ECEN 667
Power System Stability

Lecture 7: Synchronous Machine Modeling

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Announcements

- Read Chapter 3
- Homework 2 is due on Thursday September 19

Sinusoidal Steady-State

$$V_a = \sqrt{2}V_s \cos(\omega_s t + \theta_{vs})$$

$$V_b = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} - \frac{2\pi}{3}\right)$$

$$V_c = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} + \frac{2\pi}{3}\right)$$

$$I_a = \sqrt{2}I_s \cos(\omega_s t + \theta_{is})$$

$$I_b = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} - \frac{2\pi}{3}\right)$$

$$I_c = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} + \frac{2\pi}{3}\right)$$

Here we consider the application to balanced, sinusoidal conditions

Simplifying Using δ

- Define $\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$
- Hence
$$V_d = V_s \sin(\delta - \theta_{vs})$$
$$V_q = V_s \cos(\delta - \theta_{vs})$$
$$I_d = I_s \sin(\delta - \theta_{is})$$
$$I_q = I_s \cos(\delta - \theta_{is})$$
- These algebraic equations can be written as complex equations
$$\left(V_d + jV_q\right)e^{j(\delta - \pi/2)} = V_s e^{j\theta_{vs}}$$
$$\left(I_d + jI_q\right)e^{j(\delta - \pi/2)} = I_s e^{j\theta_{is}}$$

If we know δ, then we can easily relate the phase to the dq values!

Summary So Far

- The model as developed so far has been derived using the following assumptions
  - The stator has three coils in a balanced configuration, spaced 120 electrical degrees apart
  - Rotor has four coils in a balanced configuration located 90 electrical degrees apart
  - Relationship between the flux linkages and currents must reflect a conservative coupling field
  - The relationships between the flux linkages and currents must be independent of θ_{shaft} when expressed in the dq0 coordinate system

Assuming a Linear Magnetic Circuit

- From the book we have

$$L_{ss}(\theta_{\text{shaft}}) \triangleq \begin{bmatrix} L_{\ell s} + L_A - L_B \cos P\theta_{\text{shaft}} & -\frac{1}{2}L_A - L_B\cos(P\theta_{\text{shaft}} - \frac{2\pi}{3}) & -\frac{1}{2}L_A - L_B\cos(P\theta_{\text{shaft}} + \frac{2\pi}{3}) \\ -\frac{1}{2}L_A - L_B\cos(P\theta_{\text{shaft}} - \frac{2\pi}{3}) & L_{\ell s} + L_A - L_B\cos(P\theta_{\text{shaft}} + \frac{2\pi}{3}) & -\frac{1}{2}L_A - L_B \cos P\theta_{\text{shaft}} \\ -\frac{1}{2}L_A - L_B\cos(P\theta_{\text{shaft}} + \frac{2\pi}{3}) & -\frac{1}{2}L_A - L_B \cos P\theta_{\text{shaft}} & L_{\ell s} + L_A - L_B\cos(P\theta_{\text{shaft}} - \frac{2\pi}{3}) \end{bmatrix} \quad (3.95)$$

$$L_{sr}(\theta_{\text{shaft}}) = L_{rs}^t(\theta_{\text{shaft}}) \triangleq \begin{bmatrix} L_{sfd}\sin\frac{P}{2}\theta_{\text{shaft}} & L_{s1d}\sin\frac{P}{2}\theta_{\text{shaft}} & L_{s1q}\cos\frac{P}{2}\theta_{\text{shaft}} & L_{s2q}\cos\frac{P}{2}\theta_{\text{shaft}} \\ L_{sfd}\sin(\frac{P}{2}\theta_{\text{shaft}} - \frac{2\pi}{3}) & L_{s1d}\sin(\frac{P}{2}\theta_{\text{shaft}} - \frac{2\pi}{3}) & L_{s1q}\cos(\frac{P}{2}\theta_{\text{shaft}} - \frac{2\pi}{3}) & L_{s2q}\cos(\frac{P}{2}\theta_{\text{shaft}} - \frac{2\pi}{3}) \\ L_{sfd}\sin(\frac{P}{2}\theta_{\text{shaft}} + \frac{2\pi}{3}) & L_{s1d}\sin(\frac{P}{2}\theta_{\text{shaft}} + \frac{2\pi}{3}) & L_{s1q}\cos(\frac{P}{2}\theta_{\text{shaft}} + \frac{2\pi}{3}) & L_{s2q}\cos(\frac{P}{2}\theta_{\text{shaft}} + \frac{2\pi}{3}) \end{bmatrix} \quad (3.96)$$

$$L_{rr}(\theta_{\text{shaft}}) \triangleq \begin{bmatrix} L_{fdfd} & L_{fd1d} & 0 & 0 \\ L_{fd1d} & L_{1d1d} & 0 & 0 \\ 0 & 0 & L_{1q1q} & L_{1q2q} \\ 0 & 0 & L_{1q2q} & L_{2q2q} \end{bmatrix} \quad (3.97)$$

Note that the first three matrices depend upon θ_{shaft}; the rotor self-inductance matrix L_{rr} is independent of θ_{shaft}

Assuming a Linear Magnetic Circuit

- With this assumption of a linear magnetic circuit then we can write

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \hline \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} L_{ss}\left(\theta_{shaft}\right) & L_{sr}\left(\theta_{shaft}\right) \\ \hline L_{rs}\left(\theta_{shaft}\right) & L_{rr}\left(\theta_{shaft}\right) \end{array}\right] \begin{bmatrix} i_a \\ i_b \\ i_c \\ \hline i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

Conversion to dq0 for Angle Independence

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \\ \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} T_{dqo} L_{ss} T_{dqo}^{-1} & T_{dqo} L_{sr} \\ \hline L_{rs} T_{dqo}^{-1} & L_{rr} \end{array}\right] \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

Conversion to dq0 for Angle Independence

$$\lambda_d = \left(L_{\ell s} + L_{md}\right) i_d + L_{sfd} i_{fd} + L_{s1d} i_{1d}$$

$$\lambda_{fd} = \frac{3}{2} L_{sfd} i_d + L_{fdfd} i_{fd} + L_{fd1d} i_{1d}$$

$$\lambda_{1d} = \frac{3}{2} L_{s1d} i_d + L_{fd1d} i_{fd} + L_{1d1d} i_{1d}$$

$$\lambda_q = \left(L_{\ell s} + L_{mq}\right) i_q + L_{s1q} i_{1q} + L_{s2q} i_{2q}$$

$$\lambda_{1q} = \frac{3}{2} L_{s1q} i_q + L_{1q1q} i_{1q} + L_{1q2q} i_{2q}$$

$$\lambda_{2q} = \frac{3}{2} L_{s2q} i_q + L_{1q2q} i_{1q} + L_{2q2q} i_{2q}$$

$$\lambda_o = L_{\ell s} i_o$$

$$L_{md} = \frac{3}{2}\left(L_A + L_B\right),$$

$$L_{mq} = \frac{3}{2}\left(L_A - L_B\right)$$

For a round rotor machine L_B is small and hence L_{md} is close to L_{mq}. For a salient pole machine L_{md} is substantially larger

Convert to Normalized at f = ω_s

- Convert to per unit, and assume frequency of ω_s
- Then define new per unit reactance variables

$$X_{\ell s} = \frac{\omega_s L_{\ell s}}{Z_{BDQ}}, \quad X_{md} = \frac{\omega_s L_{md}}{Z_{BDQ}}, \quad X_{mq} = \frac{\omega_s L_{mq}}{Z_{BDQ}}$$

$$X_{fd} = \frac{\omega_s L_{fdfd}}{Z_{BFD}}, \quad X_{1d} = \frac{\omega_s L_{1d1d}}{Z_{B1D}}, \quad X_{fd1d} = \frac{\omega_s L_{fd1d} L_{sfd}}{Z_{BFD} L_{s1d}}$$

$$X_{1q} = \frac{\omega_s L_{1q1q}}{Z_{B1Q}}, \quad X_{2q} = \frac{\omega_s L_{2q2q}}{Z_{B2Q}}, \quad X_{1q2q} = \frac{\omega_s L_{1q2q} L_{s1q}}{Z_{B1Q} L_{s2q}}$$

$$X_{\ell fd} = X_{fd} - X_{md}, \quad X_{\ell 1d} = X_{1d} - X_{md}$$

$$X_{\ell 1q} = X_{1q} - X_{mq}, \quad X_{\ell 2q} = X_{2q} - X_{mq}$$

$$X_d = X_{\ell s} + X_{md}, \quad X_q = X_{\ell s} + X_{mq}$$

Key Simulation Parameters

- The key parameters that occur in most models can then be defined as

$$X'_d = X_{\ell s} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}} = X_d - \frac{X_{md}^2}{X_{fd}}$$

$$X'_q = X_{\ell s} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}} = X_q - \frac{X_{mq}^2}{X_{1q}}$$

$$T'_{do} = \frac{Xfd}{\omega_s R_{fd}}, \quad T'_{qo} = \frac{X_{1q}}{\omega_s R_{1q}}$$

