ECEN 667
Power System Stability

Lecture 6: Synchronous Machine Modeling

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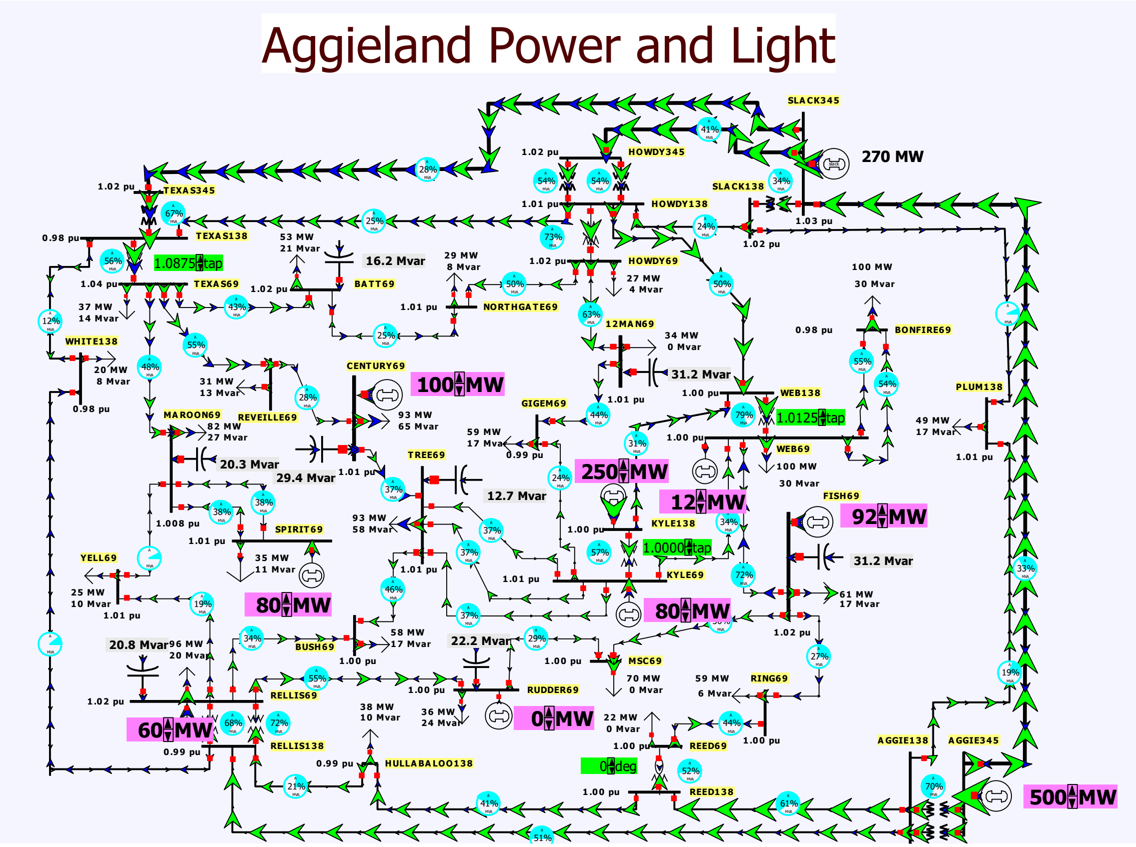
Announcements

- Read Chapter 3
- Homework 1 is due today
- Homework 2 is due on Thursday September 19

37 Bus System

- Next we consider a slightly larger, ten generator, 37 bus system. To view this system open case **AGL37_TS**. The system one-line is shown below.



To see summary listings of the transient stability models in this case select “Stability Case Info” from the ribbon, and then either “TS Generator Summary” or “TS Case Summary”

Transient Stability Case and Model Summary Displays

Models in Use | Generators | Load Characteristics | Load Summary

Records ▾ Set ▾ Columns ▾ f(x) ▾

Filter Advanced ▾ TSModelSummaryObject ▾ Find... Remove Quic

	Model Class	Object Type	Active and Online Count	Active Count	Inactive Count	Fully Supported
1	Machine Model	GENSAL	1	1	0	YES
2	Machine Model	GENROU	9	9	0	YES
3	Exciter	IEEET1	10	10	0	YES
4	Governor	TGOV1	10	10	0	YES

Right click on a line and select "Show Dialog" for more information.

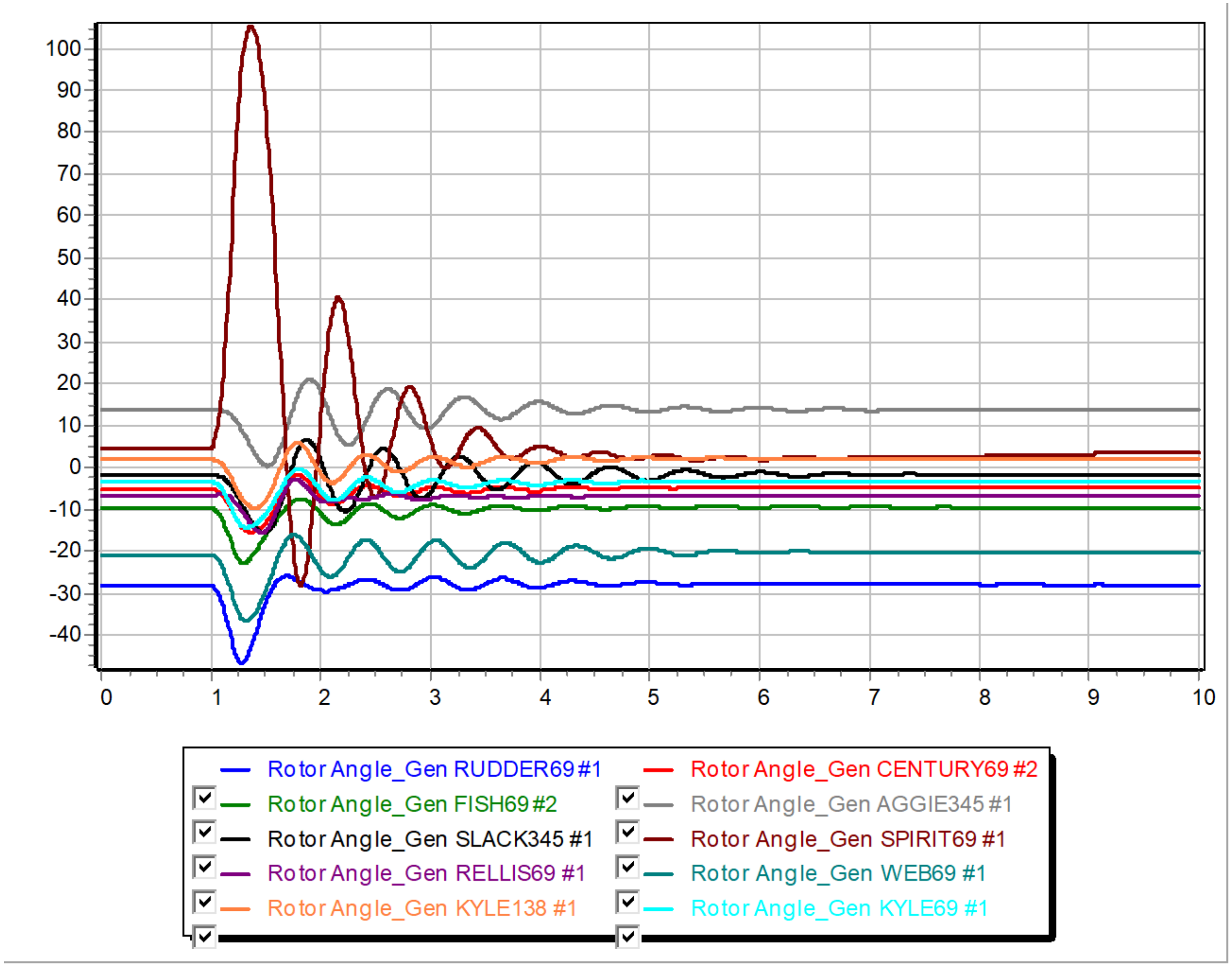
Generator Model Use | Model Summary | Generators | Load Characteristics | Load Summary

Records ▾ Geo ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

Filter Advanced ▾ Generator ▾ Find... Remove Quick Filter ▾

	Number of Bus	Name of Bus	ID	Status	Gen MW	MVA Base	Machine	Exciter	Governor	Stabilizer	Other Model	Governor Response Limits	H (system base)	TS Rcom (system b
1	14	RUDDER69	1	Closed	0.00	50.00	GENROU	IEEET1	TGOV1			Normal	1.50000	0.0
2	16	CENTURY69	2	Closed	100.00	120.00	GENROU	IEEET1	TGOV1			Normal	3.60000	0.0
3	20	FISH69	2	Closed	91.75	130.00	GENROU	IEEET1	TGOV1			Normal	3.90000	0.0
4	28	AGGIE345	1	Closed	500.00	600.00	GENROU	IEEET1	TGOV1			Normal	36.00000	0.0
5	31	SLACK345	1	Closed	270.20	600.00	GENROU	IEEET1	TGOV1			Normal	36.00000	0.0
6	37	SPIRIT69	1	Closed	80.00	90.00	GENSAL	IEEET1	TGOV1			Normal	2.70000	0.0
7	44	RELLIS69	1	Closed	60.00	80.00	GENROU	IEEET1	TGOV1			Normal	2.40000	0.0
8	48	WEB69	1	Closed	12.30	80.00	GENROU	IEEET1	TGOV1			Normal	2.40000	0.0
9	53	KYLE138	1	Closed	250.00	300.00	GENROU	IEEET1	TGOV1			Normal	9.00000	0.0
10	54	KYLE69	1	Closed	80.00	100.00	GENROU	IEEET1	TGOV1			Normal	3.00000	0.0

