2020 PSERC Summer Tutorials

Power System Application of Measurement-Based Modal Analysis

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  - Special thanks to Bernie Lesieutre, Alex Borden and Jim Gronquist!

It is a Great Time to be a Power and Energy Engineer!

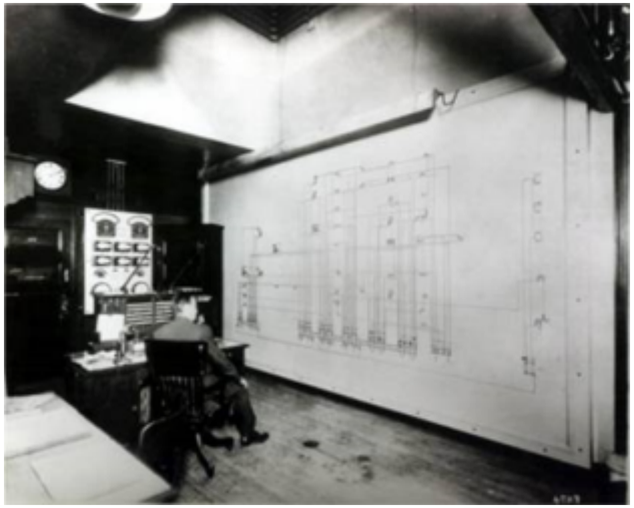
- Electric grids are in a time of rapid transition, with lots of positive developments and lots of engineering challenges!
- It is good to keep in mind the essence of engineering, which is defined by Merriam-Webster's as "The **application** of science and mathematics by which the properties of matter and the sources of energy in nature are **made useful to people**."

Overview

- To meet the challenges of today, we need to widely leverage tools from other domains and make them useful
- This tutorial presents one such tool, the application of measurement-based modal analysis techniques for large-scale electric grids

A Few Initial Thoughts

- "If I have seen further, it is by standing on the shoulders of giants."
  - Isaac Newton 1676
- The grid we inherited from the past was smart; our challenge to make it smarter!



Left: control center in early 1900's, right: ISO New England control center

A Few Initial Thoughts, cont.

- While the grid of 2000 was named the top engineering technology of the 20th century, the grid of 2020 is even more complex
  - There is question of whether anyone really fully understands it!
- My passion is to do research and develop tools to make large-scale electric grid analysis as easy as possible… But it can still be quite complex!!
- Today's focus is to show how measurement-based modal analysis can be a part of every day power systems engineering analysis

Modeling Cautions!

- "All models are wrong but some are useful," George Box, Empirical Model-Building and Response Surfaces, (1987, p. 424)
  - Models are an approximation to reality, not reality, so they always have some degree of approximation
  - Box went on to say that the practical question is how wrong to they have to be to not be useful
- A good part of engineering is deciding what is the appropriate level of modeling, and knowing under what conditions the model will fail

Signals

- Throughout the talk I'll be using the term "signal," which has several definitions
- A definition from Merrian-Webster is
  - "A detectable physical quantity or impulse by which messages or information can be transmitted."
- A common electrical engineering definition is "any time-varying quantity"
- Our focus today is on such time-varying signals, particularly associated with oscillations

Oscillations

- An oscillation is just a repetitive motion that can be either undamped, positively damped (decaying with time) or negatively damped (growing with time)
- If the oscillation can be written as a sinusoid then

$$e^{\alpha t}\left(a\cos(\omega t)+b\sin(\omega t)\right)=e^{\alpha t}C\cos(\omega t+\theta)$$

$$\text{where } C=\sqrt{A^2+B^2} \text{ and } \theta=\tan\left(\frac{-b}{a}\right)$$

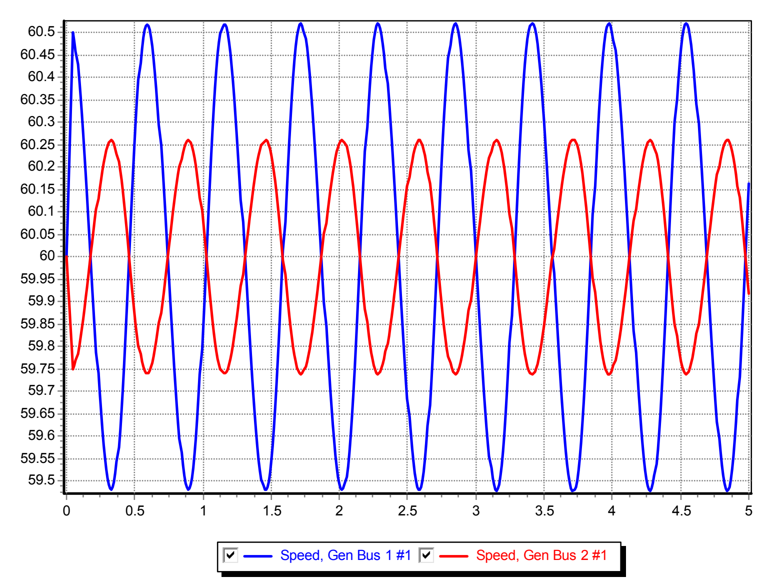
- The damping ratio is

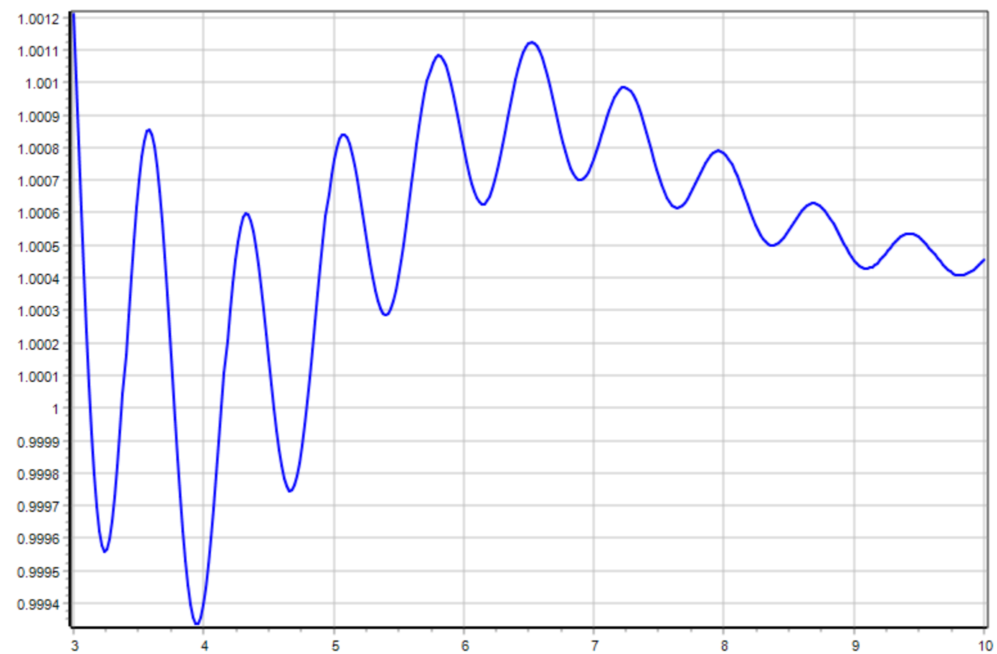
$$\xi=\frac{-\alpha}{\sqrt{\alpha^2+\omega^2}}$$

The percent damping is just the damping ratio multiplied by 100; goal is sufficiently positive damping

Types of Oscillations

- There are several different types of oscillations, including simple ones with just a single frequency; under-damped oscillations have zero frequency



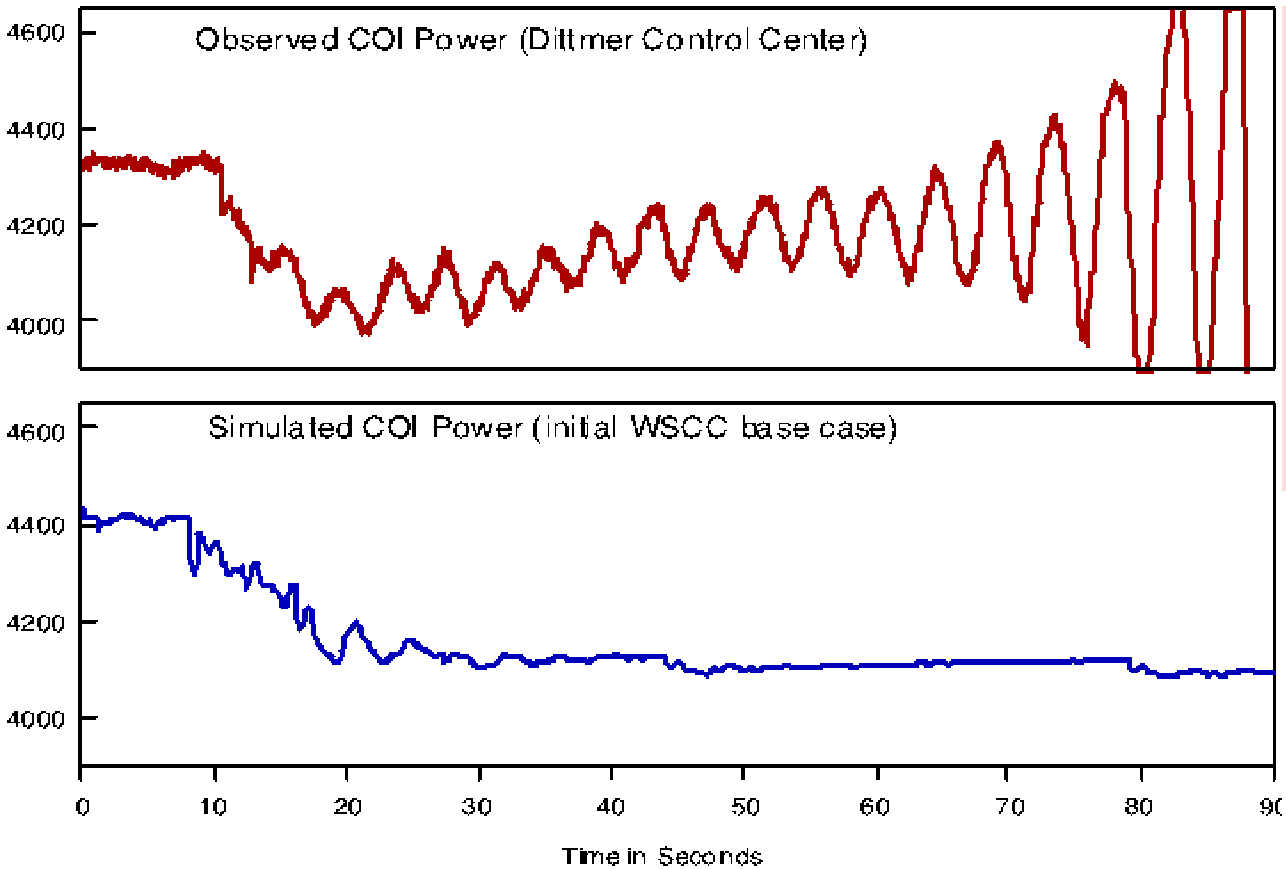


Power System Oscillations

- Power systems can experience a wide range of oscillations, ranging from highly damped and high frequency switching transients to sustained low frequency (< 2 Hz) inter-area oscillations affecting an entire interconnect
- Types of oscillations include
  - Transients: Usually high frequency and highly damped
  - Local plant: Usually from 1 to 5 Hz
  - Inter-area oscillations: From 0.15 to 1 Hz
  - Slower dynamics: Such as AGC, less than 0.15 Hz
  - Subsynchronous resonance: 10 to 50 Hz (less than synchronous)

Example Oscillations

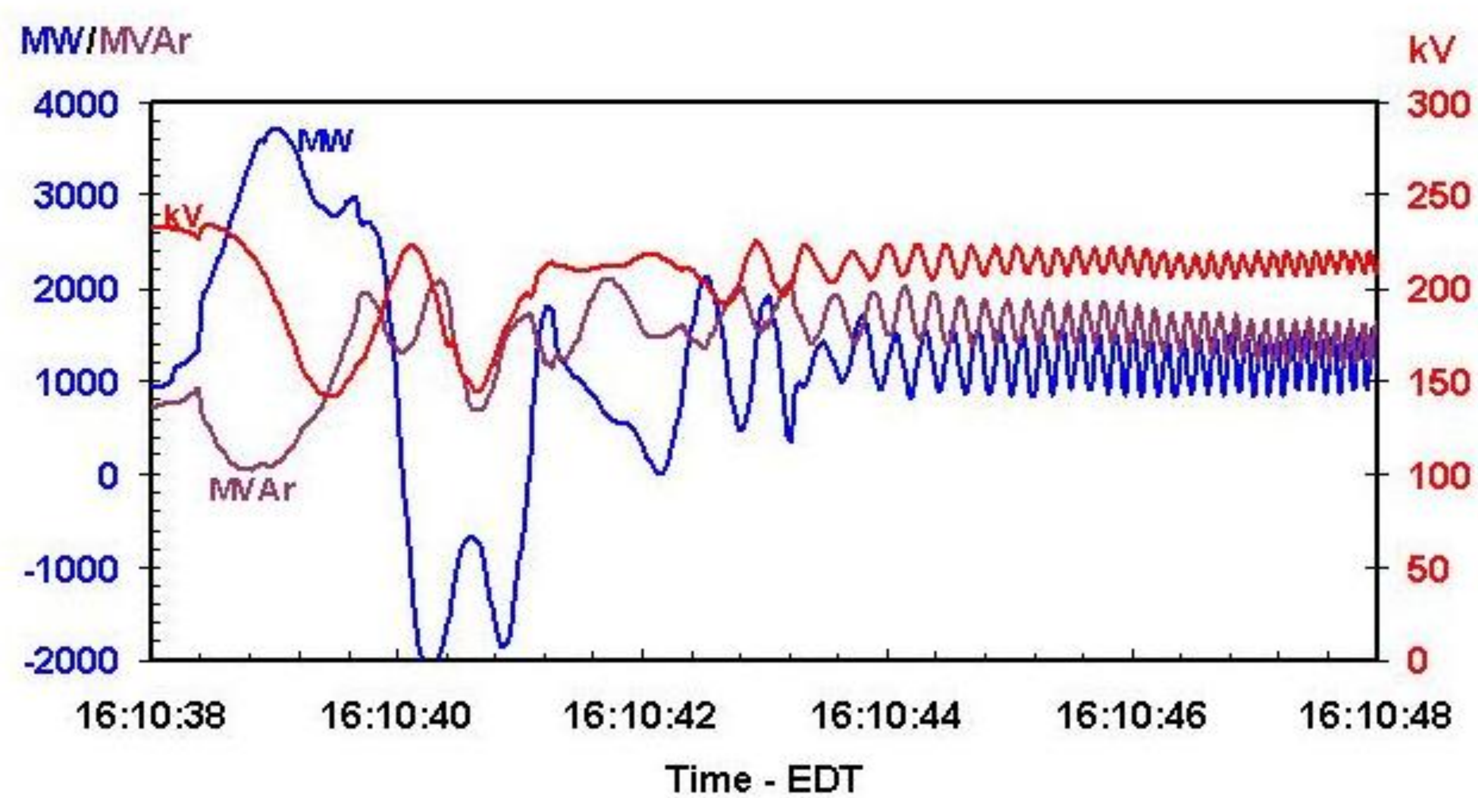
- The below graph shows an oscillation that was observed during a 1996 WECC Blackout



The electric grid and electric grid modeling has changed substantially since 1996!

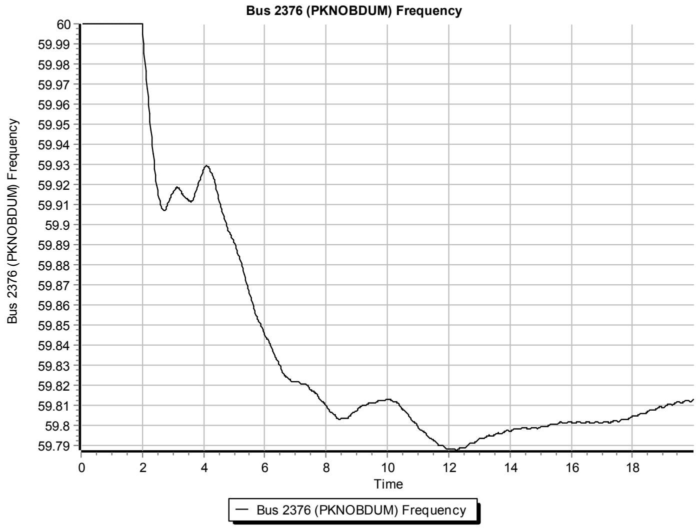
Example Oscillations

- This graph shows oscillations on the Michigan/ Ontario Interface on 8/14/03



More General Signal Analysis

- More generally we may wish to better understand the dynamic behavior of the power grid, either following a disturbance or during ambient conditions
  - Events are more common in studies



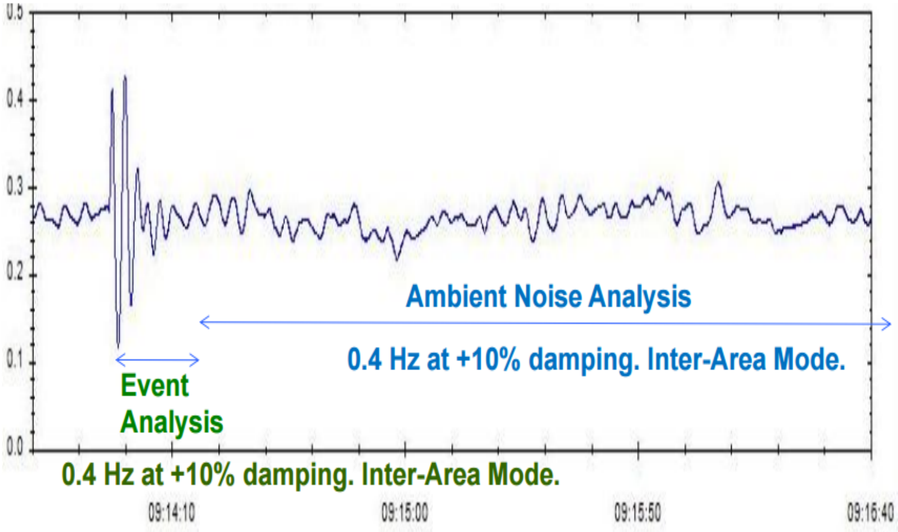


Image Source: M. Venkatasubramanian, "Oscillation Monitoring System", June 2015 http://www.energy.gov/sites/prod/files/2015/07/f24/3.%20Mani%20Oscillation%20Monitoring.pdf

Small Signal Analysis and Measurement-Based Modal Analysis

- Small signal analysis has been used for decades to determine power system frequency response
  - It is a model-based approach that considers the properties of a power system, linearized about an operating point
- Measurement-based modal analysis determines the observed dynamic properties of a system
  - Input can either be measurements from devices (such as PMUs) or dynamic simulation results
  - The same approach can be used regardless of the measurement source