These values will be used in all the synchronous machine models

In a salient rotor machine X_{mq} is small so $X_q = X'_q$; also X_{1q} is small so T'_{q0} is small

Key Simulation Parameters

- And the subtransient parameters

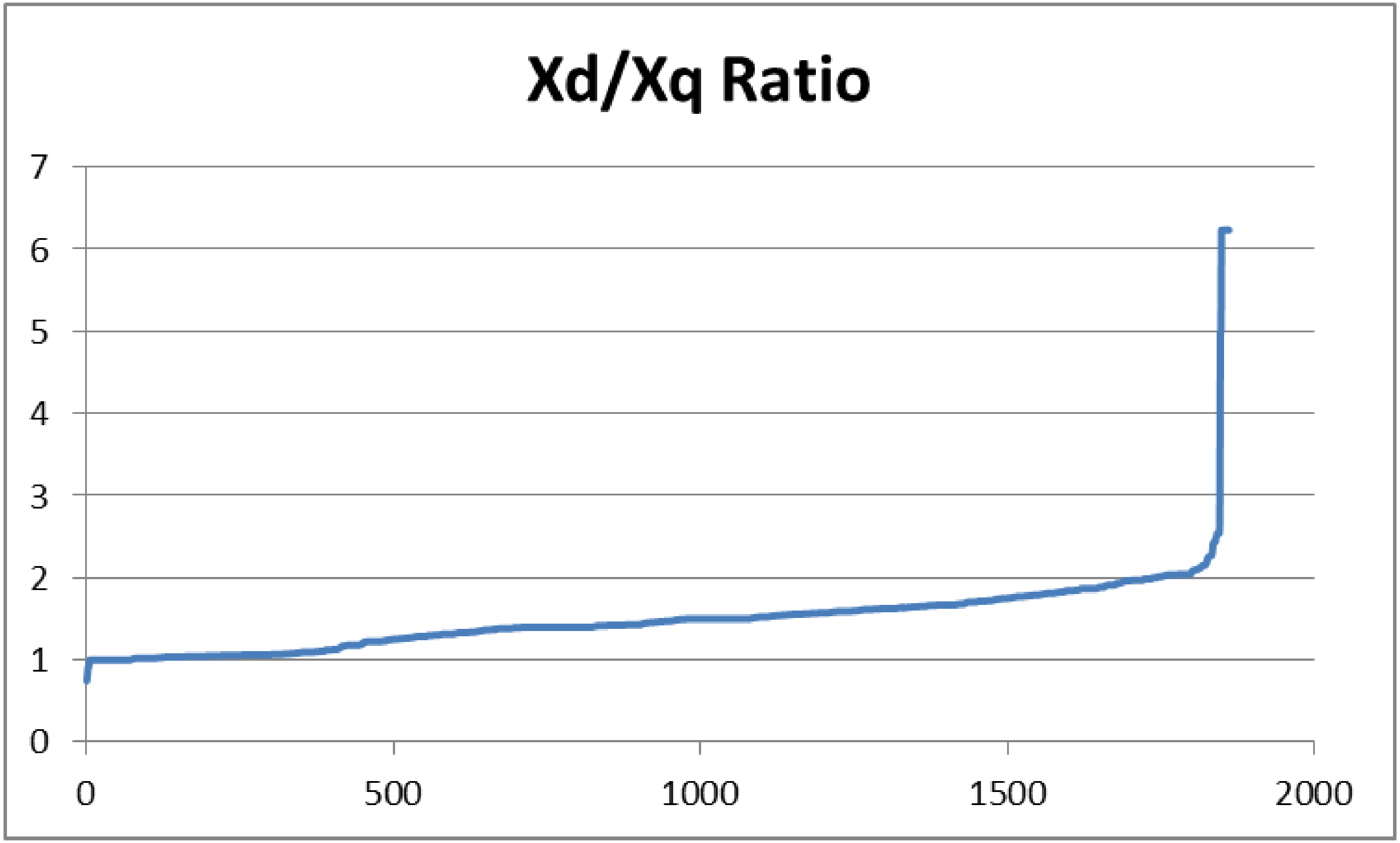
$$X''_d = X_{\ell s} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}} + \frac{1}{X_{\ell 1d}}}$$

$$X''_q = X_{\ell s} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}} + \frac{1}{X_{\ell 2q}}}$$

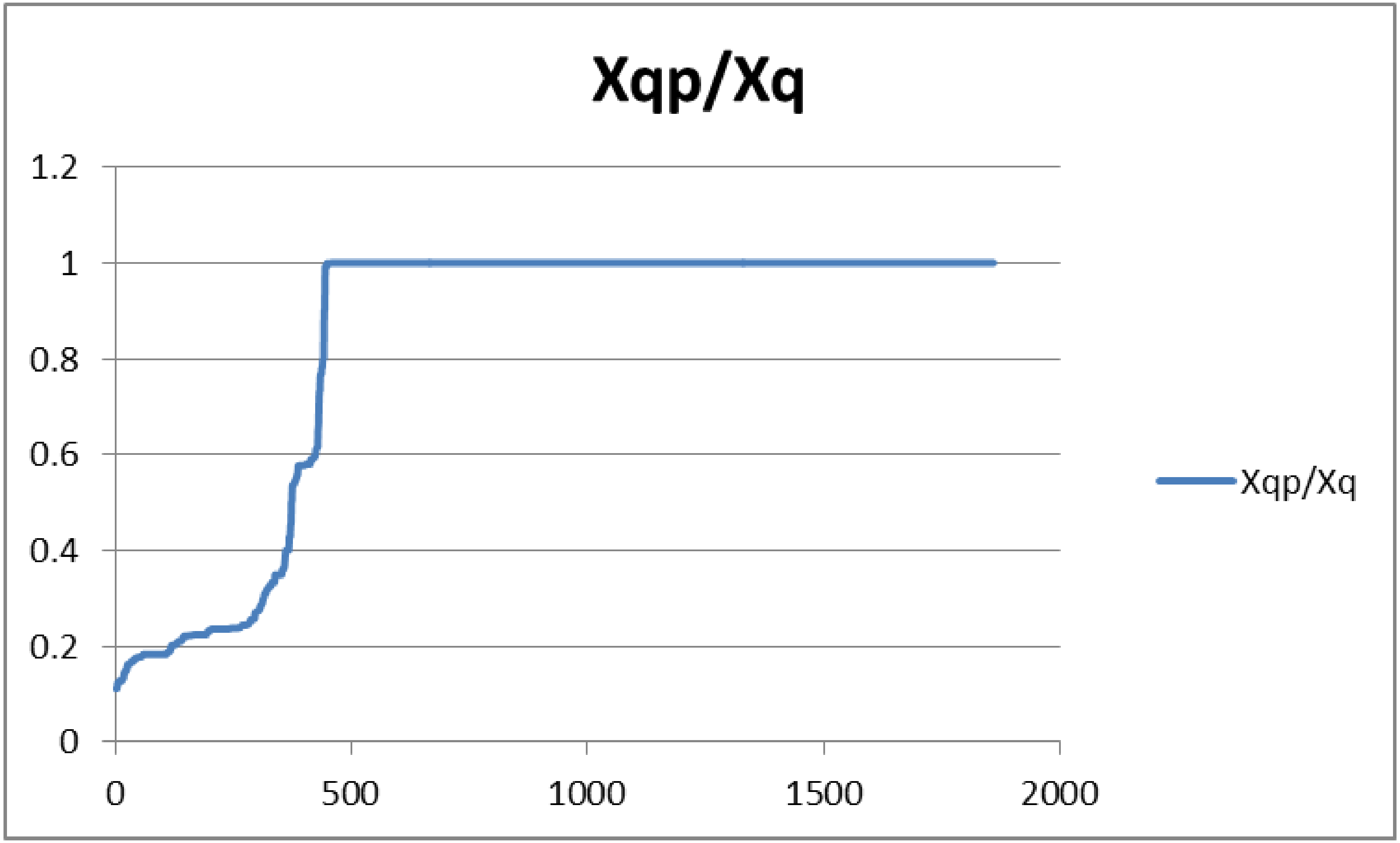
These values will be used in the subtransient machine models. It is common to assume X"$_d$ = X"$_q$

$$T''_{do} = \frac{1}{\omega_s R_{1d}} \left(X_{\ell 1d} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell 1d}}} \right), \quad T''_{qo} = \frac{1}{\omega_s R_{2q}} \left(X_{\ell 2q} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}} \right)$$

Example Xd/Xq Ratios for a WECC Case



Example X'q/Xq Ratios for a WECC Case



About 75% are Clearly Salient Pole Machines!