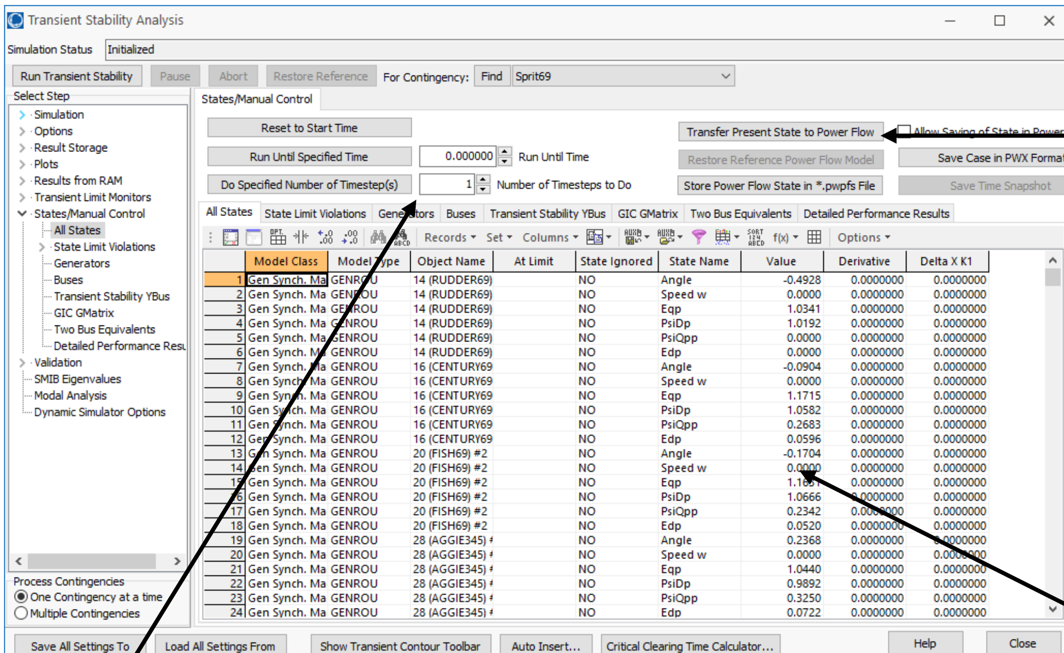
37 Bus Case Solution



Graph shows the rotor angles following a line fault

Stepping Through a Solution

- Simulator provides functionality to make it easy to see what is occurring during a solution. This functionality is accessed on the States/Manual Control Page

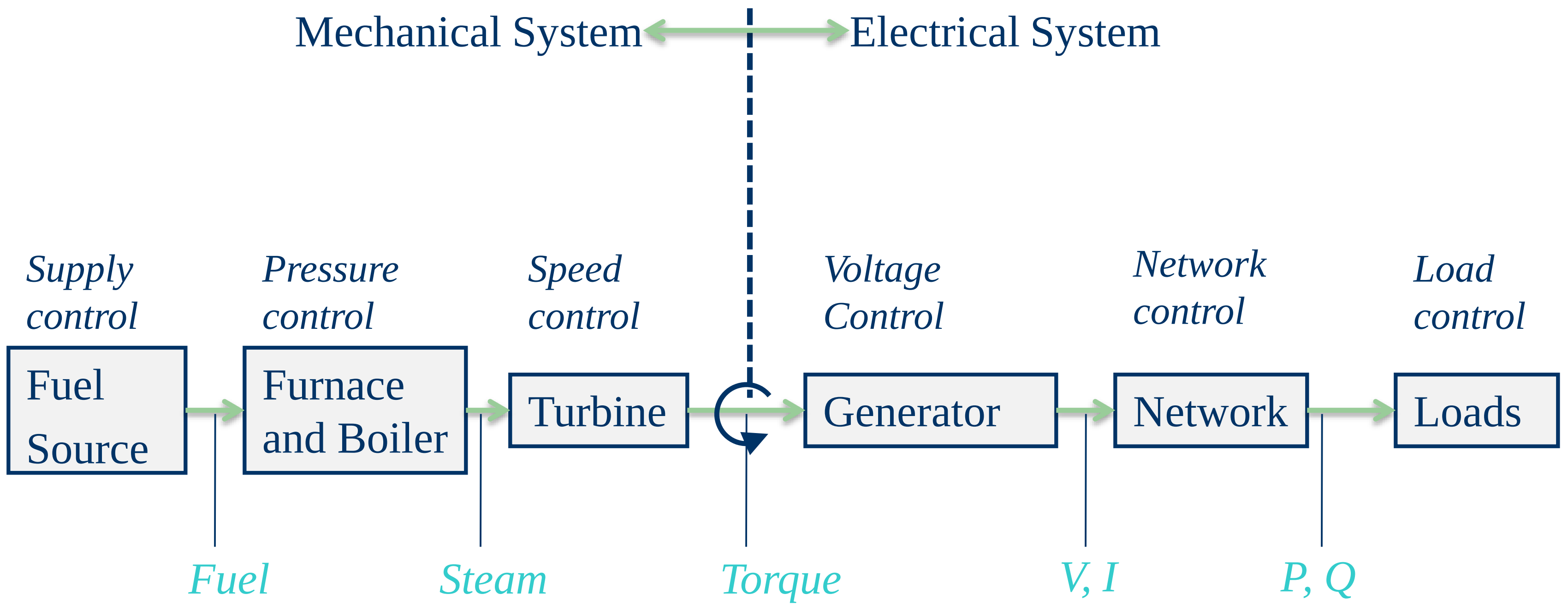


Transfer results to Power Flow to view using standard PowerWorld displays and one-lines

Run a Specified Number of Timesteps or Run Until a Specified Time, then Pause.

