ECEN 615
Methods of Electric Power Systems Analysis

Lecture 17: Sensitivity Methods

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Announcements

- Read Chapter 7 from the book (the term reliability is now used instead of security)
- Homework 4 is due on Thursday October 31.

Sensitivity Problem Formulation

- Denote the system state by

$$\mathbf{x} \triangleq \begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{V} \end{bmatrix} \qquad \boldsymbol{\theta} \triangleq [\theta^1, \theta^2, \cdots, \theta^N]^T$$

$$\mathbf{V} \triangleq [V^1, V^2, \cdots, V^N]^T$$

- Denote the conditions corresponding to the existing commitment/dispatch by $\mathbf{s}^{(0)}$, $\mathbf{p}^{(0)}$ and $\mathbf{f}^{(0)}$ so that

$$\begin{cases} \mathbf{g}(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}) = \mathbf{0} \\ \mathbf{f}^{(0)} = \mathbf{h}(\mathbf{x}^{(0)}) \end{cases}$$

the power flow equations

line real power flow vector

Sensitivity Problem Formulation

$$\mathbf{g(x,p)} = \begin{bmatrix} \mathbf{g}^{P}\mathbf{(x,p)} \\ \mathbf{g}^{Q}\mathbf{(x,p)} \end{bmatrix}$$

g includes the real and reactive power balance equations

$$g_k^P(\underline{s}, \underline{p}) = V^k \sum_{m=1}^{N} \left(V^m \left[G_{km} cos\left(\theta^k - \theta^m\right) + B_{km} sin\left(\theta^k - \theta^m\right) \right] \right) - p^k$$

$$g_k^Q(\underline{s}, \underline{p}) = V^m \sum_{m=1}^{N} \left(V^m \left[G_{km} sin\left(\theta^k - \theta^m\right) - B_{km} cos\left(\theta^k - \theta^m\right) \right] \right) - q^k$$

$$h_\ell(\underline{s}) = g_\ell \left[\left(V^i\right)^2 - V^i V^j cos(\theta^i - \theta^j) \right] - b_\ell V^i V^j sin(\theta^i - \theta^j), \;\; \ell = \left(i, j\right)$$

Sensitivity Problem Formulation

- For a small change, $\Delta\mathbf{p}$, that moves the injection from $\mathbf{p}^{(0)}$ to $\mathbf{p}^{(0)} + \Delta\mathbf{p}$, we have a corresponding change in the state $\Delta\mathbf{x}$ with

$$\mathbf{g}\,(\mathbf{x}^{(0)} + \Delta\mathbf{x}, p^{(0)} + \Delta\mathbf{p}) = \mathbf{0}$$

- We then apply a first order Taylor's series expansion

$$\mathbf{g}\left(\mathbf{x}^{(0)} + \Delta\mathbf{x}, \mathbf{p}^{(0)} + \Delta\mathbf{p}\right) = \mathbf{g}\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right) + \left.\frac{\partial\mathbf{g}}{\partial\mathbf{x}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta\mathbf{x}$$

$$+ \left.\frac{\partial\mathbf{g}}{\partial\mathbf{p}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta p + h.o.t.$$

Sensitivity Problem Formulation

- We consider this to be a "small signal" change, so we can neglect the higher order terms (h.o.t.) in the expansion
- Hence we should still be satisfying the power balance equations with this perturbation; so

$$\left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta \mathbf{x} + \left.\frac{\partial \mathbf{g}}{\partial \mathbf{p}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta \boldsymbol{p} \approx \mathbf{0}$$

Sensitivity Problem Formulation

- Also, from the power flow equations, we obtain

$$\frac{\partial \mathbf{g}}{\partial \mathbf{p}} = \begin{bmatrix} \dfrac{\partial \mathbf{g}^{P}}{\partial \mathbf{p}} \\ \hdashline \dfrac{\partial \mathbf{g}^{Q}}{\partial \mathbf{p}} \end{bmatrix} = \begin{bmatrix} -\mathbf{I} \\ \hdashline \mathbf{0} \end{bmatrix}$$

and then just the power flow Jacobian

$$\frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{g}^{P}}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{g}^{P}}{\partial \mathbf{V}} \\ \dfrac{\partial \mathbf{g}^{Q}}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{g}^{Q}}{\partial \mathbf{V}} \end{bmatrix} = \mathbf{J}(\mathbf{x},\mathbf{p})$$

Sensitivity Problem Formulation

- With the standard assumption that the power flow Jacobian is nonsingular, then

$$\Delta \mathbf{x} \;\approx\; \left[\mathbf{J}\left(\mathbf{x}^{(0)},\mathbf{p}^{(0)}\right)\right]^{-1}\begin{bmatrix}\mathbf{I}\\ \mathbf{0}\end{bmatrix}\Delta \mathbf{p}$$

- We can then compute the change in the line real power flow vector

$$\Delta \mathbf{f} \;\approx\; \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right]^{T}\Delta \mathbf{s} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right]^{T}\left[J\left(\mathbf{x}^{(0)},\mathbf{p}^{(0)}\right)\right]^{-1}\begin{bmatrix}\mathbf{I}\\ \mathbf{0}\end{bmatrix}\Delta \mathbf{p}$$

Sensitivity Comments

- Sensitivities can easily be calculated even for large systems
  - If $\Delta\mathbf{p}$ is sparse (just a few injections) then we can use a fast forward; if sensitivities on a subset of lines are desired we could also use a fast backward
- Sensitivities are dependent upon the operating point
  - They also include the impact of marginal losses
- Sensitivities could easily be expanded to include additional variables in $\mathbf{x}$ (such as phase shifter angle), or additional equations, such as reactive power flow

Sensitivity Comments, cont.

- Sensitivities are used in the optimal power flow; in that context a common application is to determine the sensitivities of an overloaded line to injections at all the buses
- In the below equation, how could we quickly get these values?

$$\Delta \mathbf{f} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right]^T \Delta f \approx \left[\frac{\partial \mathbf{h}}{\partial x}\right]^T \left[\boldsymbol{J}\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right)\right]^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p}$$

  - A useful reference is O. Alsac, J. Bright, M. Prais, B. Stott, “Further Developments in LP-Based Optimal Power Flow,” IEEE. Trans. on Power Systems, August 1990, pp. 697-711; especially see equation 3.

Sensitivity Example in PowerWorld

- Open case **B5_DistFact** and then Select **Tools, Sensitivities, Flow and Voltage Sensitivities**
  - Select **Single Meter, Multiple Transfers**, **Buses** page
  - Select the **Device Type (Line/XFMR)**, **Flow Type (MW)**, then select the line (from Bus 2 to Bus 3)
  - Click **Calculate Sensitivities**; this shows impact of a single injection going to the slack bus (Bus 1)
  - For our example of a transfer from 2 to 3 the value is the result we get for bus 2 (0.5440) minus the result for bus 3 (-0.1808) = 0.7248
  - With a flow of 118 MW, we would hit the 150 MW limit with (150-118)/0.7248 =44.1MW, close to the limit we found of 45MW

Sensitivity Example in PowerWorld

- If we change the conditions to the anticipated maximum loading (changing the load at 2 from 118 to 118+44=162 MW) and we re-evaluate the sensitivity we note it has changed little (from -0.7248 to -0.7241)
  - Hence a linear approximation (at least for this scenario) could be justified
- With what we know so far, to handle the contingency situation, we would have to simulate the contingency, and reevaluate the sensitivity values
  - We'll be developing a quicker (but more approximate) approach next

Linearized Sensitivity Analysis

- By using the approximations from the fast decoupled power flow we can get sensitivity values that are independent of the current state. That is, by using the **B**' and **B**'' matrices
- For line flow we can approximate

$$h_\ell(\underline{s}) = g_\ell\left[\left(V^i\right)^2 - V^i V^j \cos(\theta^i - \theta^j)\right] - b_\ell V^i V^j \sin(\theta^i - \theta^j),\ \ell = (i,j)$$

By using the FDPF appxomations

$$h_\ell(\underline{s}) \approx -b_\ell(\theta^i - \theta^j) = \frac{(\theta^i - \theta^j)}{X_\ell},\ \ell = (i,j)$$

Linearized Sensitivity Analysis

- Also, for each line ℓ

$$\frac{\partial \boldsymbol{h}_\ell}{\partial \boldsymbol{\theta}} \approx -\boldsymbol{b}_\ell \mathbf{a}_\ell \qquad\qquad \frac{\partial \boldsymbol{h}_\ell}{\partial \mathbf{V}} \approx \mathbf{0}$$

and so,

$$\frac{\partial \mathbf{h}}{\partial \mathbf{x}} = \left[\begin{array}{c} \dfrac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}} \\ \hdashline \dfrac{\partial \mathbf{h}}{\partial \mathbf{V}} \end{array}\right] = -\left[\begin{array}{c:c:c} \boldsymbol{b}_{\ell_1} \mathbf{a}_1 & \cdots & \boldsymbol{b}_{\ell_L} \mathbf{a}_L \\ & \cdots & \\ \hdashline \mathbf{0} & & \mathbf{0} \end{array}\right] \approx \left[\begin{array}{c} \mathbf{A}^T \tilde{\mathbf{B}} \\ \mathbf{0} \\ \hdashline \end{array}\right]$$