Internal Variables

- Define the following variables, which are quite important in subsequent models

$$E'_q \triangleq \frac{X_{md}}{X_{fd}} \psi_{fd}$$

$$E'_d \triangleq \frac{X_{mq}}{X_{1q}} \psi_{1q}$$

$$E_{fd} \triangleq \frac{X_{md}}{R_{fd}} V_{fd}$$

Hence E'q and E'd are scaled flux linkages and Efd is the scaled field voltage

Dynamic Model Development

- In developing the dynamic model not all of the currents and fluxes are independent
  - In this formulation only seven out of fourteen are independent
- Approach is to eliminate the rotor currents, retaining the terminal currents (I_d, I_q, I_0) for matching the network boundary conditions

Rotor Currents

- Use new variables to solve for the rotor currents

$$\psi_d = -X''_d I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$I_{fd} = \frac{1}{X_{md}} \left[E'_q + \left(X_d - X'_d\right)\left(I_d - I_{1d}\right) \right]$$

$$I_{1d} = \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2} \left[\psi_{1d} + \left(X'_d - X_{\ell s}\right) I_d - E'_q \right]$$

Rotor Currents

$$\psi_q = -X_q'' I_q - \frac{\left(X_q'' - X_{\ell s}\right)}{\left(X_q' - X_{\ell s}\right)} E_d' + \frac{\left(X_q' - X_q''\right)}{\left(X_q' - X_{\ell s}\right)} \psi_{2q}$$

$$I_{1q} = \frac{1}{X_{mq}} \left[-E_d' + \left(X_q - X_q'\right)\left(I_q - I_{2q}\right) \right]$$

$$I_{2q} = \frac{X_q' - X_q''}{\left(X_q' - X_{\ell s}\right)^2} \left[\psi_{2q} + \left(X_q' - X_{\ell s}\right) I_q + E_d' \right]$$

$$\psi_o = X_{\ell s}\left(-I_o\right)$$

Final Complete Model

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + Vo$$

These first three equations define what are known as the stator transients; we will shortly approximate them as algebraic constraints

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)\left[I_d - \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left(\psi_{1d} + \left(X'_d - X_{\ell s}\right)I_d - E'_q\right)\right] + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\left[I_q - \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

Final Complete Model

$$T''_{do}\frac{d\psi_{1d}}{dt}=-\psi_{1d}+E'_q-\left(X'_d-X_{\ell s}\right)I_d$$

$$T''_{qo}\frac{d\psi_{2q}}{dt}=-\psi_{2q}-E'_d-\left(X'_q-X_{\ell s}\right)I_q$$

$$\frac{d\delta}{dt}=\omega-\omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt}=T_M-\left(\psi_d I_q-\psi_q I_d\right)-T_{FW}$$

$$\psi_d=-X''_d I_d+\frac{\left(X''_d-X_{\ell s}\right)}{\left(X'_d-X_{\ell s}\right)}E'_q+\frac{\left(X'_d-X_{\ell s}\right)}{\left(X'_d-X_{\ell s}\right)}\psi_{1d}$$

$$\psi_q=-X''_q I_q-\frac{\left(X''_q-X_{\ell s}\right)}{\left(X'_q-X_{\ell s}\right)}E'_d+\frac{\left(X'_q-X''_q\right)}{\left(X'_q-X_{\ell s}\right)}\psi_{2q}$$

$$\psi_o=-X_{\ell s}I_o$$

T_{FW} is the friction and windage component

Single-Machine Steady-State

$$0 = R_s I_d + \psi_q + V_d \qquad (\omega = \omega_s)$$

$$0 = R_s I_q - \psi_d + V_q$$

$$0 = R_s I_o + V_o$$

$$0 = -E'_q - \left(X_d - X'_d\right) I_d + E_{fd}$$

$$0 = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right) I_d$$

$$0 = -E'_d + \left(X_q - X'_q\right) I_q$$

$$0 = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right) I_q$$

$$0 = \omega - \omega_s$$

$$0 = T_m - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

$$\psi_d = E'_q - X''_d I_d$$

$$\psi_q = -X''_q I_q - E'_d$$

$$\psi_o = -X_{\ell s} I_o$$

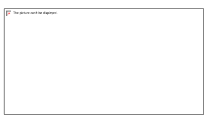
The key variable we need to determine the initial conditions is actually δ, which doesn't appear explicitly in these equations!

Field Current

- The field current, I_{fd}, is defined in steady-state as

$$I_{fd} = E_{fd} / X_{md}$$

- However, what is usually used in transient stability simulations for the field current is the product



- So the value of X_{md} is not needed

Single-Machine Steady-State

- Previous derivation was done assuming a linear magnetic circuit
- We'll consider the nonlinear magnetic circuit later but will first do the steady-state condition (3.6)
- In steady-state the speed is constant (equal to ω_s), δ is constant, and all the derivatives are zero
- Initial values are determined from the terminal conditions: voltage magnitude, voltage angle, real and reactive power injection

Determining δ without Saturation

- In order to get the initial values for the variables we need to determine δ
- We'll eventually consider two approaches: the simple one when there is no saturation, and then later a general approach for models with saturation
- To derive the simple approach we have

$$V_d = R_s I_d + E'_d + X'_q I_q$$

$$V_q = -R_s I_q + E'_q - X'_d I_d$$

Determining δ without Saturation

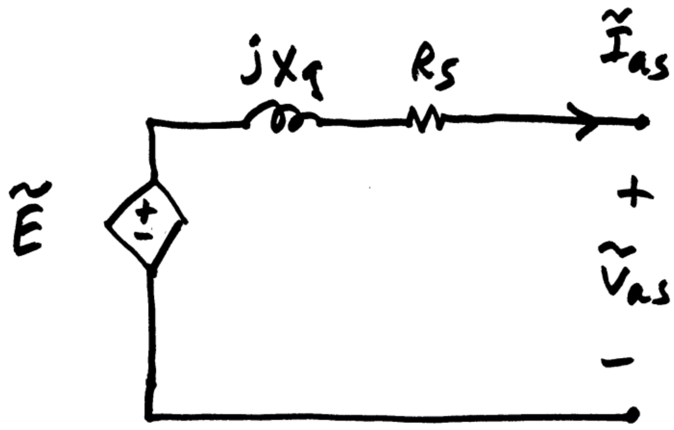
Since $j = e^{j(\pi/2)}$

$$\tilde{E} = \left[\left(X_q - X'_d\right)I_d + E'_q\right]e^{j\delta}$$

- In terms of the terminal values

$$\tilde{E} = \tilde{V}_{as} + (R_s + jX_q)\tilde{I}_{as}$$

The angle on $\tilde{E} = \delta$

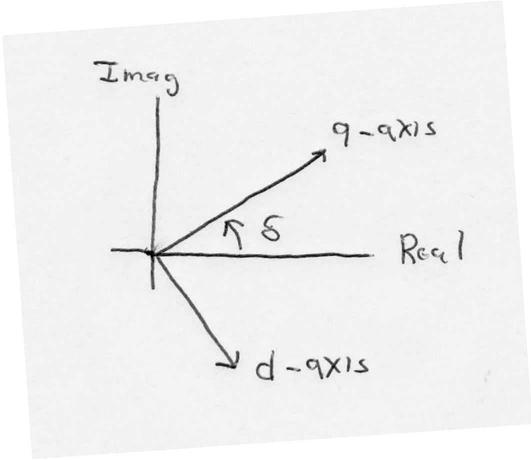


D-q Reference Frame

- Machine voltage and current are "transformed" into the d-q reference frame using the rotor angle, δ
  - Terminal voltage in network (power flow) reference frame are $V_S = V_t = V_r + jV_i$