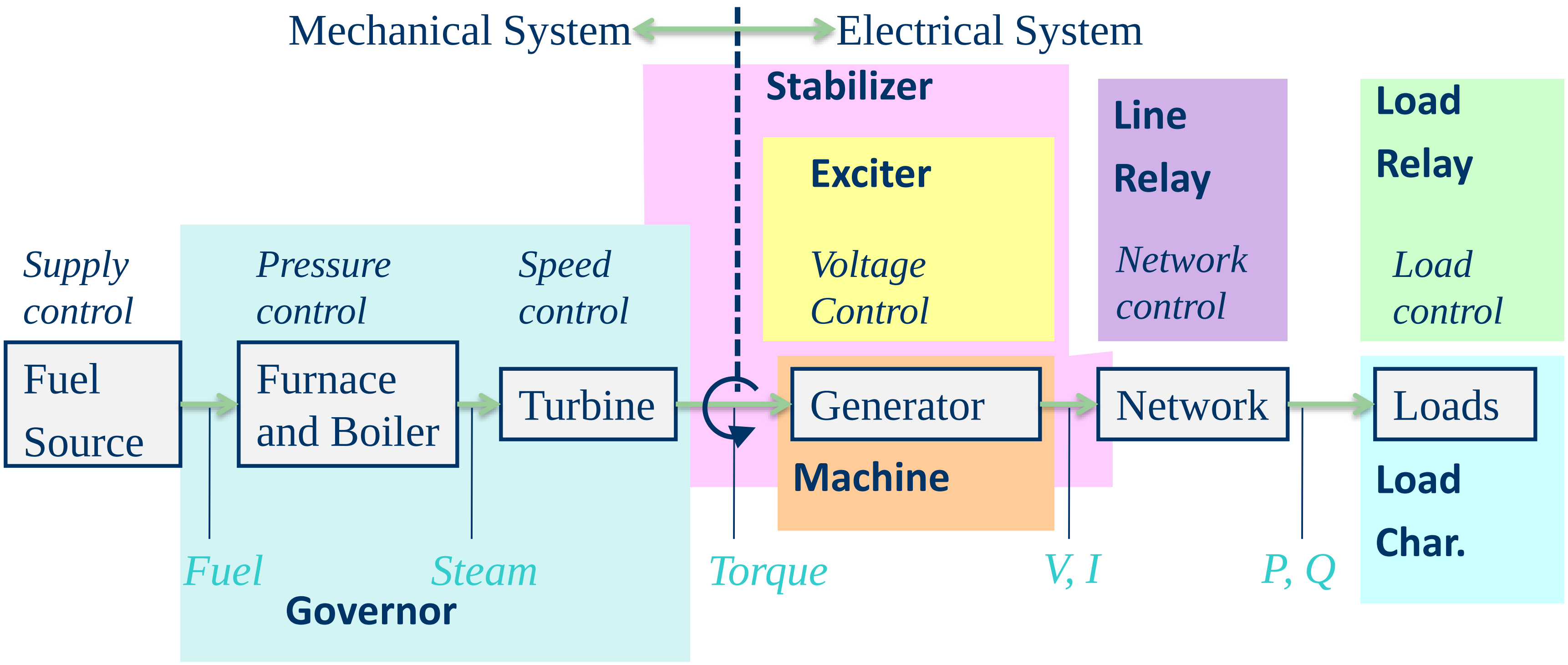
See detailed results at the Paused Time

Physical Structure
Power System Components



P. Sauer and M. Pai, *Power System Dynamics and Stability*, Stipes Publishing, 2006.

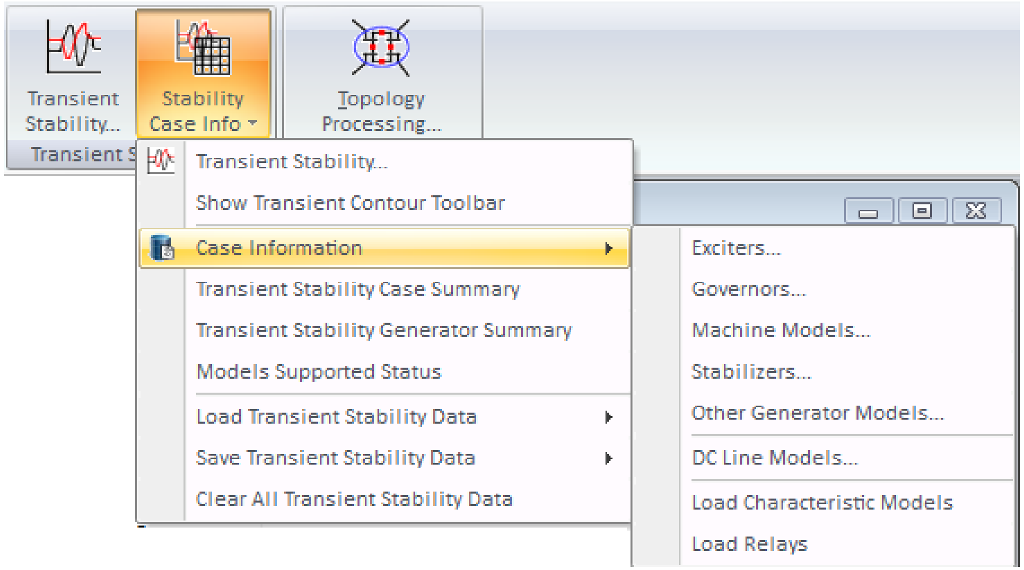
Dynamic Models in the Physical Structure



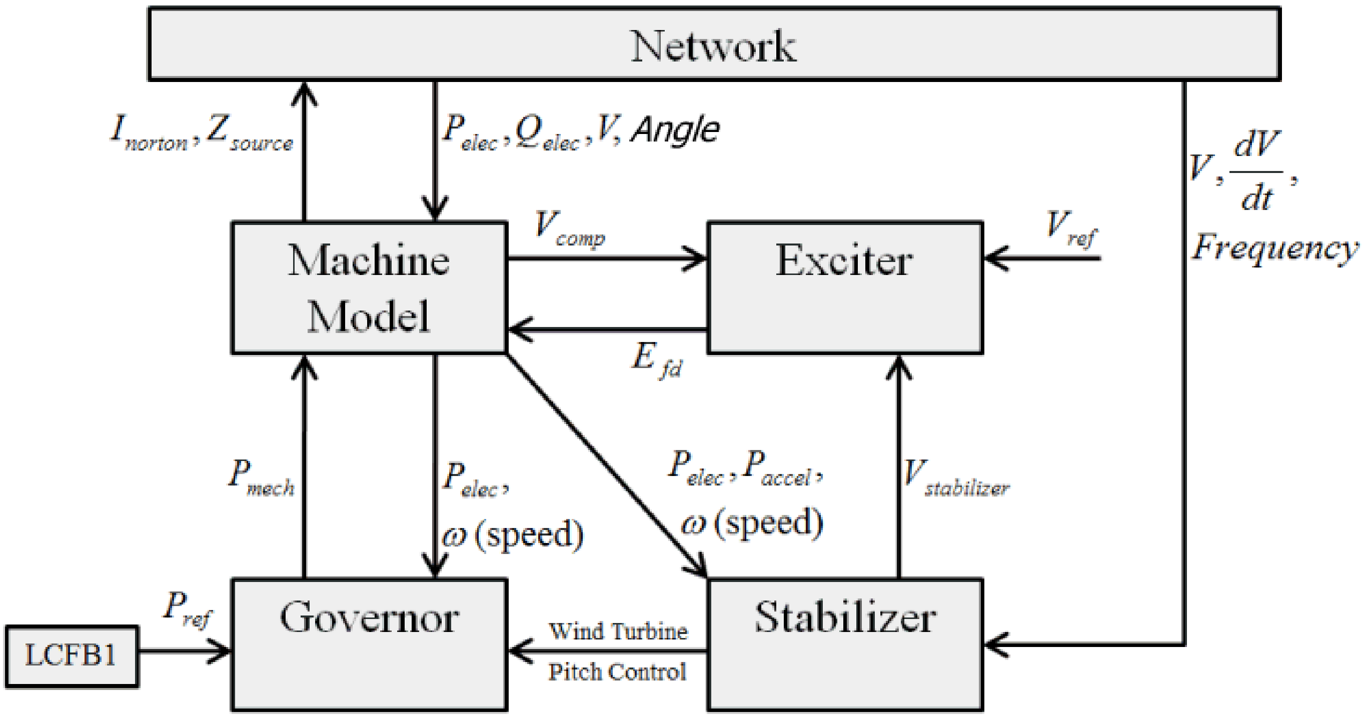
P. Sauer and M. Pai, *Power System Dynamics and Stability*, Stipes Publishing, 2006.

Generator Models

- Generators can have several classes of models assigned to them
  - Machine Models
  - Exciter
  - Governors
  - Stabilizers
- Others also available
  - Excitation limiters, voltage compensation, turbine load controllers, and generator relay model



Generator Models



P_{elec} = Electrical Power

Q_{elec} = Electrical Reactive Power

V = Voltage at Terminal Bus

$\frac{dV}{dt}$ = Derivate of Voltage

V_{comp} = Compensated Voltage

P_{mech} = Mechanical Power

ω(speed) = Rotor Speed (often it's deviation from nominal speed)

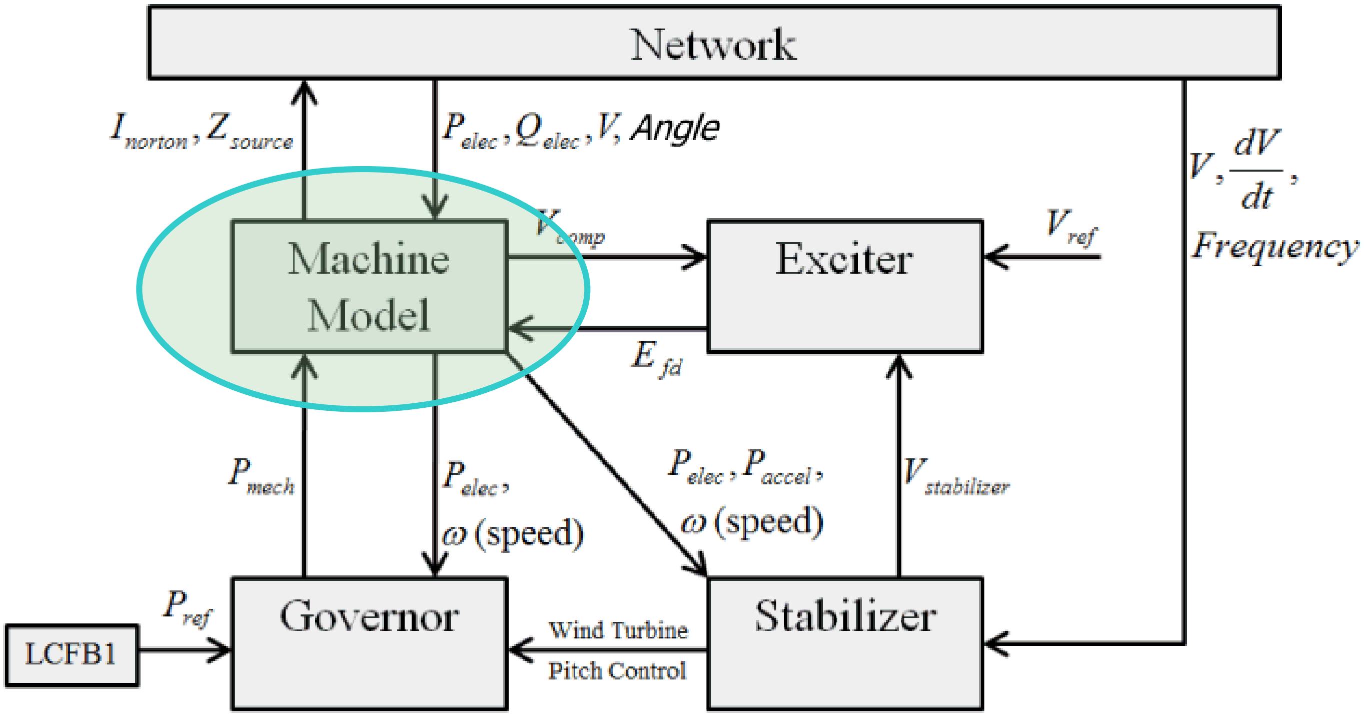
P_{accel} = Accelerating Power

$V_{stabilizer}$ = Output of Stabilizer

V_{ref} = Exciter Control Setpoint (determined during initialization)