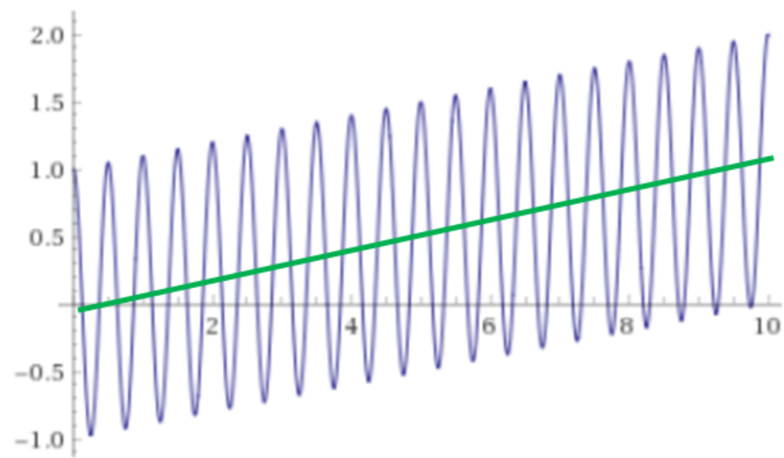
Ring-down Modal Analysis

- Ring-down analysis seeks to determine the frequency and damping of key power system modes following some disturbance
- There are several different techniques, with the Prony approach the oldest (from 1795)
- Regardless of technique, the goal is to represent the response of a sampled signal as a set of exponentially damped sinusoidals (modes)

$$y(t) = \sum_{i=1}^{q} A_i e^{\sigma_i t} \cos\left(\omega_i t + \phi_i\right) \qquad \text{Damping (\%)} = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \omega_i^2}} \times 100$$

Where We Are Going: Extracting the Modes from Signals

- The goal is to gain information about the electric grid by extracting modal information from its signals
  - The frequency and damping of the modes is key
- The premise is we'll be able to reproduce a complex signal, over a period of time, as a set a of sinusoidal modes
  - We'll also allow for linear detrending



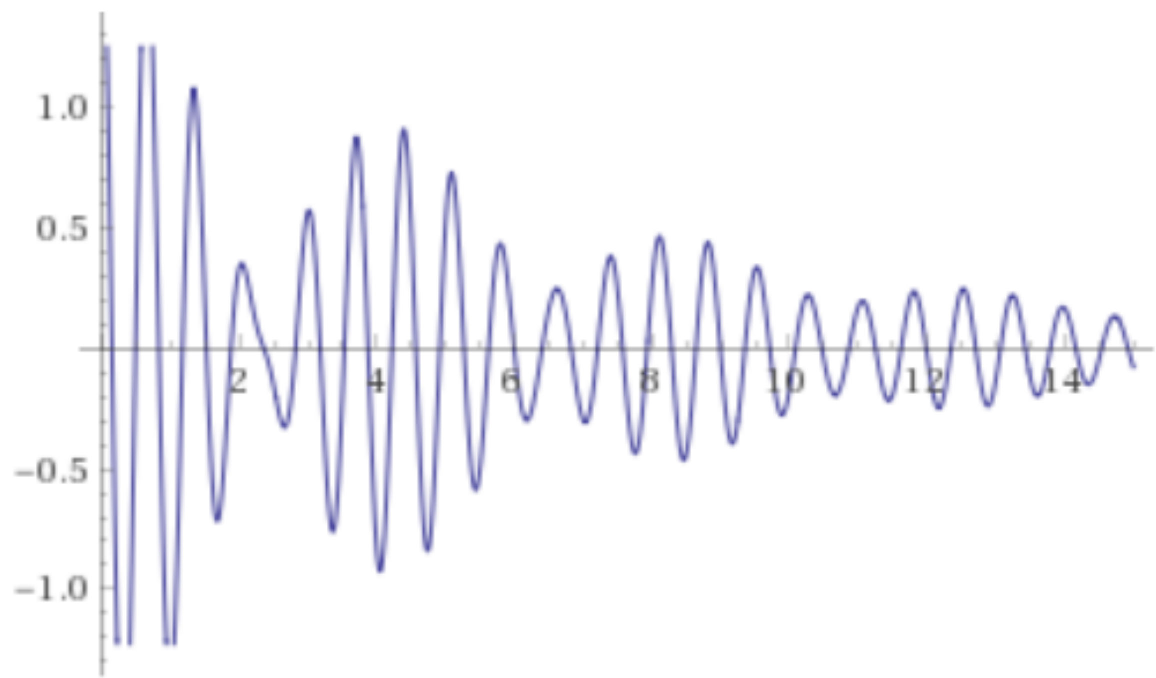
$$0.1t+\cos(2\pi 2t)$$

Example: The Summation of two damped exponentials

- This example was created by going from the modes to a signal
- We'll be going in the opposite direction (i.e., from a measured signal to the modes)

plot	$e^{-0.25x}\cos(10x) + e^{-0.125x}\cos\left(8.5x + \frac{\pi}{8}\right)$

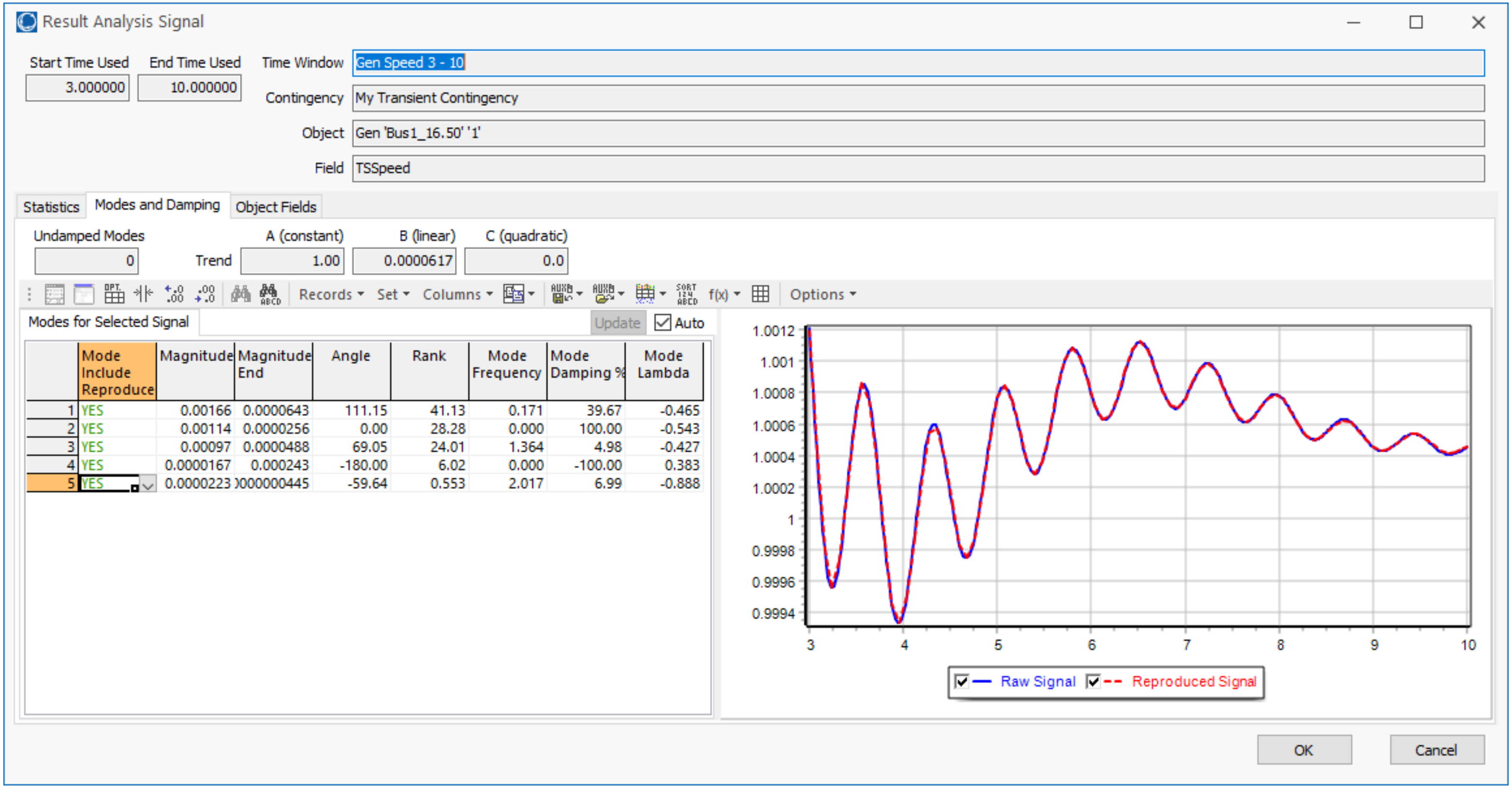
Plot:



Some Reasonable Expectations

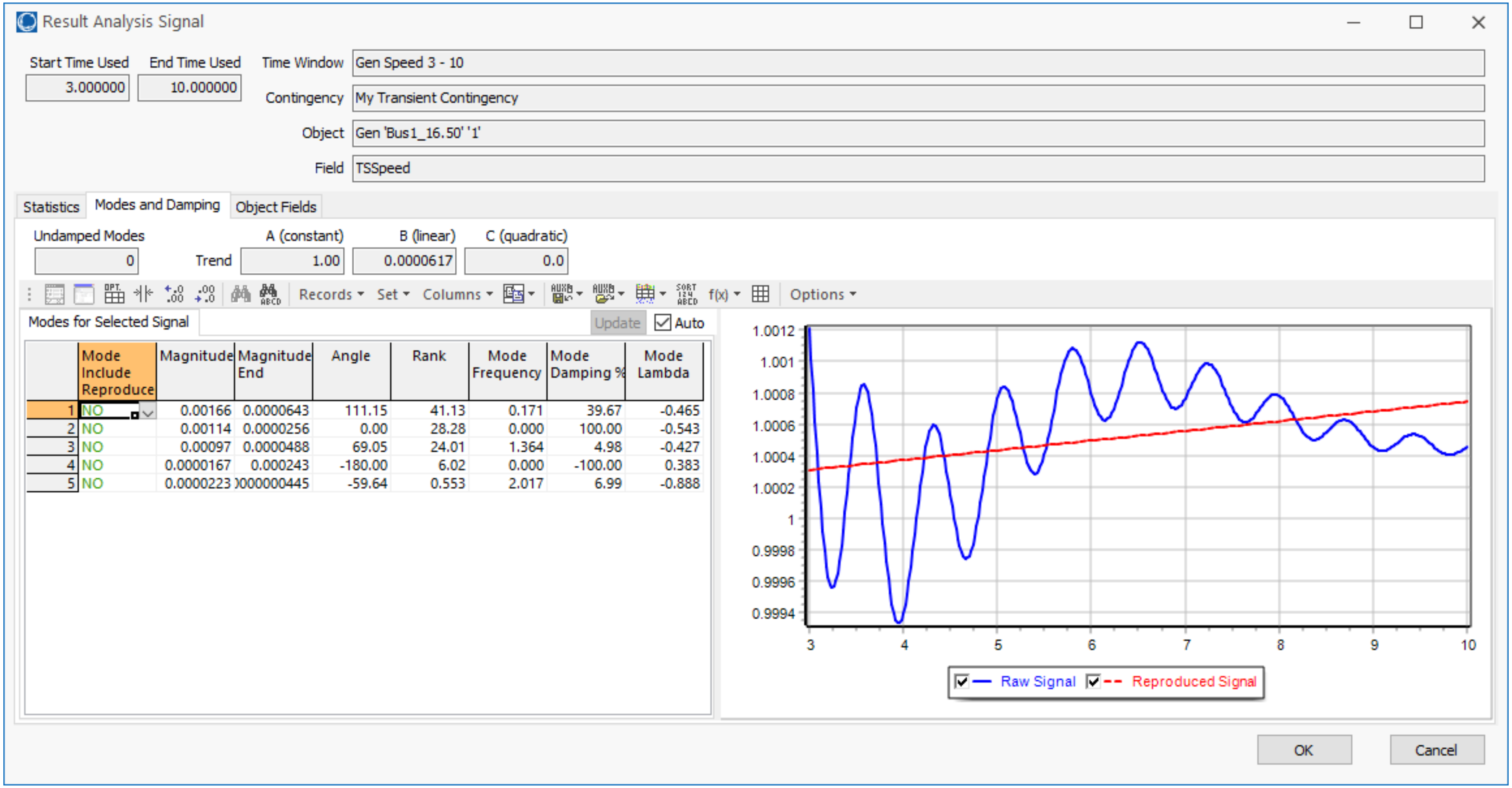
- “Trust but verify” (going back to Reagan using a Russian proverb)
  - We should be able to show how well the modes match the original signal(s)
- Flexible to handle between one and many signals
  - We’ll go up to simultaneously considering 40,000 signals
- Fast
  - What is presented will be, with a discussion of the computational scaling
- Easy to use
  - This is software implementation specific

Example: One Signal

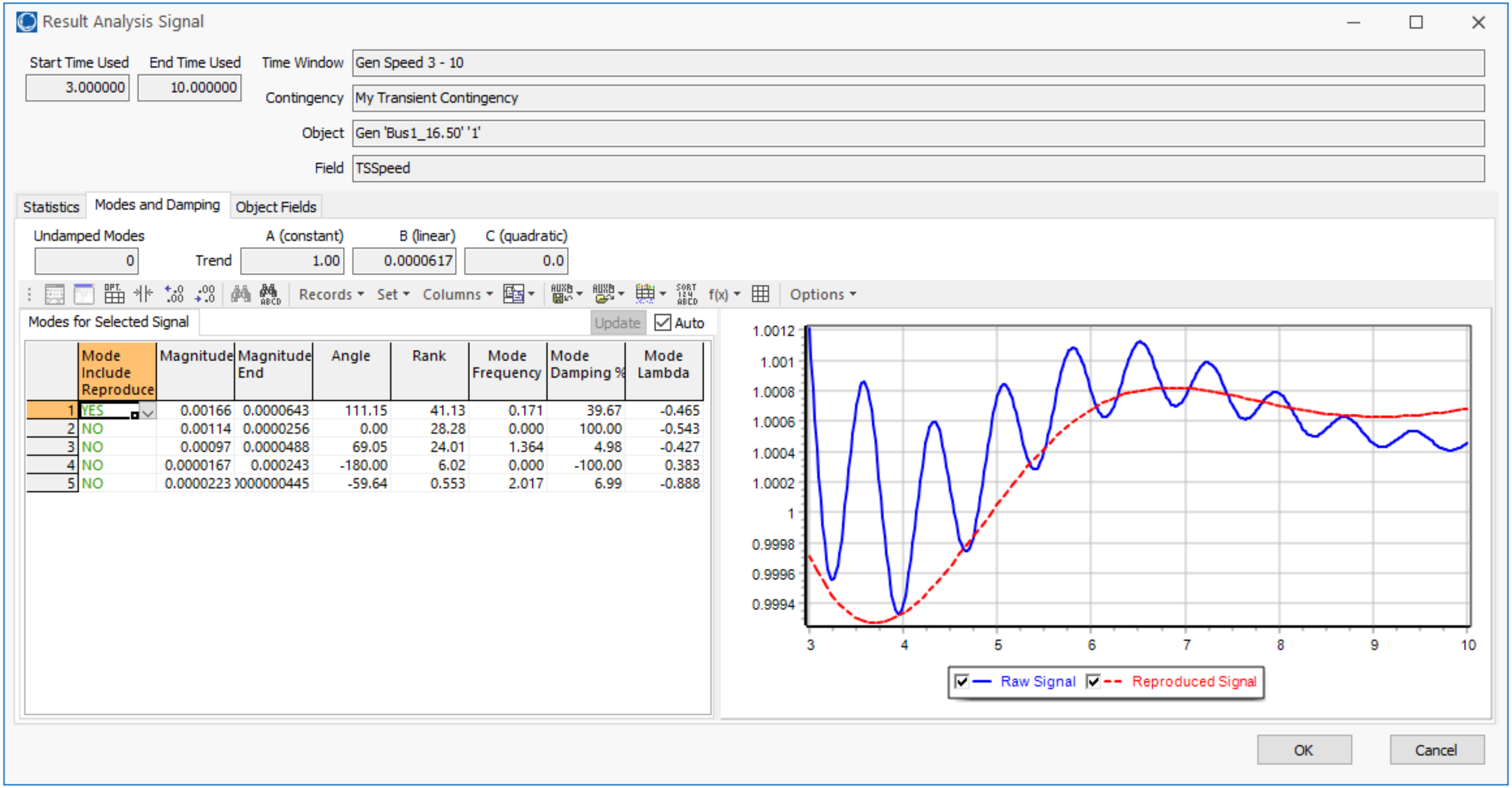


	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	YES	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	YES	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888