Sensitivity Analysis: Recall the Matrix Notation

- The series admittance of line ℓ is $g_\ell + jb_\ell$ and we define $\tilde{\mathbf{B}} \triangleq -diag\{b_1, b_2, \cdots, b_L\}$
- We define the L×N incidence matrix

$$\mathbf{A} \triangleq \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_L^T \end{bmatrix}$$

where the component j of $\mathbf{a}_i$ is nonzero whenever line ℓ_i is coincident with node j. Hence **A** is quite sparse, with at most two nonzeros per row

Linearized Active Power Flow Model

- Under these assumptions the change in the real power line flows are given as

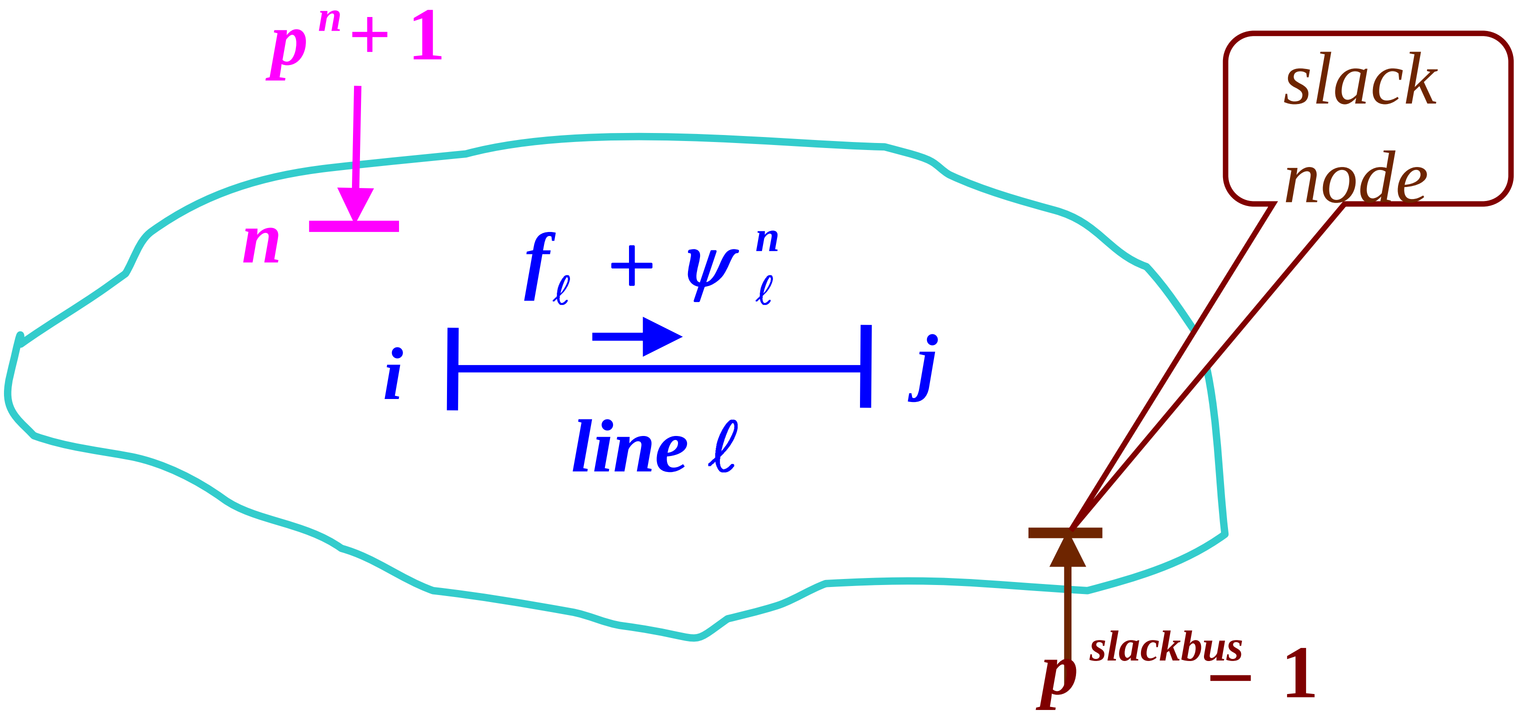
$$\Delta \mathbf{f} \approx \begin{bmatrix} \tilde{\mathbf{B}}\mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{B}' & \mathbf{0} \\ \mathbf{0} & \mathbf{B}'' \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p} = \underbrace{\tilde{\mathbf{B}}\mathbf{A}\left[\mathbf{B}'\right]^{-1} \Delta \mathbf{p}} = \mathbf{\Psi} \Delta \mathbf{p}$$

- The constant matrix $\mathbf{\Psi} \triangleq \tilde{\mathbf{B}}\mathbf{A}\left[\mathbf{B}'\right]^{-1}$ is called the injection shift factor matrix (ISF)

Injection Shift Factors (ISFs)

- The element ψ_{ℓ}^{n} in row ℓ and column n of Ψ is called the injection shift factor (*ISF)* of line ℓ with respect to the injection at node *n*
  - Absorbed at the slack bus, so it is slack bus dependent
- Terms generation shift factor (GSF) and load shift factor (LSF) are also used (such as by NERC)
  - Same concept, just a variation in the sign whether it is a generator or a load
  - Sometimes the associated element is not a single line, but rather a combination of lines (an interface)
- Terms used in North America are defined in the NERC glossary (http://www.nerc.com/files/glossary_of_terms.pdf)

ISF Interpretation



ψ_ℓ^n is the fraction of the additional 1 *MW* injection at node n that goes though line ℓ

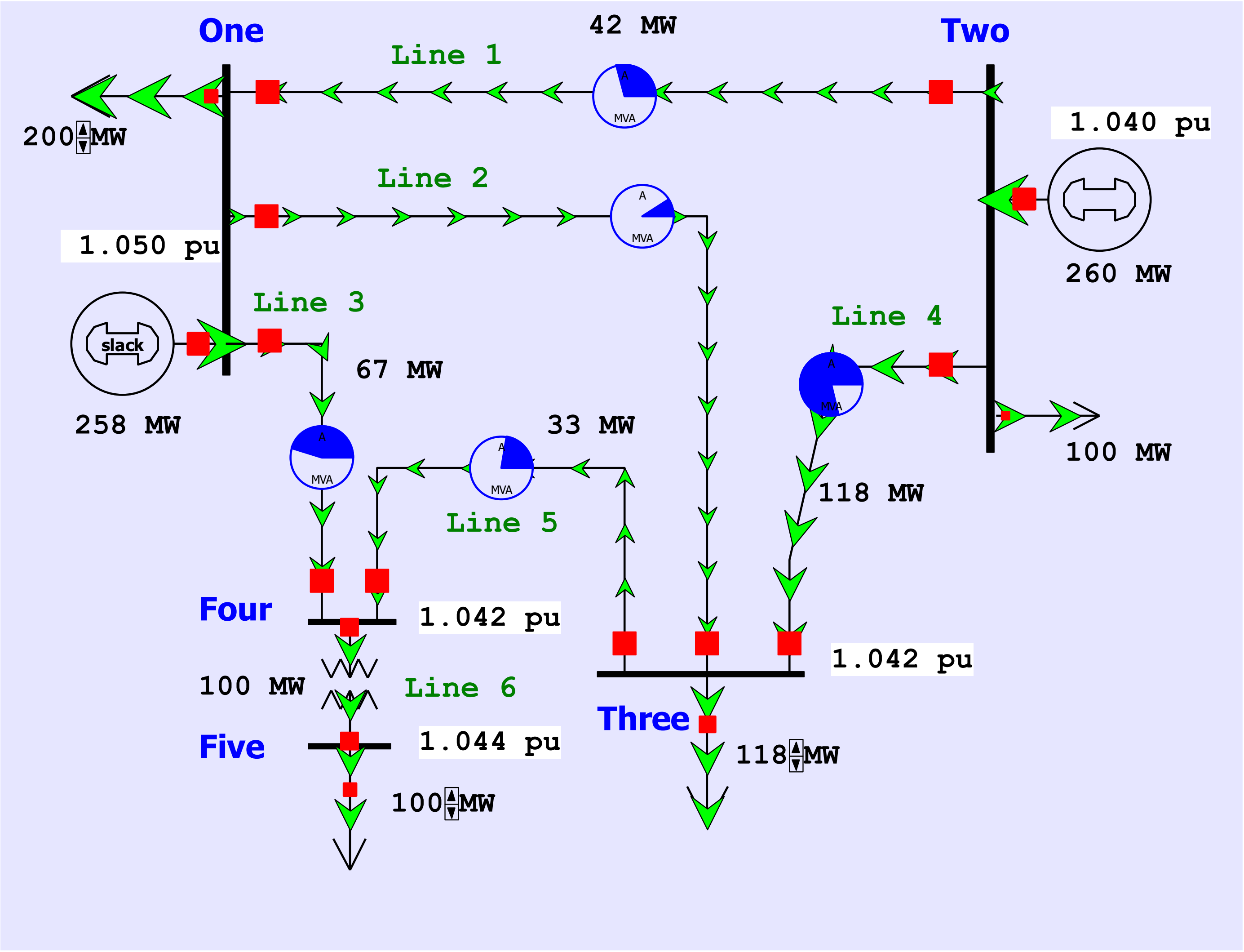
ISF Properties

- By definition, ψ_{ℓ}^{n} depends on the location of the slack bus
- By definition, $\psi_{\ell}^{slackbus} \equiv 0$ for $\forall \ell \in L$ since the injection and withdrawal buses are identical in this case and, consequently, no flow arises on any line ℓ
- The magnitude of ψ_{ℓ}^{n} is at most 1 since

