mvar Min | Mvar Max | Rem Regs (gen) | Rem Regs (XFMR) | Rem Regs (SS) | Rem Regs (VSC DC) | HDR::Bus_Bus | Rem Reg Gen Bus 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | TEXAS345 | 1 | 1.01689 | | | | 0.0 | 0.0 | 0.0 | | | | | | |
| 2 | 10 | HOWDY69 | 1 | 1.02108 | | | | 0.0 | 0.0 | 0.0 | | LTC: 10 TO 39 | | | | |
| 3 | 12 | TEXAS69 | 1 | 1.01865 | | | | 0.0 | 0.0 | 0.0 | | LTC: 12 TO 40 | | | | |
| 4 | 13 | 12MAN69 | 1 | 0.99348 | | | | 0.0 | 0.0 | 0.0 | | | Switched Shu | | | |
| 5 | 14 | RUDDER69 | 1 | 1.02000 | | | | 0.0 | 0.0 | 0.0 | Generator: 14 | | Switched Shu | | | 14 |
| 6 | 15 | TREE69 | 1 | 1.00058 | | | | 0.0 | 0.0 | 0.0 | | | Switched Shu | | | |
| 7 | 16 | CENTURY69 | 1 | 1.00286 | | | | 0.0 | 0.0 | 0.0 | Generator: 16 | | Switched Shu | | | 16 |
| 8 | 17 | BATT69 | 1 | 1.00165 | | | | 0.0 | 0.0 | 0.0 | | | Switched Shu | | | |
| 9 | 18 | MAROON69 | 1 | 1.00636 | | | | 0.0 | 0.0 | 0.0 | | | | | | |
| 10 | 20 | FISH69 | 1 | 1.03000 | | | | 0.0 | 0.0 | 0.0 | Generator: 20 | | Switched Shu | | | 20 |
| 11 | 28 | AGGIE345 | 1 | 1.03000 | | | | 0.0 | 0.0 | 0.0 | Generator: 28 | | | | | 28 |
| 12 | 31 | SLACK345 | 1 | 1.03000 | | | | 0.0 | 0.0 | 0.0 | Generator: 31 | | | | | 31 |
| 13 | 33 | REED69 | 1 | 1.00564 | | | | 0.0 | 0.0 | 0.0 | | | | | | |
| 14 | 35 | SLACK138 | 1 | 1.01706 | | | | 0.0 | 0.0 | 0.0 | | | | | | |
| 15 | 37 | SPIRIT69 | 1 | 1.01000 | | | | 0.0 | 0.0 | 0.0 | Generator: 37 | | | | | 37 |
| 16 | 39 | HOWDY138 | 1 | 0.99759 | | | | 0.0 | 0.0 | 0.0 | | LTCs: 39 TO 3 | | | | |
| 17 | 44 | RELLIS69 | 1 | 1.02000 | | | | 0.0 | 0.0 | 0.0 | Generator: 44 | LTCs: 44 TO 4 | | | | 44 |
| 18 | 48 | WEB69 | 1 | 1.02134 | | | | 0.0 | 0.0 | 0.0 | | LTC: 48 TO 47 | | | | |
| 19 | 53 | KYLE138 | 1 | 0.98467 | | | | 0.0 | 0.0 | 0.0 | Generator: 53 | | | | | 53 |

# Coordinated Reactive Control

- The challenge with implementing tap control in the power flow is it is quite common for at least some of the taps to reach their limits
  - Keeping in mind a large case may have thousands of LTCs!
- If this control was directly included in the power flow equations then every time a limit was encountered the Jacobian would change
  - Also taps are discrete variables, so voltages must be a range
- Doing an outer loop control can more directly include the limit impacts; often time sensitivity values are used
- We’ll return to this once we discuss sparse matrices and sensitivity calculations

# Phase-Shifting Transformers

- Phase shifters are transformers in which the phase angle across the transformer can be varied in order to control real power flow
  - Sometimes they are called phase angle regulars (PAR)
  - Quadrature booster (evidently British though I've never heard this term)
- They are constructed by include a delta-connected winding that introduces a 90º phase shift that is added to the output voltage