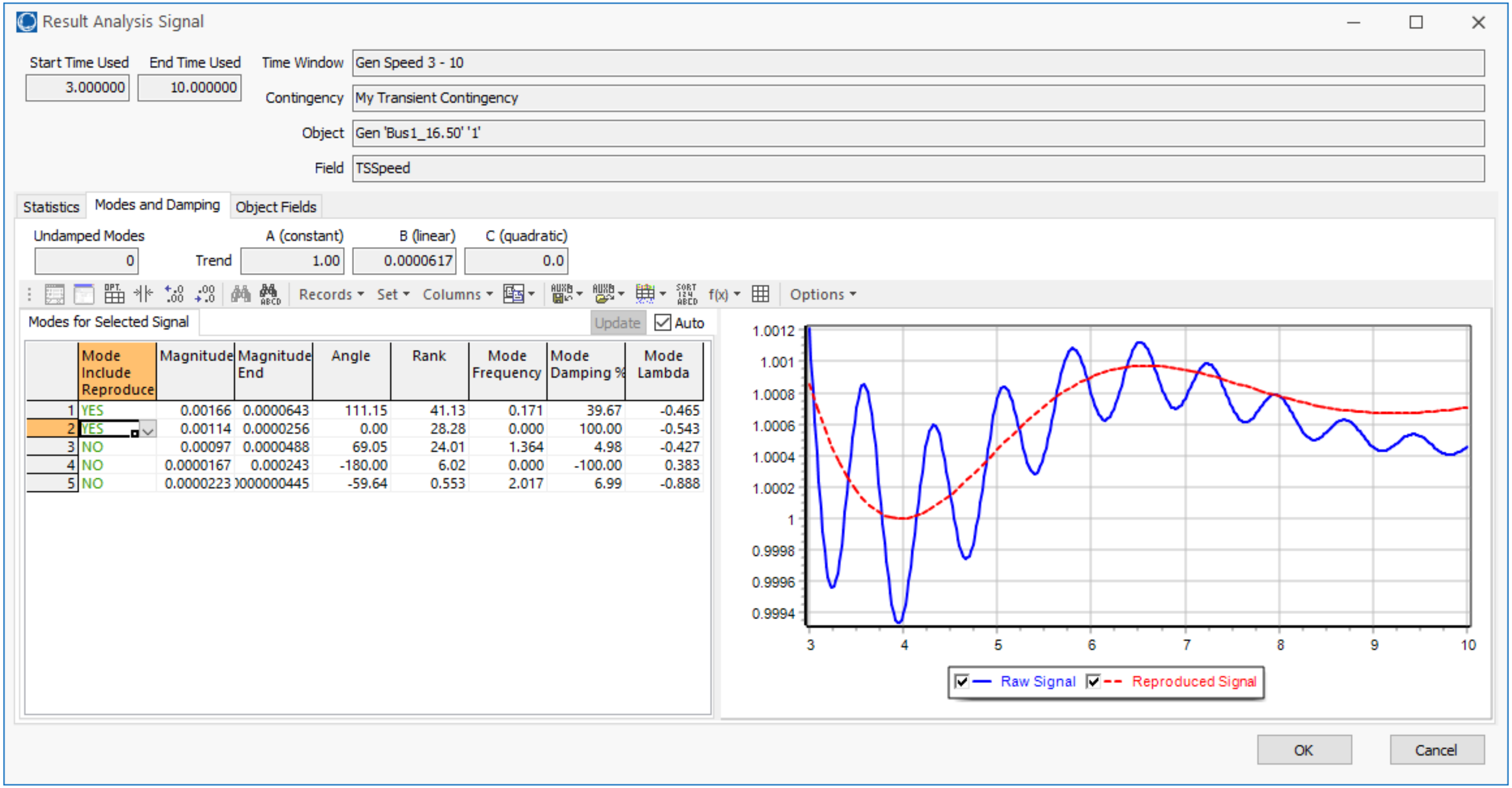
Verification:
Linear Trend Line Only



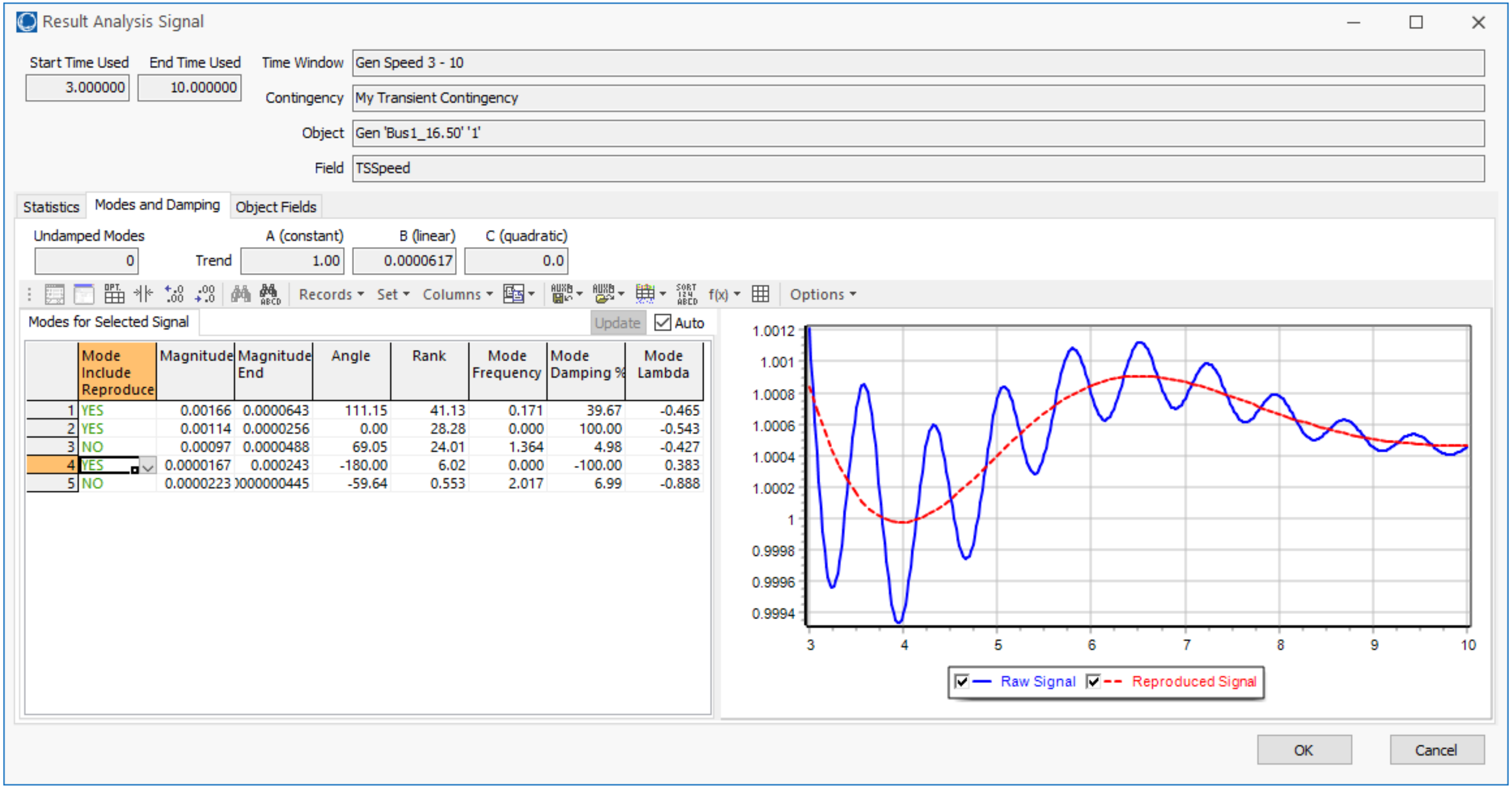
Verification: Linear Trend Line + One Mode



Verification:
Linear Trend Line + Two Modes



Verification:
Linear Trend Line + Three Modes



	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	NO	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888

Verification: Linear Trend Line + Four Modes

Result Analysis Signal

Start Time Used	End Time Used
3.000000	10.000000

Time Window: Gen Speed 3 - 10

Contingency: My Transient Contingency

Object: Gen 'Bus1_16.50' '1'

Field: TSSpeed

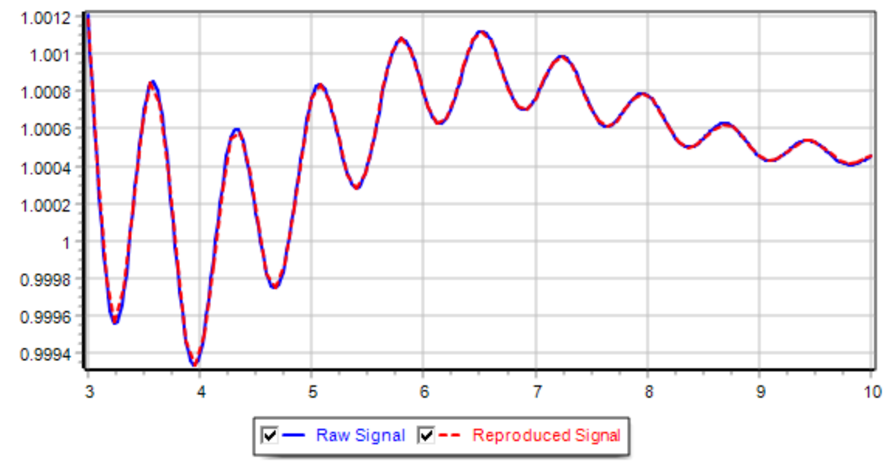
Statistics | Modes and Damping | Object Fields

Undamped Modes		A (constant)	B (linear)	C (quadratic)
0	Trend	1.00	0.0000617	0.0

Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

Modes for Selected Signal — Update — Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	YES	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	000000445	-59.64	0.553	2.017	6.99	-0.888



OK Cancel

Verification:
Linear Trend Line + Five Modes

Result Analysis Signal

Start Time Used	End Time Used
3.000000	10.000000

Time Window: Gen Speed 3 - 10
Contingency: My Transient Contingency
Object: Gen 'Bus1_16.50' '1'
Field: TSSpeed

Statistics | Modes and Damping | Object Fields

Undamped Modes: 0
Trend — A (constant): 1.00 — B (linear): 0.0000617 — C (quadratic): 0.0

Records Set Columns f(x) Options

Modes for Selected Signal — Update — Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
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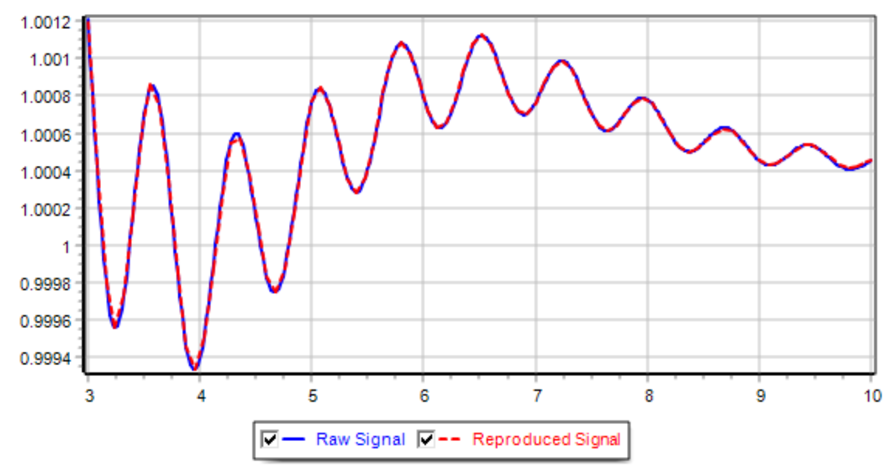
5	YES	0.0000223	000000445	-59.64	0.553	2.017	6.99	-0.888

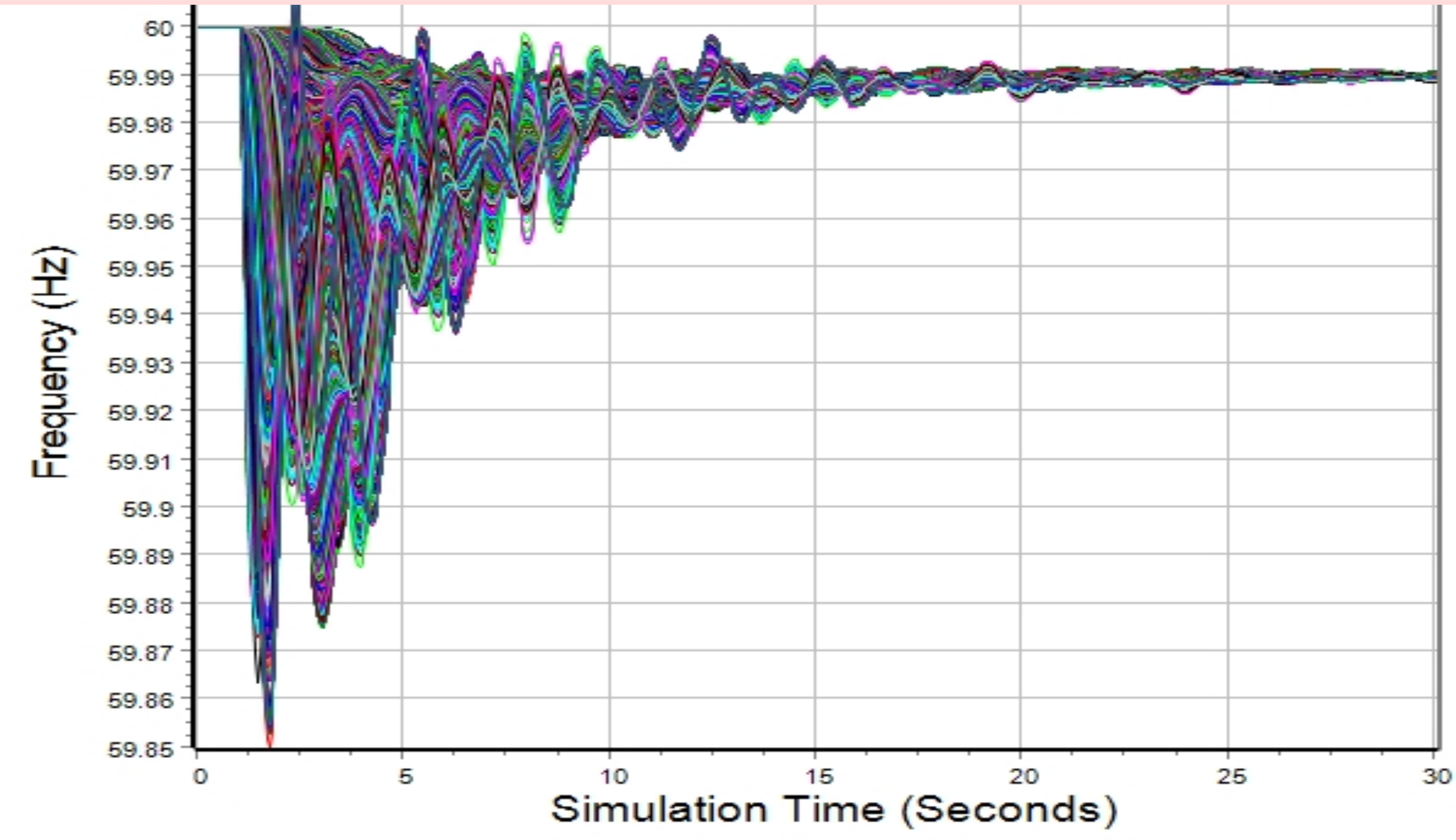


OK Cancel

It is hard to tell a difference on this one, illustrating that modes manifest differently in different signals

A Larger Example We'll Finish With

Applying the developed techniques to the response of all 43,400 substation frequencies from an 110,000 bus electric grid(20 million plus values)



Measurement-Based Modal Analysis

- There are a number of different approaches
- The idea of all techniques is to approximate a signal, $y_{org}(t)$, by the sum of other, simpler signals (basis functions)
  - Basis functions are usually exponentials, with linear and quadratic functions used to detrend the signal
  - Properties of the original signal can be quantified from basis function properties
    - Examples are frequency and damping
  - Signal is considered over time with t=0 as the start
- Approaches sample the original signal $y_{org}(t)$

Measurement-Based Modal Analysis

- Vector **y** consists of m uniformly sampled points from $y_{org}(t)$ at a sampling value of DT, starting with t=0, with values y_j for j=1…m
  - Times are then t_j= (j-1)DT
  - At each time point j, the approximation of y_j is

$$\hat{y}_j(\mathrm{t}_j,\boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j,\boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}$ is a vector with the real and imaginary eigenvalue components,

with $\phi_i(t_j,\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ for α_{i} corresponding to a real eigenvalue, and

$\phi_i(t_j,\boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{\mathrm{i+1}}\mathrm{t}_j)$ and $\phi_{i+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{\mathrm{i+1}}\mathrm{t}_j)$

for a complex eigenvector value

Measurement-Based Modal Analysis

- Error (residual) value at each point j is

$$r_j(t_j,\boldsymbol{\alpha}) = y_j - \hat{y}_j(t_j,\boldsymbol{\alpha})$$

- The closeness of the fit can be quantified using the Euclidean norm of the residuals

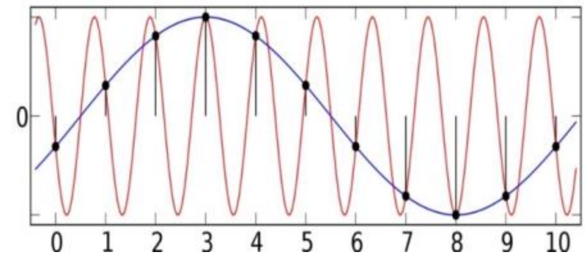
$$\frac{1}{2}\sum_{j=1}^{m}(y_j - \hat{y}_j(t_j,\boldsymbol{\alpha}))^2 = \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\alpha})\right\|_2^2$$

- Hence we need to determine **a** and **b**
  – Recall

$$\hat{y}_j(\mathrm{t}_j,\boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i\phi_i(t_j,\boldsymbol{\alpha})$$

Sampling Rate and Aliasing

- The Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency
  - For example, to see a 5 Hz frequency we need to sample the signal at a rate of at least 10 Hz
- Sampling shifts the frequency spectrum by 1/T (where T is the sample time), which causes frequency overlap
- This is known as aliasing, which can cause a high frequency signal to appear to be a lower frequency signal



  - Aliasing can be reduced by fast sampling and/or low pass filters

Image: upload.wikimedia.org/wikipedia/commons/thumb/2/28/AliasingSines.svg/2000px-AliasingSines.svg.png

One Solution Approach: The Matrix Pencil Method

- There are several algorithms for finding the modes. We'll use the Matrix Pencil Method
  - This is a newer technique for determining modes from noisy signals (from about 1990, introduced to power system problems in 2005); it is an alternative to the Prony Method (which dates back to 1795, introduced into power in 1990 by Hauer, Demeure and Scharf)
- Given m samples, with L=m/2, the first step is to form the Hankel Matrix, **Y** such that

This not a sparse matrix

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{L+1} \\ y_2 & y_3 & \cdots & y_{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L} & y_{m-L+1} & \cdots & y_m \end{bmatrix}$$

Refernece: A. Singh and M. Crow, "The Matrix Pencil for Power System Modal Extraction," IEEE Transactions on Power Systems, vol. 20, no. 1, pp. 501-502, Institute of Electrical and Electronics Engineers (IEEE), Feb 2005.

Algorithm Details, cont.

- Then calculate **Y**'s singular values using an economy singular value decomposition (SVD)

$$\mathbf{Y} = \mathbf{U\Sigma V}^{\mathbf{T}}$$

- The ratio of each singular value is then compared to the largest singular value s_c; retain the ones with a ratio > than a threshold
  - This determines the modal order, M
  - Assuming **V** is ordered by singular values (highest to lowest), let $\mathbf{V}_p$ be then matrix with the first M columns of **V**

The computational complexity increases with the cube of the number of measurements!

This threshold is a value that can be changed; decrease it to get more modes.