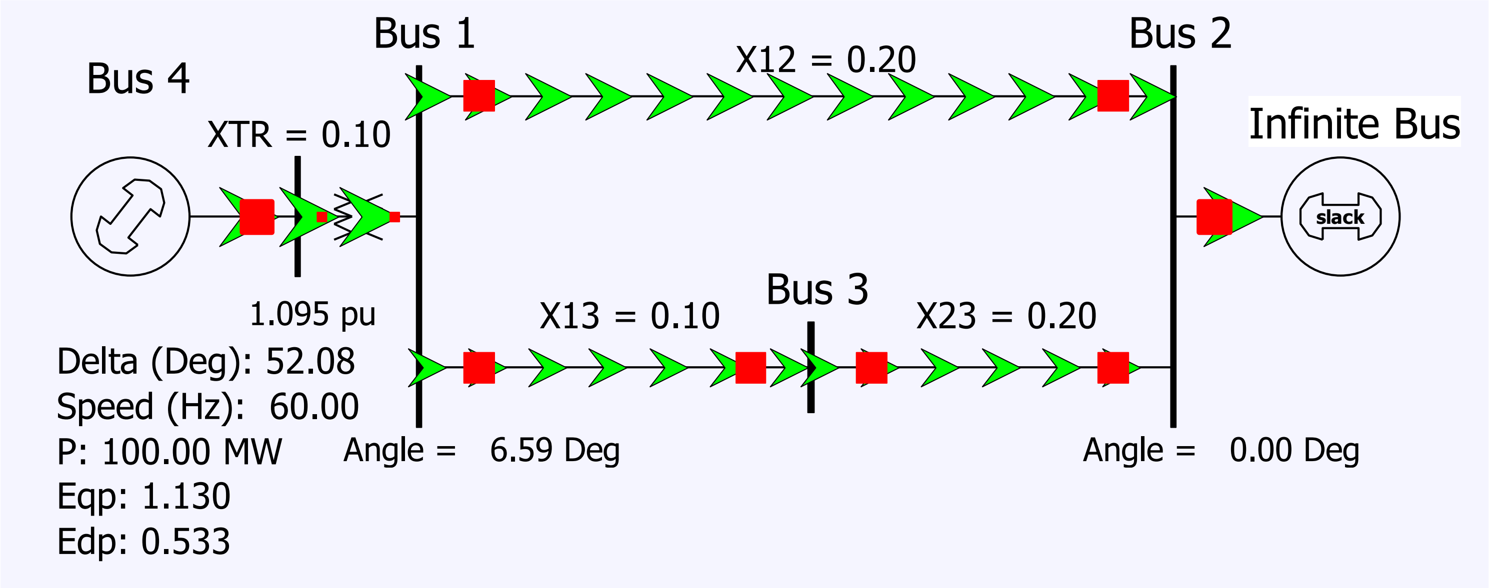
$$\begin{bmatrix} V_r \\ V_i \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix}$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$



A Steady-State Example

- Assume a generator is supplying 1.0 pu real power at 0.95 pf lagging into an infinite bus at 1.0 pu voltage through the below network. Generator pu values are $R_s=0$, $X_d=2.1$, $X_q=2.0$, $X'_d=0.3$, $X'_q=0.5$



A Steady-State Example, cont.

- First determine the current out of the generator from the initial conditions, then the terminal voltage

$$\tilde{I} = 1.0526\angle -18.20° = 1 - j0.3288$$

$$\tilde{V}_s = 1.0\angle 0° + \left(j0.22 \right)\left(1.0526\angle -18.20° \right)$$

$$= 1.0946\angle 11.59° = 1.0723 + j0.220$$

A Steady-State Example, cont.

- We can then get the initial angle and initial dq values

$$\tilde{E} = 1.0946\angle 11.59° + (j2.0)(1.052\angle -18.2°) = 2.814\angle 52.1°$$

$$\rightarrow \delta = 52.1°$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

$$V_d + jV_q = V_s e^{j\theta} e^{j(\pi/2-\delta)} = 1.0945\angle(11.6 + 90 - 52.1)$$

$$= 1.0945\angle 49.5° = 0.710 + j0.832$$

A Steady-State Example, cont

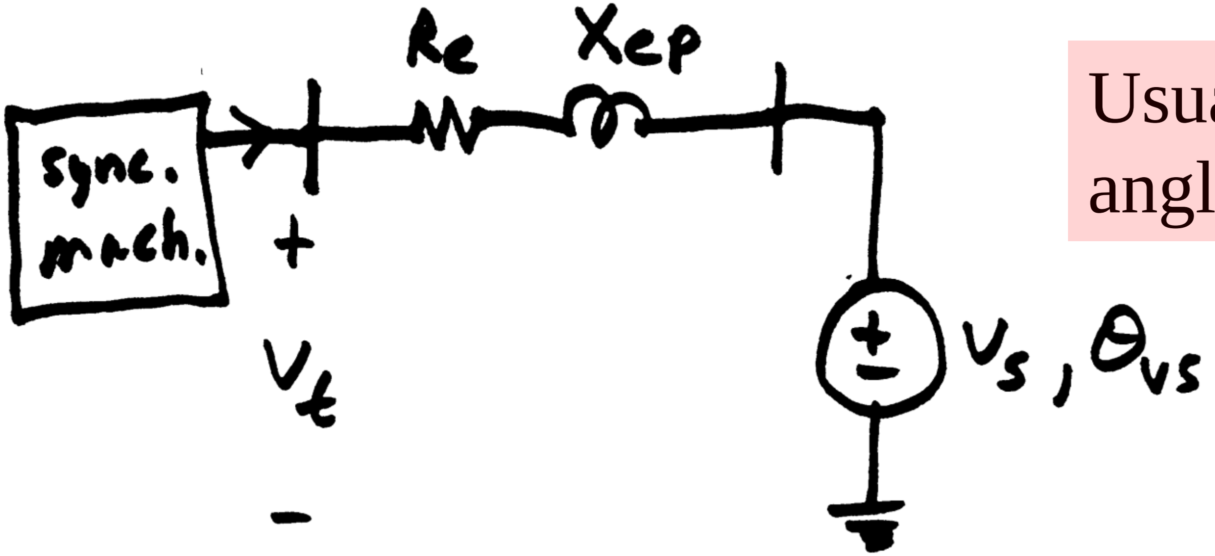
- The initial state variable are determined by solving with the differential equations equal to zero.

$$E_q' = V_q + R_s I_q + X_d' I_d = 0.8326 + \left(0.3\right)\left(0.9909\right) = 1.1299$$

$$E_d' = V_d - R_s I_d - X_q' I_q = 0.7107 - (0.5)(0.3553) = 0.5330$$

$$E_{fd} = E_q' + (X_d - X_d') I_d = 1.1299 + (2.1 - 0.3)(0.9909) = 2.9135$$

Single Machine, Infinite Bus System (SMIB)



Usually infinite bus angle, θ_{vs}, is zero

This example can be simplified by combining machine values with line values

$$\psi_{de} = \psi_d + \psi_{ed}$$

$$X_{de} = X_d + X_{ep}$$

$$R_{se} = R_s + R_e$$

etc

Introduce New Constants

$$\omega_t = T_s(\omega - \omega_s)$$

"Transient Speed"

$$T_s = \sqrt{\frac{2H}{\omega_s}}$$

Mechanical time constant

$$\varepsilon = \frac{1}{\omega_s}$$

A small parameter

We are ignoring the exciter and governor for now; they will be covered in detail later

Stator Flux Differential Equations

$$\varepsilon \frac{d\psi_{de}}{dt} = R_{se} I_d + \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_{qe} + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$\varepsilon \frac{d\psi_{qe}}{dt} = R_{se} I_q - \left(1 + \frac{\varepsilon}{T_s}\omega_t\right)\psi_{de} + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$\varepsilon \frac{d\psi_{oe}}{dt} = R_{se} I_o$$

