ECEN 615
Methods of Electric Power Systems Analysis

Lecture 4: Power Flow

Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

TEXAS A&M UNIVERSITY

Announcements

- Start reading Chapters 1 to 3 from the book (mostly background material)
- Homework 1 is assigned today. It is due on Thursday September 3
- Public website is
- https://overbye.engr.tamu.edu/ecen-615-fall-2020/

Using the Y_{bus}

If the voltages are known then we can solve for the current injections:

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

If the current injections are known then we can solve for the voltages:

$$\mathbf{Y}_{bus}^{-1}\mathbf{I} = \mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

where $\mathbf{Z}_{\text{bus}}$ is the bus impedance matrix

However, this requires that $\mathbf{Y}_{\text{bus}}$ not be singular; note it will be singular if there are no shunt connections!

Solving for Bus Currents

For example, in previous case assume

$$\mathbf{V} = \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is

$$\mathrm{S}_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$

Solving for Bus Voltages

For example, in previous case assume

$$\mathbf{I} = \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}^{-1} \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j1.098 \end{bmatrix}$$

Therefore the power injected is

$$\mathrm{S}_1 = V_1 I_1^* = (0.0738 - j0.902) \times 5 = 0.37 - j4.51$$

$$S_2 = V_2 I_2^* = (-0.0738 - j1.098) \times (-4.8) = 0.35 + j5.27$$

Power Flow Analysis

- When analyzing power systems we know neither the complex bus voltages nor the complex current injections
- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes
- Therefore we can not directly use the Y_{bus} equations, but rather must use the power balance equations

Power Balance Equations

From KCL we know at each bus i in an n bus system the current injection, I_i, must be equal to the current that flows into the network

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} I_{ik}$$

Since $\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V}$ we also know

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} Y_{ik} V_k$$

The network power injection is then $\mathrm{S_i} = V_i I_i^*$

Power Balance Equations, cont'd

$$\mathrm{S_i} = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

This is an equation with complex numbers.

Sometimes we would like an equivalent set of real power equations. These can be derived by defining

$$Y_{ik} = G_{ik} + jB_{ik}$$

$$V_i = |V_i| e^{j\theta_i} = |V_i| \angle \theta_i$$

$$\theta_{ik} = \theta_i - \theta_k$$

Recall $\mathrm{e}^{\mathrm{j}\theta} = \cos\theta + j\sin\theta$

Real Power Balance Equations

$$S_i = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} = \sum_{k=1}^{n} |V_i||V_k| e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts

$$P_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Power Flow Analysis

- When analyzing power systems we know neither the complex bus voltages nor the complex current injections
- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes
- Therefore we can not directly use the Y_{bus} equations, but rather must use the power balance equations

Power Flow Analysis

- Classic paper for this lecture is W.F. Tinney and C.E. Hart, "Power Flow Solution by Newton's Method," *IEEE Power App System*, Nov 1967
- Basic power flow is also covered in essentially any power system analysis textbooks.
- We use the term "power flow" not "load flow" since power flows not load. Also, the power flow usage is not new (see title of Tinney's 1967 paper, and note Tinney references Ward's 1956 power flow paper)
  - A nice history of the power flow is given in an insert by Alvarado and Thomas in T.J. Overbye, J.D. Weber, "Visualizing the Electric Grid," *IEEE Spectrum*, Feb 2001.

Early Power Flow System Size

- In 1957 Bill Tinney, in a paper titled "Digital Solutions for Large Power Networks," studied a 100 bus, 200 branch system (with 2 KB of memory)!
- In Tinney's 1963 "Techniques for Exploiting Sparsity of the Network Admittance Matrix" paper (which gave us the Tinney Schemes 1, 2, and 3), uses 32 kB for 1000 nodes.
- In Tinney's classic 1967 "Power Flow Solution by Newton's Method" paper he applies his method to systems with up to about 1000 buses (with 32 kB of memory) and provides a solution time of 51 seconds for a 487 bus system.

Slack Bus

- We can not arbitrarily specify S at all buses because total generation must equal total load + total losses
- We also need an angle reference bus.
- To solve these problems we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, and a varying real/reactive power injection.
- In an actual power system the slack bus does not really exist; frequency changes locally when the power supplied does not match the power consumed

Three Types of Power Flow Buses

- There are three main types of power flow buses
  - Load (PQ) at which P/Q are fixed; iteration solves for voltage magnitude and angle.
  - Slack at which the voltage magnitude and angle are fixed; iteration solves for P/Q injections
  - Generator (PV) at which P and |V| are fixed; iteration solves for voltage angle and Q injection

Newton-Raphson Algorithm

- Most common technique for solving the power flow problem is to use the Newton-Raphson algorithm
- Key idea behind Newton-Raphson is to use sequential linearization

General form of problem: Find an **x** such that

$$\mathbf{f}(\hat{\mathbf{x}}) = 0$$

Newton-Raphson Power Flow

In the Newton-Raphson power flow we use Newton's method to determine the voltage magnitude and angle at each bus in the power system.

We need to solve the power balance equations

$$P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Power Flow Variables

Assume the slack bus is the first bus (with a fixed voltage angle/magnitude). We then need to determine the voltage angle/magnitude at the other buses.

$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_{\mathrm{n}} \\ |V_2| \\ \vdots \\ |V_n| \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_{G2} + P_{D2} \\ \vdots \\ P_n(\mathbf{x}) - P_{Gn} + P_{Dn} \\ Q_2(\mathbf{x}) - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n(\mathbf{x}) - Q_{Gn} + Q_{Dn} \end{bmatrix}$$

N-R Power Flow Solution

The power flow is solved using the same procedure discussed with the general Newton-Raphson:

Set $v = 0$; make an initial guess of $\mathbf{x}$, $\mathbf{x}^{(v)}$

While $\left\| \mathbf{f}(\mathbf{x}^{(v)}) \right\| > \varepsilon$ Do

$$\begin{aligned} \mathbf{x}^{(v+1)} &= \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(v)})^{-1}\mathbf{f}(\mathbf{x}^{(v)}) \\ v &= v+1 \end{aligned}$$

End While

Power Flow Jacobian Matrix

The most difficult part of the algorithm is determining and inverting the n by n Jacobian matrix, $\mathbf{J}(\mathbf{x})$

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \frac{\partial f_n(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}$$

Power Flow Jacobian Matrix, cont'd

Jacobian elements are calculated by differentiating each function, $f_i(\mathbf{x})$, with respect to each variable.

For example, if $f_i(\mathbf{x})$ is the bus i real power equation

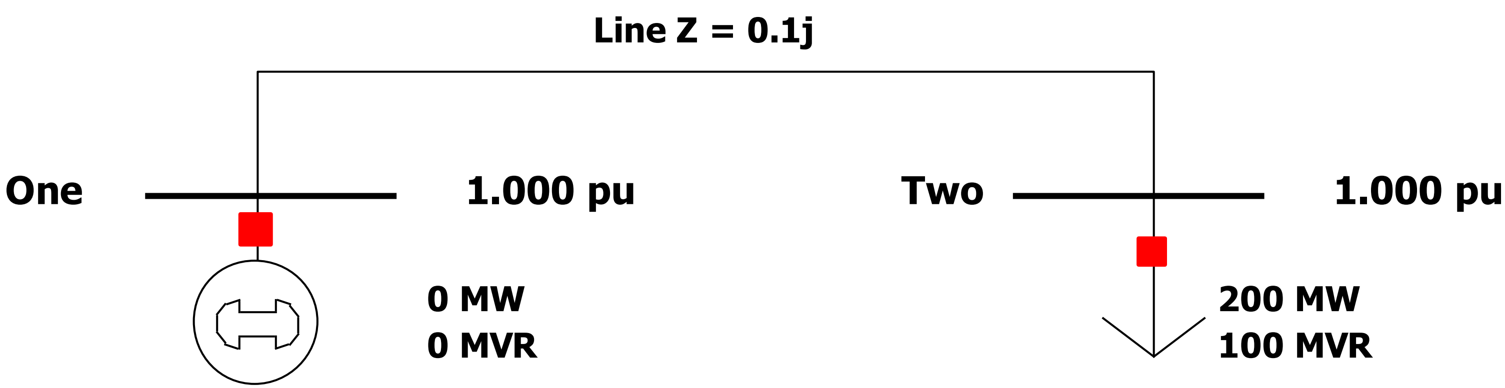
$$f_i(x) = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) - P_{Gi} + P_{Di}$$

$$\frac{\partial f_i(x)}{\partial \theta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(-G_{ik}\sin\theta_{ik} + B_{ik}\cos\theta_{ik})$$

$$\frac{\partial f_i(x)}{\partial \theta_j} = |V_i||V_j|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \quad (j \neq i)$$

Two Bus Newton-Raphson Example

- For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and S_{Base} = 100 MVA.