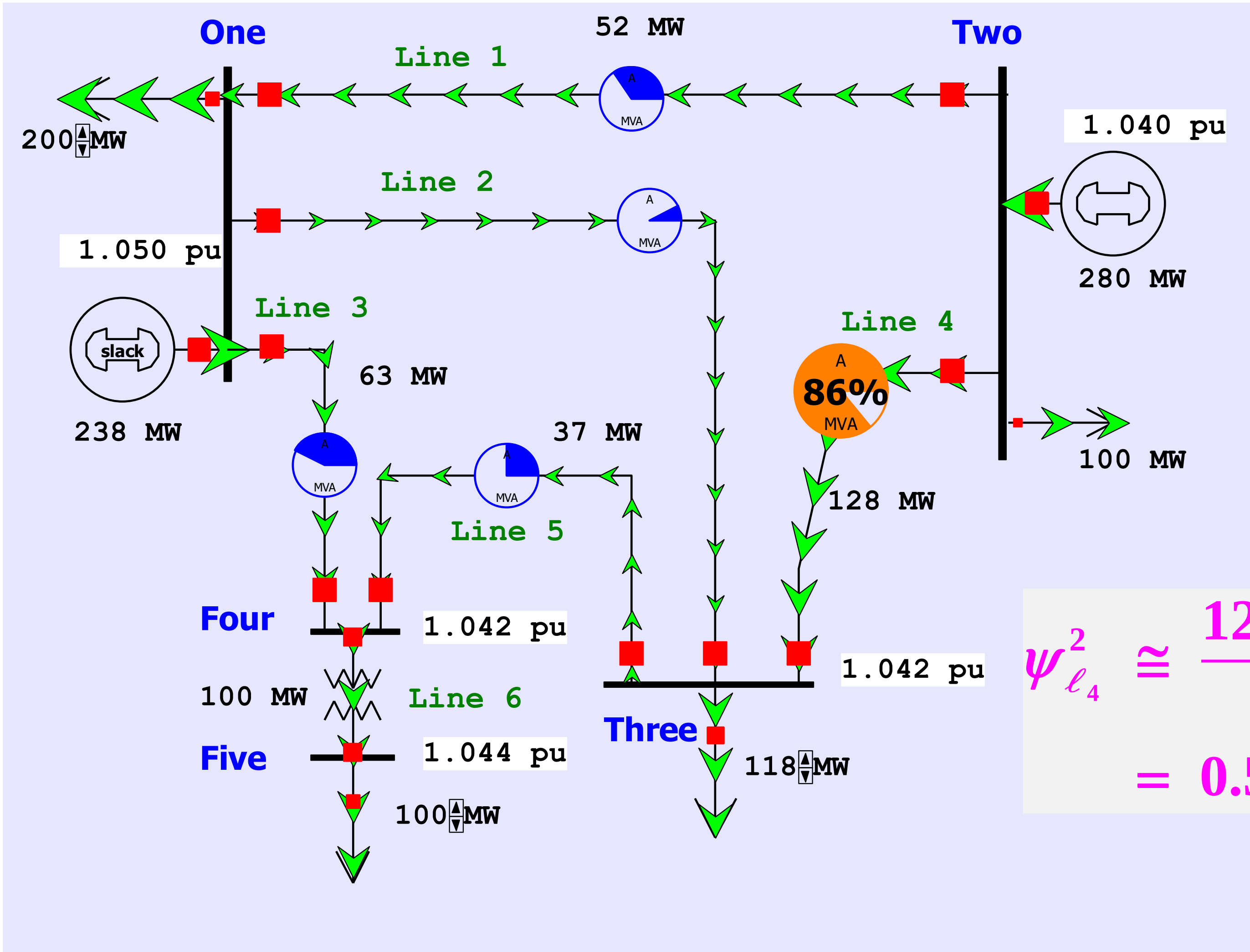
$$-1 \leq \psi_{\ell}^{n} \leq 1$$

Note, this is strictly true only for the linear (lossless) case. In the nonlinear case, it is possible that a transaction decreases losses. Hence a 1 MW injection could change a line flow by more than 1 MW.

Five Bus Example Reference



Five Bus ISF, Line 4, Bus 2 (to Slack)



$$\psi_{\ell_4}^{2} \cong \frac{128-118}{20} = 0.5$$

Five Bus Example

$$\tilde{\mathbf{B}} = -diag\{6.25, \quad 12.5, \quad 12.5, \quad 12.5, \quad 12.5, \quad 10\}$$

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

The row of **A** correspond to the lines and transformers, the columns correspond to the non-slack buses (buses 2 to 5); for each line there is a 1 at one end, a -1 at the other end (hence an assumed sign convention!). Here we put a 1 for the lower numbered bus, so positive flow is assumed from the lower numbered bus to the higher number

Five Bus Example

$$\mathbf{B}' = \mathbf{A}^T \tilde{\mathbf{B}} \mathbf{A} = \begin{bmatrix} -18.75 & 12.5 & 0 & 0 \\ 12.5 & -37.5 & 12.5 & 0 \\ 0 & 12.5 & -35 & 10 \\ 0 & 0 & 10 & -10 \end{bmatrix}$$

$$\underline{\Psi} = \underline{\tilde{B}}\,\underline{A}\left[\underline{B}'\right]^{-1} = \begin{bmatrix} -0.4545 & -0.1818 & -0.0909 & -0.0909 \\ -0.3636 & -0.5455 & -0.2727 & -0.2727 \\ -0.1818 & -0.2727 & -0.6364 & -0.6364 \\ 0.5455 & -0.1818 & -0.0909 & -0.0909 \\ 0.1818 & 0.2727 & -0.3636 & -0.3636 \\ 0 & 0 & 0 & -1.0000 \end{bmatrix}$$

With bus 1 as the slack, the buses (columns) go for 2 to 5

Five Bus Example Comments

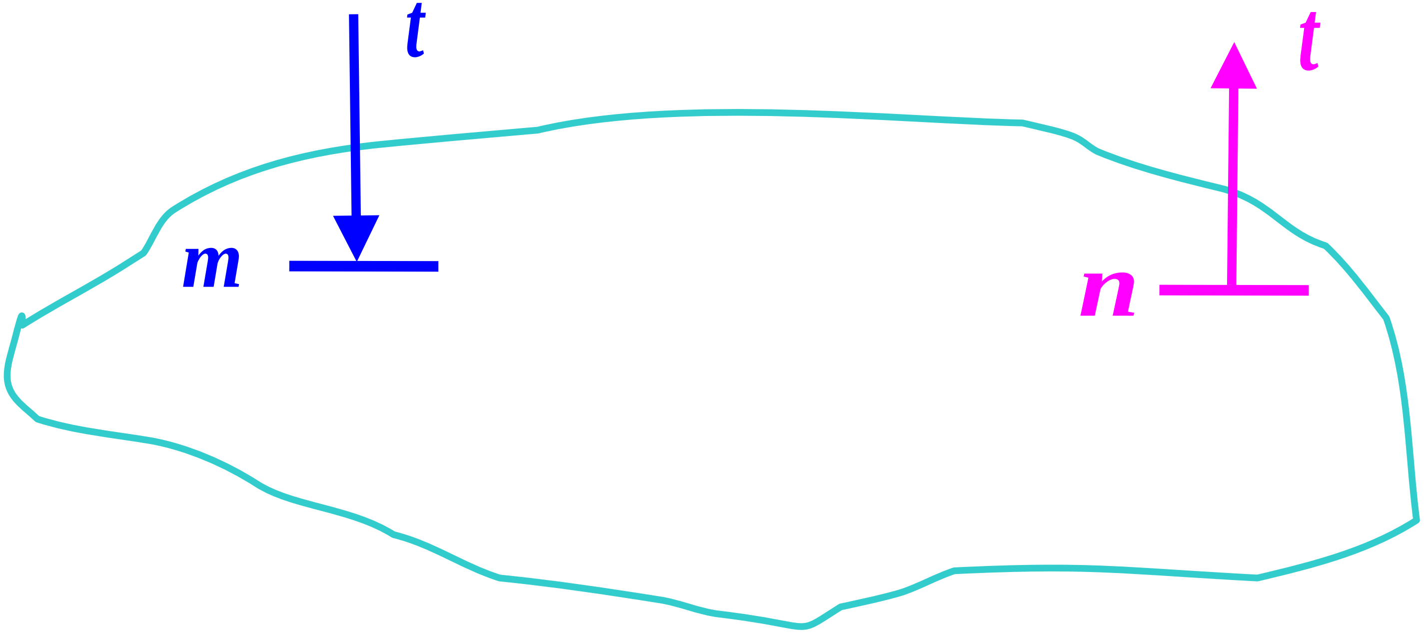
- At first glance the numerically determined value of (128-118)/20=0.5 does not match closely with the analytic value of 0.5455; however, in doing the subtraction we are losing numeric accuracy
  - Adding more digits helps (128.40 – 117.55)/20 = 0.5425
- The previous matrix derivation isn't intended for actual computation; Ψ is a full matrix so we would seldom compute all of its values
- Sparse vector methods can be used if we are only interested in the ISFs for certain lines and certain buses