P_{ref} = Governor Control Setpoint (determined during initialization)

Machine Models



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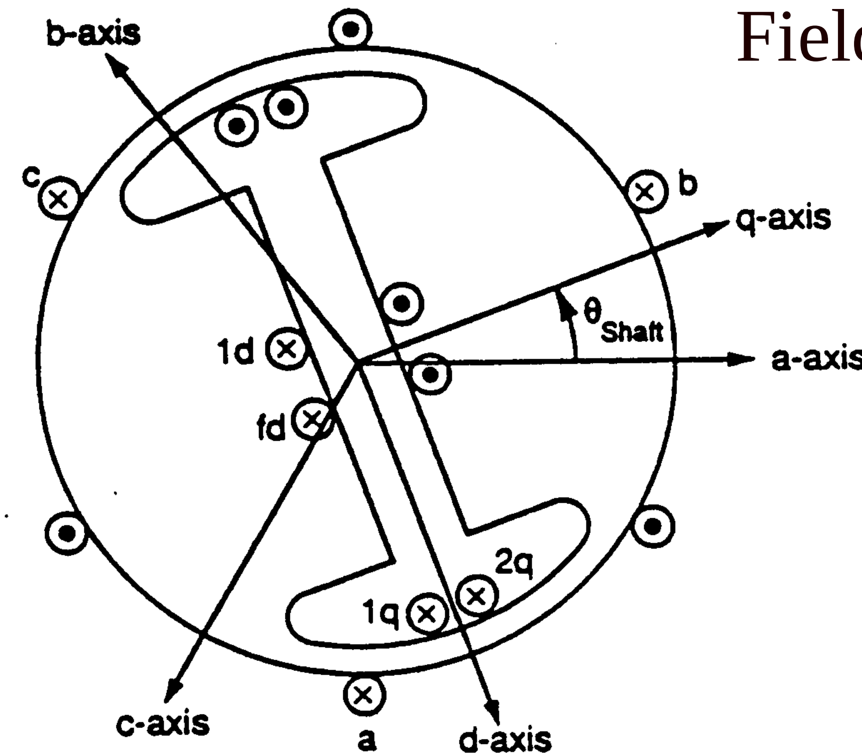
Synchronous Machine Modeling

- Electric machines are used to convert mechanical energy into electrical energy (generators) and from electrical energy into mechanical energy (motors)
  - Many devices can operate in either mode, but are usually customized for one or the other
- Vast majority of electricity is generated using synchronous generators and some is consumed using synchronous motors, so we'll start there
- There is much literature on subject, and sometimes it is overly confusing with the use of different conventions and nomenclature

Synchronous Machine Modeling

3ϕ bal. windings (a,b,c) – stator



Field winding (fd) on rotor

Damper in "d" axis (1d) on rotor

Two dampers in "q" axis (1q, 2q) on rotor

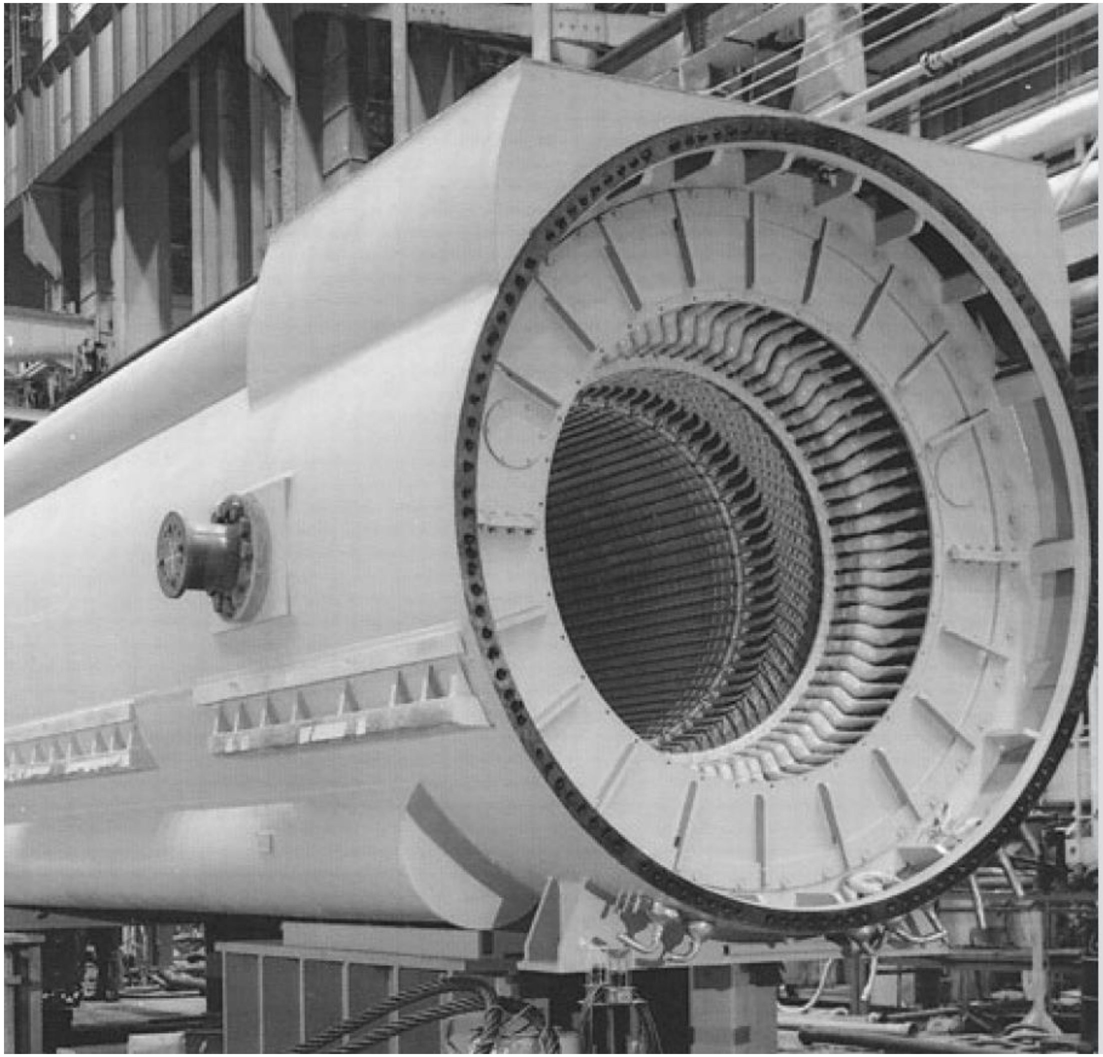
Two Main Types of Synchronous Machines

- Round Rotor
  - Air-gap is constant, used with higher speed machines
- Salient Rotor (often called Salient Pole)
  - Air-gap varies circumferentially
  - Used with many pole, slower machines such as hydro
  - Narrowest part of gap in the d-axis and the widest along the q-axis

Dq0 Reference Frame

- Stator is stationary, rotor is rotating at synchronous speed
- Rotor values need to be transformed to fixed reference frame for analysis
- Done using Park's transformation into what is known as the dq0 reference frame (direct, quadrature, zero)
  - Parks' 1929 paper voted 2nd most important power paper of 20th century (1st was Fortescue's sym. components)
- Convention used here is the q-axis leads the d-axis (which is the IEEE standard)

Synchronous Machine Stator

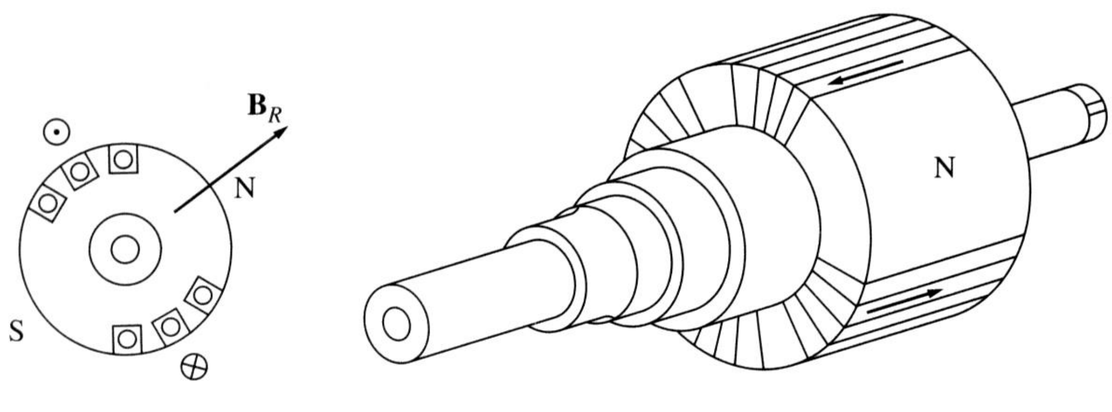


Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric.)