$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix} \qquad \mathbf{Y}_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

Two Bus Example, cont'd

General power balance equations

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Bus two power balance equations

$$|V_2||V_1|(10\sin\theta_2) + 2.0 = 0$$

$$|V_2||V_1|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 = 0$$

Two Bus Example, cont'd

$$P_2(\mathbf{x}) = |V_2|(10\sin\theta_2) + 2.0 = 0$$

$$Q_2(\mathbf{x}) = |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 = 0$$

Now calculate the power flow Jacobian

$$J(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial P_2(\mathbf{x})}{\partial \theta_2} & \dfrac{\partial P_2(\mathbf{x})}{\partial |V|_2} \\ \dfrac{\partial Q_2(\mathbf{x})}{\partial \theta_2} & \dfrac{\partial Q_2(\mathbf{x})}{\partial |V|_2} \end{bmatrix}$$

$$= \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| \end{bmatrix}$$

Two Bus Example, First Iteration

$$\text{Set } v = 0\text{, guess } \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Calculate

$$\mathrm{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\theta_2) + 2.0 \\ |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$\text{Solve } \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$$

Two Bus Example, Next Iterations

$$\mathrm{f}(\mathbf{x}^{(1)}) = \begin{bmatrix} 0.9(10\sin(-0.2)) + 2.0 \\ 0.9(-10\cos(-0.2)) + 0.9^2 \times 10 + 1.0 \end{bmatrix} = \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(1)}) = \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}$$

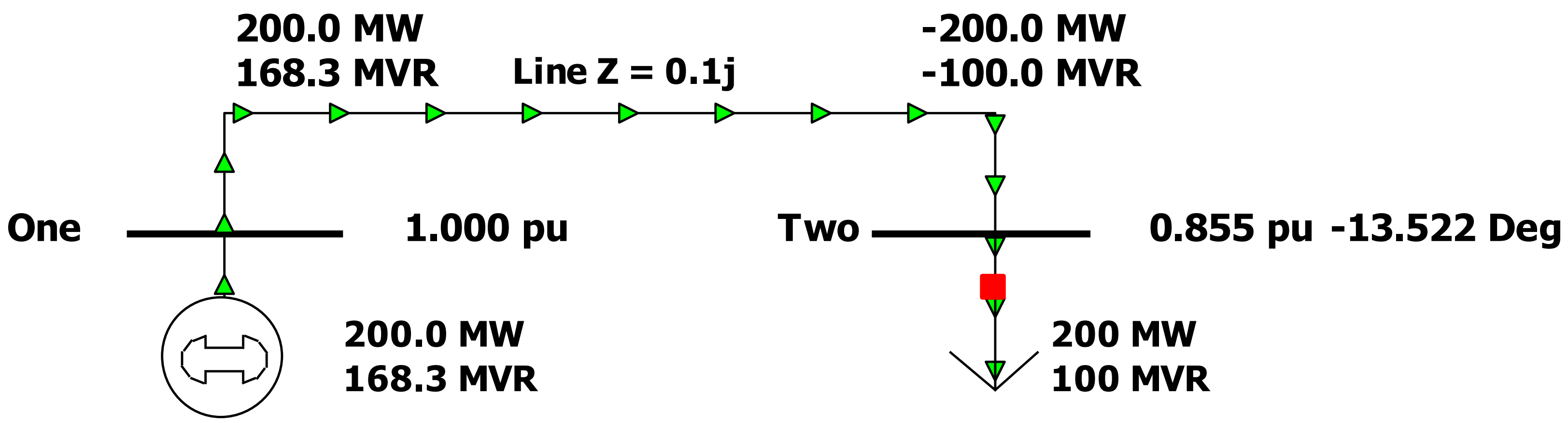
$$\mathbf{x}^{(2)} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}^{-1} \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix} = \begin{bmatrix} -0.233 \\ 0.8586 \end{bmatrix}$$

$$\mathrm{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.0145 \\ 0.0190 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} -0.236 \\ 0.8554 \end{bmatrix}$$

$$\mathrm{f}(\mathbf{x}^{(3)}) = \begin{bmatrix} 0.0000906 \\ 0.0001175 \end{bmatrix} \qquad \text{Done!} \qquad V_2 = 0.8554\angle -13.52^\circ$$

Two Bus Solved Values

- Once the voltage angle and magnitude at bus 2 are known we can calculate all the other system values, such as the line flows and the generator reactive power



PowerWorld Case Name: Bus2_Intro
Note, most PowerWorld cases will be available on the course website

Two Bus Case Low Voltage Solution

This case actually has two solutions! The second "low voltage" is found by using a low initial guess.

Set $v = 0$, guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$

Calculate

$$\mathrm{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\theta_2) + 2.0 \\ |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2 \\ -0.875 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 2.5 & 0 \\ 0 & -5 \end{bmatrix}$$

Low Voltage Solution, cont'd

$$\text{Solve } \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix} - \begin{bmatrix} 2.5 & 0 \\ 0 & -5 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -0.875 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0.075 \end{bmatrix}$$

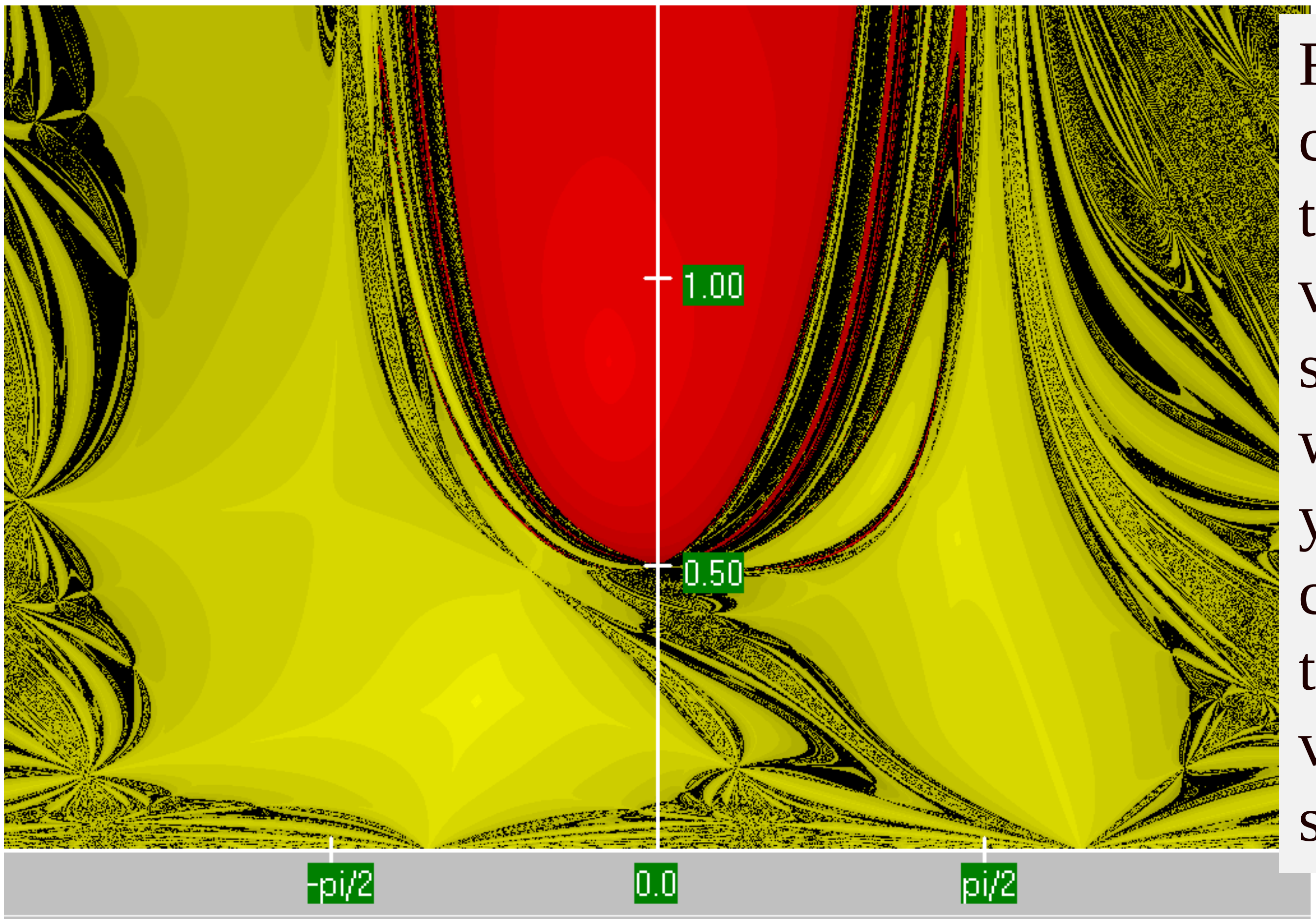
$$\mathbf{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 1.462 \\ 0.534 \end{bmatrix} \quad \mathbf{x}^{(2)} = \begin{bmatrix} -1.42 \\ 0.2336 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} -0.921 \\ 0.220 \end{bmatrix}$$