Distribution Factors

- Various additional distribution factors may be defined
  - power transfer distribution factor (PTDF)
  - line outage distribution factor (LODF)
  - line addition distribution factor (LADF)
  - outage transfer distribution factor (OTDF)
- These factors may be derived from the ISFs making judicious use of the superposition principle

Definition: Basic Transaction

- A basic transaction involves the transfer of a specified amount of power t from an injection node m to a withdrawal node n



Definition: Basic Transaction

- We use the notation

$$\boldsymbol{w} \triangleq \{\boldsymbol{m}, \boldsymbol{n}, \boldsymbol{t}\}$$

injection node

withdrawal node

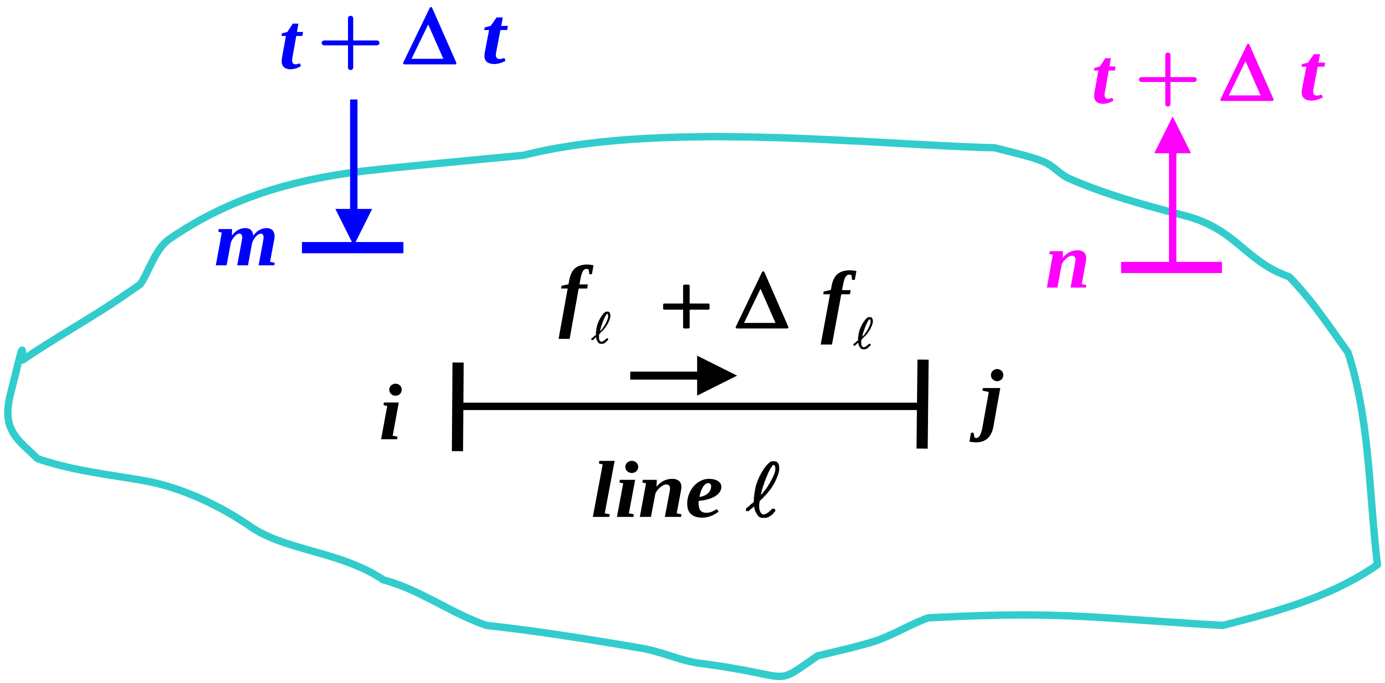
quantity

to denote a basic transaction

Definition: PTDF

- NERC defines a PTDF as
  - "In the pre-contingency configuration of a system under study, a measure of the responsiveness or change in electrical loadings on transmission system Facilities due to a change in electric power transfer from one area to another, expressed in percent (up to 100%) of the change in power transfer"
  - Transaction dependent
- We'll use the notation $\varphi_{\ell}^{(w)}$ to indicate the PTDF on line ℓ with respect to basic transaction w
- In the lossless formulation presented here (and commonly used) it is slack bus independent