Image Source: Glover/Overbye/Sarma Book, Sixth Edition, Beginning of Chapter 8 Photo

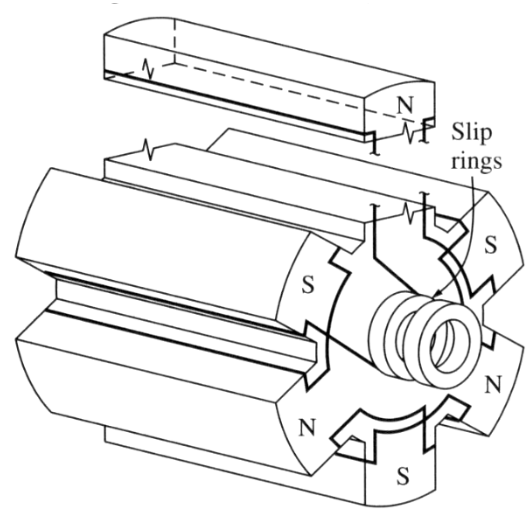
Synchronous Machine Rotors

- Rotors are essentially electromagnets



End view

Side view



Two pole (P) round rotor

Six pole salient rotor

Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Synchronous Machine Rotor

High pole salient rotor

Shaft

Part of exciter, which is used to control the field current

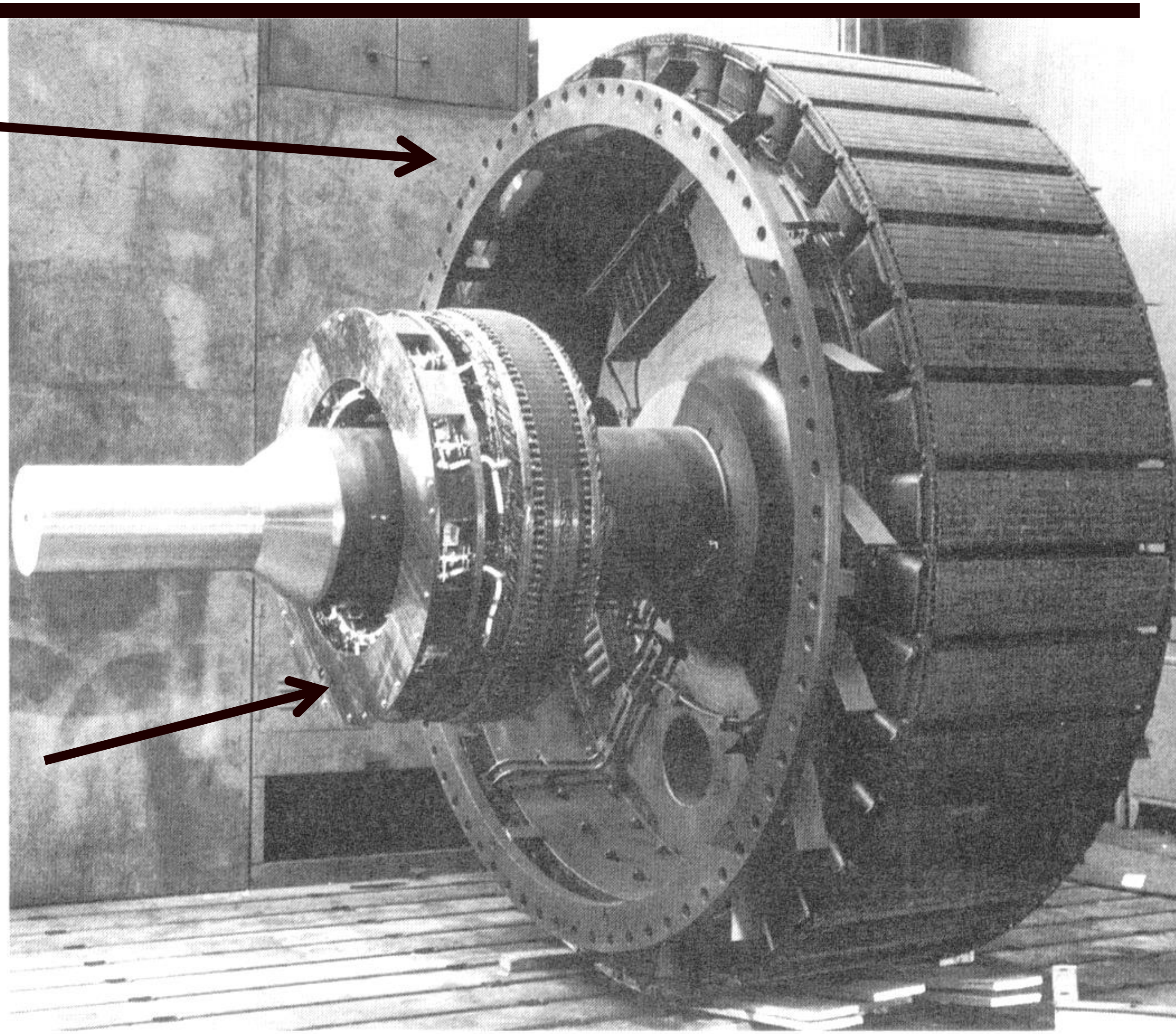


Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Fundamental Laws

- Kirchhoff's Voltage Law, Ohm's Law, Faraday's Law, Newton's Second Law

Stator

$$v_a = i_a r_s + \frac{d\lambda_a}{dt}$$

$$v_b = i_b r_s + \frac{d\lambda_b}{dt}$$

$$v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

Rotor

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{\text{shaft}}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

Dq0 Transformations

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \triangleq T_{dqo} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \text{or} \quad i, \lambda$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = T_{dqo}^{-1} \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix}$$

In the next few slides we'll quickly go through how these basic equations are transformed into the standard machine models. The point is to show the physical basis for the models. And there is NO quiz at the end!!

Dq0 Transformations

$$T_{dqo} \triangleq \frac{2}{3}\begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \cos\frac{P}{2}\theta_{shaft} & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

with the inverse,

$$T_{dqo}^{-1} = \begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \cos\frac{P}{2}\theta_{shaft} & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

Note that the transformation depends on the shaft angle.

Transformed System

Stator

$$v_d = r_s i_d - \omega \lambda_q + \frac{d\lambda_d}{dt}$$

$$v_q = r_s i_q + \omega \lambda_d + \frac{d\lambda_q}{dt}$$

$$v_o = r_s i_o + \frac{d\lambda_o}{dt}$$

Rotor

$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = r_{1d} i_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = r_{1q} i_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = r_{2q} i_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega$$

$$J \frac{2}{P} \frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

Electrical & Mechanical Relationships

Electrical system:

$$v = iR + \frac{d\lambda}{dt} \quad \text{(voltage)}$$

$$vi = i^2 R + i\frac{d\lambda}{dt} \quad \text{(power)}$$

Mechanical system:

$$J\left(\frac{2}{P}\right)\frac{d\omega}{dt} = T_m - T_e - T_{fw} \quad \text{(torque)}$$

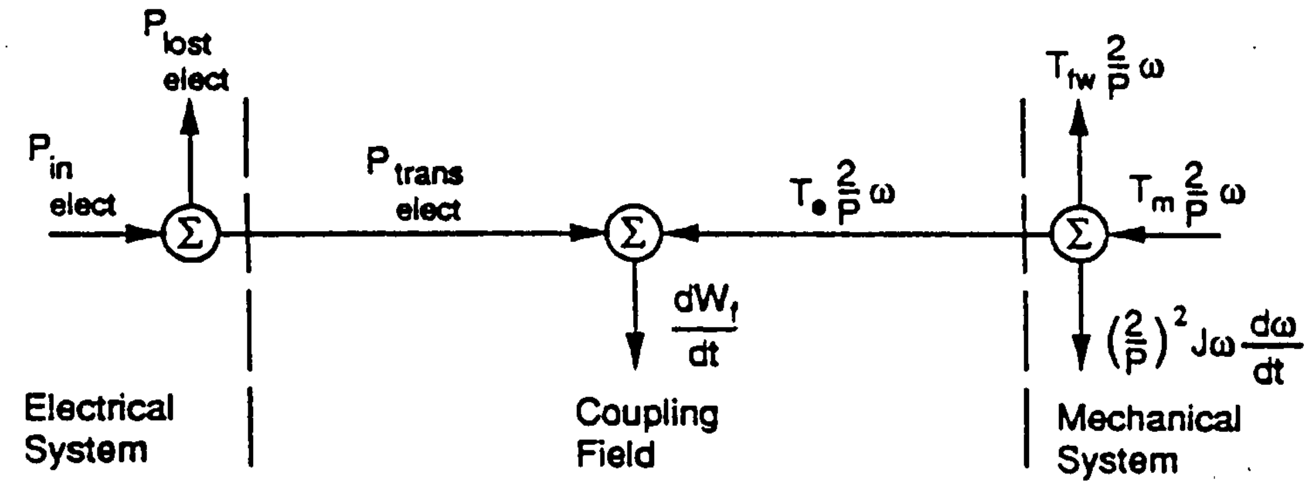
$$J\left(\frac{2}{P}\right)^2 \omega\frac{d\omega}{dt} = \frac{2}{P}\omega T_m - \frac{2}{P}\omega T_e - \frac{2}{P}\omega T_{fw} \quad \text{(power)}$$