Practical Power Flow Software Note

- Most commercial software packages have built in defaults to prevent convergence to low voltage solutions.
  - One approach is to automatically change the load model from constant power to constant current or constant impedance when the load bus voltage gets too low
  - In PowerWorld these defaults can be modified on the Tools, Simulator Options, Advanced Options page; note you also need to disable the "Initialize from Flat Start Values" option
  - The PowerWorld case Bus2_Intro_Low is set solved to the low voltage solution
  - Initial bus voltages can be set using the Bus Information Dialog

Two Bus Region of Convergence

Slide shows the region of convergence for different initial guesses of bus 2 angle (x-axis) and magnitude (y-axis)



Red region converges to the high voltage solution, while the yellow region converges to the low voltage solution

Power Flow Fractal Region of Convergence

- Earliest paper showing fractal power flow regions of convergence is by C.L DeMarco and T.J. Overbye, “Low Voltage Power Flow Solutions and Their Role in Exit Time Bases Security Measures for Voltage Collapse,” *Proc. 27th IEEE CDC*, December 1988
- A more widely known paper is J.S. Thorp, S.A. Naqavi, “Load-Flow Fractals Draw Clues to Erratic Behavior,” *IEEE Computer Applications in Power*, January 1997

PV Buses

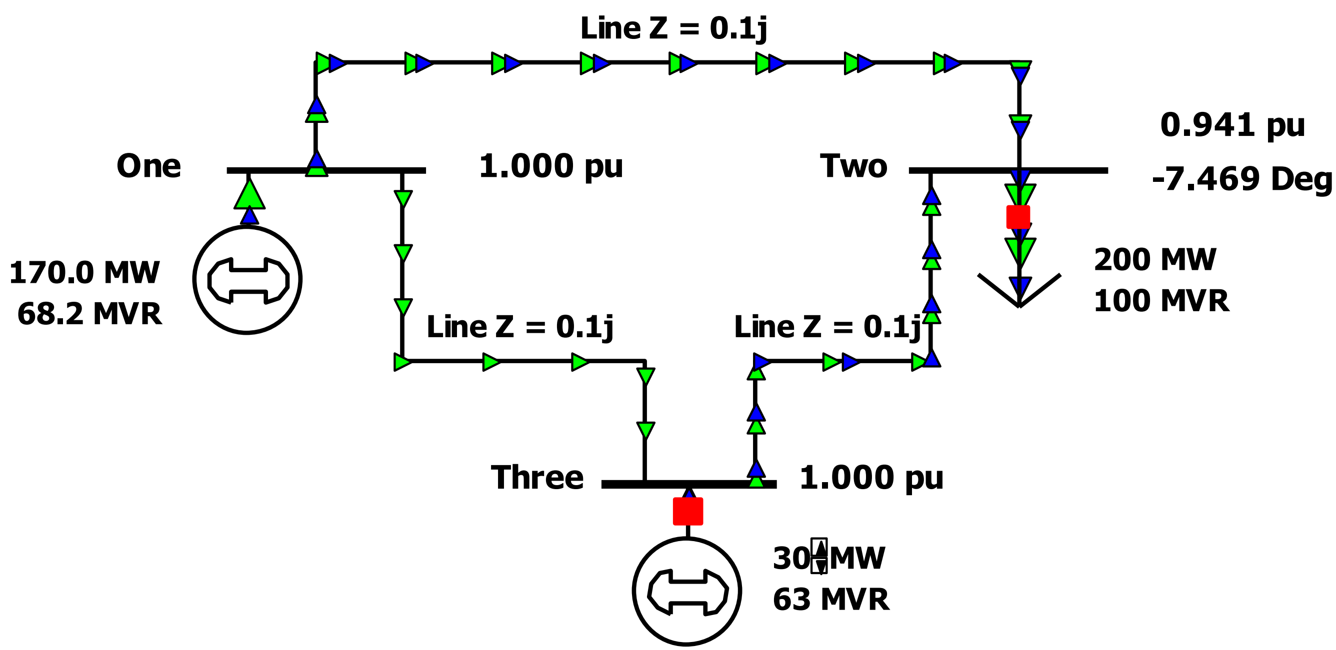
- Since the voltage magnitude at PV buses is fixed there is no need to explicitly include these voltages in **x** or write the reactive power balance equations
  - the reactive power output of the generator varies to maintain the fixed terminal voltage (within limits)
  - optionally these variations/equations can be included by just writing the explicit voltage constraint for the generator bus

$$|V_i| - V_{i\ setpoint} = 0$$

Three Bus PV Case Example

For this three bus case we have

$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ \theta_3 \\ |V_2| \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_{G2} + P_{D2} \\ P_3(\mathbf{x}) - P_{G3} + P_{D3} \\ Q_2(\mathbf{x}) + Q_{D2} \end{bmatrix} = 0$$



Modeling Voltage Dependent Load

So far we've assumed that the load is independent of the bus voltage (i.e., constant power). However, the power flow can be easily extended to include voltage depedence with both the real and reactive load. This is done by making P_{Di} and Q_{Di} a function of $|V_i|$:

$$\sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) - P_{Gi} + P_{Di}(|V_i|) = 0$$

$$\sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) - Q_{Gi} + Q_{Di}(|V_i|) = 0$$

Voltage Dependent Load Example

In previous two bus example now assume the load is constant impedance, so

$$P_2(\mathbf{x}) = |V_2|(10\sin\theta_2) + 2.0|V_2|^2 = 0$$

$$Q_2(\mathbf{x}) = |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0|V_2|^2 = 0$$

Now calculate the power flow Jacobian

$$J(\mathbf{x}) = \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 + 4.0|V_2| \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| + 2.0|V_2| \end{bmatrix}$$

Voltage Dependent Load, cont'd

Again set $v = 0$, guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Calculate