Elimination of Stator Transients

- If we assume the stator flux equations are much faster than the remaining equations, then letting ε go to zero allows us to replace the differential equations with algebraic equations

$$0 = R_{se} I_d + \psi_{qe} + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$0 = R_{se} I_q - \psi_{de} + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$0 = R_{se} I_o$$

This assumption might not be valid if we are considering faster dynamics on other devices (such as converter dynamics)

Impact on Studies

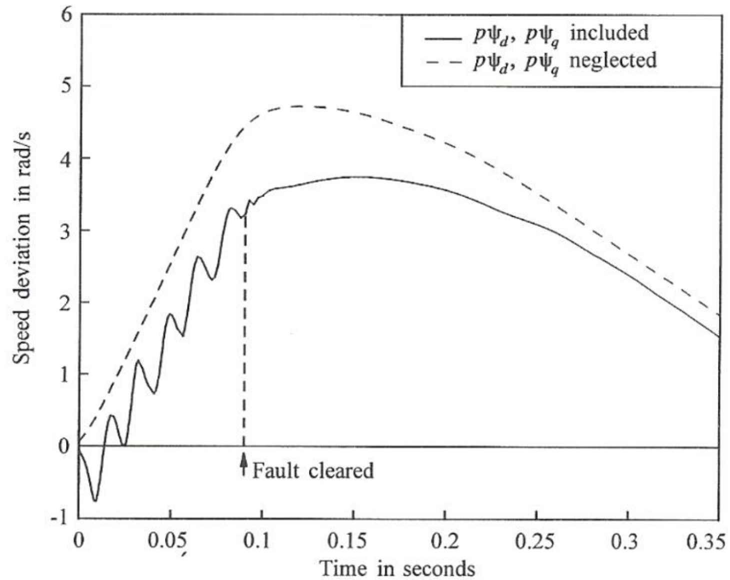


Figure 5.3 Effect of neglecting stator transients on speed deviation

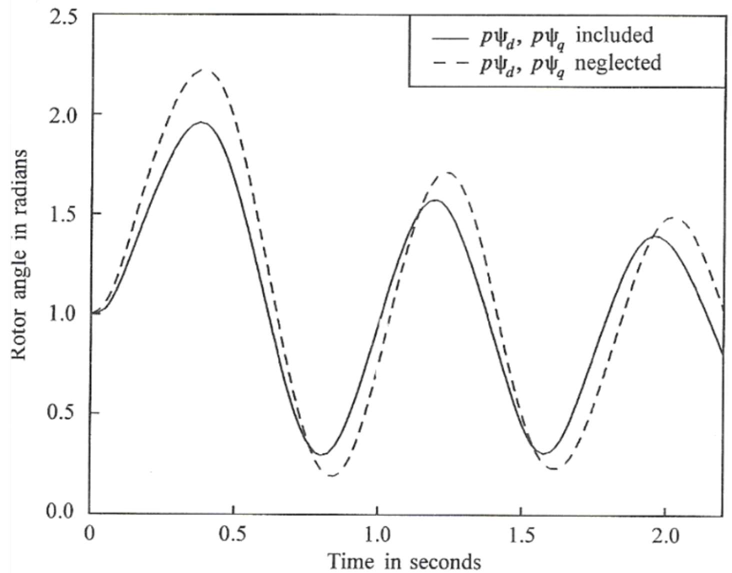


Figure 5.4 Effect of neglecting stator transients on rotor angle swings

Stator transients are not usually considered in transient stability studies

Image Source: P. Kundur, *Power System Stability and Control, EPRI, McGraw-Hill, 1994*

Machine Variable Summary

- Three fast dynamic states, now eliminated

$$\psi_{de},\ \psi_{qe},\ \psi_{oe}$$

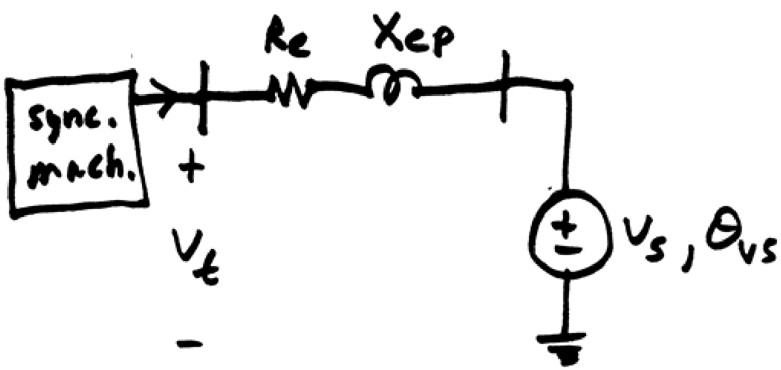
- Seven not so fast dynamic states

$$E'_q, \psi_{1d}, E'_d, \psi_{2q}, \delta, \omega_t E_{fd}$$

- Eight algebraic states

$$I_d, I_q, I_o, V_d, V_q, V_t, \psi_{ed}, \psi_{eq}$$

We'll get to the exciter and governor shortly



$$V_t = \sqrt{V_d^2 + V_q^2}$$

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin(\delta - \theta_{vs})$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos(\delta - \theta_{vs})$$

Network Expressions

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos\left(\delta - \theta_{vs}\right)$$

These two equations can be written as one complex equation.

$$\left(V_d + jV_q\right)e^{j(\delta-\pi/2)} = \left(R_e + jX_{ep}\right)\left(I_d + jI_q\right)e^{j(\delta-\pi/2)} + V_s e^{j\theta_{vs}}$$

Machine Variable Summary

Three fast dynamic states, now eliminated

$$\psi_{de},\ \psi_{qe},\ \psi_{oe}$$

Seven not so fast dynamic states

$$E'_q, \psi_{1d}, E'_d, \psi_{2q}, \delta, \omega_t E_{fd}$$

Eight algebraic states

$$I_d, I_q, I_o, V_d, V_q, V_t, \psi_{ed}, \psi_{eq}$$

We'll get to the exciter and governor shortly

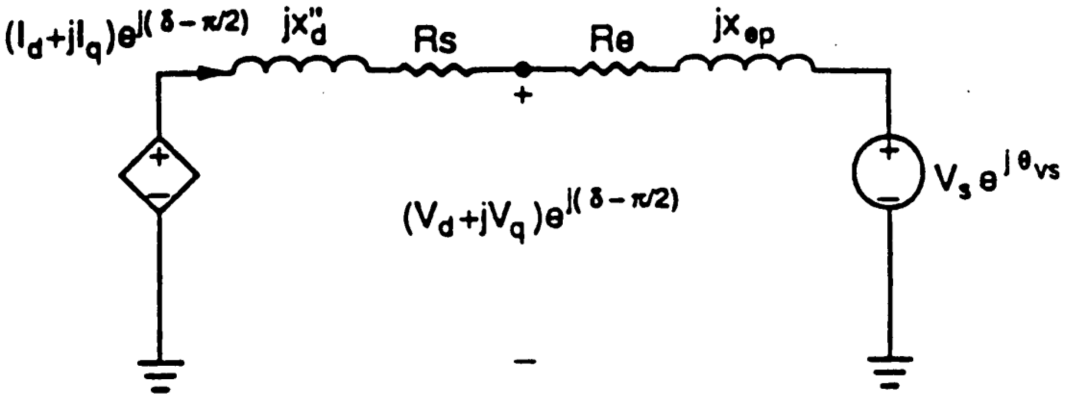
Stator Flux Expressions

$$\psi_{de} = -X''_{de} I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$\psi_{qe} = -X''_{qe} I_q - \frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$\psi_{oe} = -X_{oe} I_o$$

Subtransient Algebraic Circuit



$$\left[\left(\frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)}E'_d - \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)}\psi_{2q} + \left(X''_q - X''_d\right)I_q\right)\right.$$

$$\left.+j\left(\frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)}E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)}\psi_{1d}\right)\right]e^{j(\delta-\pi/2)}$$

Network Reference Frame

- In transient stability the initial generator values are set from a power flow solution, which has the terminal voltage and power injection
  - Current injection is just conjugate of Power/Voltage
- These values are on the network reference frame, with the angle given by the slack bus angle

$$\bar{V}_j = V_{r,j} + jV_{i,j} \quad \text{or} \quad \bar{V}_j = V_{Dj} + jV_{Qj}$$