P is the number of poles (e.g., 2,4,6); Tfw is the friction and windage torque

Torque Derivation

- Torque is derived by looking at the overall energy balance in the system
- Three systems: electrical, mechanical and the coupling magnetic field
  - Electrical system losses are in the form of resistance
  - Mechanical system losses are in the form of friction
- Coupling field is assumed to be lossless, hence we can track how energy moves between the electrical and mechanical systems

Energy Conversion



Look at the instantaneous power:

$$v_a i_a + v_b i_b + v_c i_c = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o$$

Change to Conservation of Power

$$P_{in\ elect} = v_a i_a + v_b i_b + v_c i_c + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q}$$

$$+ v_{2q} i_{2q}$$

$$P_{lost\ elect} = r_s \left(i_a^2 + i_b^2 + i_c^2 \right) + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

$$P_{trans\ elect} = i_a \frac{d\lambda_a}{dt} + i_b \frac{d\lambda_b}{dt} + i_c \frac{d\lambda_c}{dt} + i_{fd} \frac{d\lambda_{fd}}{dt} + i_{1d} \frac{d\lambda_{1d}}{dt}$$

$$+ i_{1q} \frac{d\lambda_{1q}}{dt} + i_{2q} \frac{d\lambda_{2q}}{dt}$$

We are using
v = dλ/dt here

With the Transformed Variables

$$P_{\substack{in\\elect}} = \frac{3}{2}v_d i_d + \frac{3}{2}v_q i_q + 3v_o i_o + v_{fd} i_{fd} + v_{1d} i_{1d}$$

$$+ v_{1q} i_{1q} + v_{2q} i_{2q}$$

$$P_{\substack{lost\\elect}} = \frac{3}{2}r_s i_d^2 + \frac{3}{2}r_s i_q^2 + 3r_s i_o^2 + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2$$

$$+ r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

With the Transformed Variables

$$P_{\substack{trans\\elect}} = -\frac{3}{2}\frac{P}{2}\frac{d\theta_{shaft}}{dt}\lambda_q i_d + \frac{3}{2} i_d \frac{d\lambda_d}{dt} + \frac{3}{2}\frac{P}{2}\frac{d\theta_{shaft}}{dt}\lambda_d i_q$$

$$+\frac{3}{2} i_q \frac{d\lambda_q}{dt} + 3 i_o \frac{d\lambda_o}{dt} + i_{fd}\frac{d\lambda_{fd}}{dt} + i_{1d}\frac{d\lambda_{1d}}{dt}$$

$$+ i_{1q}\frac{d\lambda_{1q}}{dt} + i_{2q}\frac{d\lambda_{2q}}{dt}$$

Change in Coupling Field Energy

$$\frac{dW_f}{dt} = \boxed{T_e \frac{2}{P}} \frac{d\theta}{dt} + \boxed{i_a} \frac{d\lambda_a}{dt} + \boxed{i_b} \frac{d\lambda_b}{dt}$$

$$+ \boxed{i_c} \frac{d\lambda_c}{dt} + \boxed{i_{fd}} \frac{d\lambda_{fd}}{dt} + \boxed{i_{1d}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{i_{1q}} \frac{d\lambda_{1q}}{dt} + \boxed{i_{2q}} \frac{d\lambda_{2q}}{dt}$$

First term on right is what is going on mechanically, other terms are what is going on electrically

This requires the lossless coupling field assumption

Change in Coupling Field Energy

For independent states $\theta, \lambda_a, \lambda_b, \lambda_c, \lambda_{fd}, \lambda_{1d}, \lambda_{1q}, \lambda_{2q}$

$$\frac{dW_f}{dt} = \boxed{\frac{\partial W_f}{\partial \theta}} \frac{d\theta}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_a}} \frac{d\lambda_a}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_b}} \frac{d\lambda_b}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_c}} \frac{d\lambda_c}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{fd}}} \frac{d\lambda_{fd}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{1d}}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_{1q}}} \frac{d\lambda_{1q}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{2q}}} \frac{d\lambda_{2q}}{dt}$$

Equate the Coefficients

$$T_e \frac{2}{P} = \frac{\partial W_f}{\partial \theta} \qquad i_a = \frac{\partial W_f}{\partial \lambda_a} \qquad \text{etc.}$$

There are eight such “reciprocity conditions for this model.

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

Equate the Coefficients

$$\frac{\partial W_f}{\partial \theta_{shaft}} = \frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right) + T_e$$

$$\frac{\partial W_f}{\partial \lambda_d} = \frac{3}{2} i_d \,, \qquad \frac{\partial W_f}{\partial \lambda_q} = \frac{3}{2} i_q, \qquad \frac{\partial W_f}{\partial \lambda_o} = 3 i_o$$

$$\frac{\partial W_f}{\partial \lambda_{fd}} = i_{fd} \,, \qquad \frac{\partial W_f}{\partial \lambda_{1d}} = i_{1d}, \qquad \frac{\partial W_f}{\partial \lambda_{1q}} = i_{1q}, \qquad \frac{\partial W_f}{\partial \lambda_{2q}} = i_{2q}$$

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

Coupling Field Energy

- The coupling field energy is calculated using a path independent integration
  - For integral to be path independent, the partial derivatives of all integrands with respect to the other states must be equal

    For example, $\frac{3}{2}\frac{\partial i_d}{\partial \lambda_{fd}} = \frac{\partial i_{fd}}{\partial \lambda_d}$
- Since integration is path independent, choose a convenient path
  - Start with a de-energized system so variables are zero
  - Integrate shaft position while other variables are zero
  - Integrate sources in sequence with shaft at final value

Define Unscaled Variables

$$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$

ω_s is the rated synchronous speed

δ plays an important role!

$$\frac{d\lambda_d}{dt} = -r_s i_d + \omega\lambda_q + v_d$$

$$\frac{d\lambda_q}{dt} = -r_s i_q - \omega\lambda_d + v_q$$

$$\frac{d\lambda_o}{dt} = -r_s i_o + v_o$$

$$\frac{d\lambda_{fd}}{dt} = -r_{fd} i_{fd} + v_{fd}$$

$$\frac{d\lambda_{1d}}{dt} = -r_{1d} i_{1d} + v_{1d}$$

$$\frac{d\lambda_{1q}}{dt} = -r_{1q} i_{1q} + v_{1q}$$

$$\frac{d\lambda_{2q}}{dt} = -r_{2q} i_{2q} + v_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$J\frac{2}{p}\frac{d\omega}{dt} = T_m + \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_d i_q - \lambda_q i_d\right) - T_{f\omega}$$

Synchronous Machine Equations in Per Unit

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + V_o$$

$$\frac{1}{\omega_s}\frac{d\psi_{fd}}{dt} = -R_{fd} I_{fd} + V_{fd}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1d}}{dt} = -R_{1d} I_{1d} + V_{1d}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1q}}{dt} = -R_{1q} I_{1q} + V_{1q}$$

$$\frac{1}{\omega_s}\frac{d\psi_{2q}}{dt} = -R_{2q} I2 + V_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

Units of H are seconds

Sinusoidal Steady-State

$$V_a = \sqrt{2}V_s \cos(\omega_s t + \theta_{vs})$$

$$V_b = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} - \frac{2\pi}{3}\right)$$

$$V_c = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} + \frac{2\pi}{3}\right)$$

$$I_a = \sqrt{2}I_s \cos(\omega_s t + \theta_{is})$$

$$I_b = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} - \frac{2\pi}{3}\right)$$

$$I_c = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} + \frac{2\pi}{3}\right)$$

Here we consider the application to balanced, sinusoidal conditions

Simplifying Using δ

- Define $\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$
- Hence
$$V_d = V_s \sin(\delta - \theta_{vs})$$
$$V_q = V_s \cos(\delta - \theta_{vs})$$
$$I_d = I_s \sin(\delta - \theta_{is})$$
$$I_q = I_s \cos(\delta - \theta_{is})$$
- These algebraic equations can be written as complex equations
$$\left(V_d + jV_q\right)e^{j(\delta - \pi/2)} = V_s e^{j\theta_{vs}}$$
$$\left(I_d + jI_q\right)e^{j(\delta - \pi/2)} = I_s e^{j\theta_{is}}$$

The conclusion is if we know δ, then we can easily relate the phase to the dq values!