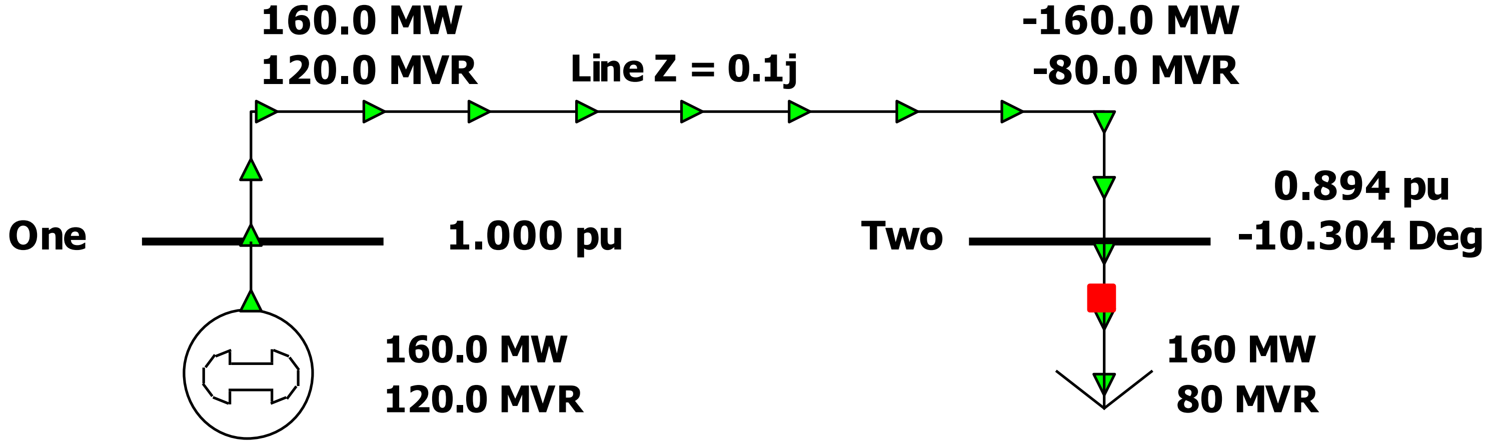
$$\mathrm{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\theta_2) + 2.0|V_2|^2 \\ |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0|V_2|^2 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10 & 4 \\ 0 & 12 \end{bmatrix}$$

Solve $\mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 4 \\ 0 & 12 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -0.1667 \\ 0.9167 \end{bmatrix}$

Voltage Dependent Load, cont'd

With constant impedance load the MW/Mvar load at bus 2 varies with the square of the bus 2 voltage magnitude. This if the voltage level is less than 1.0, the load is lower than 200/100 MW/Mvar



PowerWorld Case Name: **Bus2_Intro_Z**

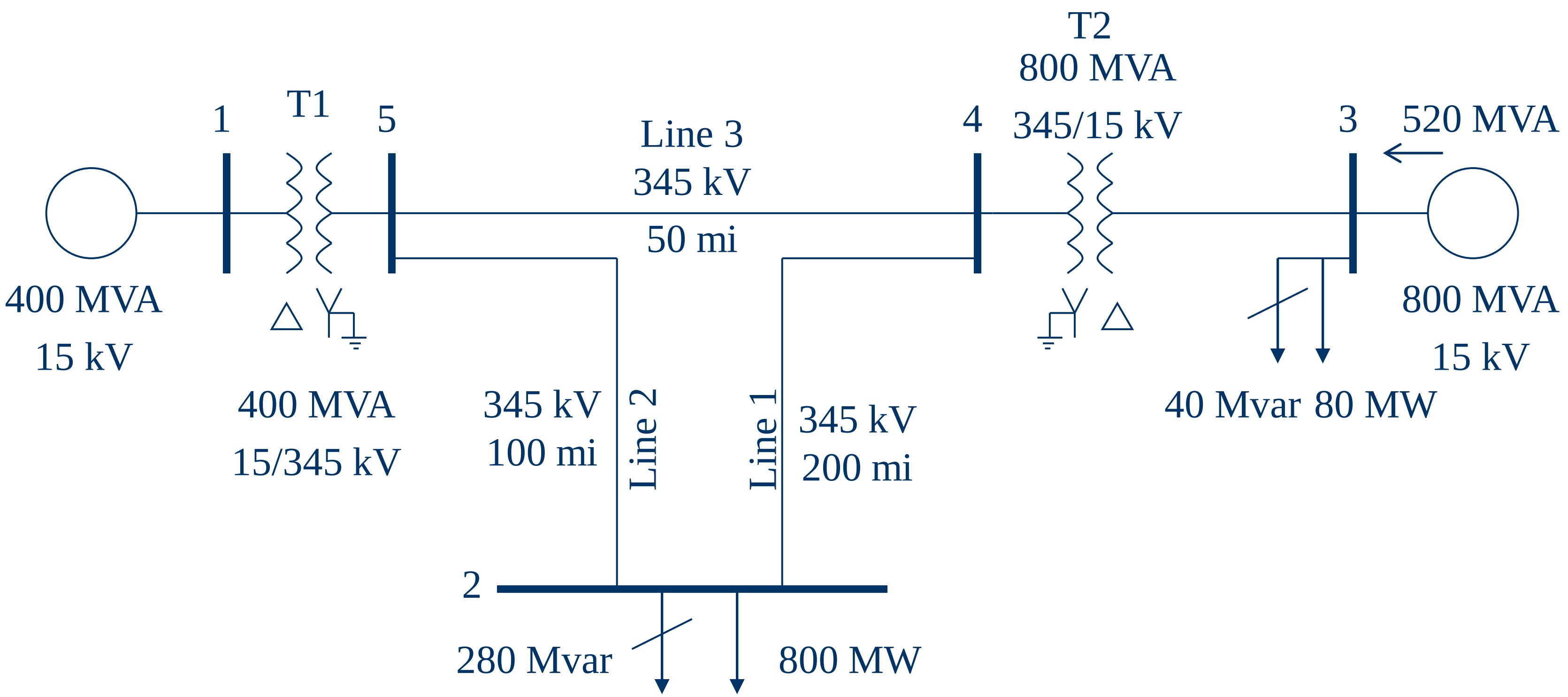
Generator Reactive Power Limits

- The reactive power output of generators varies to maintain the terminal voltage; on a real generator this is done by the exciter
- To maintain higher voltages requires more reactive power
- Generators have reactive power limits, which are dependent upon the generator's MW output
- These limits must be considered during the power flow solution.

Generator Reactive Limits, cont'd

- During the power flow once a solution is obtained there is a check to make sure the generator reactive power output is within its limits
- If the reactive power is outside of the limits, fix Q at the max or min value, and resolve treating the generator as a PQ bus
  - this is know as "type-switching"
  - also need to check if a PQ generator can again regulate
- Rule of thumb: to raise system voltage we need to supply more vars

The N-R Power Flow: 5-bus Example



Single-line diagram

This five bus example is taken from Chapter 6 of Power System Analysis and Design by Glover, Overbye, and Sarma, 6th Edition, 2016

The N-R Power Flow: 5-bus Example

Table 1. Bus input data

Bus	Type	V per unit	δ degrees	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	-2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

Table 2. Line input data

Bus-to-Bus	R' per unit	X' per unit	G' per unit	B' per unit	Maximum MVA per unit
2-4	0.0090	0.100	0	1.72	12.0
2-5	0.0045	0.050	0	0.88	12.0
4-5	0.00225	0.025	0	0.44	12.0