- Voltages at bus j converted to d-q reference by

$$\begin{bmatrix} V_{d,j} \\ V_{q,j} \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_{r,j} \\ V_{i,j} \end{bmatrix} \quad \begin{bmatrix} V_{r,j} \\ V_{i,j} \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_{d,j} \\ V_{q,j} \end{bmatrix}$$

Network Reference Frame

- Issue of calculating δ, which is key, will be considered for each model
- Starting point is the per unit stator voltages

$$V_d = -\psi_q \omega - R_s I_d$$

$$V_q = \psi_d \omega - R_s I_q$$

Equivalently, $\left(\mathrm{V_d} + \mathrm{jV_q}\right) + R_s\left(I_d + \mathrm{jI_q}\right) = \omega\left(-\psi_q + j\psi_d\right)$

- Sometimes the scaling of the flux by the speed is neglected, but this can have a major solution impact
- In per unit the initial speed is unity

Simplified Machine Models

- Often more simplified models were used to represent synchronous machines
- These simplifications are becoming much less common but they are still used in some situations and can be helpful for understanding generator behavior
- Next several slides go through how these models can be simplified, then we'll cover the standard industrial models

Two-Axis Model

- If we assume the damper winding dynamics are sufficiently fast, then T''_{do} and T''_{qo} go to zero, so there is an integral manifold for their dynamic states

$$\psi_{1d} = E'_q - \left(X'_d - X_{\ell s} \right) I_d$$

$$\psi_{2q} = -E'_d - \left(X'_q - X_{\ell s} \right) I_q$$

Two-Axis Model

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right)I_d = 0$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)\times$$

$$\left[I_d - \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left(\psi_{1d} + \left(X'_d - X_{\ell s}\right)I_d - E'_q\right)\right] + E_{fd}$$

Note this entire term becomes zero

Which can be simplified to

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)I_d + E_{fd}$$

Two-Axis Model

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right)I_q = 0$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\times$$

$$\left[I_q - \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

Likewise this entire term becomes zero

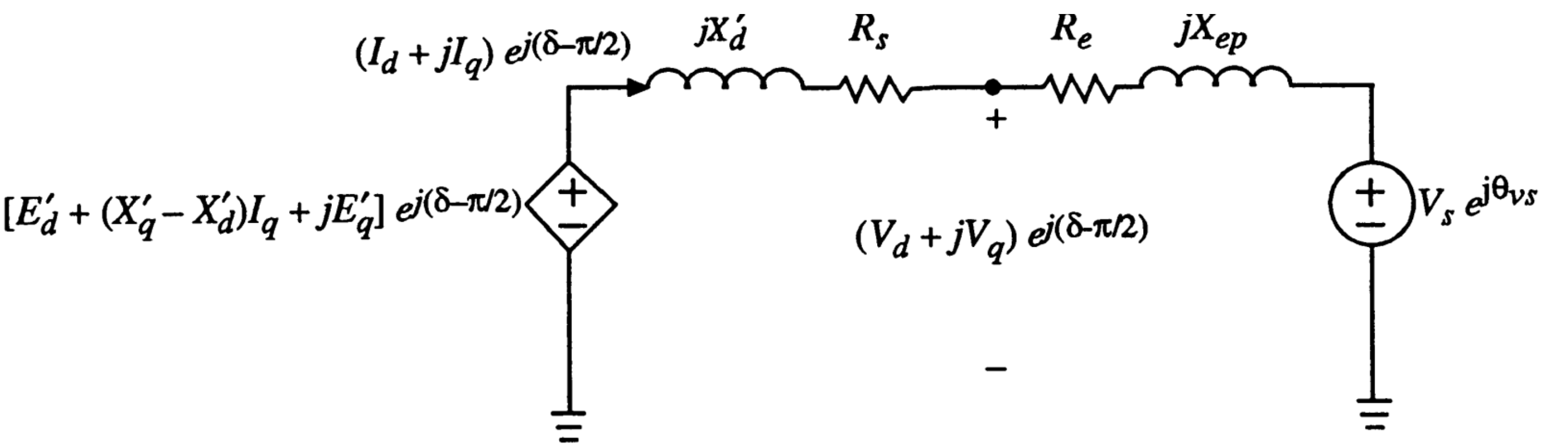
Which can simplified to

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + I_q\left(X_q - X'_q\right)$$

Two-Axis Model

$$0 = (R_s + R_e)I_d - (X'_q + X_{ep})I_q - E'_d + V_s \sin(\delta - \theta_{vs})$$

$$0 = (R_s + R_e)I_q + (X'_d + X_{ep})I_d - E'_q + V_s \cos(\delta - \theta_{vs})$$



Two-Axis Model

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)I_d + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

$$0 = \left(R_s + R_e\right)I_d - \left(X'_q + X_{ep}\right)I_q - E'_d + V_s \sin\left(\delta - \theta_{vs}\right)$$

$$0 = \left(R_s + R_e\right)I_q + \left(X'_d + X_{ep}\right)I_d - E'_q + V_s \cos\left(\delta - \theta_{vs}\right) \backslash$$

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin\left(\delta - \theta_{vs}\right)$$

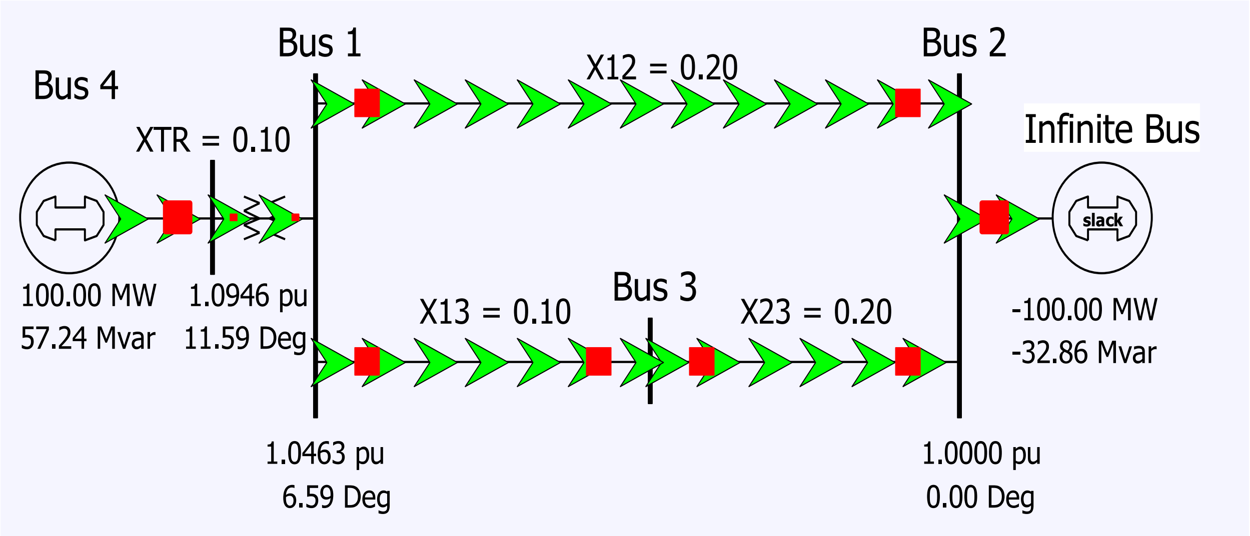
$$V_q = R_e I_q + X_{ep} I_d + V_s \cos\left(\delta - \theta_{vs}\right)$$

$$V_t = \sqrt{V_d^2 + V_q^2}$$

No saturation effects are included with this model

Example (Used for All Models)

- Below example will be used with all models. Assume a 100 MVA base, with gen supplying 1.0+j0.3286 power into infinite bus with unity voltage through network impedance of j0.22
  - Gives current of 1.0 - j0.3286 = 1.0526∠-18.19 °
  - Generator terminal voltage of 1.072+j0.22 = 1.0946 ∠11.59 °



Sign convention on current is out of the generator is positive

Two-Axis Example

- For the two-axis model assume H = 3.0 per unit-seconds, R_s=0, X_d = 2.1, X_q = 2.0, X'_d= 0.3, X'_q = 0.5, T'_{do} = 7.0, T'_{qo} = 0.75 per unit using the 100 MVA base.
- Solving we get

$$\bar{E} = 1.0946\angle 11.59° + (j2.0)(1.0526\angle -18.19°) = 2.81\angle 52.1°$$

$$\rightarrow \delta = 52.1°$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