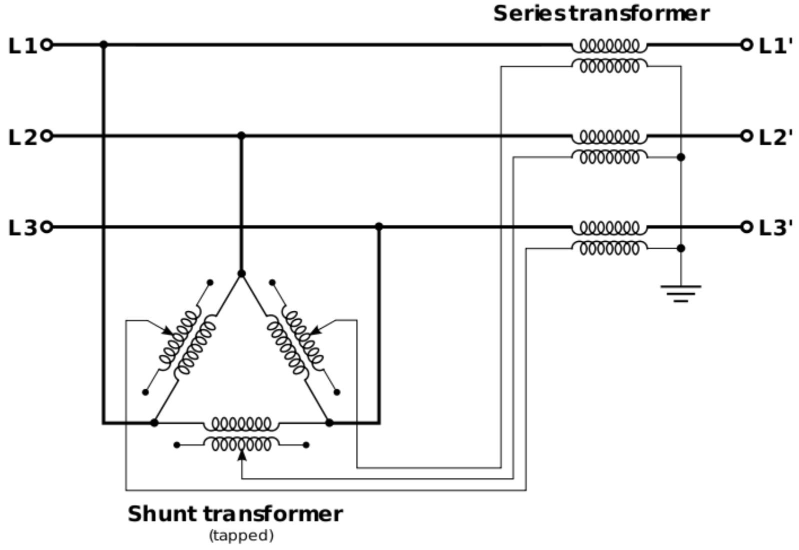



Image: http://en.wikipedia.org/wiki/Quadrature_booster

# Phase-Shifter Model

- We develop the mathematical model of a phase shifting transformer as a first step toward our study of its simulation
- Let buses *k* and *m* be the terminals of the phase–shifting transformer, then define the phase shift angle as $\Phi_{km}$
- The latter differs from an off–nominal turns ratio LTC transformer in that its tap ratio is a complex quantity, i.e., a complex number, $t_{km}\angle\Phi_{km}$
- The phase shift angle is a discrete value, with one degree a typical increment

# Phase-Shifter Model

- For a phase shifter located on the branch (*k*, *m*), the admittance matrix representation is obtained analogously to that for the LTC

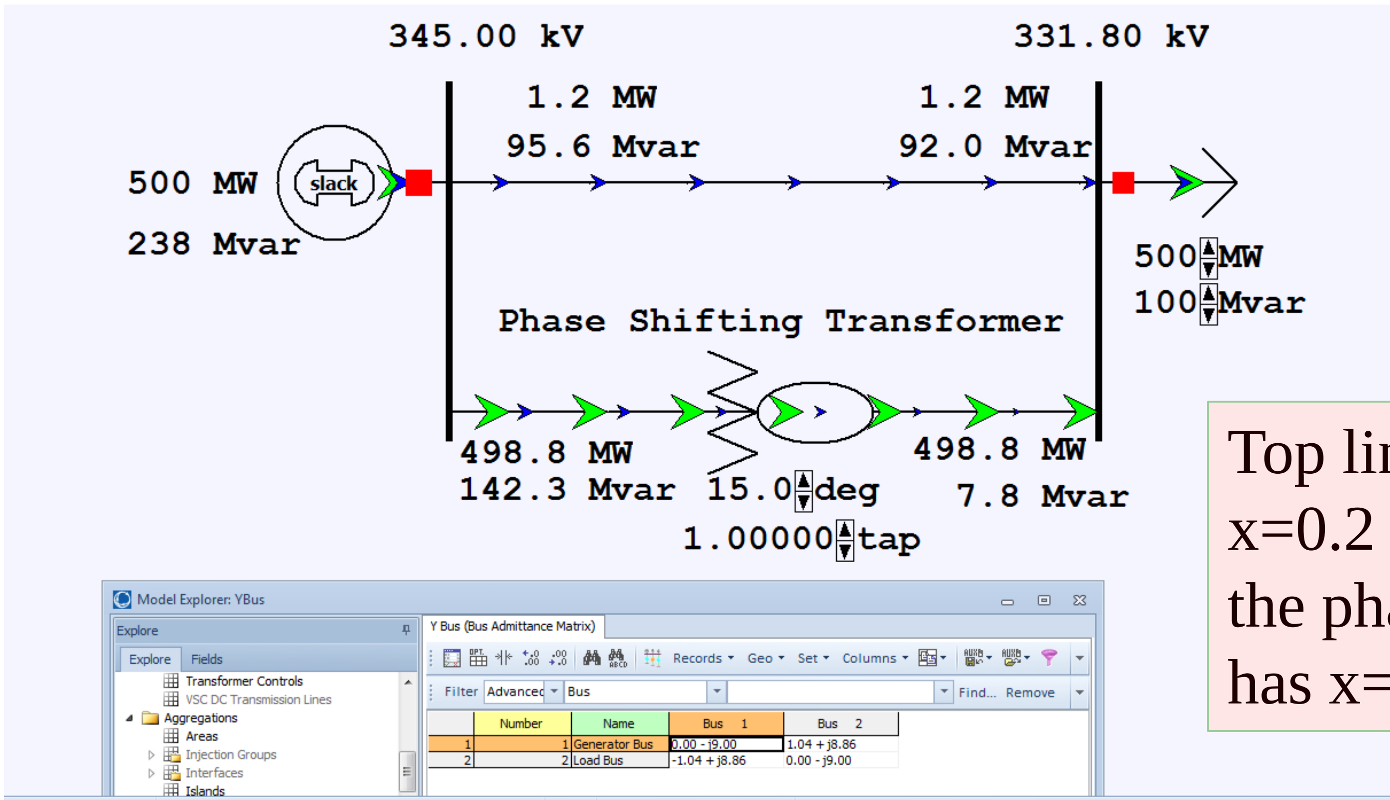
$$\begin{bmatrix} \bar{I}_k \\ \bar{I}_m \end{bmatrix} = \begin{bmatrix} \dfrac{\bar{y}_{km}}{t^2} & -\dfrac{\bar{y}_{km}}{te^{j\phi_{km}}} \\ -\dfrac{\bar{y}_{km}}{te^{-j\phi_{km}}} & \bar{y}_{km} \end{bmatrix} \begin{bmatrix} \bar{E}_k \\ \bar{E}_m \end{bmatrix}$$