Aside: The Matrix Singular Value Decomposition (SVD)

- The SVD is a factorization of a matrix that generalizes the eigendecomposition to any m by n matrix to produce

$$\mathbf{Y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{T}}$$

The original concept is more than 100 years old, but has found lots of recent applications

  where **S** is a diagonal matrix of the singular values
- The singular values are non-negative, real numbers that can be used to indicate the major components of a matrix (the gist is they provide a way to decrease the rank of a matrix)

Aside: SVD Image Compression Example

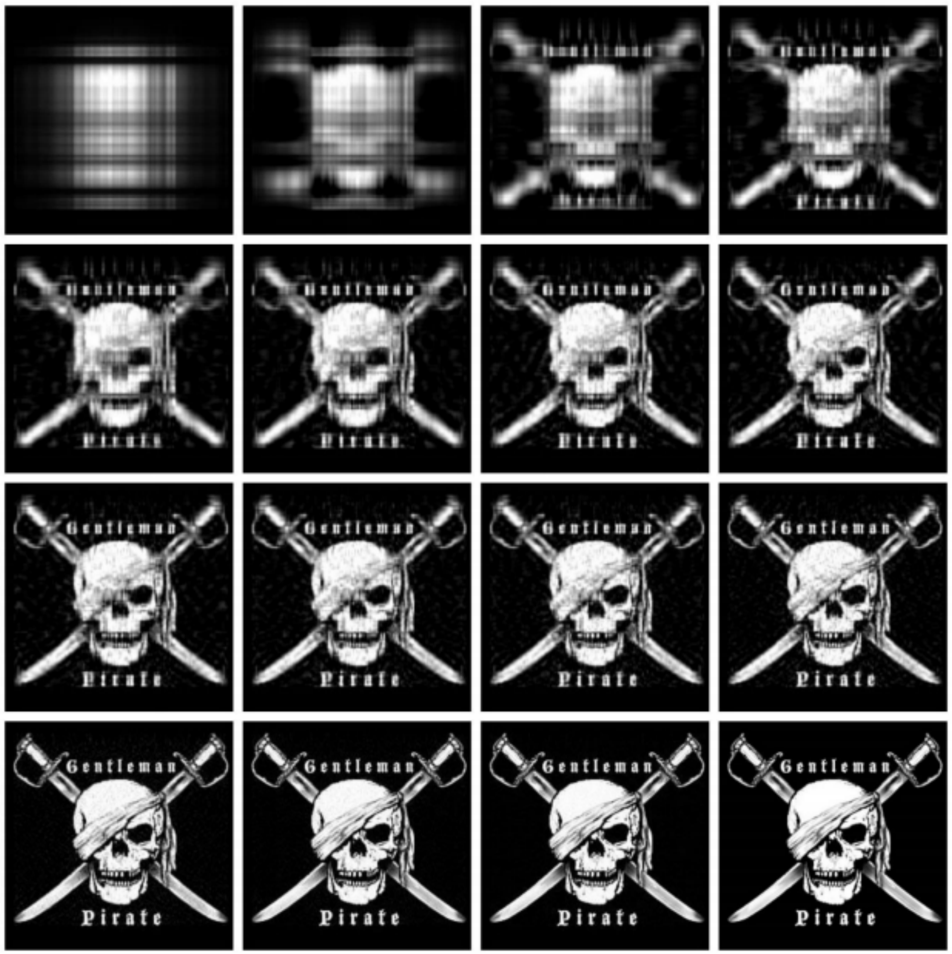


Figure 3.1: Image size 250x236 – modes used {{1,2,4,6},{8,10,12,14},{16,18,20,25},{50,75,100,original image}}

Images can be represented with matrices. When an SVD is applied and only the largest singular values are retained the image is compressed.

Image Source: www.math.utah.edu/~goller/F15_M2270/BradyMathews_SVDImage.pdf

Aside: SVD and Principle Component Analysis (PCA)

- The previous image compression example demonstrates PCA, which reduces dimensionality
  - Extracting the principle components
- The principle components are associated with the largest singular values
  - This helps to extract the key features of the data and removes redundancy
- PCA can be used to do facial recognition
- The Matrix Pencil Method is similar; that is, retaining only the largest singular values from the Hankel matrix

Matrix Pencil Algorithm Details, cont.

- Then form the matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ such that
  - – $\mathbf{V}_1$ is the matrix consisting of all but the last row of $\mathbf{V}_p$
  - – $\mathbf{V}_2$ is the matrix consisting of all but the first row of $\mathbf{V}_p$
- Discrete-time poles are found as the generalized eigenvalues of the pair $(\mathbf{V}_2^T\mathbf{V}_1, \mathbf{V}_1^T\mathbf{V}_1) = (\mathbf{A},\mathbf{B})$
- These eigenvalues are the discrete-time poles, z_i with the modal eigenvalues then

$$\lambda_i = \frac{\ln(z_i)}{\Delta T}$$

The log of a complex number z=r?? is ln(r) + j?

If **B** is nonsingular (the situation here) then the generalized eigenvalues are the eigenvalues of $\mathbf{B}^{-1}\mathbf{A}$

Matrix Pencil Method with Many Signals

- The Matrix Pencil approach can be used with one signal or with multiple signals
- Multiple signals are handled by forming a $\mathbf{Y}_k$ matrix for each signal k using the measurements for that signal and then combining the matrices

$$\mathbf{Y}_k = \begin{bmatrix} y_{1,\mathrm{k}} & y_{2,\mathrm{k}} & \cdots & y_{L+1,\mathrm{k}} \\ y_{2,\mathrm{k}} & y_{3,k} & \cdots & y_{L+2,\mathrm{k}} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L,\mathrm{k}} & y_{m-L+1,\mathrm{k}} & \cdots & y_{m,\mathrm{k}} \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_N \end{bmatrix}$$

The required computation scales linearly with the number of signals

Matrix Pencil Method with Many Signals

- However, when dealing with many signals, usually the signals are somewhat correlated, so vary few of the signals are actually need to be included to determine the desired modes
- Ultimately we are finding

$$y_j(\mathrm{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

- The **a** is common to all the signals (i.e., the system modes) while the **b** vector is signal specific (i.e., how the modes manifest in that signal)

Quickly Determining the b Vectors

- A key insight is from an approach known as the Variable Projection Method (from Borden, 2013) that for any signal k

$$\mathbf{y}_k = \mathbf{\Phi}(\boldsymbol{\alpha})\mathbf{b}_k$$

And then the residual is minimized by selecting $\mathbf{b}_k = \mathbf{\Phi}(\boldsymbol{\alpha})^+ \mathbf{y}_k$

where $\mathbf{\Phi}(\boldsymbol{\alpha})$ is the m by n matrix with values

$\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ if α_i corresponds to a real eigenvalue,

and $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{i+1} t_j)$ and $\Phi_{ji+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{i+1} t_j)$

for a complex eigenvalue; $t_j = \left(j-1\right)\Delta T$

Finally, $\mathbf{\Phi}(\boldsymbol{\alpha})^+$ is the pseudoinverse of $\mathbf{\Phi}(\boldsymbol{\alpha})$

Where m is the number of measurements and n is the number of modes

A. Borden, B.C. Lesieutre, J. Gronquist, "Power System Modal Analysis Tool Developed for Industry Use," *Proc. 2013 North American Power Symposium*, Manhattan, KS, Sept. 2013

Aside: Pseudoinverse of a Matrix

- The pseudoinverse of a matrix generalizes concept of a matrix inverse to an m by n matrix, in which m >= n
  - Specifically this is a Moore-Penrose Matrix Inverse
- Notation for the pseudoinverse of **A** is $\mathbf{A}^+$
- Satisfies $\mathbf{A}\mathbf{A}^+\mathbf{A} = \mathbf{A}$
- If **A** is a square matrix, then $\mathbf{A}^+ = \mathbf{A}^{-1}$
- Quite useful for solving the least squares problem since the least squares solution of $\mathbf{A}\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = \mathbf{A}^+\mathbf{b}$
- Can be calculated using an SVD

$$\mathbf{A} = \mathbf{U}\,\mathbf{\Sigma}\,\mathbf{V}^T$$

$$\mathbf{A}^+ = \mathbf{V}\,\mathbf{\Sigma}^+\,\mathbf{U}^T$$

Aside: Pseudoinverse Least Squares Matrix Example

- Assume we wish to fit a line (mx + b = y) to three data points: (1,1), (2,4), (6,4)
- Two unknowns, m and b; hence **x** = [m b]^T
- Setup in form of **Ax = b**

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} \quad \text{so} \quad \mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 6 & 1 \end{bmatrix}$$

Aside: Pseudoinverse Least Squares Matrix Example

- Doing an economy SVD

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T = \begin{bmatrix} -0.182 & -0.765 \\ -0.331 & -0.543 \\ -0.926 & 0.345 \end{bmatrix} \begin{bmatrix} 6.559 & 0 \\ 0 & 0.988 \end{bmatrix} \begin{bmatrix} -0.976 & -0.219 \\ 0.219 & -0.976 \end{bmatrix}$$

- Computing the pseudoinverse

$$\mathbf{A}^+ = \mathbf{V}\,\boldsymbol{\Sigma}^+\mathbf{U}^T = \begin{bmatrix} -0.976 & 0.219 \\ -0.219 & -0.976 \end{bmatrix} \begin{bmatrix} 0.152 & 0 \\ 0 & 1.012 \end{bmatrix} \begin{bmatrix} -0.182 & -0.331 & -0.926 \\ -0.765 & -0.543 & 0.345 \end{bmatrix}$$

$$\mathbf{A}^+ = \mathbf{V}\,\boldsymbol{\Sigma}^+\mathbf{U}^T = \begin{bmatrix} -0.143 & -0.071 & 0.214 \\ 0.762 & 0.548 & -0.310 \end{bmatrix}$$

In an economy SVD the **S** matrix has dimensions of m by m if m < n or n by n if n < m

Least Squares Matrix Pseudoinverse Example, cont.

- Computing $\mathbf{x} = [m\ b]^T$ gives

$$\mathbf{A}^{+}\mathbf{b} = \begin{bmatrix} -0.143 & -0.071 & 0.214 \\ 0.762 & 0.548 & -0.310 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.429 \\ 1.71 \end{bmatrix}$$

- With the pseudoinverse approach we immediately see the sensitivity of the elements of **x** to the elements of **b**
  - New values of m and b can be readily calculated if **y** changes
- Computationally the SVD is order m^2n+n^3 (with $n < m$)

Iterative Matrix Pencil Method

- When there are a large number of signals the iterative matrix pencil method works by
  – Selecting an initial signal to calculate the **a** vector
  – Quickly calculating the **b** vectors for all the signals, and getting a cost function for how closely the reconstructed signals match their sampled values
  – Selecting a signal that has a high cost function, and repeating the above adding this signal to the algorithm to get an updated **a**

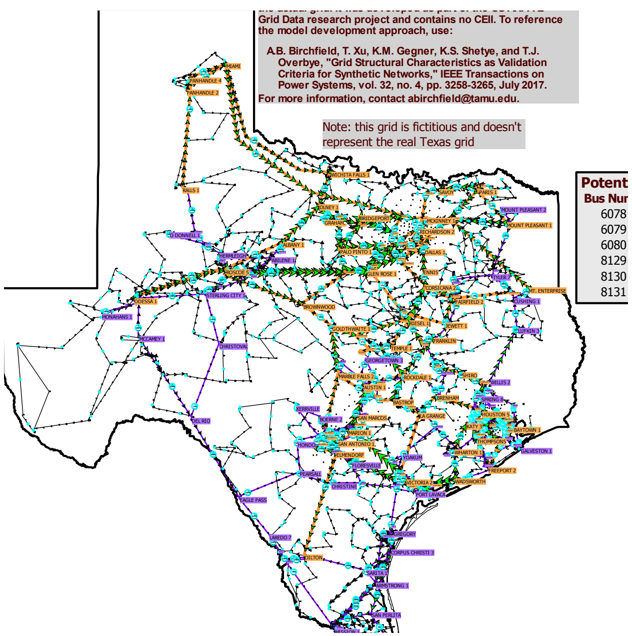
An open access paper describing this is W. Trinh, K.S. Shetye, I. Idehen, T.J. Overbye, "Iterative Matrix Pencil Method for Power System Modal Analysis," *Proc. 52nd Hawaii International Conference on System Sciences*, Wailea, HI, January 2019; available at scholarspace.manoa.hawaii.edu/handle/10125/59803

Demonstrations Using Large-Scale Synthetic Electric Grids

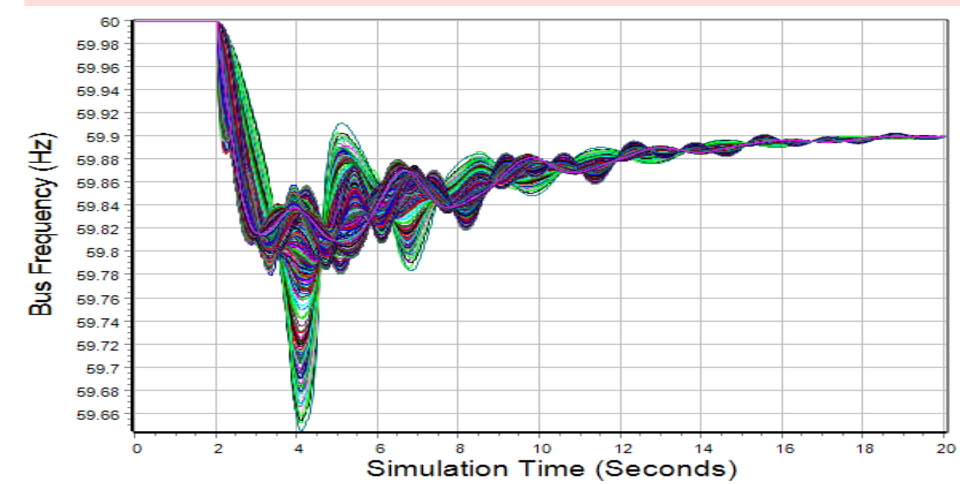
- The following examples demonstrate the approach using large-scale synthetic grids
  - Synthetic grids are designed to mimic the complexity of the actual grids, but are fictional so they contain no CEII, allowing them to be publicly disseminated
  - For those who are interested, PSERC project S-91 (Generating Value from Detailed, Realistic Synthetic Electric Grids) has just started. Additional industrial advisors are certainly welcome to join the team!
    - More details on this project are available at overbye.engr.tamu.edu/pserc-project-s-91
- Many synthetic grids, including the ones used here, are available at electricgrids.engr.tamu.edu

Texas 2000 Bus System Example

- This synthetic grids serves an electric load on the ERCOT footprint
- We'll use the Iterative Matrix Pencil Method to examine its modes
  - The contingency is the loss of two large generators

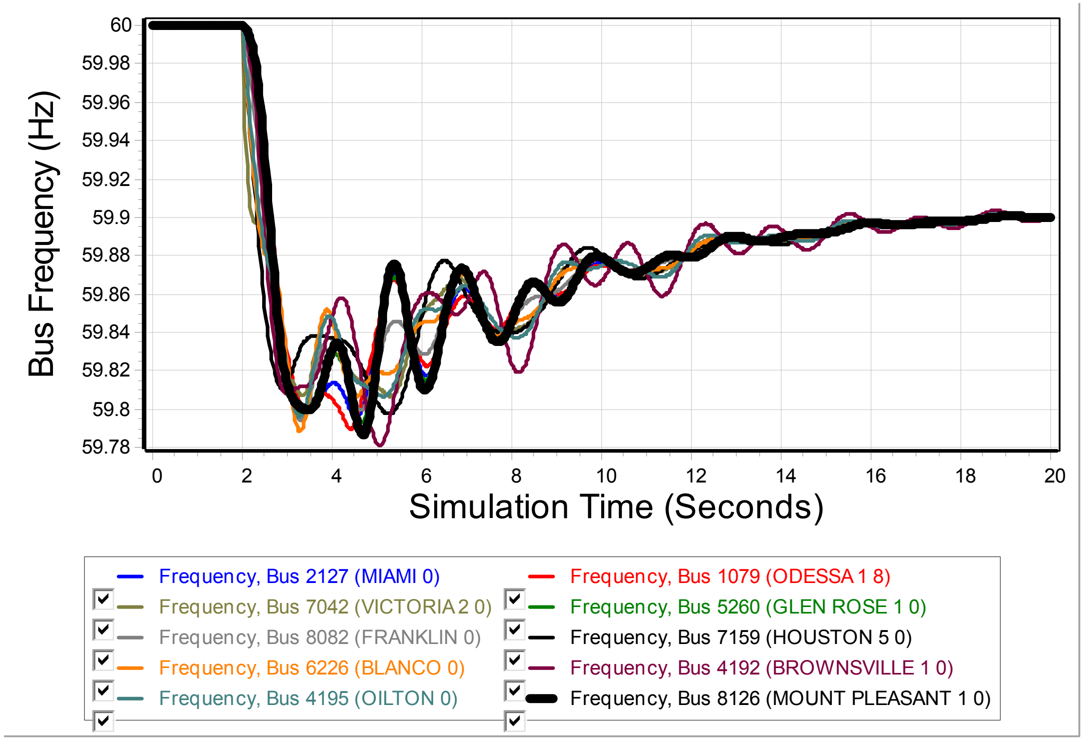


The measurements will be the frequencies at all 2000 buses



2000 Bus System Example, Initially Just One Signal

- Initially our goal is to understand the modal frequencies and their damping
- First we'll consider just one of the 2000 signals; arbitrarily I selected bus 8126 (Mount Pleasant)



Some Initial Considerations

- The input is a dynamics study running using a ½ cycle time step; data was saved every 3 steps, so at 40 Hz
  – The contingency was applied at time = 2 seconds
- We need to pick the portion of the signal to consider and the sampling frequency
  – Because of the underlying SVD, the algorithm scales with the cube of the number of time points (in a single signal)
- I selected between 2 and 17 seconds
- I sampled at ten times per second (so a total of 150 samples)

2000 Bus System Example, One Signal

- The results from the Matrix Pencil Method are

Number of Complex and Real Modes: 6

Lowest Percent Damping: 10.137

☑ Include Detrend in Reproduced Signals

☐ Subtract Reproduced from Actual

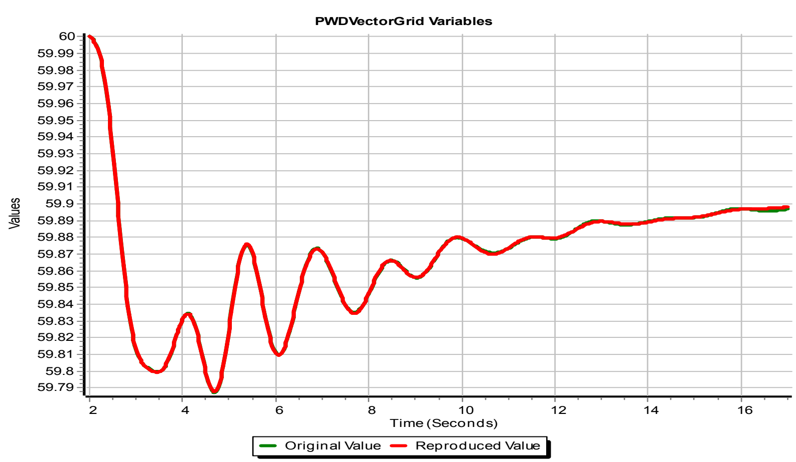
Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (%)	Largest Component Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	Include in Reproduced Signal
1	0.383	32.011	0.44275	Bus 1073 (ODE	12.224	Bus 7310 (WHA	-0.8136	YES
2	0.670	24.191	0.38466	Bus 2120 (PARI	11.549	Bus 8078 (MT. E	-1.0490	YES
3	0.665	10.705	0.23093	Bus 2115 (PARI	6.801	Bus 2115 (PARI	-0.4501	YES
4	0.312	14.397	0.16911	Bus 1073 (ODE	4.954	Bus 7310 (WHA	-0.2855	YES
5	0.971	10.137	0.08179	Bus 1051 (MON	2.551	Bus 6147 (SAN	-0.6215	YES
6	0.052	41.828	0.04603	Bus 1074 (ODE	1.063	Bus 3035 (CHE	-0.1506	YES