The N-R Power Flow: 5-bus Example

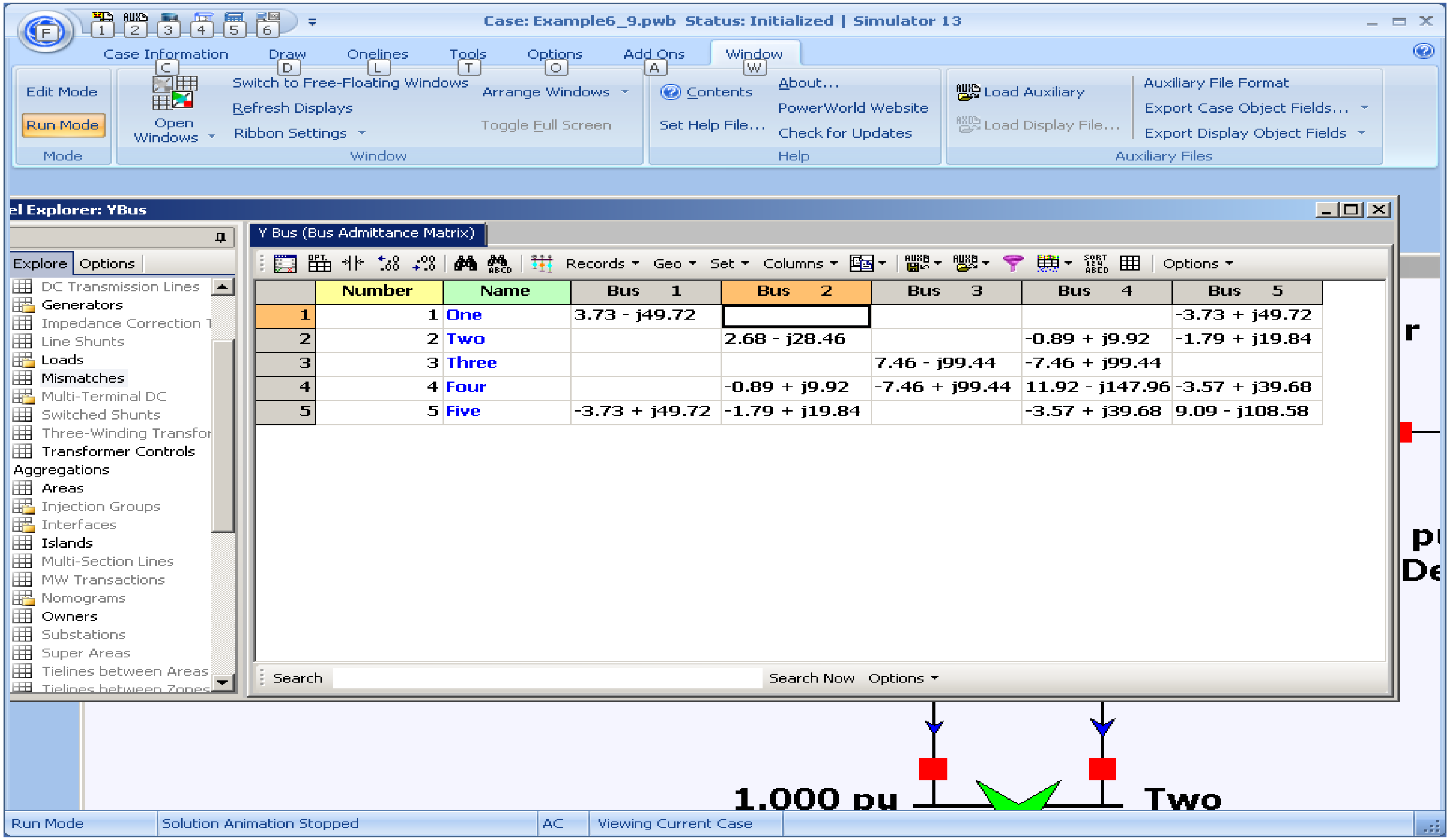
Table 3. Transformer input data

Bus-to-Bus	R per unit	X per unit	G_c per unit	B_m per unit	Maximum MVA per unit	Maximum TAP Setting per unit
1-5	0.00150	0.02	0	0	6.0	—
3-4	0.00075	0.01	0	0	10.0	—

Table 4. Input data and unknowns

Bus	Input Data	Unknowns
1	$V_1 = 1.0,\ \delta_1 = 0$	$P_1,\ Q_1$
2	$P_2 = P_{G2} - P_{L2} = -8$ $Q_2 = Q_{G2} - Q_{L2} = -2.8$	$V_2,\ \delta_2$
3	$V_3 = 1.05$ $P_3 = P_{G3} - P_{L3} = 4.4$	$Q_3,\ \delta_3$
4	$P_4 = 0,\ Q_4 = 0$	$V_4,\ \delta_4$
5	$P_5 = 0,\ Q_5 = 0$	$V_5,\ \delta_5$

Five Bus Case Ybus



	Number	Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
1	1	One	3.73 - j49.72				-3.73 + j49.72
2	2	Two		2.68 - j28.46		-0.89 + j9.92	-1.79 + j19.84
3	3	Three			7.46 - j99.44	-7.46 + j99.44	
4	4	Four		-0.89 + j9.92	-7.46 + j99.44	11.92 - j147.96	-3.57 + j39.68
5	5	Five	-3.73 + j49.72	-1.79 + j19.84		-3.57 + j39.68	9.09 - j108.58

PowerWorld Case Name: **GOS_FiveBus**

Ybus Calculation Details

Elements of Y_{bus} connected to bus 2

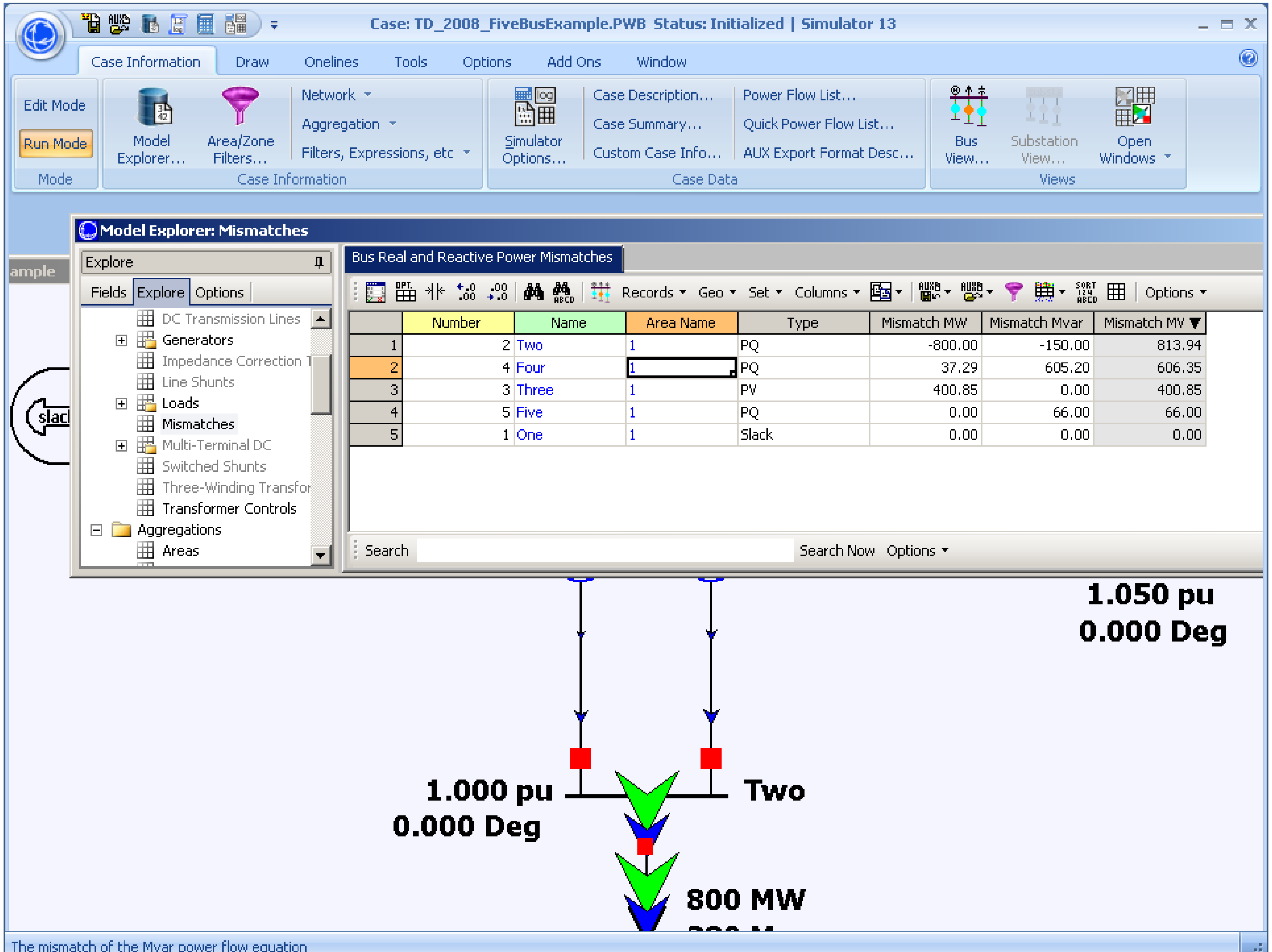
$$Y_{21} = Y_{23} = 0$$

$$Y_{24} = \frac{-1}{R_{24}^{'} + jX_{24}^{'}} = \frac{-1}{0.009 + j0.1} = -0.89276 + j9.91964 \textit{ per unit}$$