- Note, if there is a phase shift then $\mathbf{Y}_{\text{bus}}$ is no longer symmetric!! In a large case there are almost always some phase shifters. Y-Δ transformers also introduce a phase shift that is often not modeled

# Integrated Phase-Shifter Control

- Phase shifters are usually used to control the real power flow on a device
- Similar to LTCs, phase-shifter control can either be directly integrated into the power flow equations (adding an equation for the real power flow equality constraint, and a variable for the phase shifter value), or they can be handled in with an outer loop approach
- As was the case with LTCs, limit enforcement often makes the outer loop approach preferred
- Coordinated control is needed when there are multiple, close by phase shifters

# Two Bus Phase Shifter Example



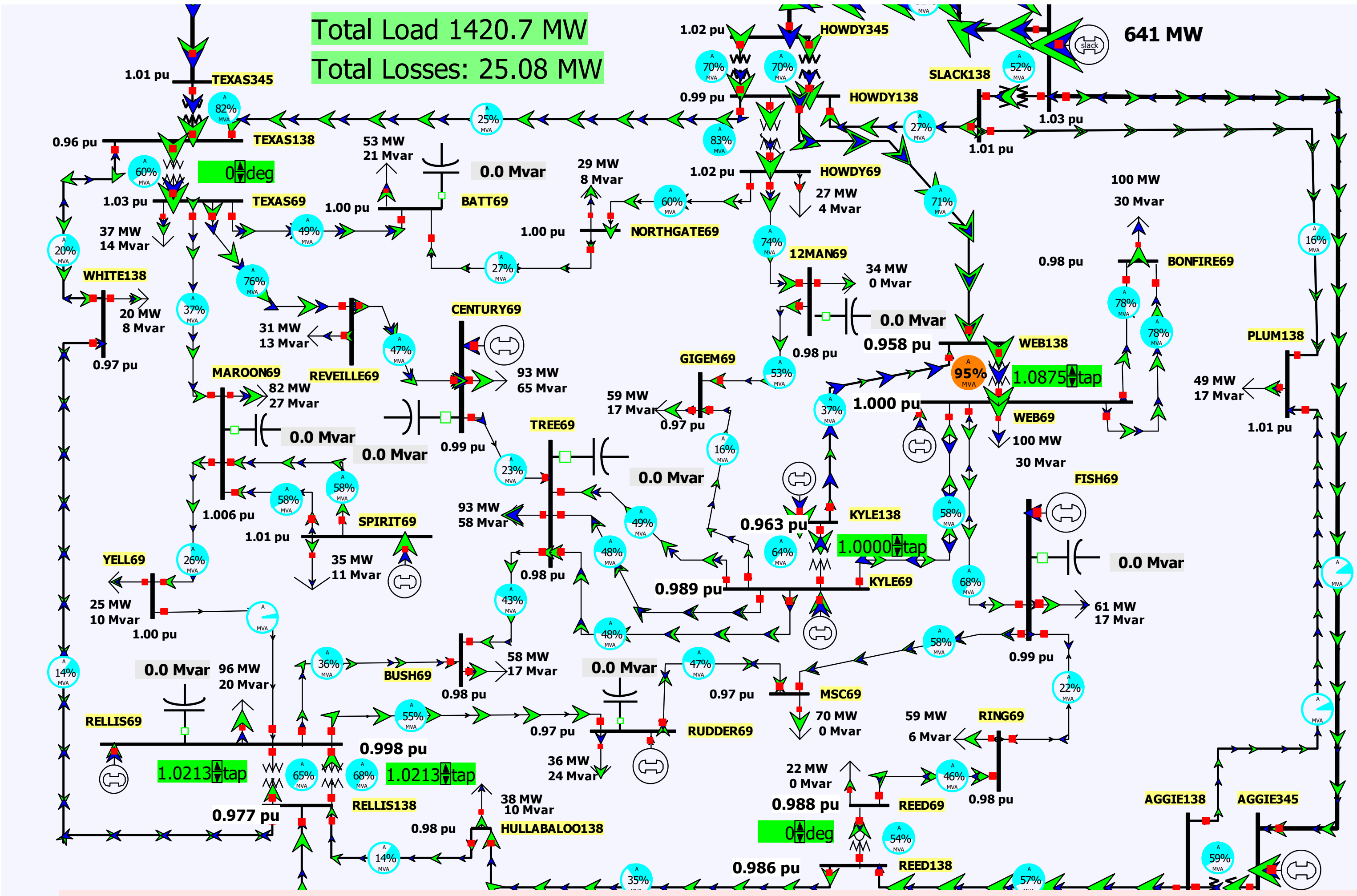

Top line has x=0.2 pu, while the phase shifter has x=0.25 pu.

$$\mathbf{Y}_{12} = -\frac{1}{j0.2} - \frac{1}{j0.25}\left(\cos\left(-15^\circ\right) + j\sin\left(-15^\circ\right)\right) = j5 + (j4)(0.966 - j0.259)$$