Summary So Far

- The model as developed so far has been derived using the following assumptions
  - The stator has three coils in a balanced configuration, spaced 120 electrical degrees apart
  - Rotor has four coils in a balanced configuration located 90 electrical degrees apart
  - Relationship between the flux linkages and currents must reflect a conservative coupling field
  - The relationships between the flux linkages and currents must be independent of θ_{shaft} when expressed in the dq0 coordinate system

Assuming a Linear Magnetic Circuit

- If the flux linkages are assumed to be a linear function of the currents then we can write

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \hline \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} L_{ss}\left(\theta_{shaft}\right) & L_{sr}\left(\theta_{shaft}\right) \\ \hline L_{rs}\left(\theta_{shaft}\right) & L_{rr}\left(\theta_{shaft}\right) \end{array}\right] \begin{bmatrix} i_a \\ i_b \\ i_c \\ \hline i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

The rotor self-inductance matrix L_{rr} is independent of θ_{shaft}

Conversion to dq0 for Angle Independence

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \\ \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} T_{dqo} L_{ss} T_{dqo}^{-1} & T_{dqo} L_{sr} \\ \hline L_{rs} T_{dqo}^{-1} & L_{rr} \end{array}\right] \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

Conversion to dq0 for Angle Independence

$$\lambda_d = \left(L_{\ell s} + L_{md}\right) i_d + L_{sfd} i_{fd} + L_{s1d} i_{1d}$$

$$\lambda_{fd} = \frac{3}{2} L_{sfd} i_d + L_{fdfd} i_{fd} + L_{fd1d} i_{1d}$$

$$\lambda_{1d} = \frac{3}{2} L_{s1d} i_d + L_{fd1d} i_{fd} + L_{1d1d} i_{1d}$$

$$\lambda_q = \left(L_{\ell s} + L_{mq}\right) i_q + L_{s1q} i_{1q} + L_{s2q} i_{2q}$$

$$\lambda_{1q} = \frac{3}{2} L_{s1q} i_q + L_{1q1q} i_{1q} + L_{1q2q} i_{2q}$$

$$\lambda_{2q} = \frac{3}{2} L_{s2q} i_q + L_{1q2q} i_{1q} + L_{2q2q} i_{2q}$$

$$\lambda_o = L_{\ell s} i_o$$

$$L_{md} = \frac{3}{2}\left(L_A + L_B\right),$$

$$L_{mq} = \frac{3}{2}\left(L_A - L_B\right)$$

For a round rotor machine L_B is small and hence L_{md} is close to L_{mq}. For a salient pole machine L_{md} is substantially larger

Convert to Normalized at f = ω_s

- Convert to per unit, and assume frequency of ω_s
- Then define new per unit reactance variables

$$X_{\ell s} = \frac{\omega_s L_{\ell s}}{Z_{BDQ}}, \quad X_{md} = \frac{\omega_s L_{md}}{Z_{BDQ}}, \quad X_{mq} = \frac{\omega_s L_{mq}}{Z_{BDQ}}$$

$$X_{fd} = \frac{\omega_s L_{fdfd}}{Z_{BFD}}, \quad X_{1d} = \frac{\omega_s L_{1d1d}}{Z_{B1D}}, \quad X_{fd1d} = \frac{\omega_s L_{fd1d} L_{sfd}}{Z_{BFD} L_{s1d}}$$

$$X_{1q} = \frac{\omega_s L_{1q1q}}{Z_{B1Q}}, \quad X_{2q} = \frac{\omega_s L_{2q2q}}{Z_{B2Q}}, \quad X_{1q2q} = \frac{\omega_s L_{1q2q} L_{s1q}}{Z_{B1Q} L_{s2q}}$$

$$X_{\ell fd} = X_{fd} - X_{md}, \quad X_{\ell 1d} = X_{1d} - X_{md}$$

$$X_{\ell 1q} = X_{1q} - X_{mq}, \quad X_{\ell 2q} = X_{2q} - X_{mq}$$

$$X_d = X_{\ell s} + X_{md}, \quad X_q = X_{\ell s} + X_{mq}$$

Key Simulation Parameters

- The key parameters that occur in most models can then be defined as

$$X'_d = X_{\ell s} + \frac{1}{\dfrac{1}{X_{md}} + \dfrac{1}{X_{\ell fd}}} = X_d - \frac{X_{md}^2}{X_{fd}}$$

$$X'_q = X_{\ell s} + \frac{1}{\dfrac{1}{X_{mq}} + \dfrac{1}{X_{\ell 1q}}} = X_q - \frac{X_{mq}^2}{X_{1q}}$$

$$T'_{do} = \frac{Xfd}{\omega_s R_{fd}}, \quad T'_{qo} = \frac{X_{1q}}{\omega_s R_{1q}}$$

These values will be used in all the synchronous machine models

In a salient rotor machine X_{mq} is small so $X_q = X'_q$; also X_{1q} is small so T'_{q0} is small

Key Simulation Parameters

- And the subtransient parameters

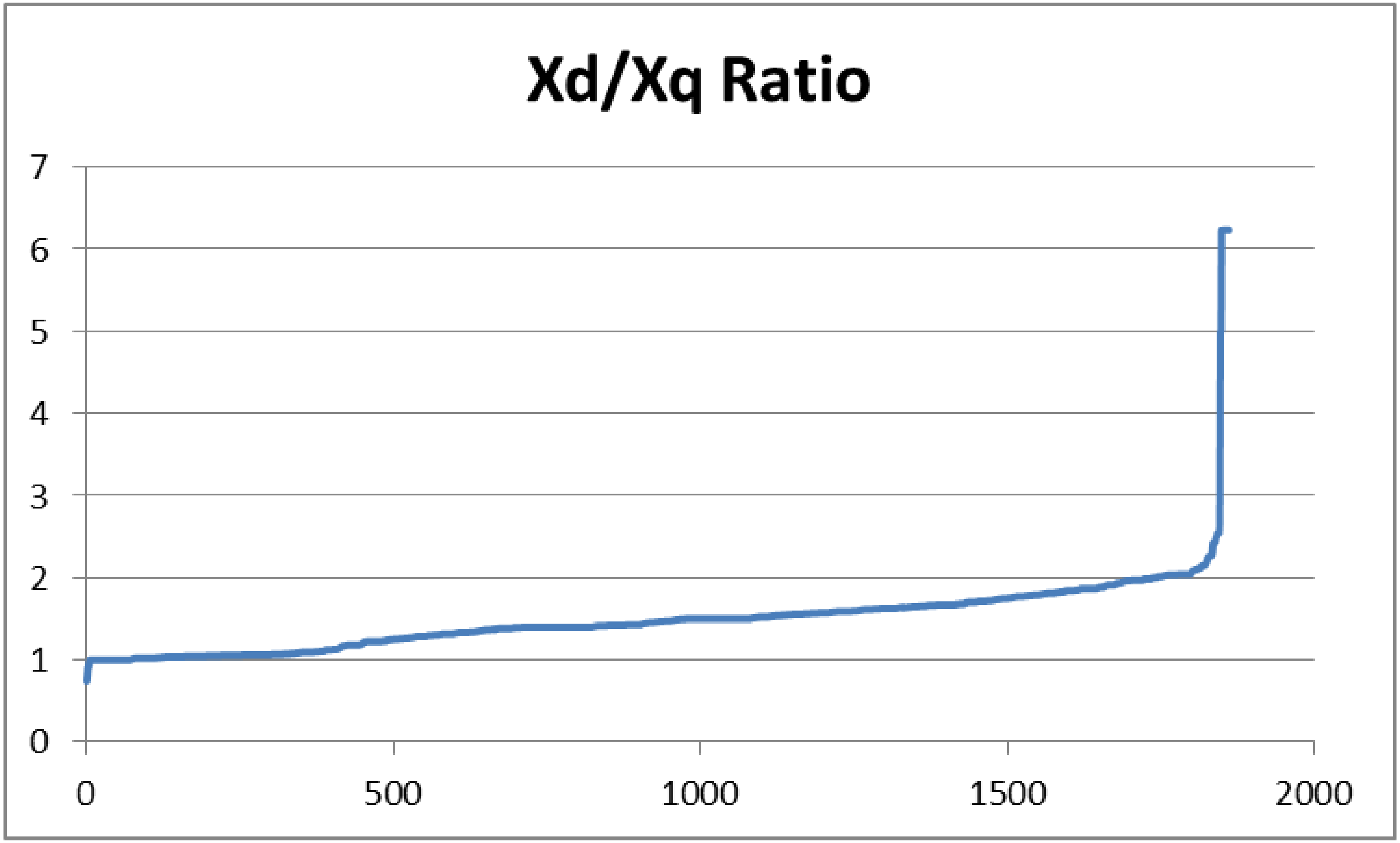
$$X''_d = X_{\ell s} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}} + \frac{1}{X_{\ell 1d}}}$$

$$X''_q = X_{\ell s} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}} + \frac{1}{X_{\ell 2q}}}$$