Calculated mode information

Trust but verify results

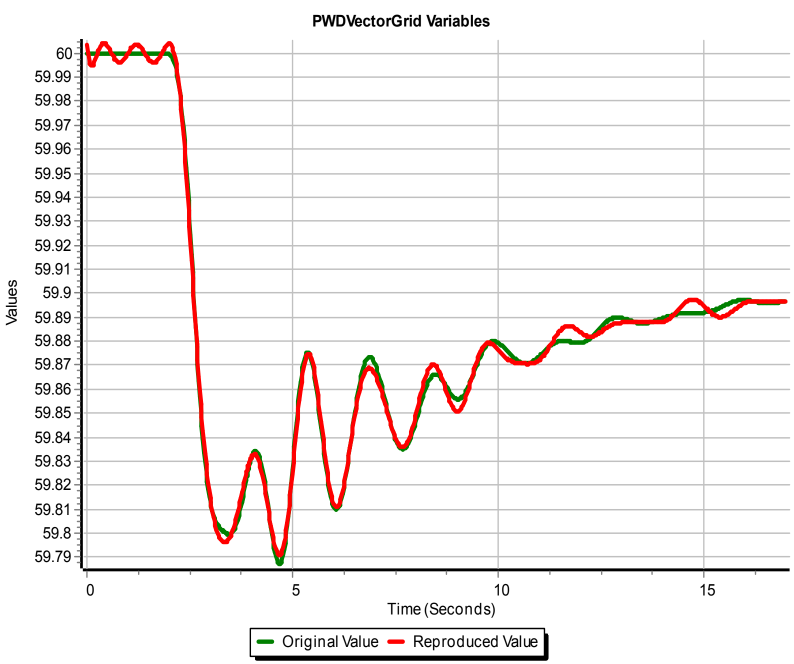


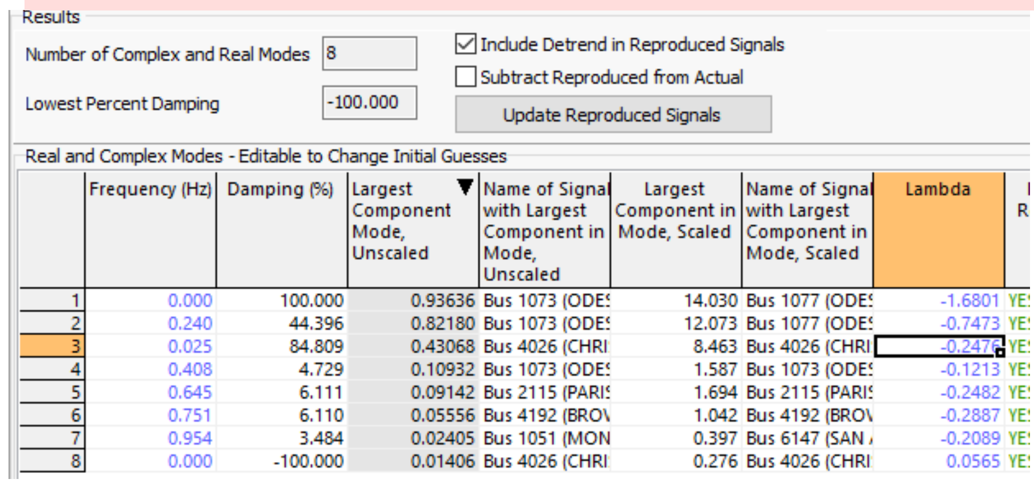
Some Observations

- These results are based on the consideration of just one signal
- The start time **should** be at or after the event!

If it isn't then…

The results show the algorithm trying to match the first two flat seconds; this should not be done!!




Results

Number of Complex and Real Modes 8

Lowest Percent Damping -100.000

☑ Include Detrend in Reproduced Signals

☐ Subtract Reproduced from Actual

Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (%)	Largest Component Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	R
1	0.000	100.000	0.93636	Bus 1073 (ODES	14.030	Bus 1077 (ODES	-1.6801	YE
2	0.240	44.396	0.82180	Bus 1073 (ODES	12.073	Bus 1077 (ODES	-0.7473	YE
3	0.025	84.809	0.43068	Bus 4026 (CHRI	8.463	Bus 4026 (CHRI	-0.2476	YE
4	0.408	4.729	0.10932	Bus 1073 (ODES	1.587	Bus 1073 (ODES	-0.1213	YE
5	0.645	6.111	0.09142	Bus 2115 (PARIS	1.694	Bus 2115 (PARIS	-0.2482	YE
6	0.751	6.110	0.05556	Bus 4192 (BROV	1.042	Bus 4192 (BROV	-0.2887	YE
7	0.954	3.484	0.02405	Bus 1051 (MON	0.397	Bus 6147 (SAN	-0.2089	YE
8	0.000	-100.000	0.01406	Bus 4026 (CHRI	0.276	Bus 4026 (CHRI	0.0565	YE

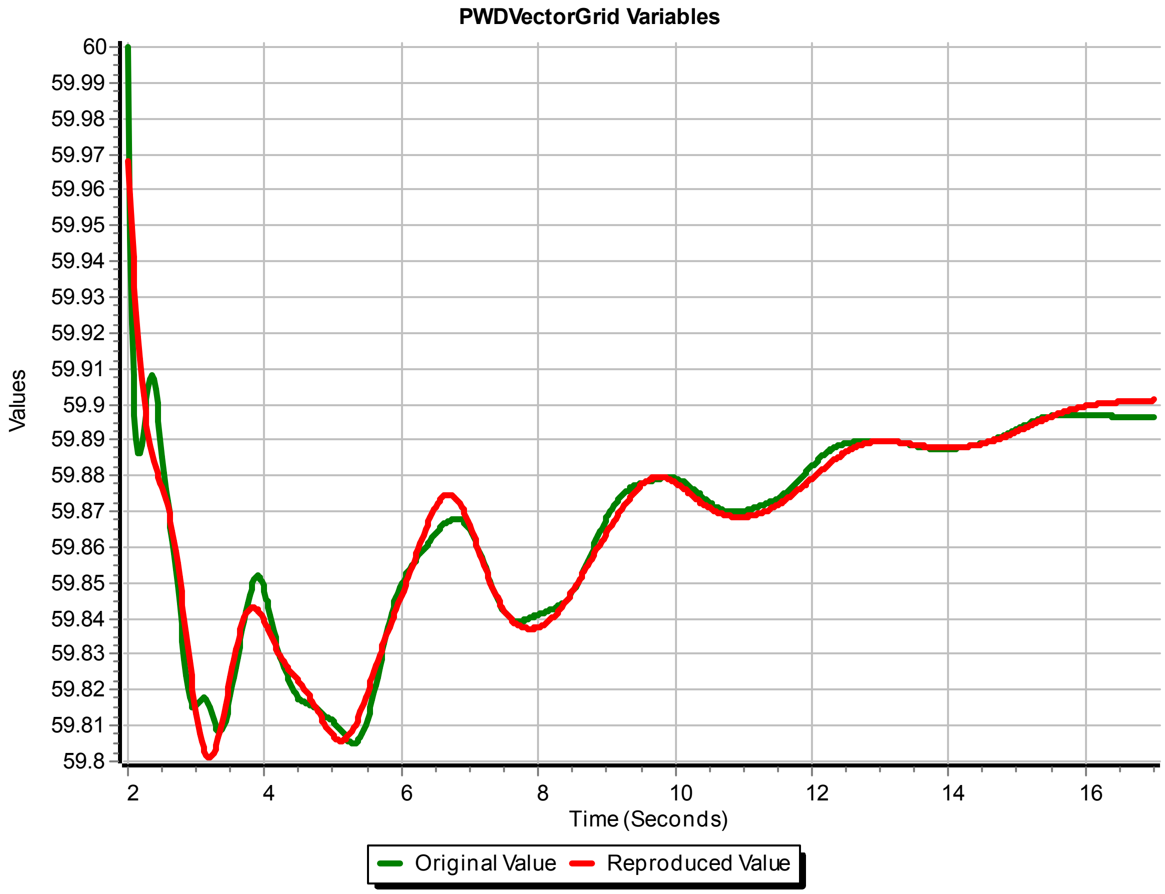
2000 Bus System Example, One Signal Included, Cost for All

- Using the previously discussed pseudoinverse approach, for a given set of modes (**a**) the $\mathbf{b}_k$ vectors for all the signals can be quickly calculated

$$\mathbf{b}_k = \boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\mathbf{y}_k$$

  – Recall that the dimensions of the pseudoinverse are the number of modes by the number of sample points for one signal
- This allows each cost function to be calculated
- The Iterative Matrix Pencil approach sequentially adds the signals with the worst match (i.e., the highest cost function)

2000 Bus System Example, the Worst Match (Bus 7061)



2000 Bus System Example, Two Signals

With two signals

Number of Complex and Real Modes: 9

Lowest Percent Damping: 7.359

☑ Include Detrend in Reproduced Signals

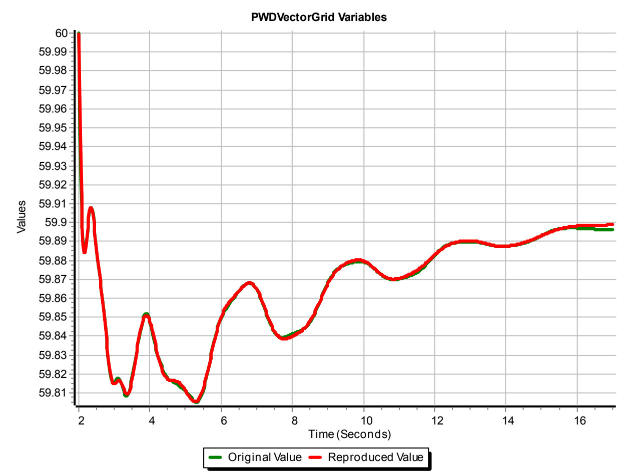
☐ Subtract Reproduced from Actual

Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (%)	Largest Component in Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	In Rep
1	2.266	17.168	0.04028	Bus 7329 (NEW	1.730	Bus 7307 (WHA	-2.4809	YES
2	1.413	21.844	0.10763	Bus 4030 (FANN	4.475	Bus 4030 (FANN	-1.9867	YES
3	0.958	7.359	0.04666	Bus 6147 (SAN /	1.801	Bus 6147 (SAN /	-0.4441	YES
4	0.701	11.705	0.21220	Bus 1051 (MON	5.762	Bus 8077 (MT. E	-0.5187	YES
5	0.630	13.361	0.20903	Bus 2120 (PARIS	6.350	Bus 4192 (BROV	-0.5337	YES
6	0.352	36.405	0.44679	Bus 1051 (MON	13.024	Bus 7311 (WHA	-0.8654	YES
7	0.322	14.403	0.19570	Bus 1073 (ODES	5.372	Bus 7311 (WHA	-0.2948	YES
8	0.000	100.000	0.09305	Bus 1051 (MON	1.767	Bus 1051 (MON	-0.6853	YES
9	0.064	36.756	0.02993	Bus 1073 (ODES	1.182	Bus 7307 (WHA	-0.1586	YES

The new match on Bus 7061 is quite good!



With one signal

Number of Complex and Real Modes: 6

Lowest Percent Damping: 10.137

☑ Include Detrend in Reproduced Signals

☐ Subtract Reproduced from Actual

Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (%)	Largest Component Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	In Re
1	0.383	32.011	0.44275	Bus 1073 (ODES	12.224	Bus 7310 (WHA	-0.8136	YES
2	0.670	24.191	0.38466	Bus 2120 (PARIS	11.549	Bus 8078 (MT. E	-1.0490	YES
3	0.665	10.705	0.23093	Bus 2115 (PARIS	6.801	Bus 2115 (PARIS	-0.4501	YES
4	0.312	14.397	0.16911	Bus 1073 (ODES	4.954	Bus 7310 (WHA	-0.2855	YES
5	0.971	10.137	0.08179	Bus 1051 (MON	2.551	Bus 6147 (SAN /	-0.6215	YES
6	0.052	41.828	0.04603	Bus 1074 (ODES	1.063	Bus 3035 (CHEF	-0.1506	YES

2000 Bus System Example, Iterative Matrix Pencil

- The Iterative Matrix Pencil intelligently adds signals until a specified number is met
  - Doing ten iterations takes about four seconds

Number of Complex and Real Modes: 11

Lowest Percent Damping: 6.082

☑ Include Detrend in Reproduced Signals

☐ Subtract Reproduced from Actual

Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (% ▲	Largest Component in Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	Include in Reproduced Signal
1	0.631	6.082	0.10313	Bus BROWNSVI	3.292	Bus BROWNSVI	-0.2415	YES
2	0.959	7.068	0.04897	Bus SAN ANTOI	1.890	Bus SAN ANTOI	-0.4269	YES
3	1.364	7.246	0.03780	Bus ODESSA 1 ;	1.420	Bus CHRISTINE	-0.6228	YES
4	0.593	7.897	0.07205	Bus BROWNSVI	2.300	Bus BROWNSVI	-0.2949	YES
5	1.602	8.562	0.04887	Bus FANNIN 2 F	2.032	Bus FANNIN 2 F	-0.8650	YES
6	0.732	11.936	0.21348	Bus MONAHAN	4.054	Bus MONAHAN	-0.5529	YES
7	0.324	14.207	0.19906	Bus ODESSA 1 ;	5.268	Bus WHARTON	-0.2917	YES
8	0.324	39.346	0.55936	Bus MONAHAN	12.994	Bus WHARTON	-0.8722	YES
9	0.060	39.972	0.03815	Bus ODESSA 1 ;	1.196	Bus POINT COI	-0.1645	YES
10	0.964	57.683	0.61264	Bus ODESSA 1 ;	18.504	Bus POINT COI	-4.2760	YES
11	0.000	100.000	0.59650	Bus ODESSA 1 ;	14.434	Bus WHARTON	-2.5257	YES

Takeaways So Far

- Modal analysis can be quickly done on a large number of signals
  - Computationally is an $O(N^3)$ process for one signal, where N is the number of sample points; it varies linearly with the number of included signals
  - The number of sample points can be automatically determined from the highest desired frequency (the Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency)
  - Determining how all the signals are manifested in the modes is quite fast!!

Getting Mode Details

- An advantage of this approach is the contribution of each mode in each signal is directly available

Modal Analysis Mode Details

Frequency (Hz) and Damping (%) 0.631 Hz, Damping = 6.082%

Transfer Results from Selected Column to Object Custom Floating Pont Field

Custom Floating Point Field 1 Transfer Results

Records Set Columns f(x) Options

	Type	Name	Units	Description	Post-Detrend Standard Deviation	Angle (Deg)	Magnitude, Unscaled	Magnitude Scaled by SD	Cost Function
1	Bus	Bus BROWNSVILLE 1 0 Frequency		Frequency	0.031	176.451	0.10313	3.29203	0.0019
2	Bus	Bus BROWNSVILLE 1 1 Frequency		Frequency	0.031	176.451	0.10248	3.27853	0.0019
3	Bus	Bus BROWNSVILLE 3 0 Frequency		Frequency	0.031	176.454	0.10148	3.25747	0.0018
4	Bus	Bus BROWNSVILLE 2 0 Frequency		Frequency	0.031	176.525	0.10041	3.23684	0.0017
5	Bus	Bus OLMITO 0 Frequency		Frequency	0.031	176.456	0.10032	3.23265	0.0018
6	Bus	Bus BROWNSVILLE 2 1 Frequency		Frequency	0.031	176.522	0.09964	3.22005	0.0017
7	Bus	Bus SAN BENITO 0 Frequency		Frequency	0.031	176.452	0.09836	3.19018	0.0017
8	Bus	Bus PORT ISABEL 0 Frequency		Frequency	0.031	176.519	0.09817	3.18788	0.0016
9	Bus	Bus LOS FRESNOS 0 Frequency		Frequency	0.031	176.480	0.09601	3.13896	0.0016
10	Bus	Bus CORPUS CHRISTI 3 3 Frequency		Frequency	0.030	177.479	0.09573	3.15533	0.0013
11	Bus	Bus CORPUS CHRISTI 3 2 Frequency		Frequency	0.030	177.619	0.09533	3.14610	0.0013
12	Bus	Bus RIO HONDO 0 Frequency		Frequency	0.030	176.500	0.09462	3.10807	0.0015
13	Bus	Bus CORPUS CHRISTI 3 5 Frequency		Frequency	0.030	177.488	0.09393	3.11626	0.0013
14	Bus	Bus SAN PERLITA 0 Frequency		Frequency	0.030	176.760	0.09338	3.08711	0.0014
15	Bus	Bus SEBASTIAN 2 1 Frequency		Frequency	0.030	176.485	0.09249	3.05864	0.0014
16	Bus	Bus SEBASTIAN 2 0 Frequency		Frequency	0.030	176.500	0.09234	3.05579	0.0014
17	Bus	Bus CORPUS CHRISTI 3 4 Frequency		Frequency	0.030	177.256	0.09203	3.06646	0.0013
18	Bus	Bus SANTA ROSA 1 4 Frequency		Frequency	0.030	176.457	0.09189	3.04368	0.0014
19	Bus	Bus SANTA ROSA 1 8 Frequency		Frequency	0.030	176.462	0.09183	3.04122	0.0014
20	Bus	Bus SEBASTIAN 1 0 Frequency		Frequency	0.030	176.504	0.09153	3.03706	0.0014
21	Bus	Bus SAN PERLITA 1 Frequency		Frequency	0.030	176.588	0.09134	3.03507	0.0014
22	Bus	Bus HARLINGEN 1 0 Frequency		Frequency	0.030	176.483	0.09114	3.02757	0.0014
23	Bus	Bus CORPUS CHRISTI 1 3 Frequency		Frequency	0.030	178.815	0.09102	3.06810	0.0019
24	Bus	Bus MERCEDES 0 Frequency		Frequency	0.030	176.459	0.09095	3.02245	0.0014
25	Bus	Bus SANTA ROSA 1 6 Frequency		Frequency	0.030	176.377	0.09081	3.01773	0.0014
26	Bus	Bus SANTA ROSA 1 5 Frequency		Frequency	0.030	176.439	0.09075	3.01600	0.0014
27	Bus	Bus SANTA MARIA 0 Frequency		Frequency	0.030	176.423	0.09065	3.01479	0.0014
28	Bus	Bus HARLINGEN 2 0 Frequency		Frequency	0.030	176.455	0.09043	3.01019	0.0014
29	Bus	Bus SANTA ROSA 1 2 Frequency		Frequency	0.030	176.315	0.09034	3.00472	0.0014
30	Bus	Bus PROGRESO 0 Frequency		Frequency	0.030	176.363	0.09016	3.00188	0.0015
31	Bus	Bus SANTA ROSA 1 9 Frequency		Frequency	0.030	176.399	0.08996	2.99744	0.0014
32	Bus	Bus SANTA ROSA 1 3 Frequency		Frequency	0.030	176.399	0.08996	2.99744	0.0014
33	Bus	Bus SANTA ROSA 1 1 Frequency		Frequency	0.030	176.399	0.08996	2.99744	0.0014
34	Bus	Bus SANTA ROSA 1 7 Frequency		Frequency	0.030	176.399	0.08996	2.99744	0.0014
35	Bus	Bus SANTA ROSA 1 0 Frequency		Frequency	0.030	176.399	0.08996	2.99744	0.0014
36	Bus	Bus GREGORY 6 Frequency		Frequency	0.030	179.245	0.08996	3.04202	0.0022
37	Bus	Bus EDCOUCH 0 Frequency		Frequency	0.030	176.406	0.08974	2.99246	0.0014
38	Bus	Bus CORPUS CHRISTI 3 0 Frequency		Frequency	0.030	177.218	0.08968	3.01155	0.0013
39	Bus	Bus CORPUS CHRISTI 1 4 Frequency		Frequency	0.030	178.224	0.08963	3.02501	0.0019
40	Bus	Bus GREGORY 4 Frequency		Frequency	0.029	179.015	0.08959	3.03974	0.0021
41	Bus	Bus HARGILL 0 Frequency		Frequency	0.030	176.406	0.08955	2.98806	0.0014