$$\mathbf{Y}_{12} = 1.036 + j8.864$$

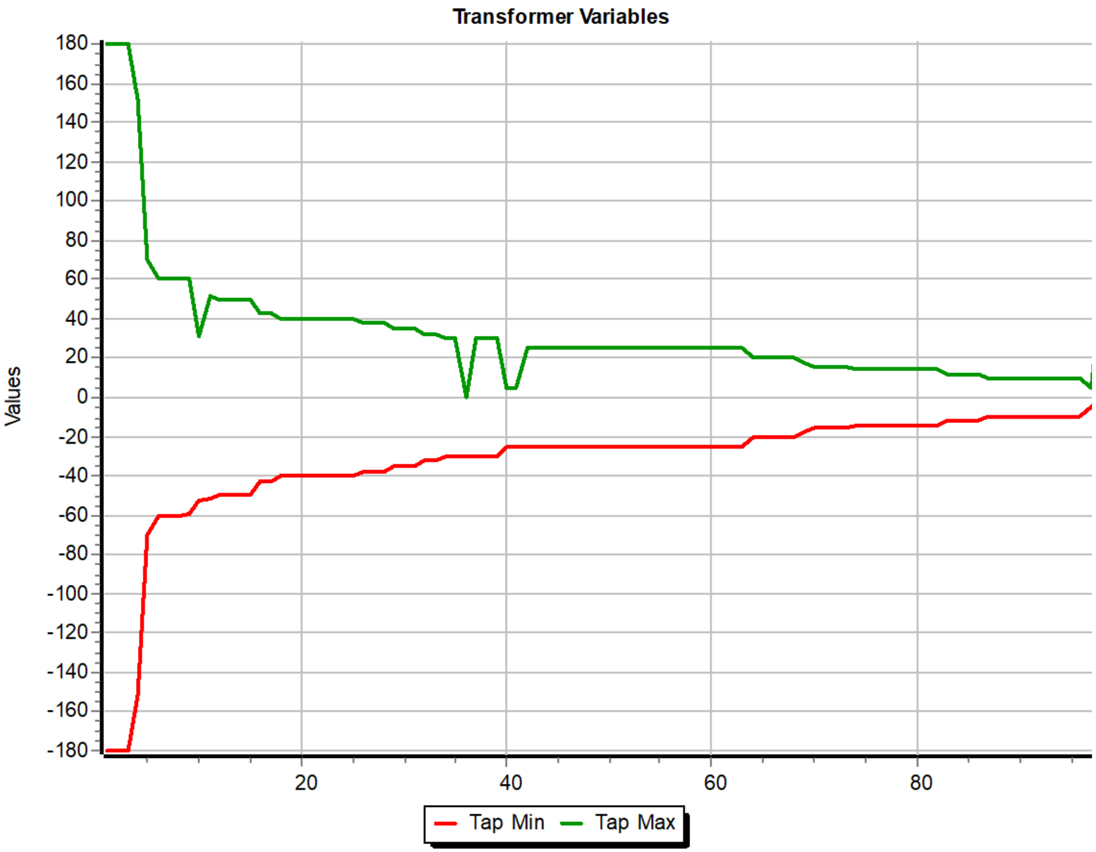
PowerWorld Case: **B2PhaseShifter**

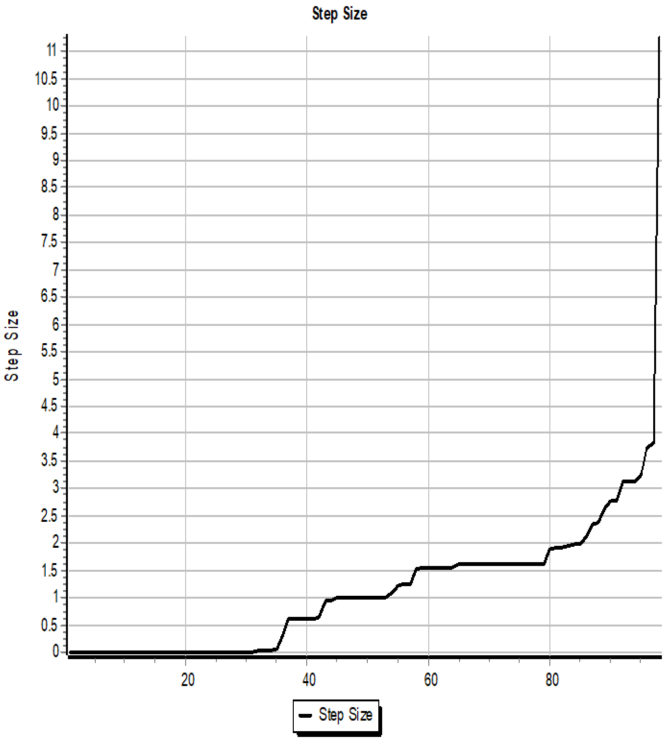
# Aggieland37 With Phase Shifters




PowerWorld Case: **Aggieland37_PhaseShifter**

# Large Case Phase Shifter Limits and Step Size



# Example of Phase Shifters in Practice

- The below report mentions issues associated with the Ontario-Michigan PARs