Two-Axis Example

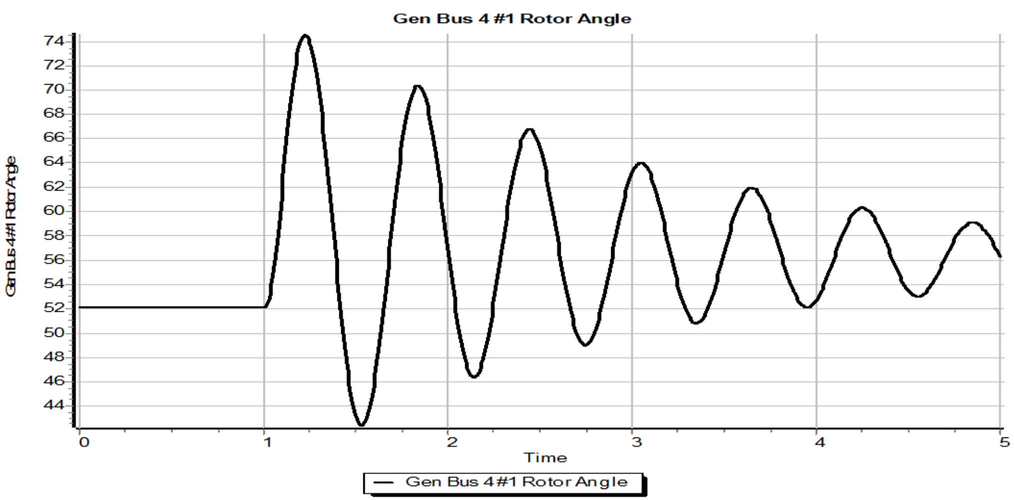
- And $E'_q = 0.8326 + (0.3)(0.9909) = 1.130$

$$E'_d = 0.7107 - (0.5)(0.3553) = 0.533$$

$$E_{fd} = 1.1299 + (2.1 - 0.3)(0.9909) = 2.913$$

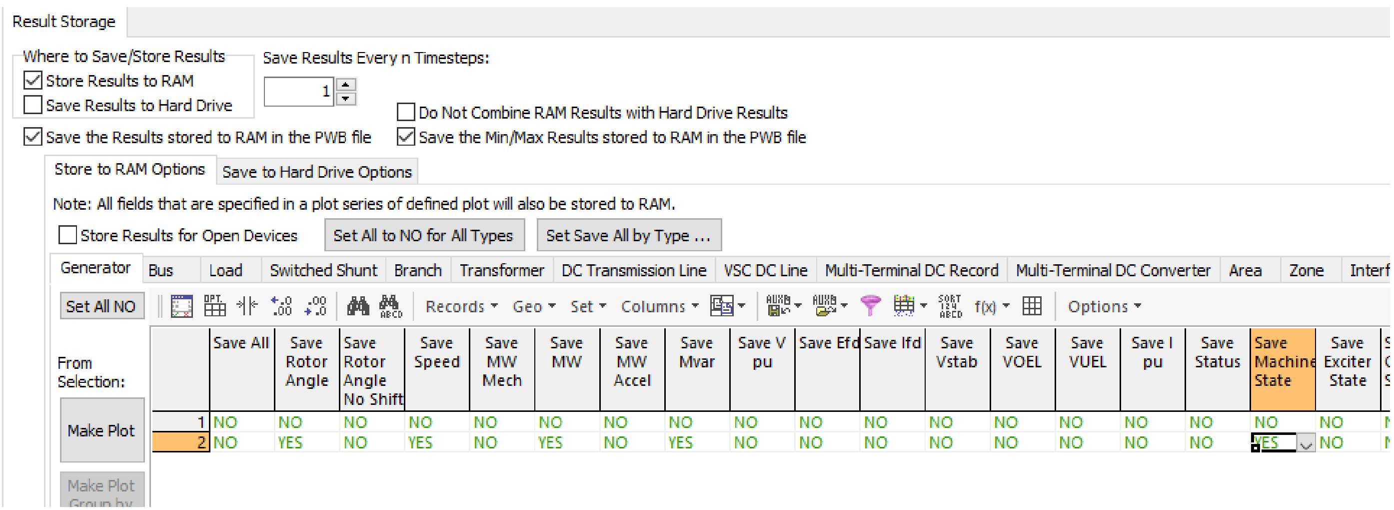
Saved as case **B4_TwoAxis**

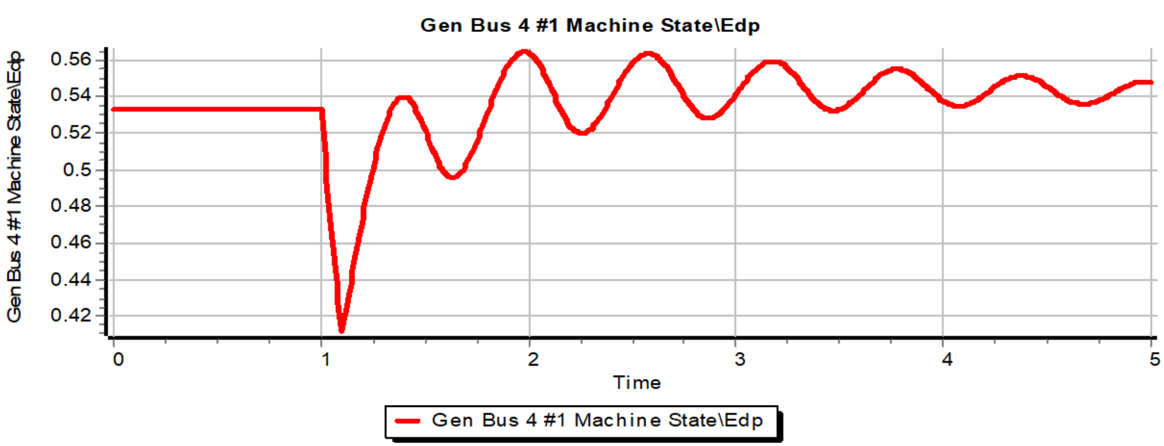
- Assume a fault at bus 3 at time t=1.0, cleared by opening both lines into bus 3 at time t=1.1 seconds



Two-Axis Example

- PowerWorld allows the gen states to be easily stored





Graph shows variation in E_d'

Flux Decay Model

- If we assume T'_{qo} is sufficiently fast that its equation becomes an algebraic constraint

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q = 0$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)I_d + E_{fd}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