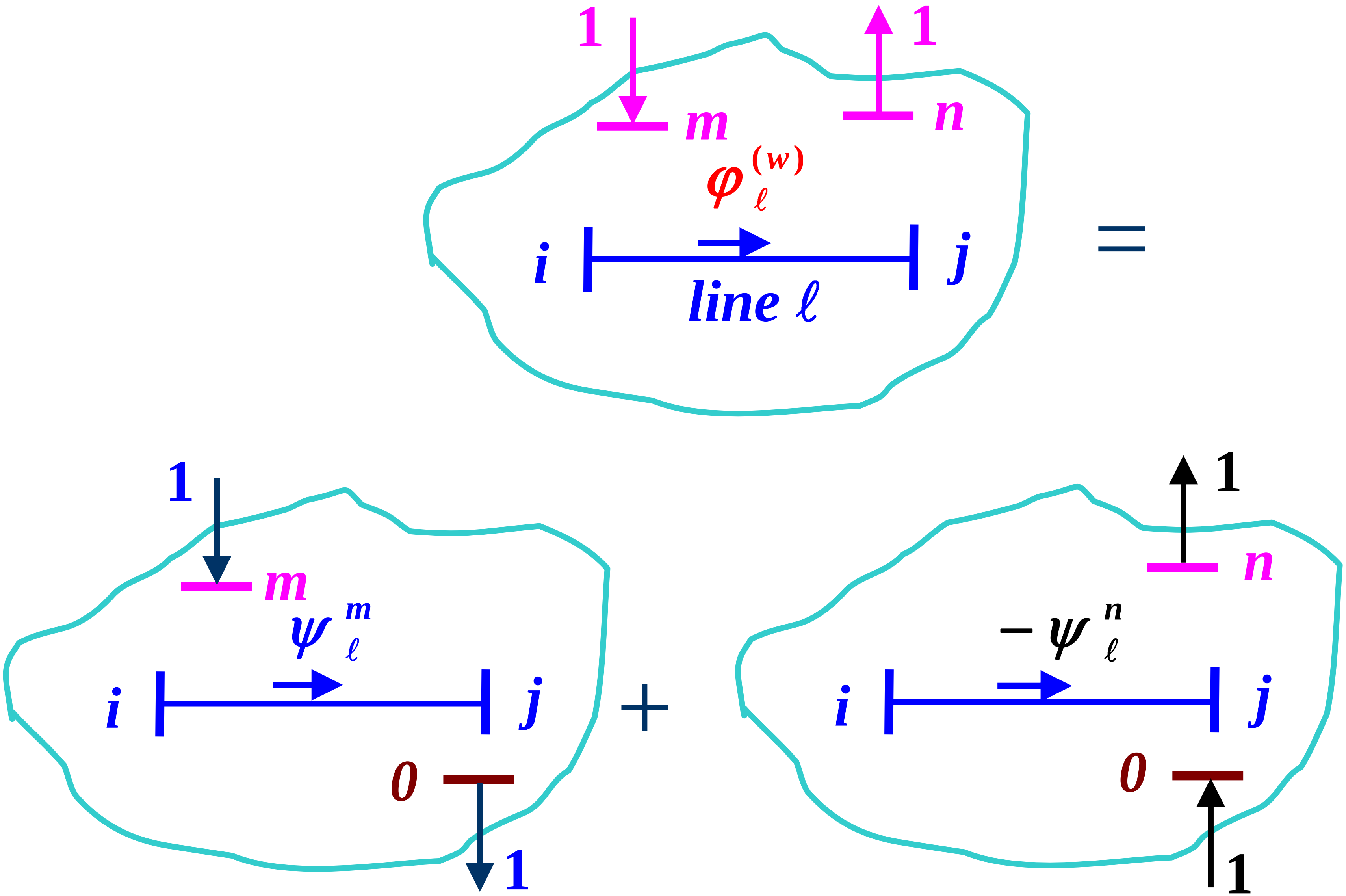
PTDFs



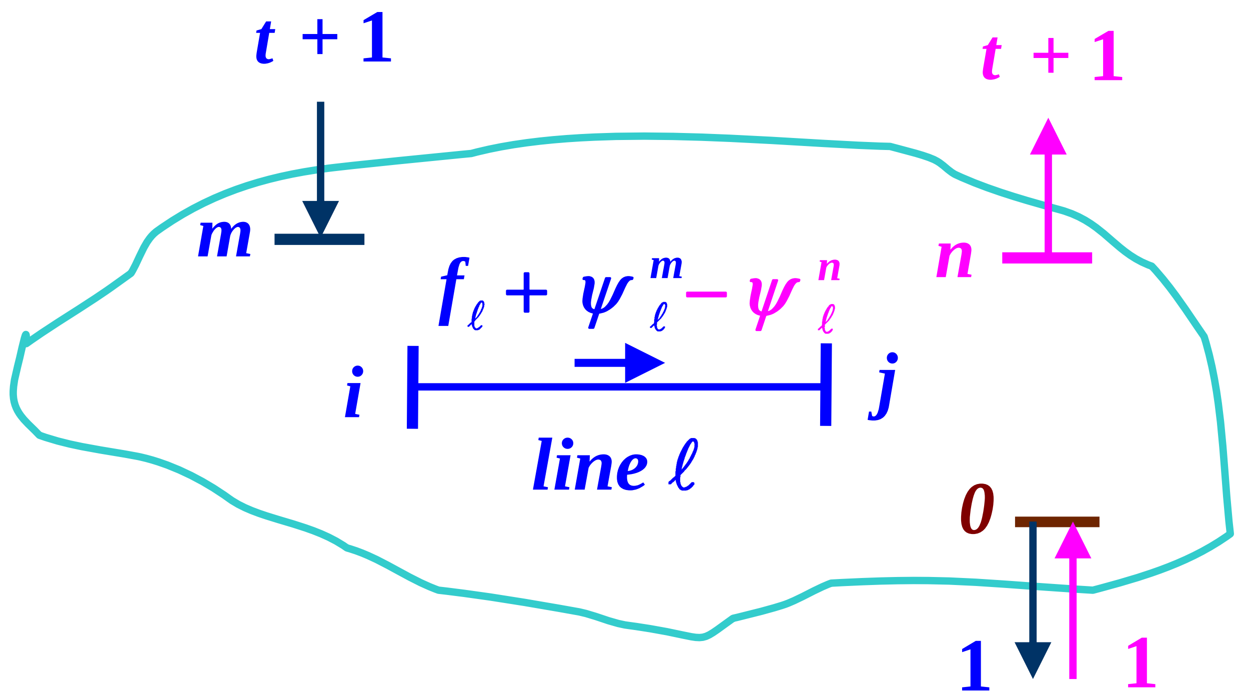
$$\varphi_\ell^{(w)} \triangleq \frac{\Delta f_\ell}{\Delta t}$$

Note, the PTDF is independent of the amount t; which is often expressed as a percent

PTDF Evaluation in Two Parts



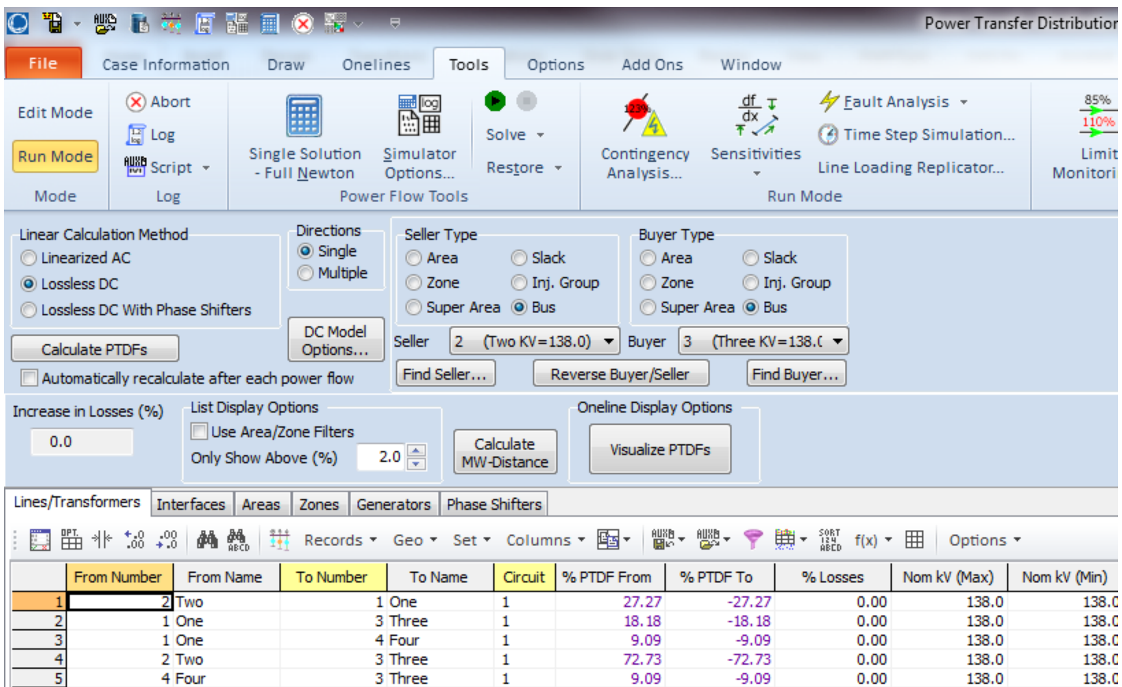
PTDF Evaluation



$$\boxed{\varphi_{\ell}^{(w)} = \psi_{\ell}^{m} - \psi_{\ell}^{n}}$$

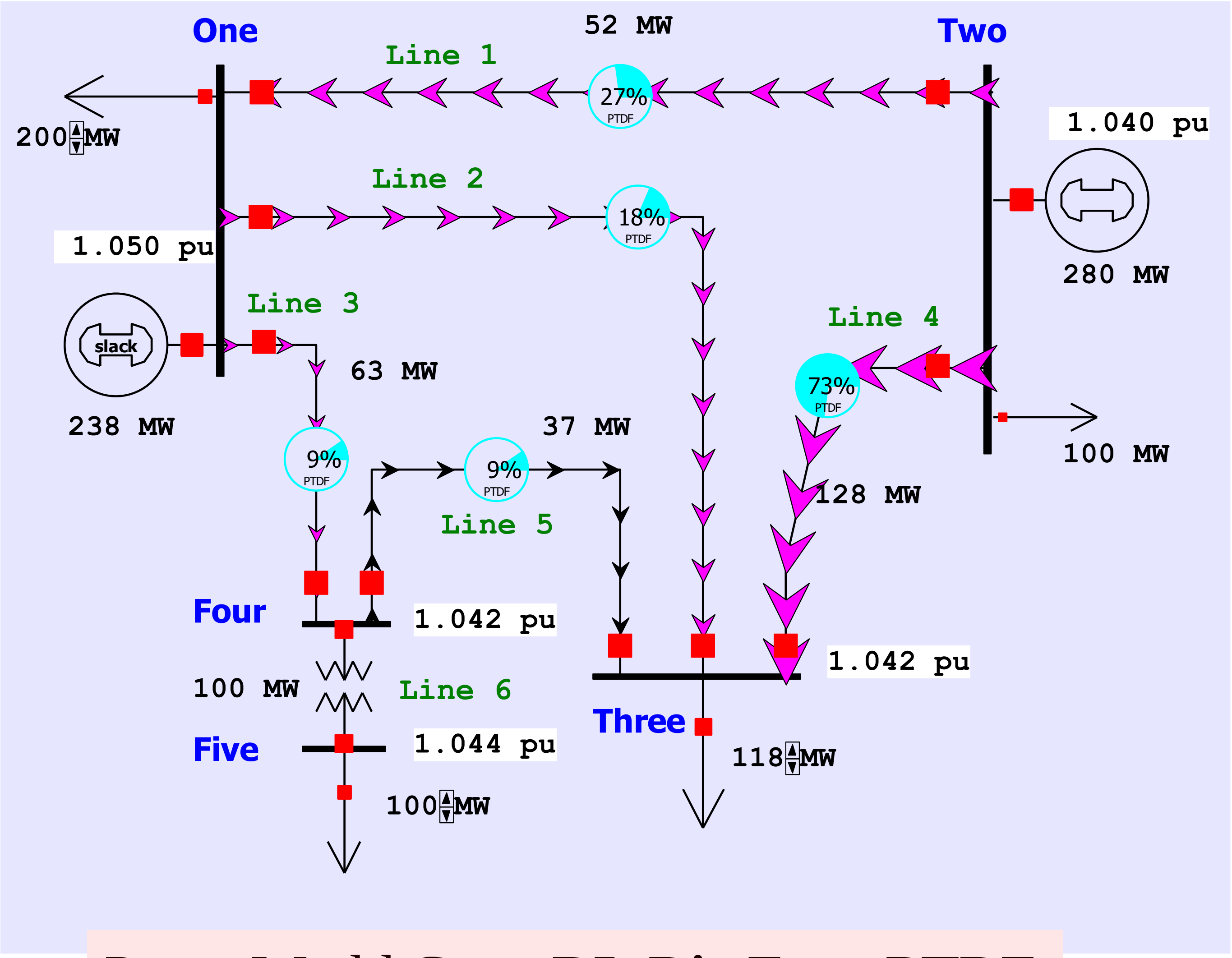
Calculating PTDFs in PowerWorld

- PowerWorld provides a number of options for calculating and visualizing PTDFs
  - Select Tools, Sensitivities, Power Transfer Distribution Factors (PTDFs)