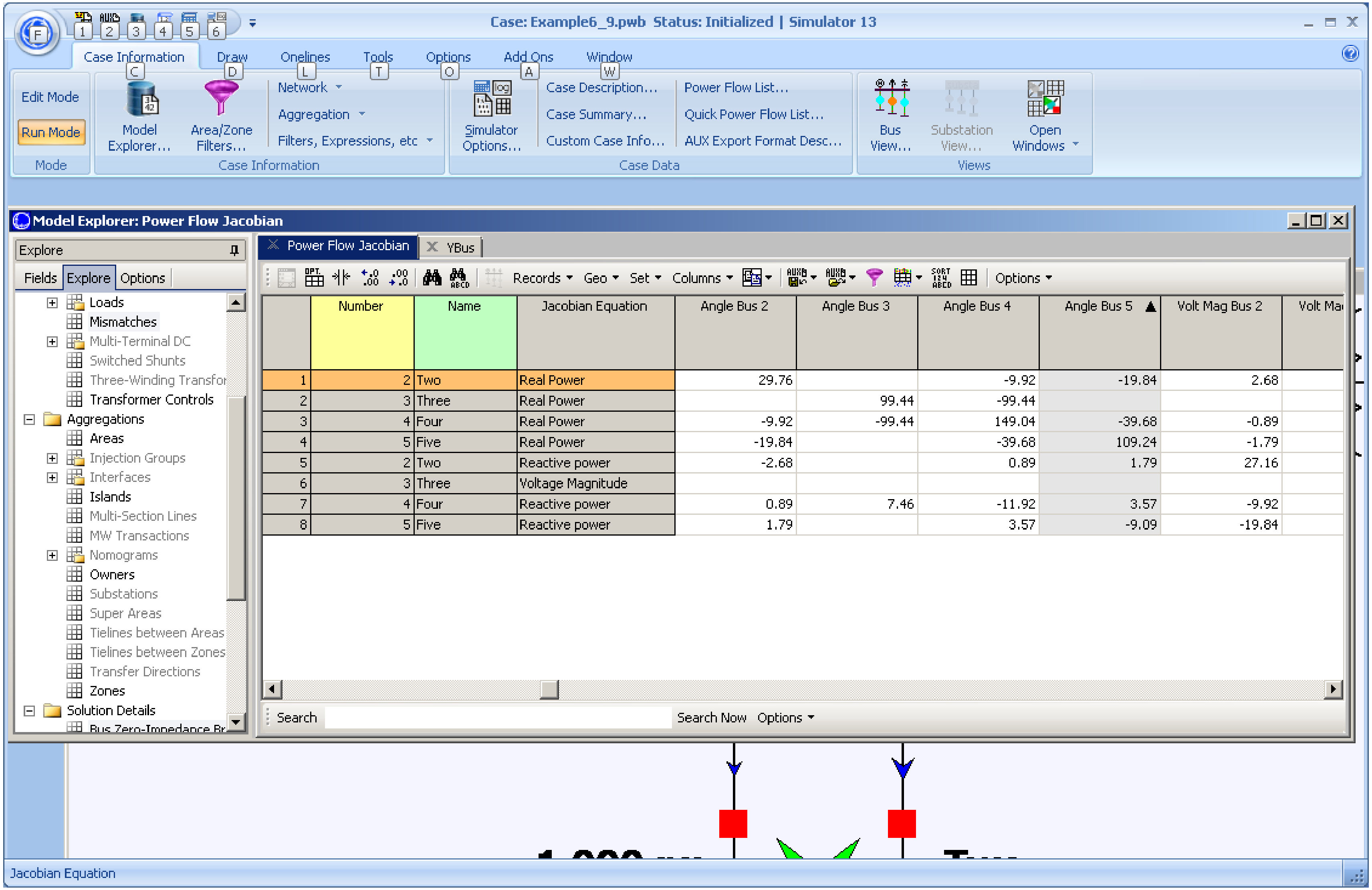
$$Y_{25} = \frac{-1}{R_{25}^{'} + jX_{25}^{'}} = \frac{-1}{0.0045 + j0.05} = -1.78552 + j19.83932 \textit{ per unit}$$

$$\begin{aligned} Y_{22} &= \frac{1}{R_{24}^{'} + jX_{24}^{'}} + \frac{1}{R_{25}^{'} + jX_{25}^{'}} + j\frac{B_{24}^{'}}{2} + j\frac{B_{25}^{'}}{2} \\ &= (0.89276 - j9.91964) + (1.78552 - j19.83932) + j\frac{1.72}{2} + j\frac{0.88}{2} \\ &= 2.67828 - j28.4590 = 28.5847\angle -84.624^{\circ} \textit{ per unit} \end{aligned}$$

Initial Bus Mismatches



Initial Power Flow Jacobian



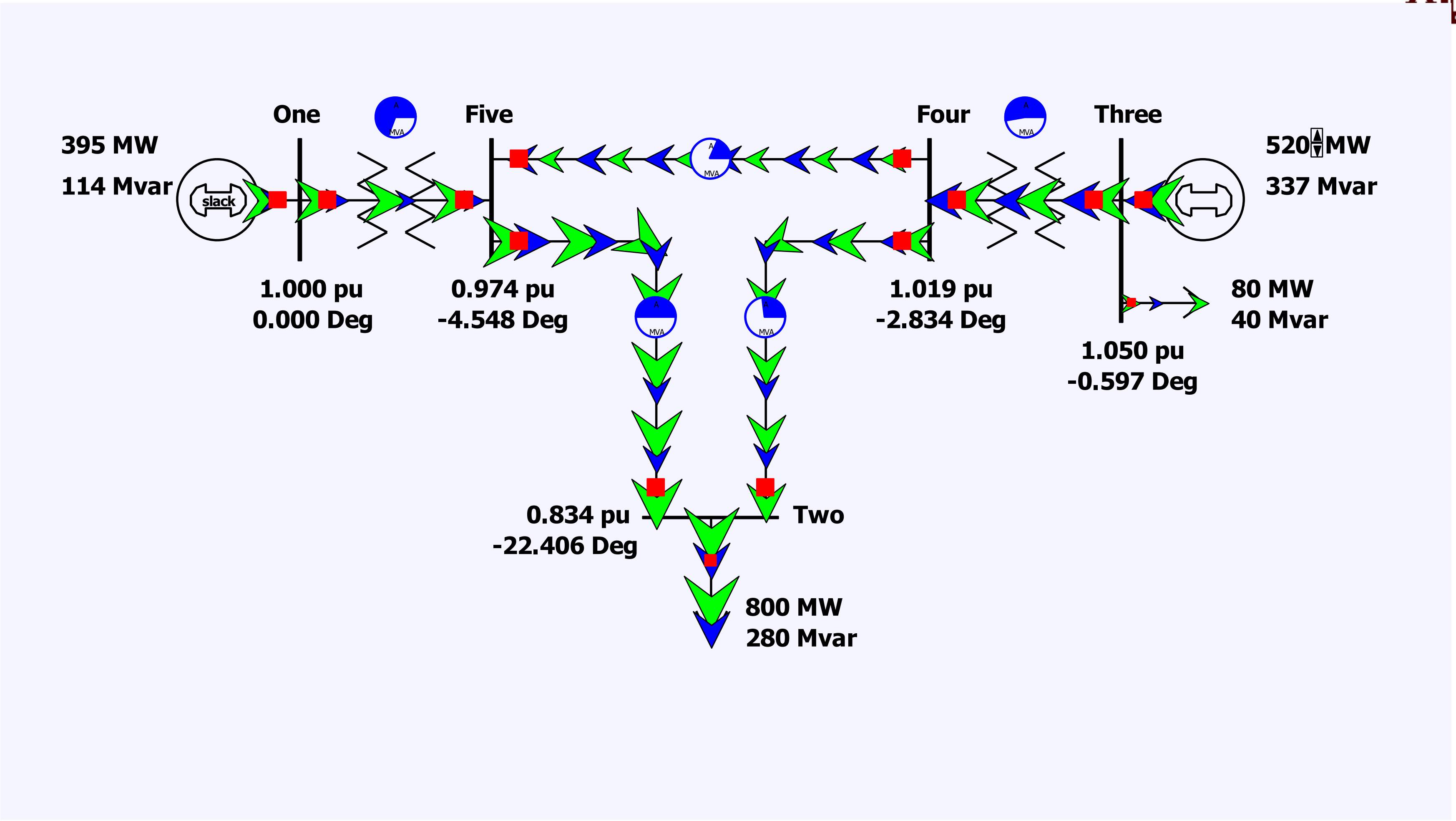
	Number	Name	Jacobian Equation	Angle Bus 2	Angle Bus 3	Angle Bus 4	Angle Bus 5	Volt Mag Bus 2
1	2	Two	Real Power	29.76		-9.92	-19.84	2.68
2	3	Three	Real Power		99.44	-99.44		
3	4	Four	Real Power	-9.92	-99.44	149.04	-39.68	-0.89
4	5	Five	Real Power	-19.84		-39.68	109.24	-1.79
5	2	Two	Reactive power	-2.68		0.89	1.79	27.16
6	3	Three	Voltage Magnitude					
7	4	Four	Reactive power	0.89	7.46	-11.92	3.57	-9.92
8	5	Five	Reactive power	1.79		3.57	-9.09	-19.84