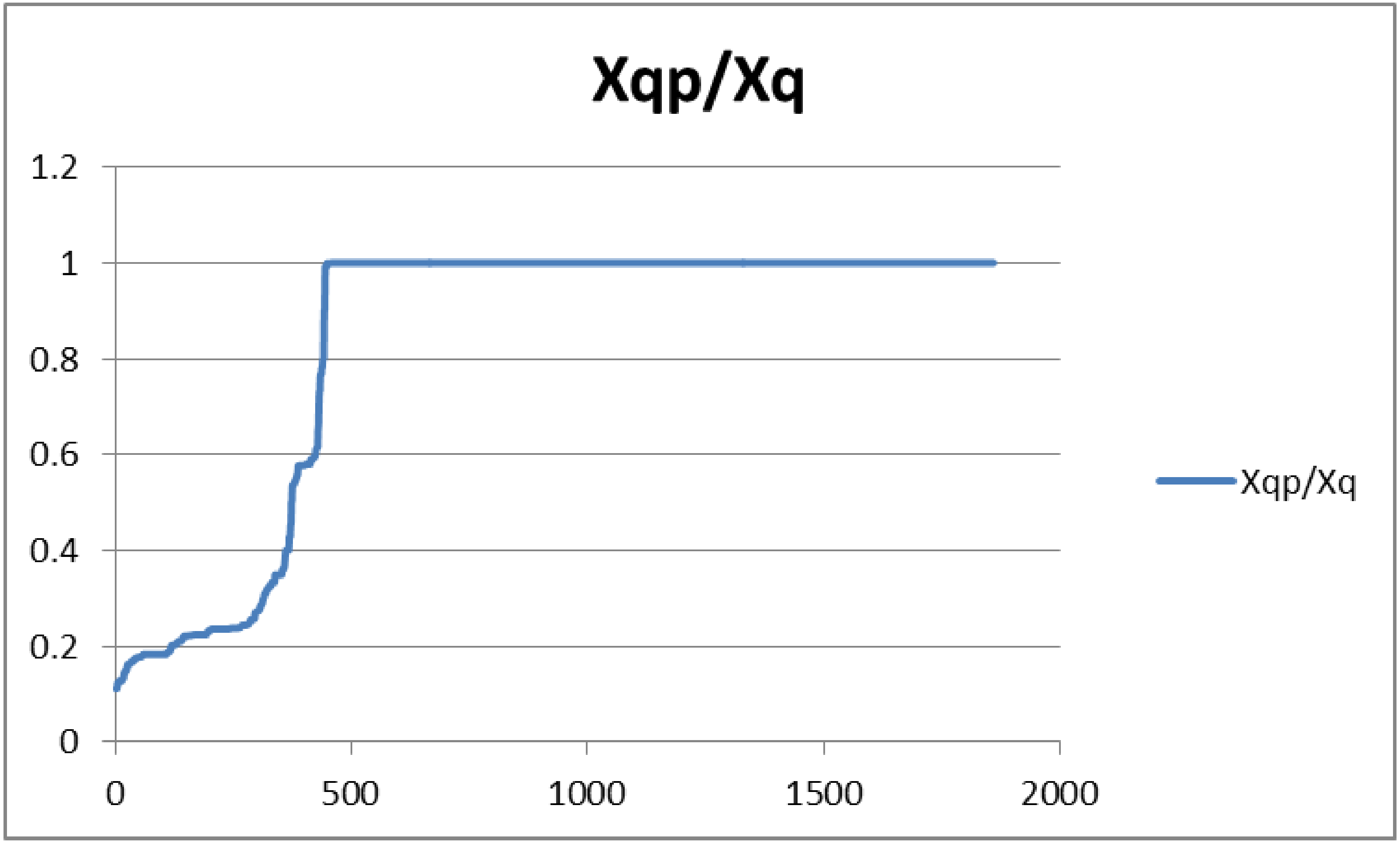
These values will be used in the subtransient machine models. It is common to assume $X''_d = X''_q$

$T''_{do} = \frac{1}{\omega_s R_{1d}} \Big(X$ [redacted] $\frac{1}{X_{md}} + \frac{1}{X}$ [redacted]

Example Xd/Xq Ratios for a WECC Case



Example X'q/Xq Ratios for a WECC Case



About 75% are Clearly Salient Pole Machines!

Internal Variables

- Define the following variables, which are quite important in subsequent models

$$E'_q \triangleq \frac{X_{md}}{X_{fd}} \psi_{fd}$$

$$E'_d \triangleq \frac{X_{mq}}{X_{1q}} \psi_{1q}$$

$$E_{fd} \triangleq \frac{X_{md}}{R_{fd}} V_{fd}$$

Hence E'q and E'd are scaled flux linkages and Efd is the scaled field voltage

Dynamic Model Development

- In developing the dynamic model not all of the currents and fluxes are independent
  - In this formulation only seven out of fourteen are independent
- Approach is to eliminate the rotor currents, retaining the terminal currents (I_d, I_q, I_0) for matching the network boundary conditions

Rotor Currents

- Use new variables to solve for the rotor currents

$$\psi_d = -X_d'' I_d + \frac{\left(X_d'' - X_{\ell s}\right)}{\left(X_d' - X_{\ell s}\right)} E_q' + \frac{\left(X_d' - X_d''\right)}{\left(X_d' - X_{\ell s}\right)} \psi_{1d}$$

$$I_{fd} = \frac{1}{X_{md}}\left[E_q' + \left(X_d - X_d'\right)\left(I_d - I_{1d}\right)\right]$$

$$I_{1d} = \frac{X_d' - X_d''}{\left(X_d' - X_{\ell s}\right)^2}\left[\psi_{1d} + \left(X_d' - X_{\ell s}\right) I_d - E_q'\right]$$

Rotor Currents

$$\psi_q = -X_q'' I_q - \frac{\left(X_q'' - X_{\ell s}\right)}{\left(X_q' - X_{\ell s}\right)} E_d' + \frac{\left(X_q' - X_q''\right)}{\left(X_q' - X_{\ell s}\right)} \psi_{2q}$$

$$I_{1q} = \frac{1}{X_{mq}} \left[-E_d' + \left(X_q - X_q'\right)\left(I_q - I_{2q}\right) \right]$$

$$I_{2q} = \frac{X_q' - X_q''}{\left(X_q' - X_{\ell s}\right)^2} \left[\psi_{2q} + \left(X_q' - X_{\ell s}\right) I_q + E_d' \right]$$

$$\psi_o = X_{\ell s}\left(-I_o\right)$$

Final Complete Model

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + Vo$$

These first three equations define what are known as the stator transients; we will shortly approximate them as algebraic constraints

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)\left[I_d - \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left(\psi_{1d} + \left(X'_d - X_{\ell s}\right)I_d - E'_q\right)\right] + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\left[I_q - \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

Final Complete Model

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right) I_d$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right) I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

$$\psi_d = -X''_d I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$\psi_q = -X''_q I_q - \frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$\psi_o = -X_{\ell s} I_o$$

TFW is the friction and windage component

Single-Machine Steady-State

$$0 = R_s I_d + \psi_q + V_d \qquad (\omega = \omega_s)$$

$$0 = R_s I_q - \psi_d + V_q$$

$$0 = R_s I_o + V_o$$

$$0 = -E'_q - \left(X_d - X'_d\right) I_d + E_{fd}$$

$$0 = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right) I_d$$

$$0 = -E'_d + \left(X_q - X'_q\right) I_q$$

$$0 = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right) I_q$$

$$0 = \omega - \omega_s$$

$$0 = T_m - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

$$\psi_d = E'_q - X''_d I_d$$

$$\psi_q = -X''_q I_q - E'_d$$

$$\psi_o = -X_{\ell s} I_o$$

The key variable we need to determine the initial conditions is actually δ, which doesn't appear explicitly in these equations!

Field Current

- The field current, I_{fd}, is defined in steady-state as

$$I_{fd} = E_{fd} / X_{md}$$

- However, what is usually used in transient stability simulations for the field current is the product

$$I_{fd} X_{md}$$

- So the value of X_{md} is not needed

Single-Machine Steady-State

- Previous derivation was done assuming a linear magnetic circuit
- We'll consider the nonlinear magnetic circuit later but will first do the steady-state condition (3.6)
- In steady-state the speed is constant (equal to ω_s), δ is constant, and all the derivatives are zero
- Initial values are determined from the terminal conditions: voltage magnitude, voltage angle, real and reactive power injection

Determining δ without Saturation

- In order to get the initial values for the variables we need to determine δ
- We'll eventually consider two approaches: the simple one when there is no saturation, and then later a general approach for models with saturation
- To derive the simple approach we have

$$V_d = R_s I_d + E'_d + X'_q I_q$$

$$V_q = -R_s I_q + E'_q - X'_d I_d$$

Determining δ without Saturation

Since $j = e^{j(\pi/2)}$

$$\tilde{E} = \left[\left(X_q - X_d'\right)I_d + E_q'\right]e^{j\delta}$$

- In terms of the terminal values

$$\tilde{E} = \tilde{V}_{as} + (R_s + jX_q)\tilde{I}_{as}$$

The angle on $\tilde{E} = \delta$