This slide shows the mode with the lowest damping, sorted by the signals with the largest magnitude in the mode

A Couple of Comments on Damping

- How damping is defined seems to depend on prior industry experience
  - Folks familiar with eigenvalue analysis will tend to define it in terms of the eigenvalues

$$e^{\alpha t}\left(a\cos(\omega t)+b\sin(\omega t)\right)=e^{\alpha t}C\cos(\omega t+\theta)$$

$$\text{where } C=\sqrt{A^2+B^2} \text{ and } \theta=\tan\left(\frac{-b}{a}\right) \qquad \xi=\frac{-\alpha}{\sqrt{\alpha^2+\omega^2}}$$

  - Multiplying this value by 100 gives a damping percentage

A Couple of Comments on Damping

- However, it can also be defined more graphically, in terms of a decrease in a signal from one peak to the next (see below for SPP)
  - In SPP, to be considered "damped", one of the following two requirements must be met
    - Peak to peak magnitude decreased 5% over one cycle
    - Peak to peak decreases by 22.6% over 5 cycles

www.spp.org/documents/28859/spp%20disturbance%20performance%20requirements%20(twg%20approved).pdf

A Couple of Comments on Damping: An Easy Conversion Between the Two

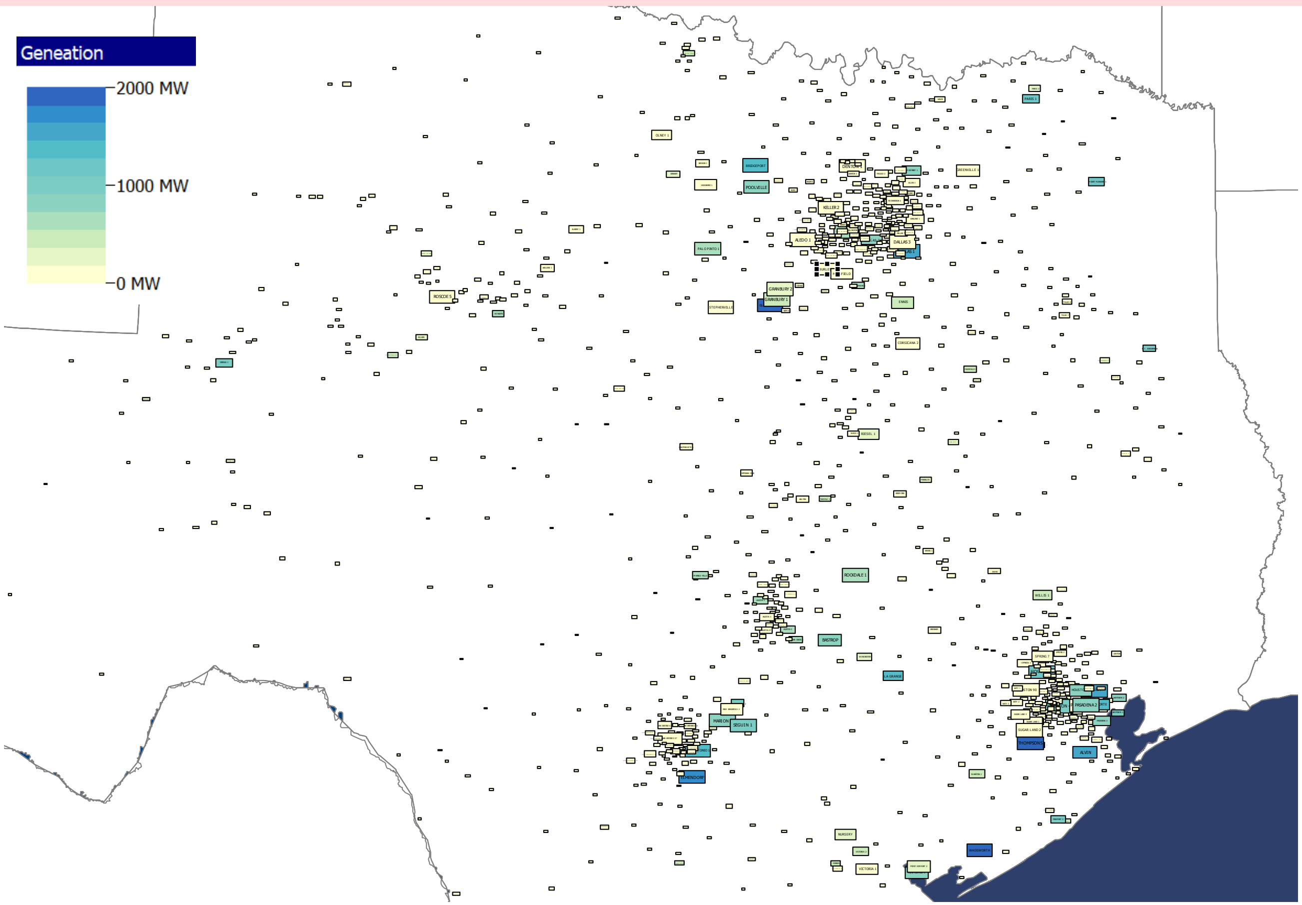
- Assume we want 5% drop peak to peak
  - $0.95 = e^{\lambda(t - T_{start})}$
  - Time for one cycle is 1/freq → $[t - T_{start} = 1/f]$
  - $0.95 = e^{\lambda/f}$ → $\ln(0.95) = \lambda/f$ → $\lambda = \ln(0.95) f$
- Plug this into Damping Ratio calculation
  - $\text{Damping Ratio} = -\ln(0.95) f / \sqrt{[\ln(0.95) f]^2 + (2\pi f)^2}$
  - The frequency cancels out in this equation
  - $\text{Damping Ratio} = -\ln(0.95) / \sqrt{[\ln(0.95)]^2 + (2\pi)^2} = 0.0081633$

Visualizing the Mode

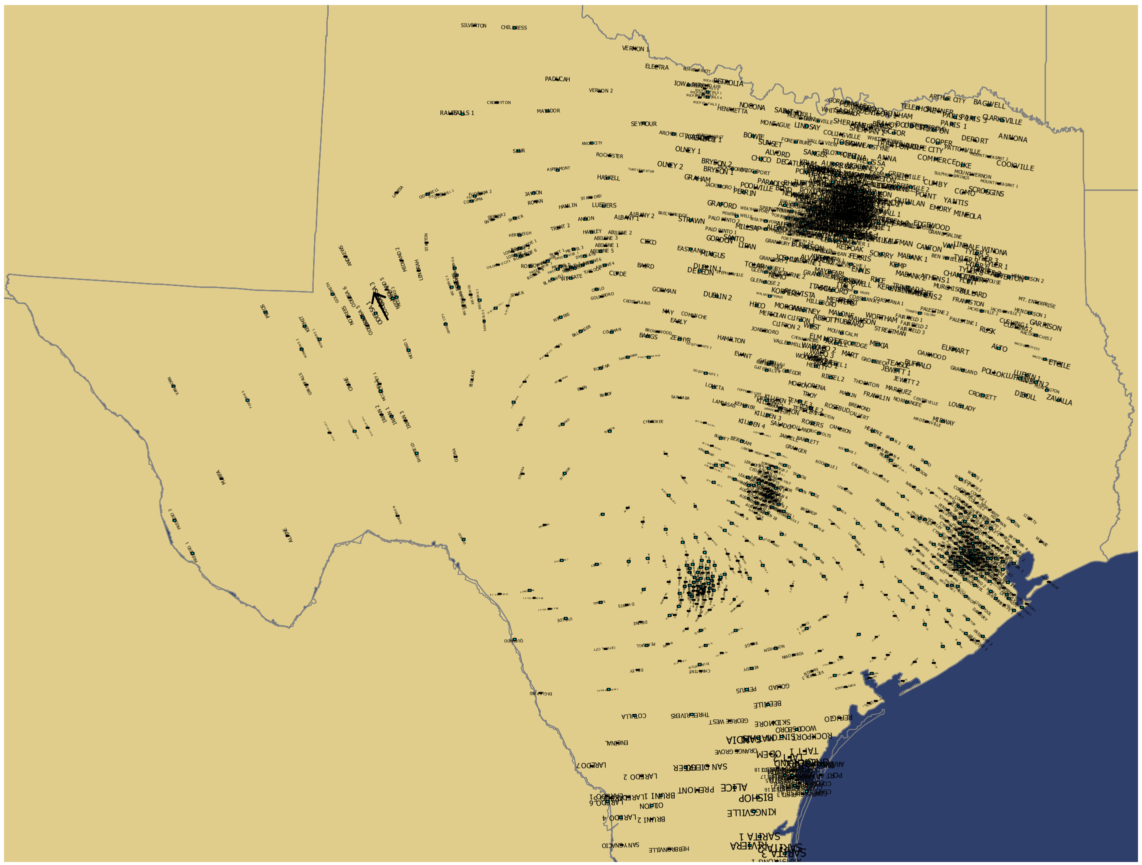
- If the grid has embedded geographic coordinates, the contributions for the mode to each signal can be readily visualized
- One approach is to utilize Geographic Data Views
  - T.J. Overbye, E.M. Rantanen, S. Judd, "Electric power control center visualizations using geographic data views," Bulk Power System Dynamics and Control -- VII. Revitalizing Operational Reliability -- 2007 IREP Symposium, Charleston, SC, August 2007, pp1-8; available at ieeexplore.ieee.org/document/4410539
- The GDVs will be used to show the geographic location of the magnitude and angle of the contribution of the mode in each signal

Texas 2000 Bus Substation GDV

Size is proportional to the substation MW throughput, while the color is based on the amount of substation generation; we'll use the same substation GDV to display damping

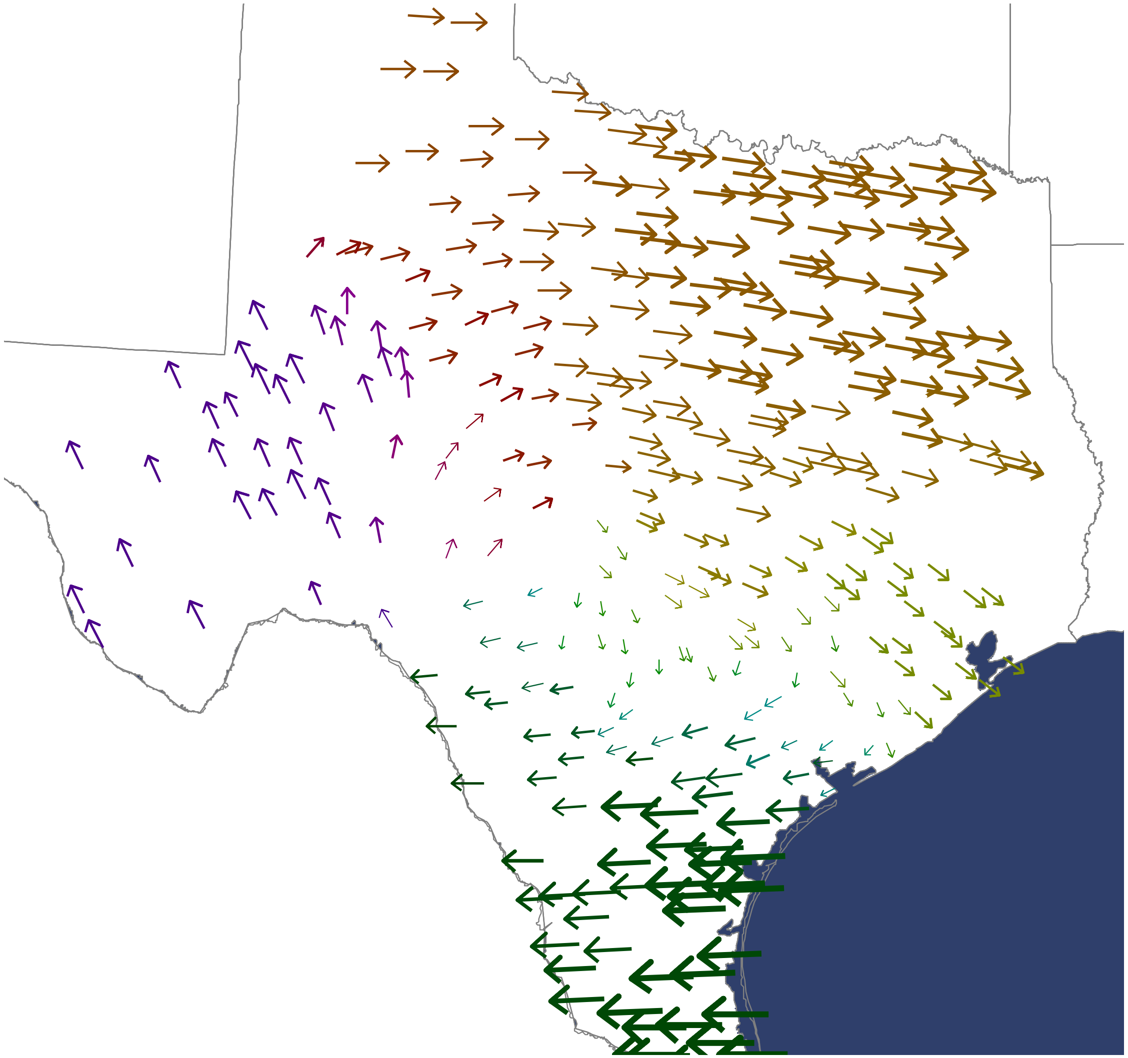


Visualization of 0.63 Hz Mode



In this display the arrows show the magnitude and angle (direction) for the mode at each substation. However, the problem is there are too many arrows! The solution it to dynamically prune the display using the GDV Options, Pruning command

Visualization of 0.63 Hz Mode with Pruning and Some Color



The display was pruned so only one arrow per geographic region is shown; the size of the arrow is proportional to its magnitude, and a color mapping is used for the angle

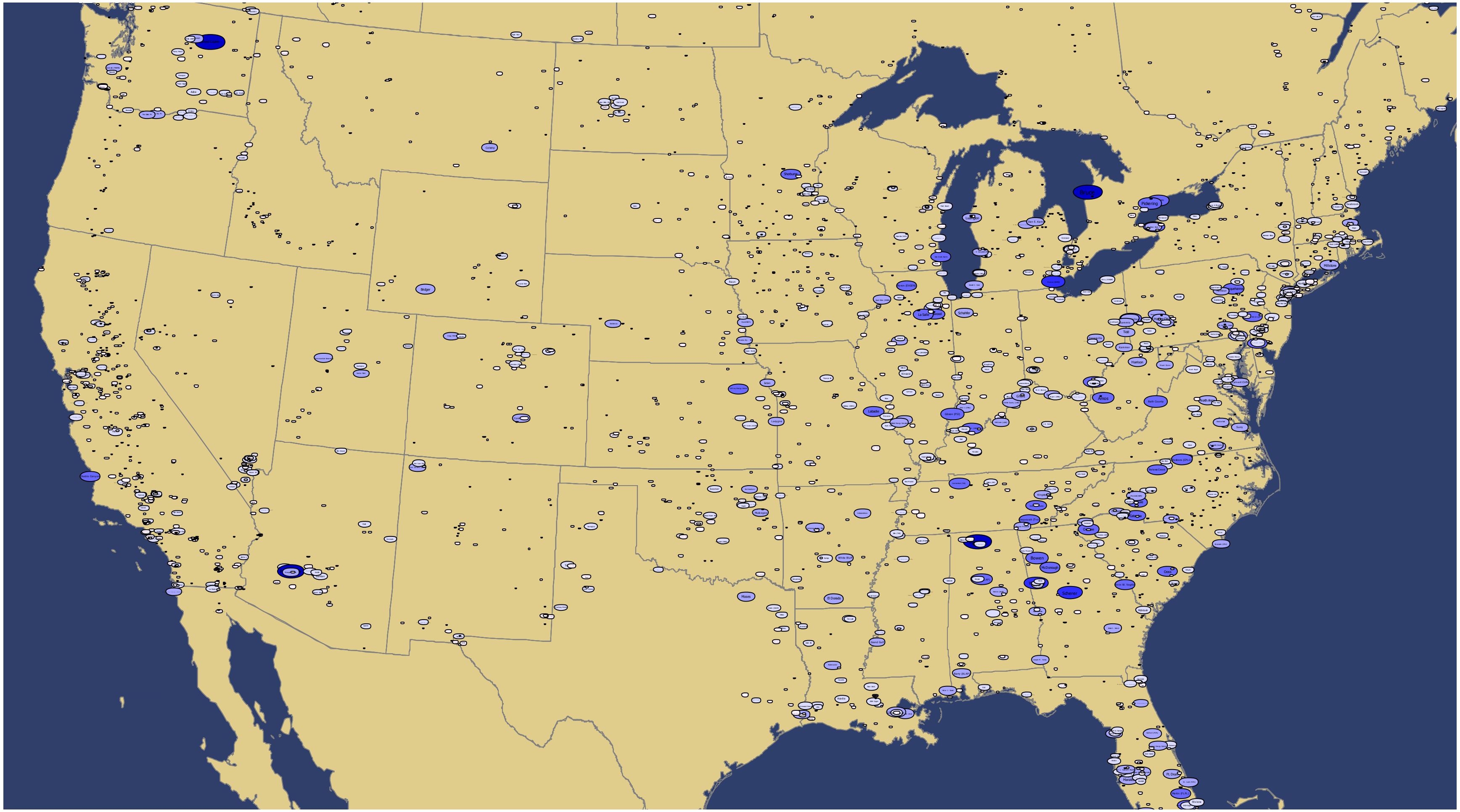
Application to a Larger System

- The following few slides show an application to a larger, real system
- The examples are from PSERC Project S-92G, which is currently looking at the dynamic aspects of interconnecting the North American Eastern and Western grids
- There are many cross-cutting issues associated with this, and additional PSERC industrial advisor involvement is welcomed!!!