Hand Calculation Details

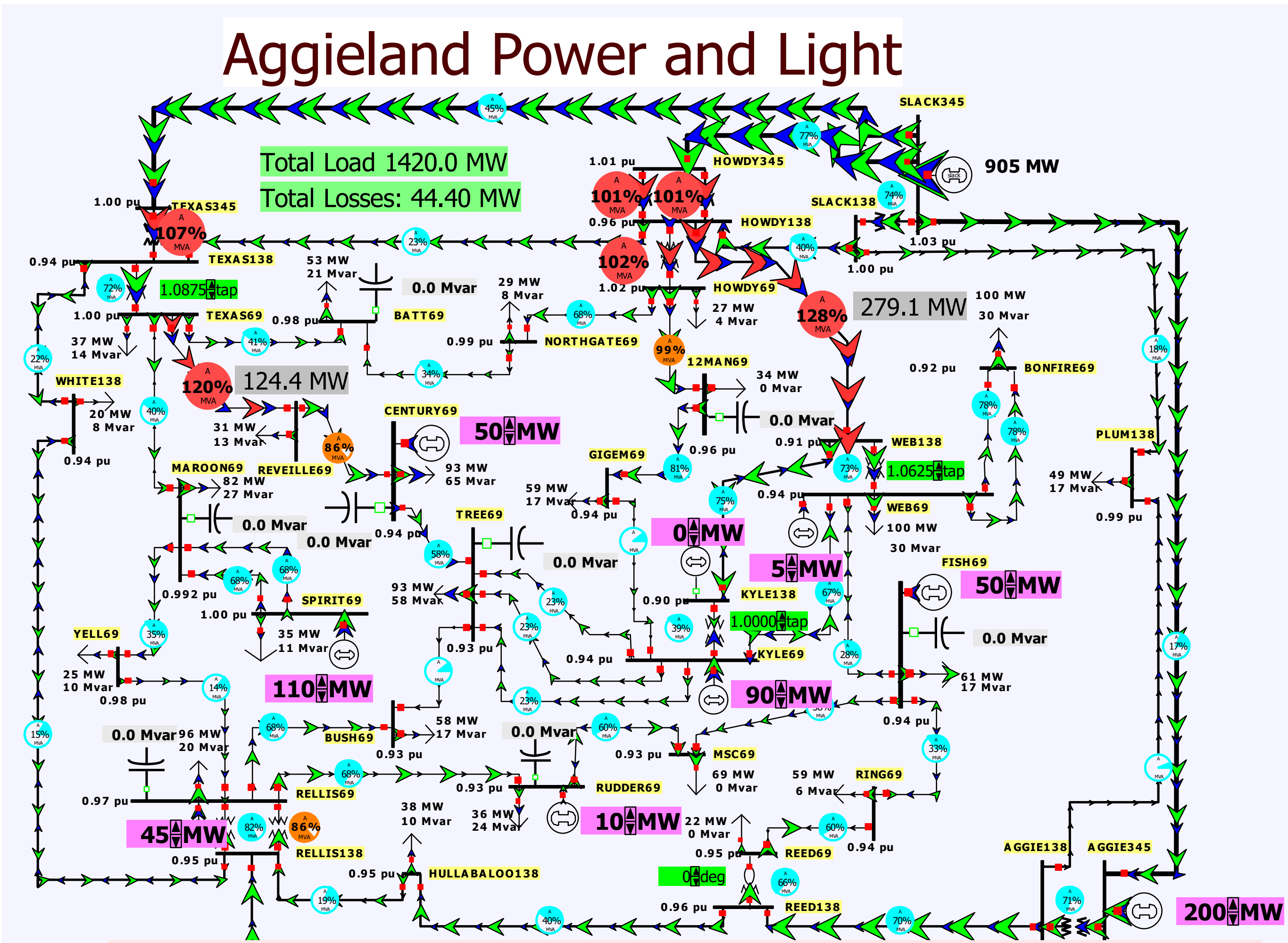
$$\begin{aligned}
\Delta P_2(0) = P_2 - P_2(x) = P_2 - V_2(0)\{ & Y_{21}V_1\cos[\delta_2(0)-\delta_1(0)-\theta_{21}] \\
& + Y_{22}V_2\cos[-\theta_{22}] + Y_{23}V_3\cos[\delta_2(0)-\delta_3(0)-\theta_{23}] \\
& + Y_{24}V_4\cos[\delta_2(0)-\delta_4(0)-\theta_{24}] \\
& + Y_{25}V_5\cos[\delta_2(0)-\delta_5(0)-\theta_{25}]\} \\
= -8.0 - 1.0\{ & 28.5847(1.0)\cos(84.624°) \\
& + 9.95972(1.0)\cos(-95.143°) \\
& + 19.9159(1.0)\cos(-95.143°)\} \\
= -8.0 - (& -2.89\times 10^{-4}) = -7.99972 \ \textit{per unit}
\end{aligned}$$

$$\begin{aligned}
J1_{24}(0) &= V_2(0)Y_{24}V_4(0)\sin[\delta_2(0)-\delta_4(0)-\theta_{24}] \\
&= (1.0)(9.95972)(1.0)\sin[-95.143°] \\
&= -9.91964 \ \textit{per unit}
\end{aligned}$$