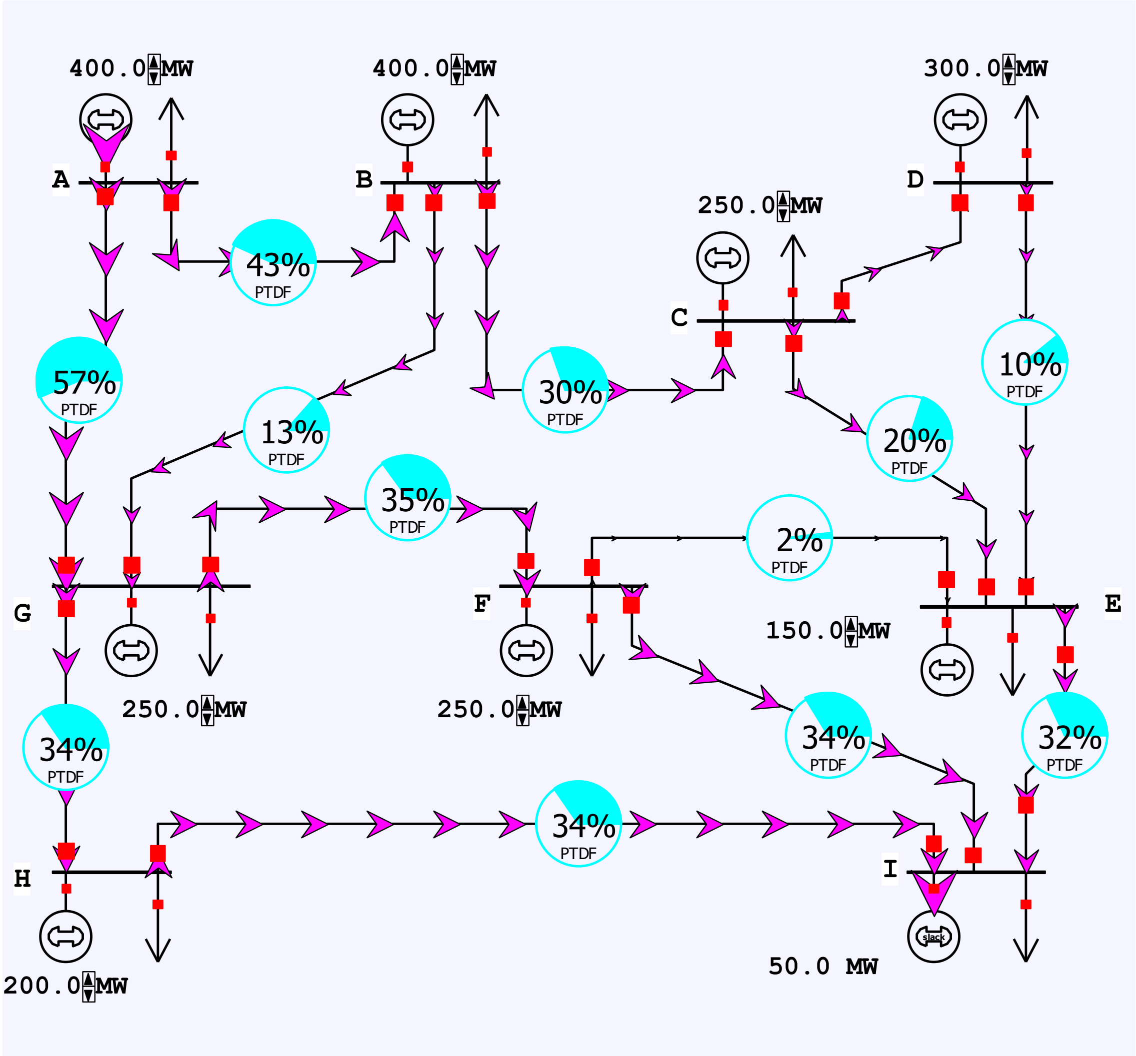
Results are shown for the five bus case for the Bus 2 to Bus 3 transaction

Five Bus PTDF Visualization



PowerWorld Case: **B5_DistFact_PTDF**

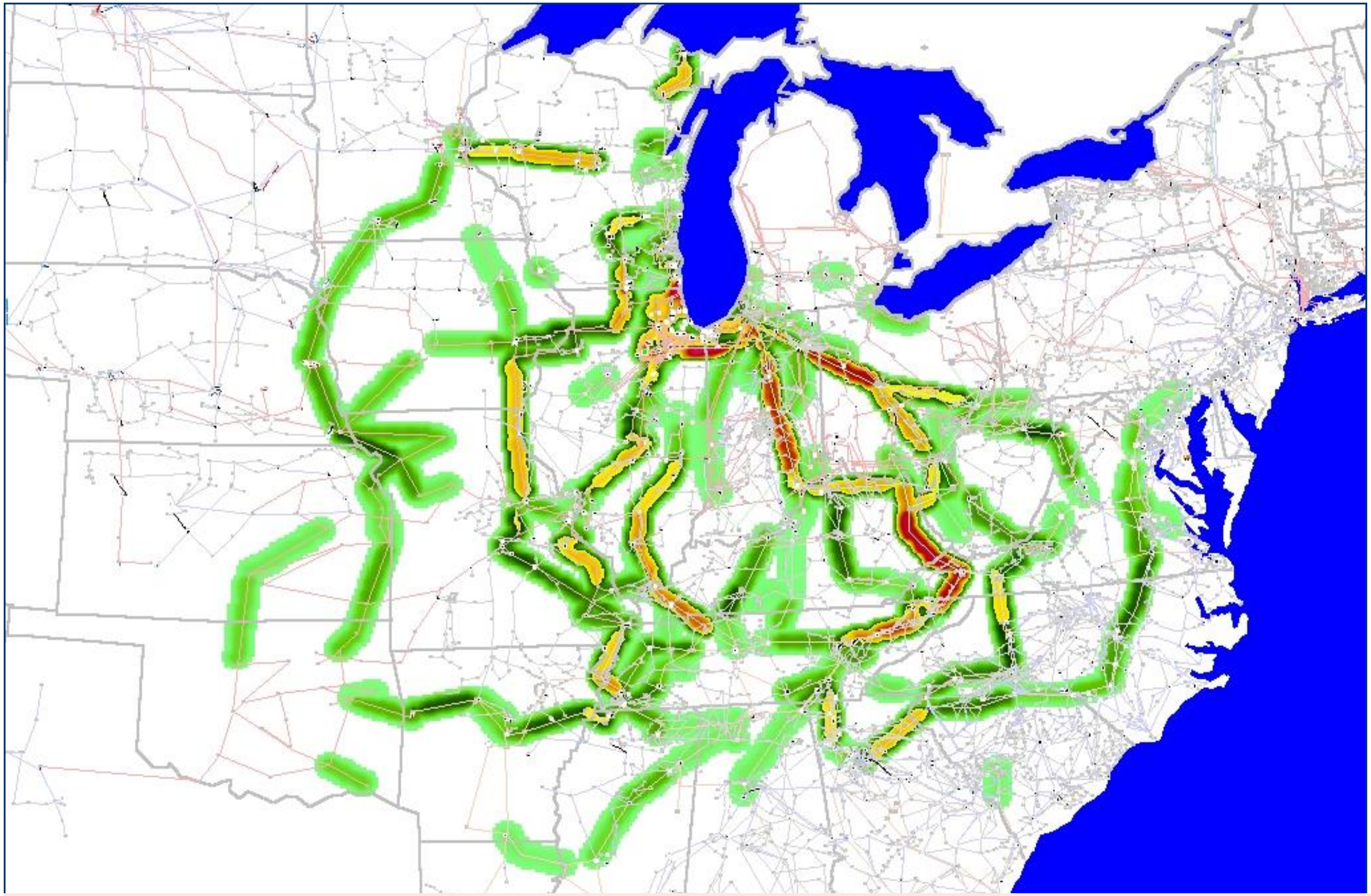
Nine Bus PTDF Example



Display shows the PTDFs for a basic transaction from Bus A to Bus I. Note that 100% of the transaction leaves Bus A and 100% arrives at Bus I