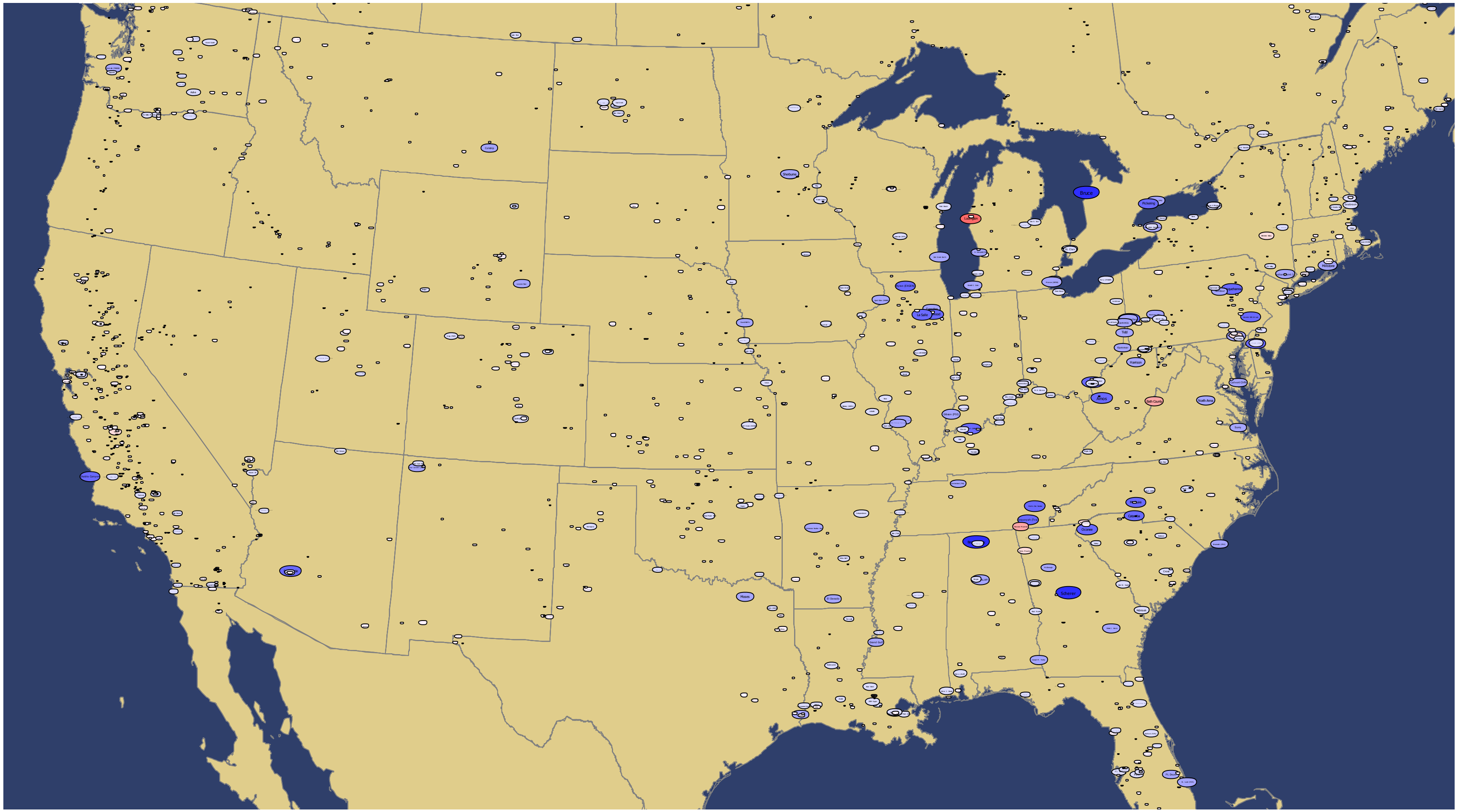
Some Preliminary Results from S-92G Germane to Modal Analysis

- The project is primarily looking at the dynamic aspects of interconnecting the grids, but is also considering static power flow and contingency analysis considerations
  - There is a public synthetic model analysis, and a not public consideration of the actual grid models
- The actual grid model was created by merging the East and West models
- It has 110,000 buses, 14,000 generators, 37,000 dynamic model devices with 243 different model types
  - Integrations are solved using a ½ cycle time step

Model 1: Heavy Load Conditions with 828 GW of Load; Substation GDV with Generation Sized and Colored by MW Value



Model 2: Light Load Conditions with 408 GW of Load; Substation GDV with Generation Sized and Colored by MW Value



Bus Frequency Results for a Generator Outage Contingency

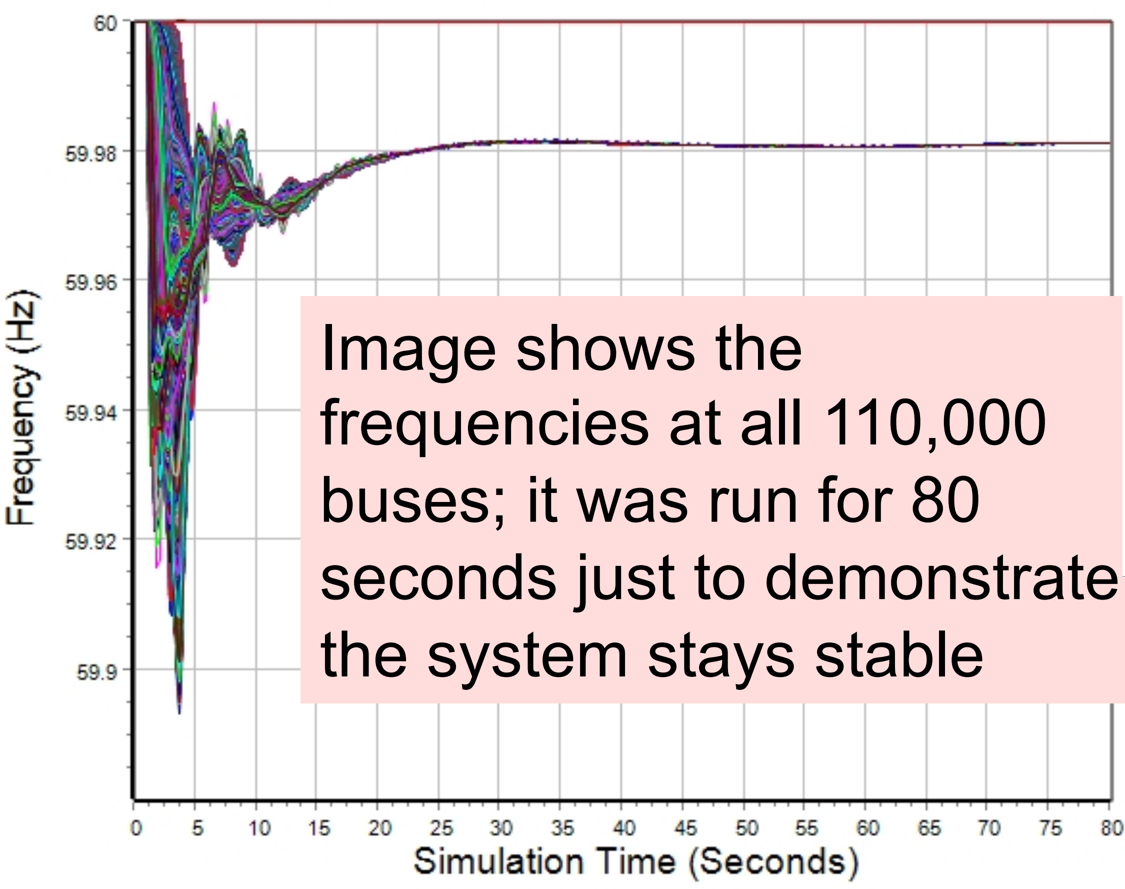
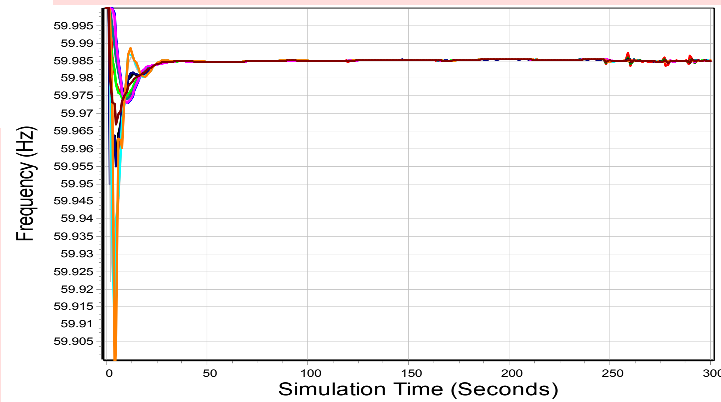
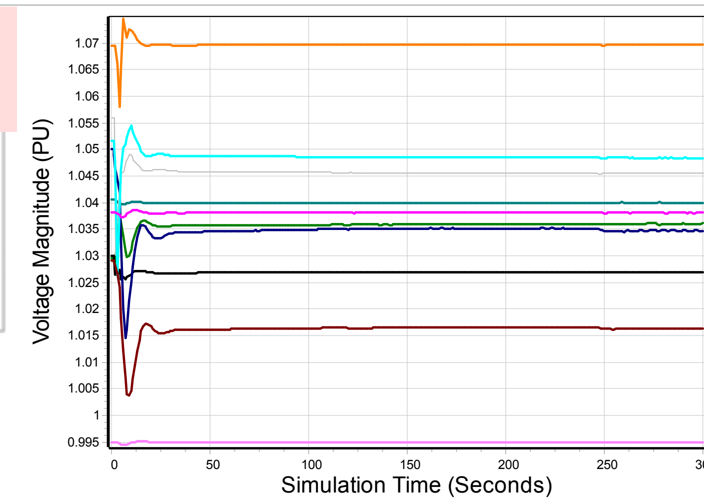


Image shows the frequencies at all 110,000 buses; it was run for 80 seconds just to demonstrate the system stays stable

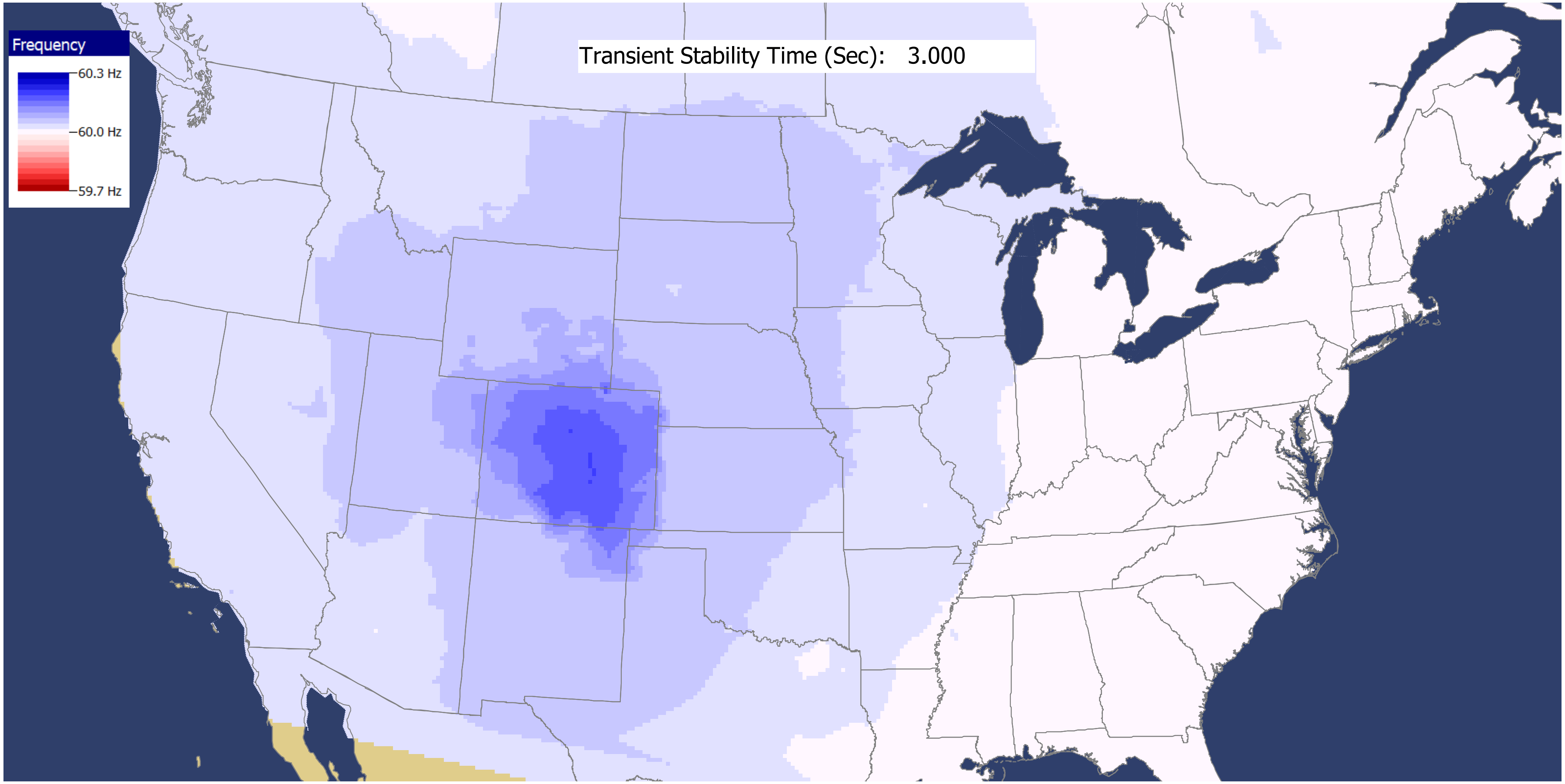
Five minute East-West simulations





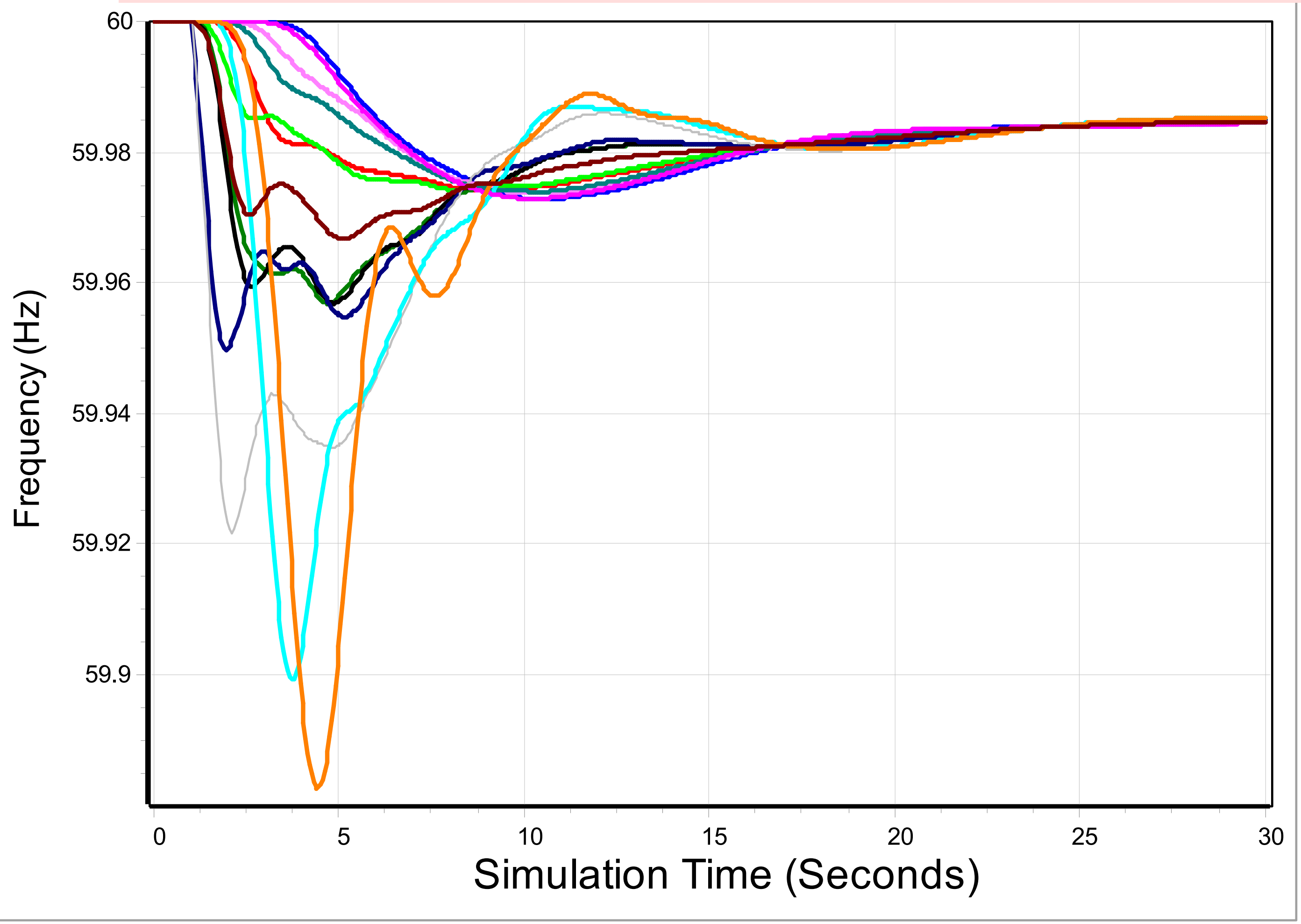
For modal analysis we'll be looking at the first 20 second

Spatial Frequency Contour (Movies Can Also be Easily Created)