**Ontario-Michigan Interface**

LEC flow is affected by several factors including PARs in multiple locations around Lake Erie (*see Figure 1*). This report considered data only for PARs on the Ontario-Michigan interface.


Figure 1 – PAR Locations Which Impact Lake Erie Circulation Flow

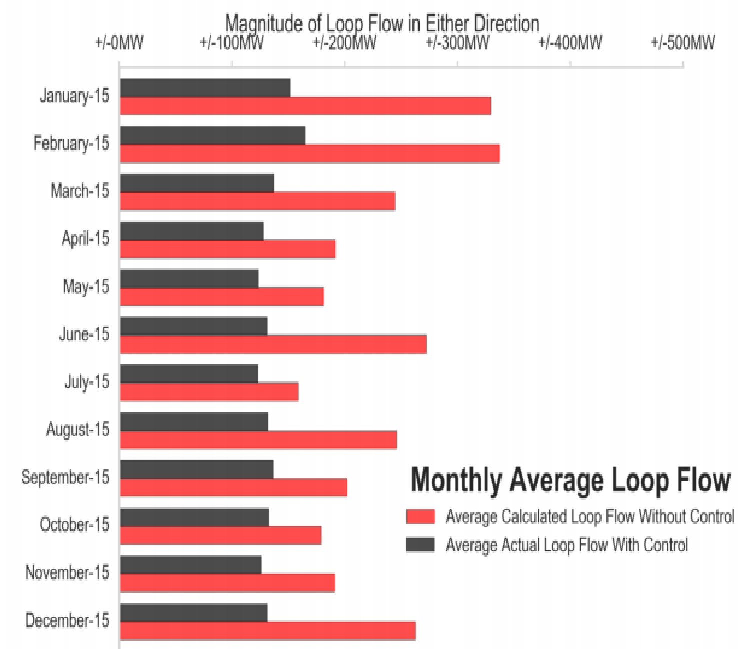



Figure 6 - Monthly Average Loop Flow

https://www.nyiso.com/public/webdocs/markets_operations/committees/bic_miwg/meeting_materials/2017-02-28/2016%20Ontario-Michigan%20Interface%20PAR%20Evaluation%20Final%20Report.pdf