PowerWorld Case: **B9_PTDF**

Eastern Interconnect Example: Wisconsin Utility to TVA PTDFs



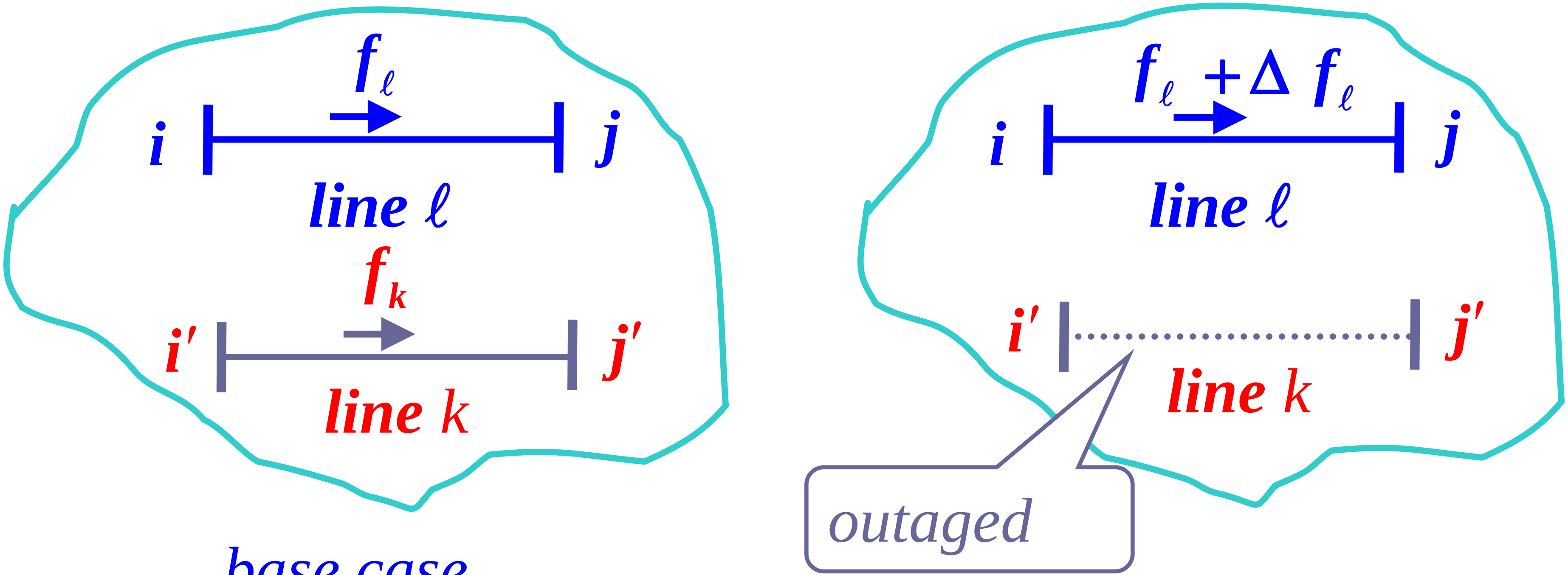
In this example multiple generators contribute for both the seller and the buyer

Contours show lines that would carry at least 2% of a power transfer from Wisconsin to TVA

Line Outage Distribution Factors (LODFs)

- Power system operation is practically always limited by contingencies, with line outages comprising a large number of the contingencies
- Desire is to determine the impact of a line outage (either a transmission line or a transformer) on other system real power flows without having to explicitly solve the power flow for the contingency
- These values are provided by the LODFs
- The LODF $\boldsymbol{d}_{\ell}^{k}$ is the portion of the pre-outage real power line flow on line k that is redistributed to line ℓ as a result of the outage of line k

LODFs



base case

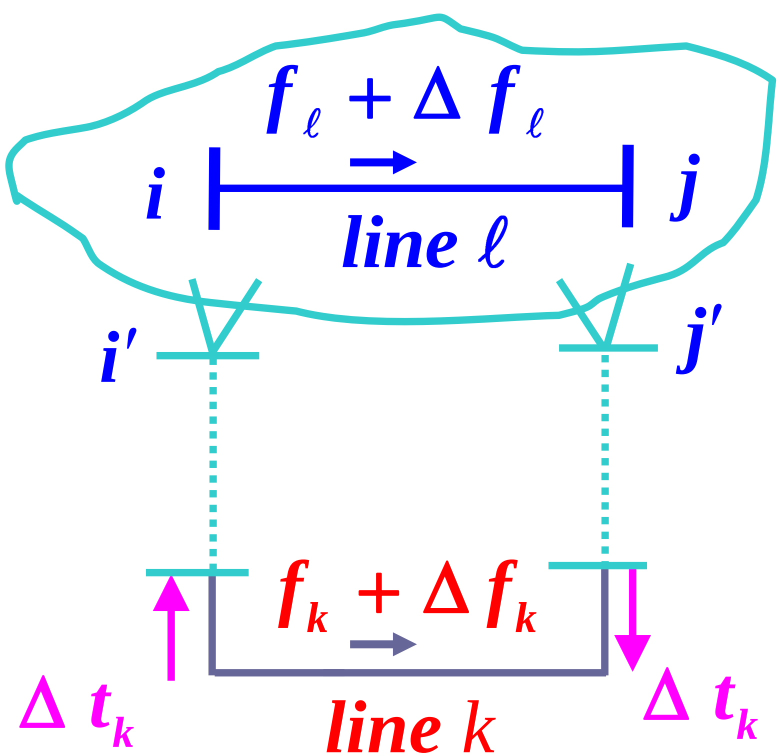
outage case

$$d_\ell^k = \frac{\Delta f_\ell}{f_k} = d_{\ell,k}$$

Best reference is Chapter 7 of the course book

LODF Evaluation

We simulate the impact of the outage of line k by adding the basic transaction $w_k = \{i', j', \Delta t_k\}$



and selecting Δt_k in such a way that the flows on the dashed lines become exactly zero

In general this Δt_k is not equal to the original line flow

LODF Evaluation

- We select Δt_k to be such that

$$f_k + \Delta f_k - \Delta t_k = 0$$

where Δf_k is the active power flow change on the line k due to the transaction w_k

- The line k flow from w_k depends on its PTDF

$$\Delta f_k = \varphi_k^{(w_k)} \Delta t_k$$

it follows that $\Delta t_k = \dfrac{f_k}{1 - \varphi_k^{(w_k)}} = \dfrac{f_k}{1 - \left(\psi_k^{i'} - \psi_k^{j'}\right)}$

LODF Evaluation

- For the rest of the network, the impacts of the outage of line k are the same as the impacts of the additional basic transaction w_k

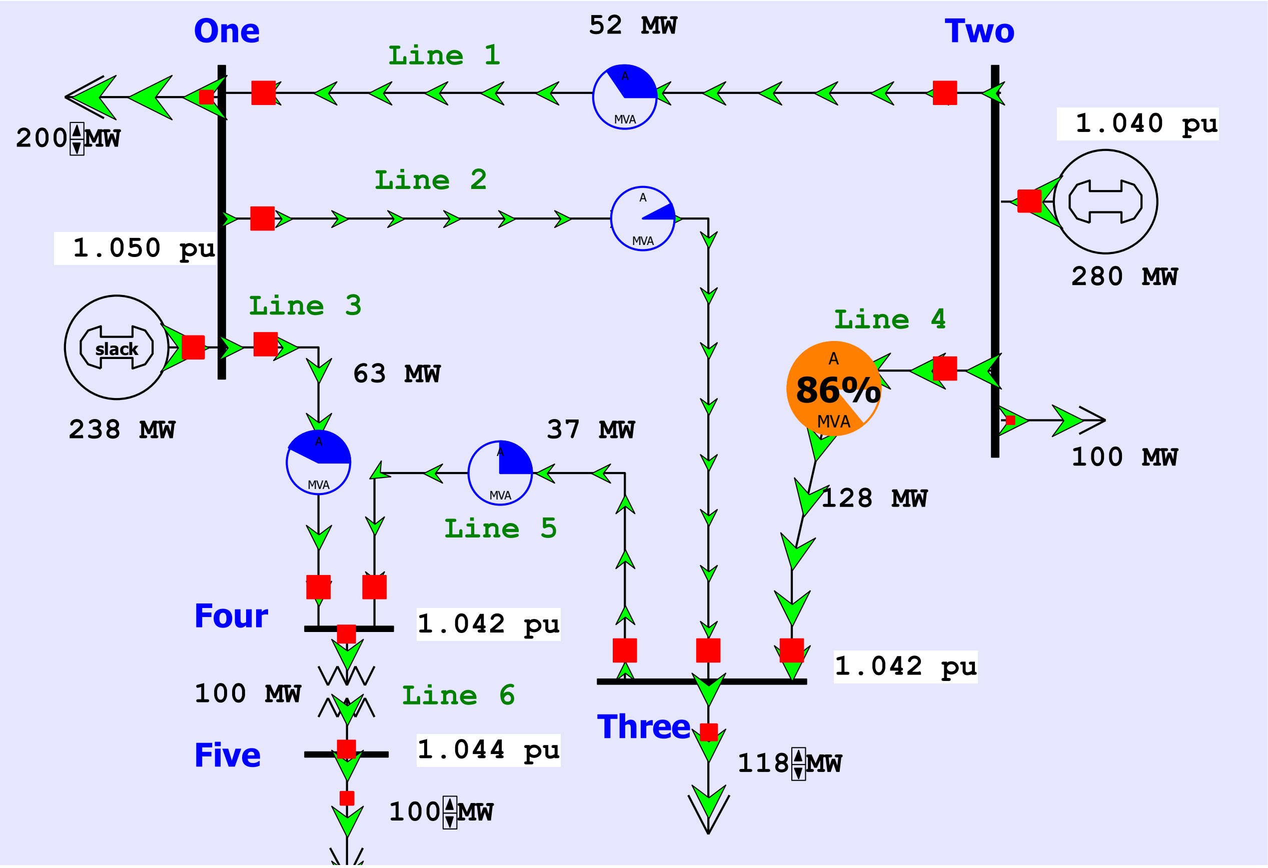
$$\Rightarrow \quad \Delta f_{\ell} = \varphi_{\ell}^{(w_k)} \Delta t_k = \frac{\varphi_{\ell}^{(w_k)}}{1 - \varphi_{k}^{(w_k)}} f_k$$