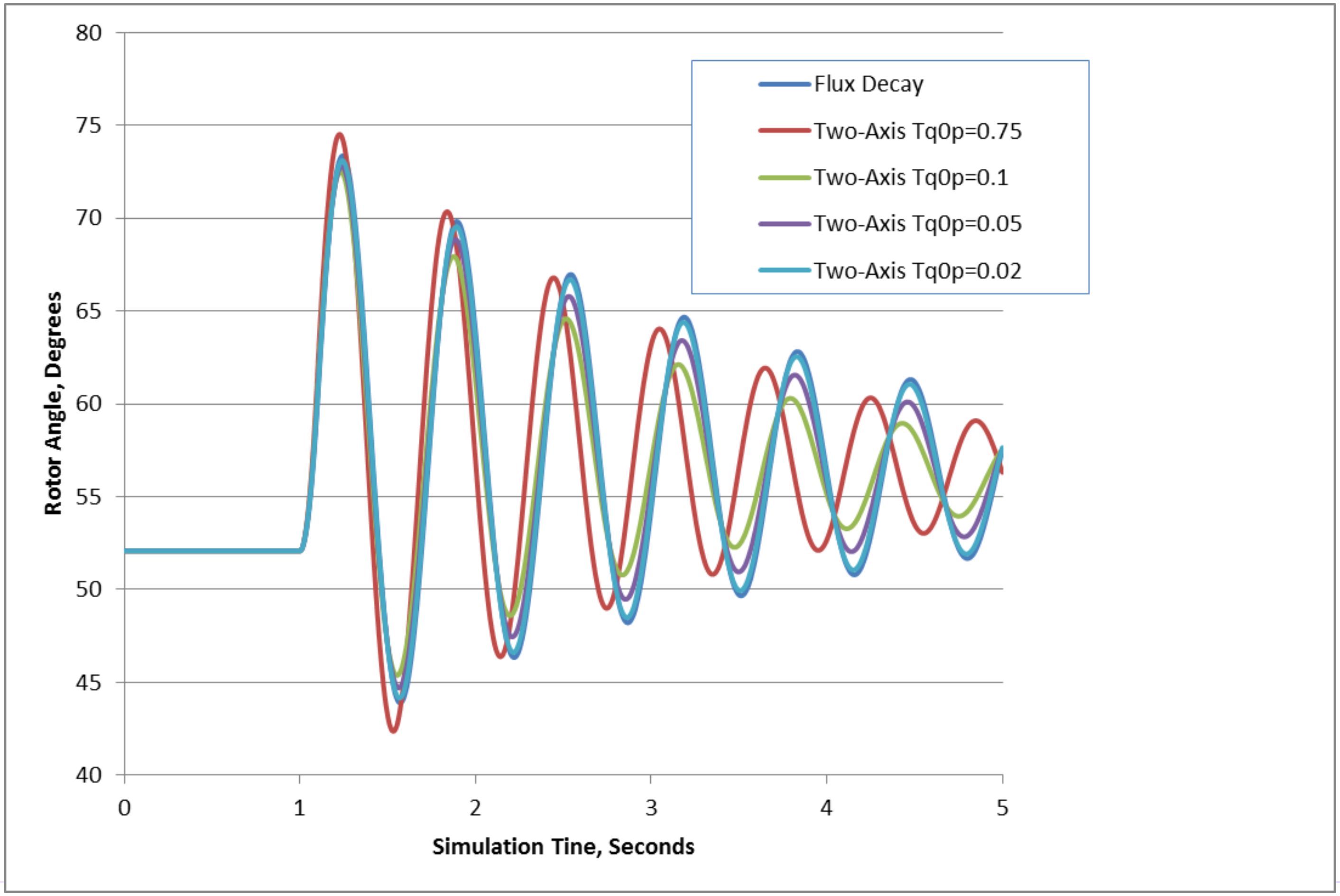
$$= T_M - \left(X_q - X'_q\right)I_q I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

$$= T_M - E'_q I_q - \left(X_q - X'_d\right)I_d I_q - T_{FW}$$

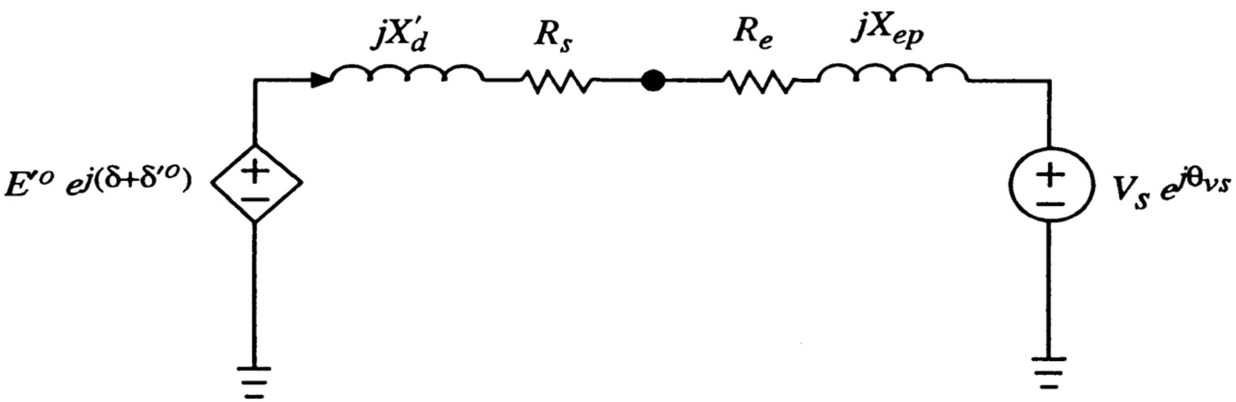
This model assumes that E_d' stays constant. In previous example T_{q0}'=0.75

Rotor Angle Sensitivity to Tqop

- Graph shows variation in the rotor angle as Tqop is varied, showing the flux decay is the same as Tqop = 0



Classical Model



$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_0}\frac{d\omega}{dt} = T_M^0 - \frac{E'^0 V_s}{X'_d + X_{ep}}\sin\left(\delta - \theta_{vs}\right) - T_{FW}$$

This is a pendulum model

The classical model had been widely used because it is simple. At best it can only approximate a very short term response. It is no longer common.

Classical Model Justification

- It is difficult to justify. One approach would be to go from the flux decay model and assume

$$X_q = X'_d \qquad T'_{do} = \infty$$

$$E' = E'_q \qquad \delta'^0 = 0$$

- Or go back to the two-axis model and assume

$$X'_q = X'_d \qquad T'_{do} = \infty \qquad T'_{qo} = \infty$$

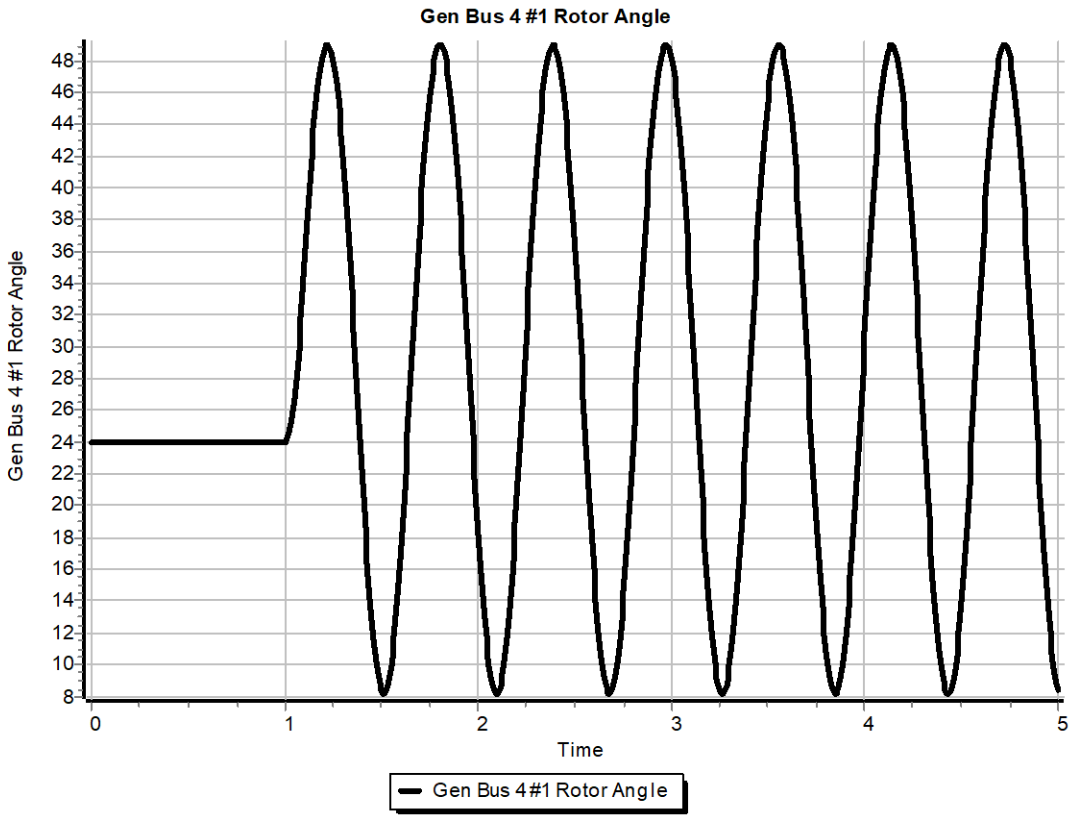
$$(E'_q = \text{const} \quad E'_d = \text{const})$$

$$E' = \sqrt{{E'^0_q}^2 + {E'^0_d}^2}$$

$$\delta'^0 = \tan^{-1}\left(E'^0_q \big/ E'^0_d \right) - \pi/2$$

Classical Model Response

- Rotor angle variation for same fault as before



Notice that even though the rotor angle is quite different, its initial increase (of about 24 degrees) is similar. However there is no damping.

Saved as case **B4_GENCLS**