Bus Frequency Results for a Generator Outage Contingency

A few selected results for the first 30 seconds



Iterative Matrix Pencil Method Applied to 43,400 Substation Signals

Processing all 43,400 signals took about 75 seconds (with 20 seconds of simulation data, sampling at 10 Hz)

Results

Number of Complex and Real Modes: 11

Lowest Percent Damping: 1.384

☑ Include Detrend in Reproduced Signals

☐ Subtract Reproduced from Actual

Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

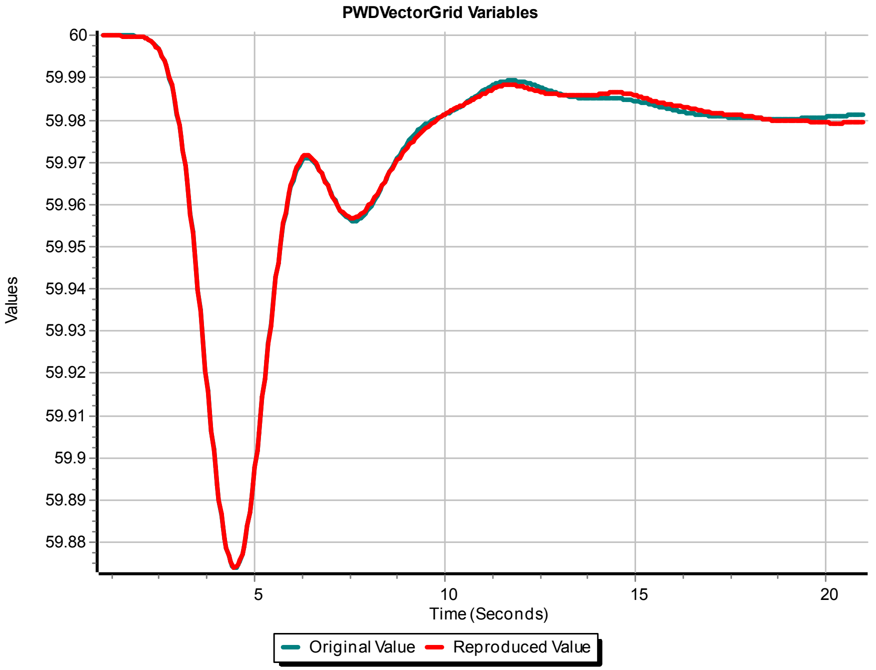
	Frequency (Hz)	Damping (%)	Largest Component Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	Include in Reproduce Signal
1	0.000	100.000	0.40738	Substation 337	33.497	Substation 337	-0.3848	YES
2	0.033	65.660	0.30063	Substation 337	24.165	Substation 337	-0.1832	YES
3	0.230	28.635	0.15452	Substation 337	6.082	Substation 337	-0.4316	YES
4	0.347	17.971	0.08249	Substation 320	3.246	Substation 320	-0.3987	YES
5	0.471	16.180	0.06326	Substation 337	2.801	Substation 337	-0.4848	YES
6	0.758	6.884	0.05116	Substation 300	3.202	Substation 300	-0.3285	YES
7	0.841	14.975	0.04579	Substation 341	3.651	Substation 337	-0.8004	YES
8	0.000	100.000	0.04051	Substation 337	8.528	Substation 347	-0.0443	YES
9	2.600	5.285	0.02356	Substation 337	1.909	Substation 337	-0.8646	YES
10	1.872	8.085	0.01473	Substation 320	1.188	Substation 320	-0.9539	YES
11	0.635	1.384	0.00376	Substation 337	0.166	Substation 337	-0.0552	YES

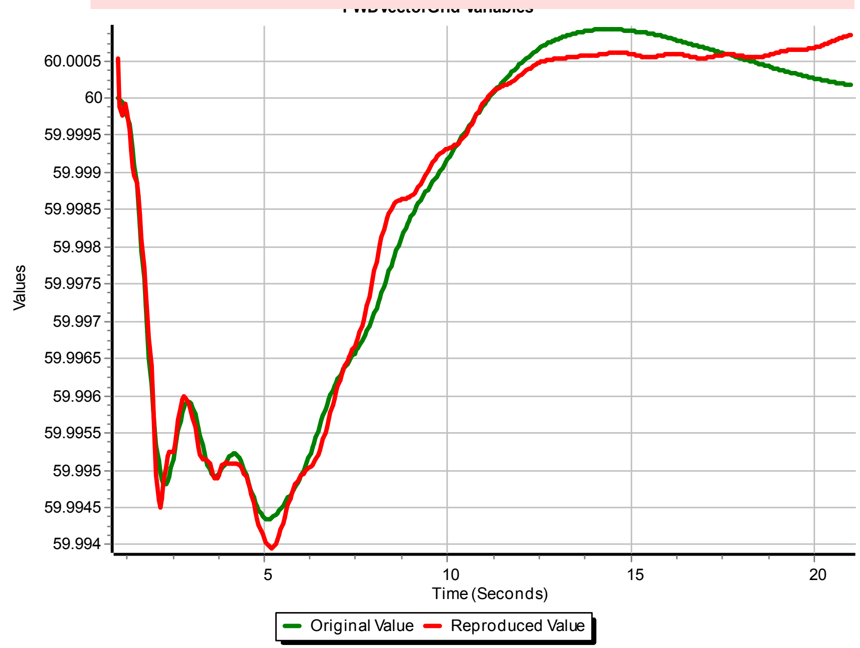
Iterative Matrix Pencil Method Applied to 43,400 Substation Signals

Trust but verify results

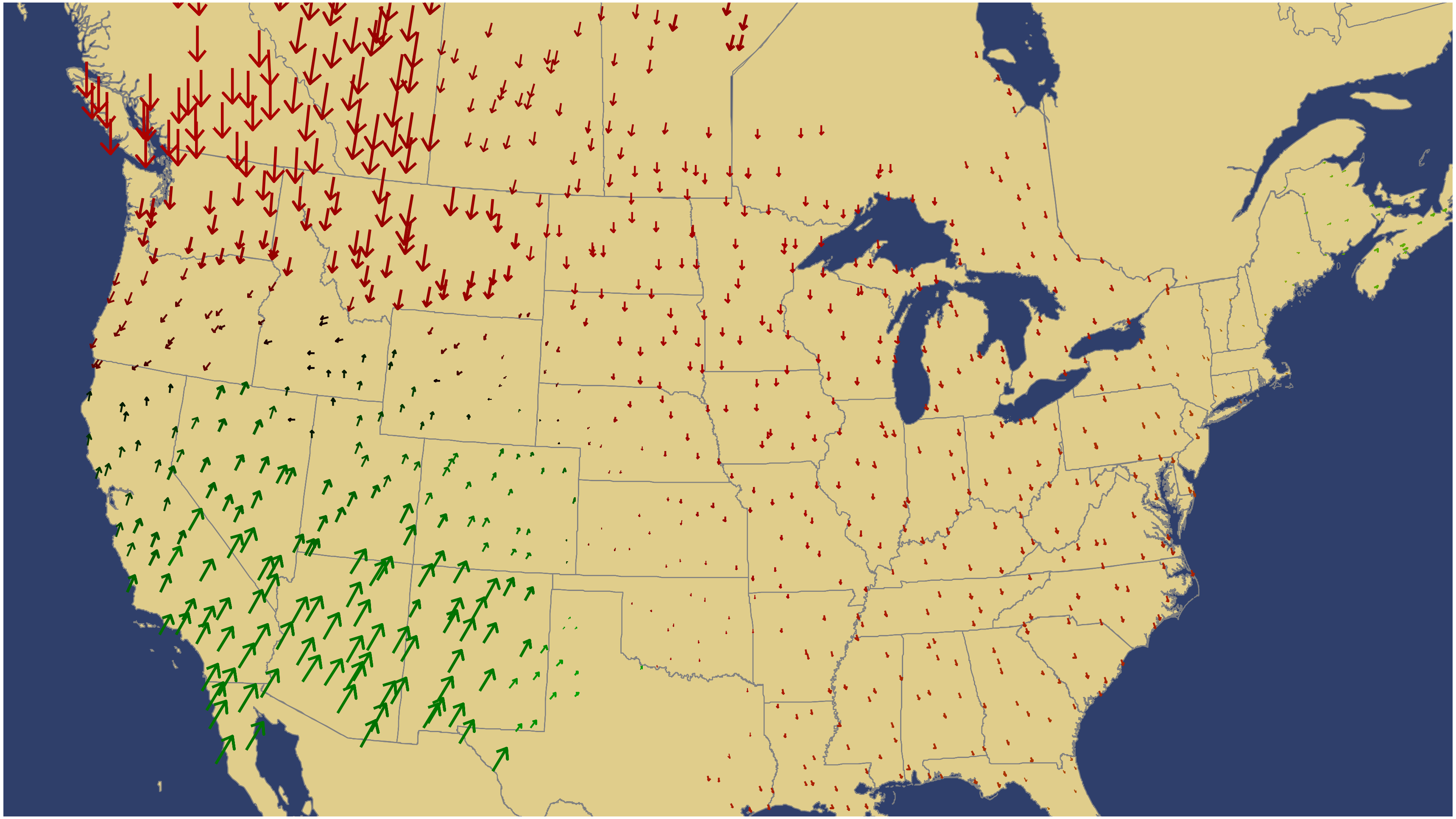
Matching for a large deviation example

The worst match (out of 43,400 signals); note the change in the y-axis

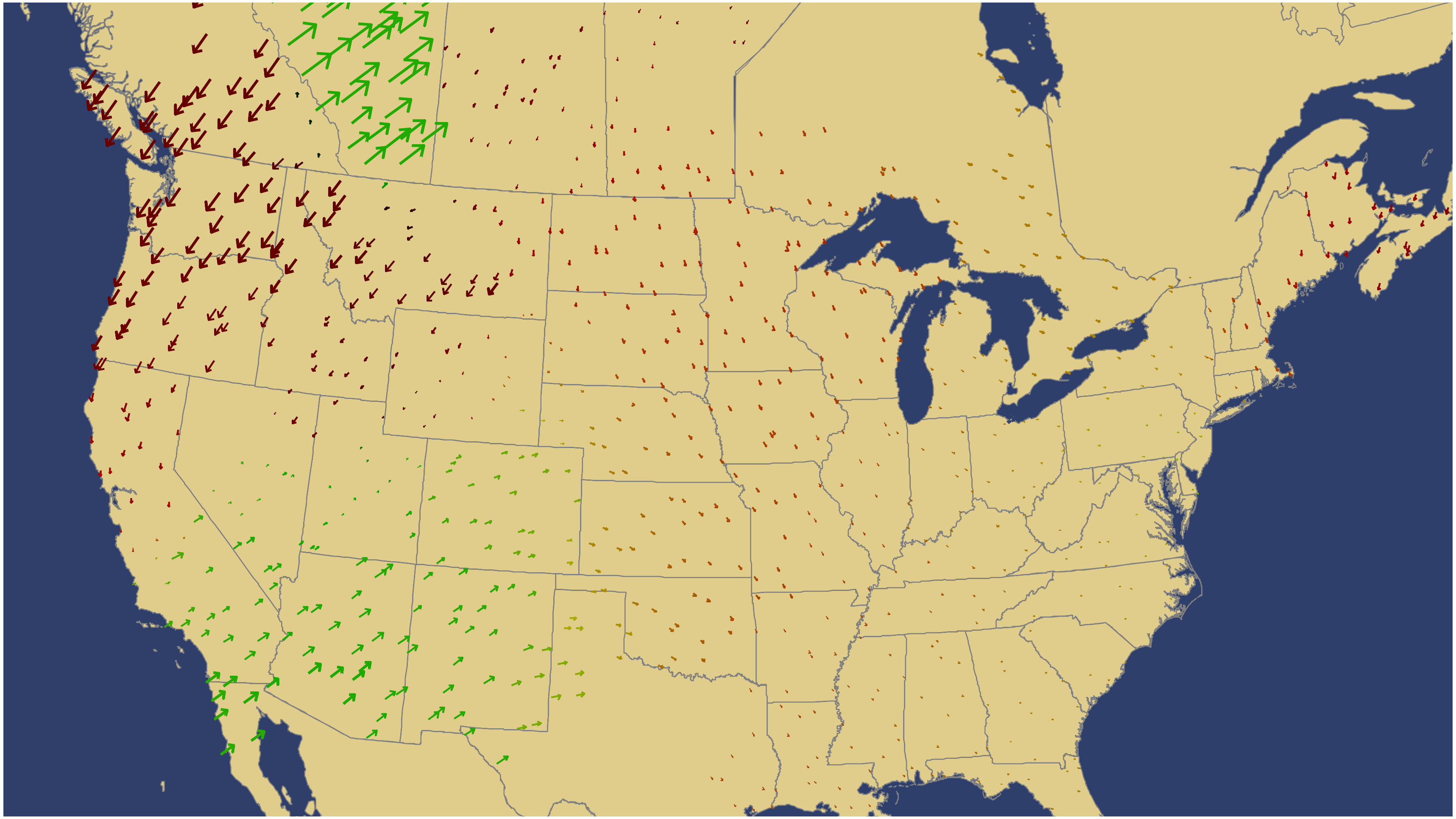




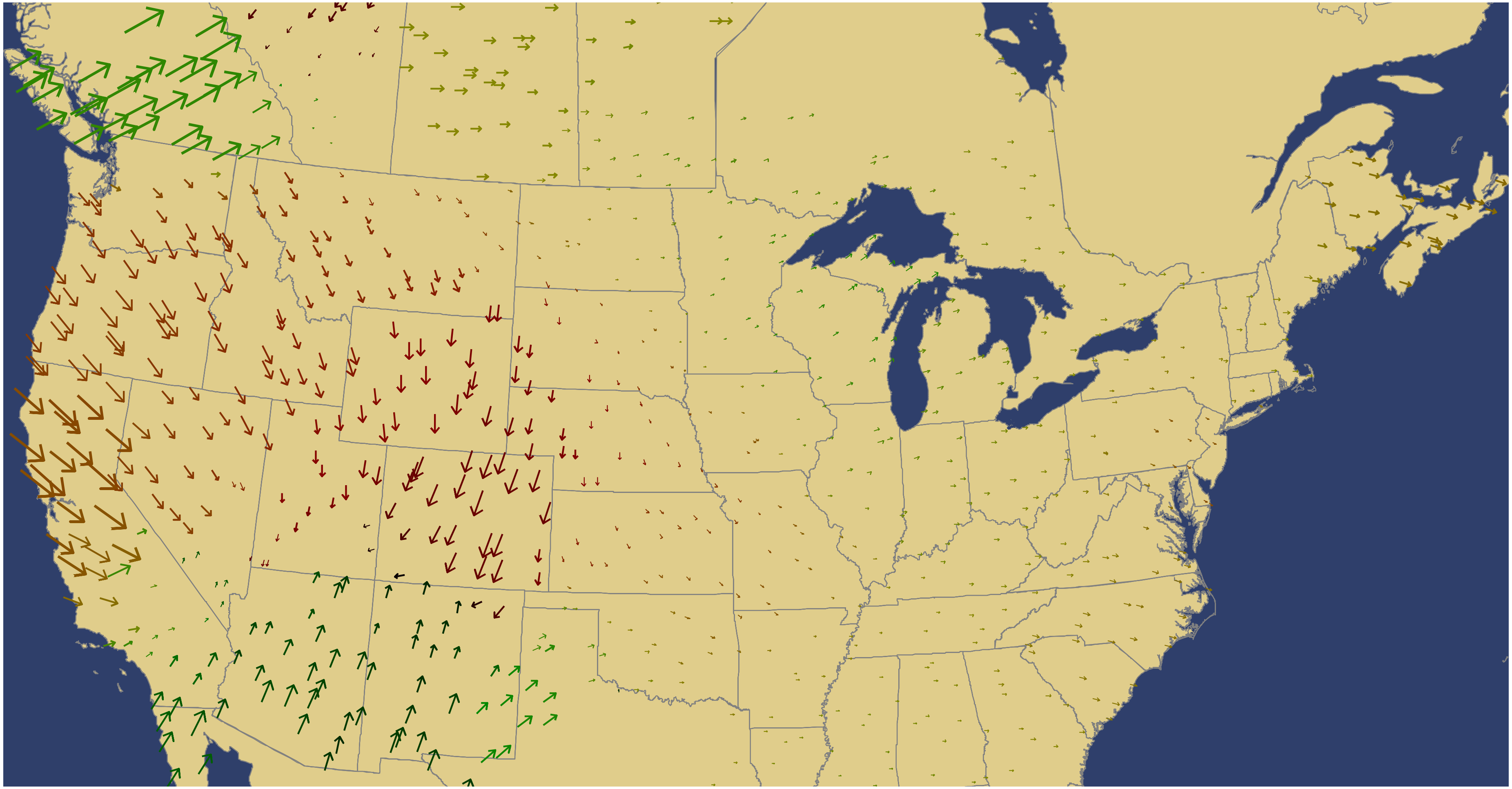
Large System Visualization of a Mode using GDVs



Large System GDV Visualization of Another Mode (Same Arrow Scale)



And a Third, Perhaps Less Familiar Mode (with 2x magnification)



Results with a Light Load

- Below are the results for the light load case. Modal analysis allows different conditions to be compared.

Results

Number of Complex and Real Modes: 12

Lowest Percent Damping: 3.494

☑ Include Detrend in Reproduced Signals

☐ Subtract Reproduced from Actual

Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (%)	Largest Component Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	Include Reprodu Signa
1	0.731	30.391	0.31420	Substation 337	20.539	Substation 337	-1.4651	YES
2	0.000	100.000	0.22750	Substation 337	21.146	Substation 348	-0.7839	YES
3	0.092	46.557	0.18418	Substation 250	10.815	Substation 250	-0.3035	YES
4	0.000	100.000	0.17542	Substation 337	14.070	Substation 337	-2.1556	YES
5	0.821	13.710	0.17379	Substation 341	8.972	Substation 341	-0.7144	YES
6	0.299	17.715	0.15336	Substation 320	4.273	Substation 320	-0.3381	YES
7	0.476	12.076	0.10477	Substation 337	3.656	Substation 337	-0.3640	YES
8	0.008	23.704	0.06188	Substation 250	6.221	Substation 347	-0.0115	YES
9	0.786	5.025	0.05632	Substation 300	2.662	Substation 300	-0.2484	YES
10	1.804	3.494	0.03150	Substation 320	1.986	Substation 320	-0.3962	YES
11	0.636	7.417	0.03124	Substation 337	1.415	Substation 337	-0.2974	YES

Summary

- The tutorial has covered the power system application of measurement-based modal analysis
- Techniques are now available that can be readily applied to both small and large sets of power system measurements, either from the actual system or from simulations
- The result is measurement-based modal analysis is now be a standard power system analysis tool
- Large-scale system results can also be readily visualized

Questions?

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