- Therefore, by definition the LODF is

$$d_{\ell}^{k} = \frac{\Delta f_{\ell}}{f_k} = \frac{\varphi_{\ell}^{(w_k)}}{1 - \varphi_{k}^{(w_k)}}$$

Five Bus Example

- Assume we wish to calculate the values for the outage of line 4 (between buses 2 and 3); this is line k



Say we wish to know the change in flow on the line 3 (Buses 3 to 4). PTDFs for a transaction from 2 to 3 are 0.7273 on line 4 and 0.0909 on line 3

Five Bus Example

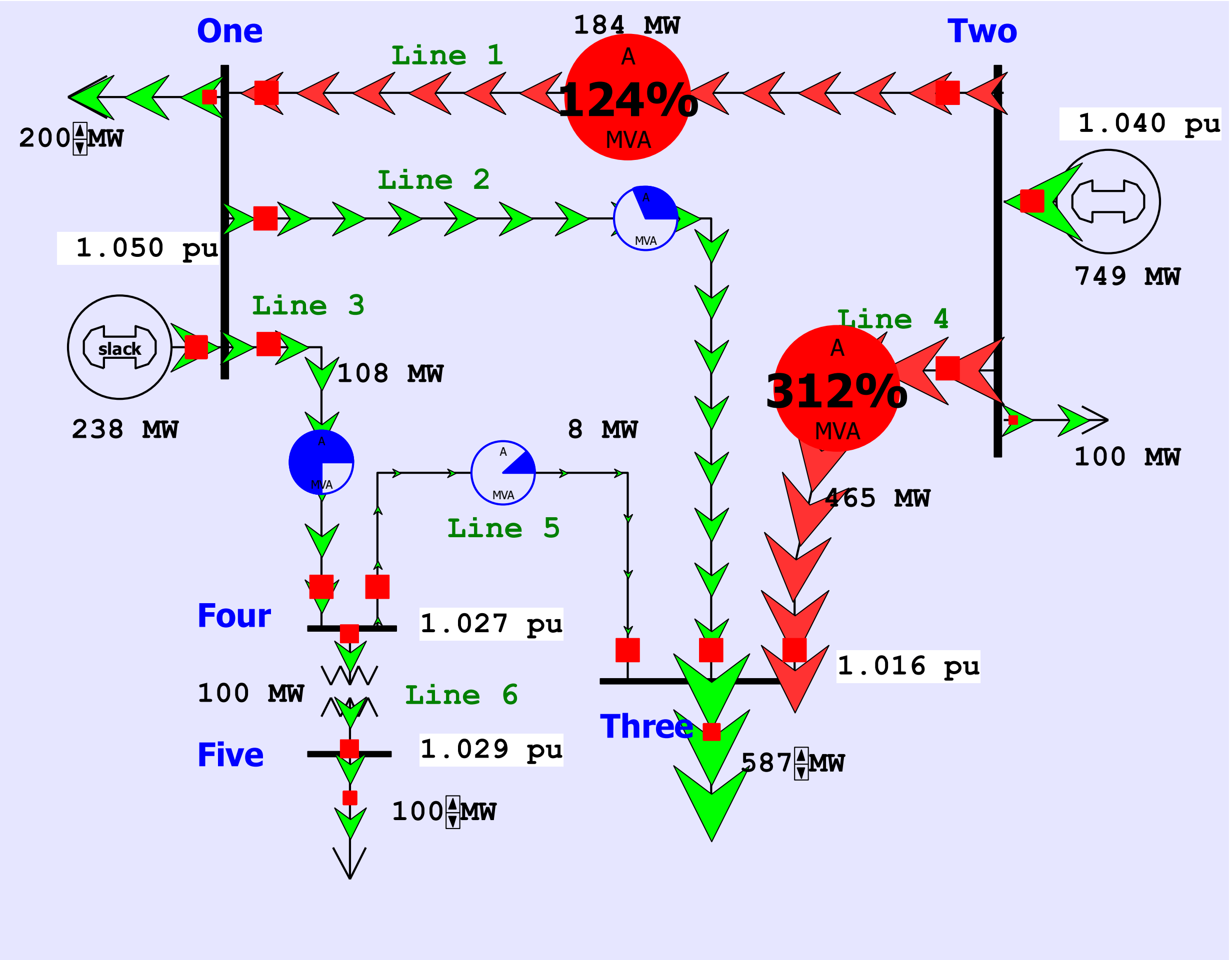
- Hence we get

$$\Delta t_k = \frac{f_k}{1 - \varphi_k^{(w_k)}} = \frac{128}{1 - 0.7273} = 469.4$$

$$d_3^4 = \frac{\Delta f_3}{f_4} = \frac{\varphi_3^{(w_4)}}{1 - \varphi_4^{(w_4)}} = \frac{0.0909}{1 - 0.7273} = 0.333$$

$$\Delta f_3 = (0.333)\, f_4 = 0.333 \times 128 = 42.66 \text{ MW}$$

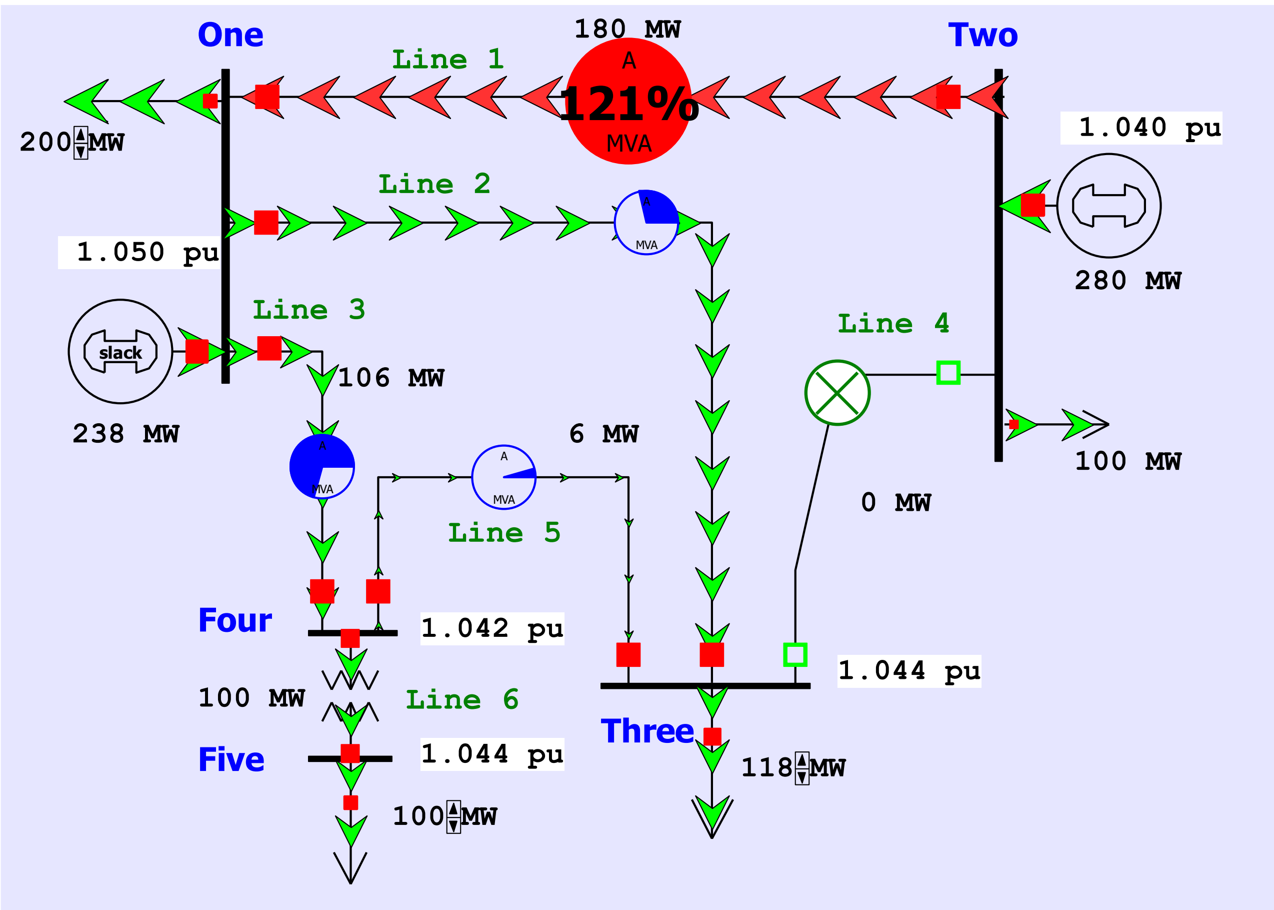
Five Bus Example Compensated



Here is the system with the compensation added to bus 2 and removed at bus 3; we are canceling the impact of the line 4 flow for the reset of the network.

Five Bus Example

- Below we see the network with the line actually outaged



The line 3 flow changed from 63 MW to 106 MW, an increase of 43 MW, matching the LODF value