Five Bus Power System Solved

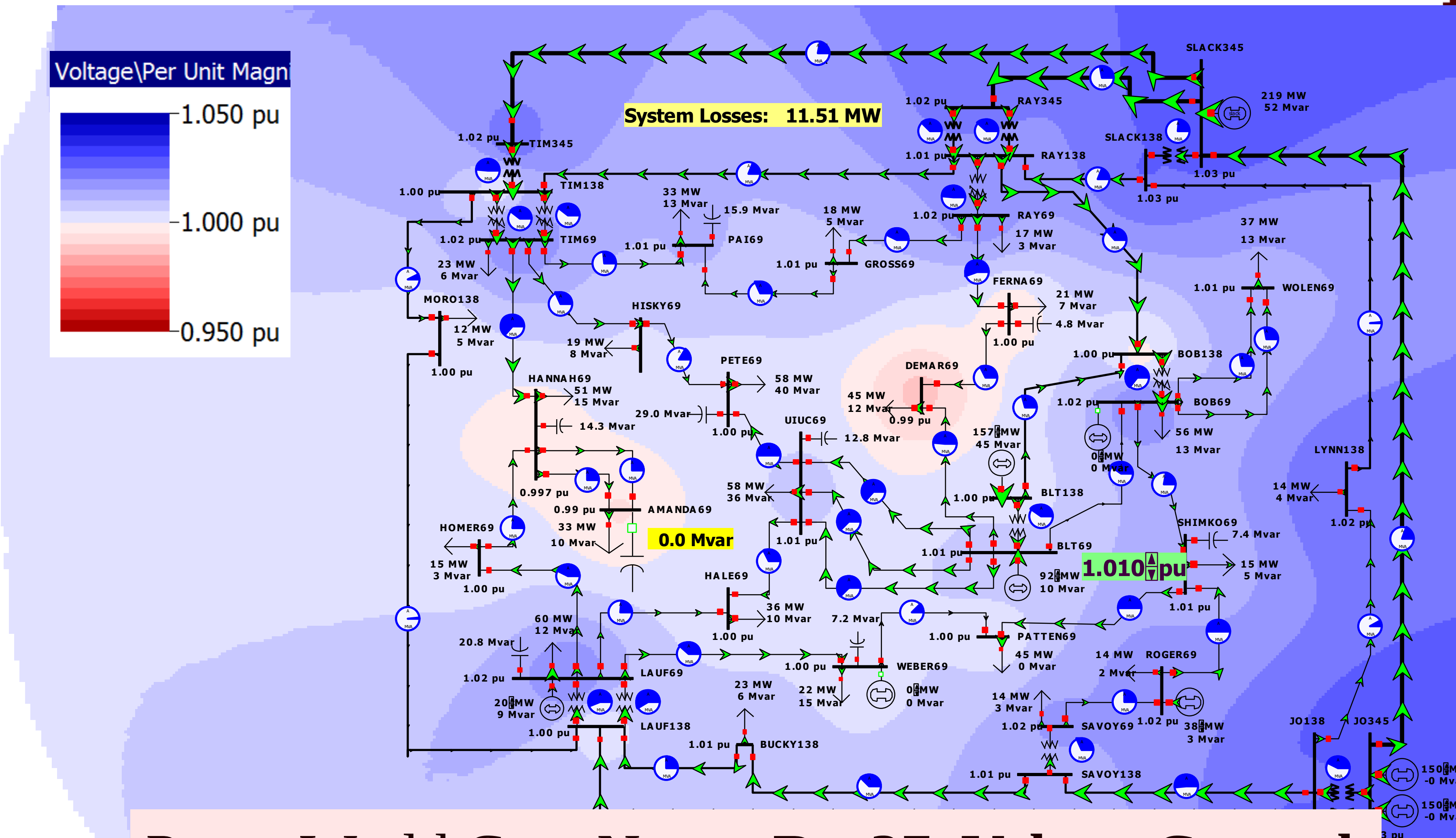


37 Bus Case Example



PowerWorld Case Name: **Aggieland37**

Voltage Control Example: 37 Buses



PowerWorld Case Name: **Bus37_VoltageControl**

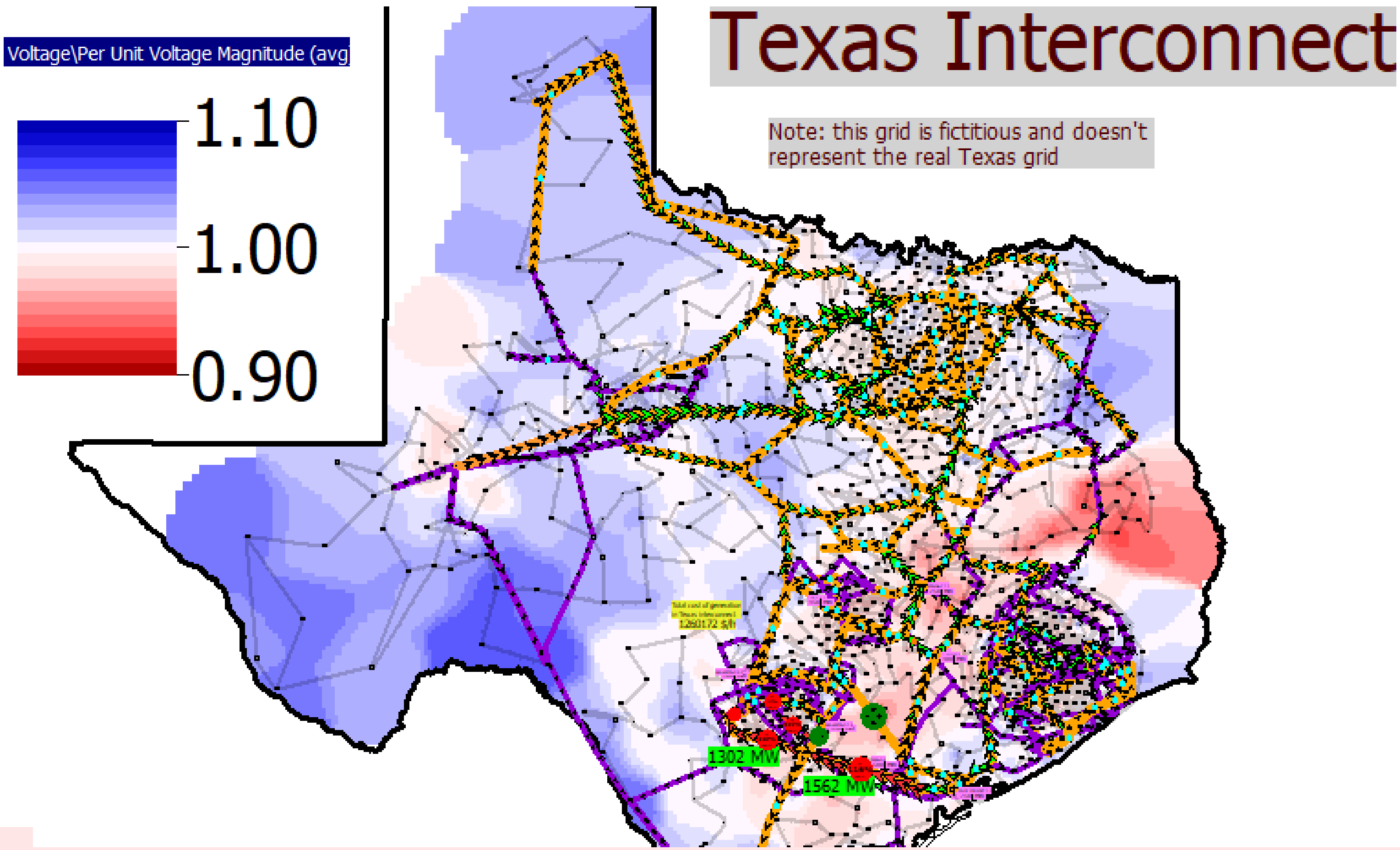
Power System Operations Overview

- Goal is to provide an intuitive feel for power system operation
- Emphasis will be on the impact of the transmission system
- Introduce basic power flow concepts through small system examples

Power System Basics

- All power systems have three major components: Generation, Load and Transmission/Distribution.
- Generation: Creates electric power.
- Load: Consumes electric power.
- Transmission/Distribution: Transmits electric power from generation to load.
  - Lines/transformers operating at voltages above 100 kV are usually called the transmission system. The transmission system is usually networked.
  - Lines/transformers operating at voltages below 100 kV are usually called the distribution system (radial).

Large System Example: Texas 2000 Bus Synthetic System

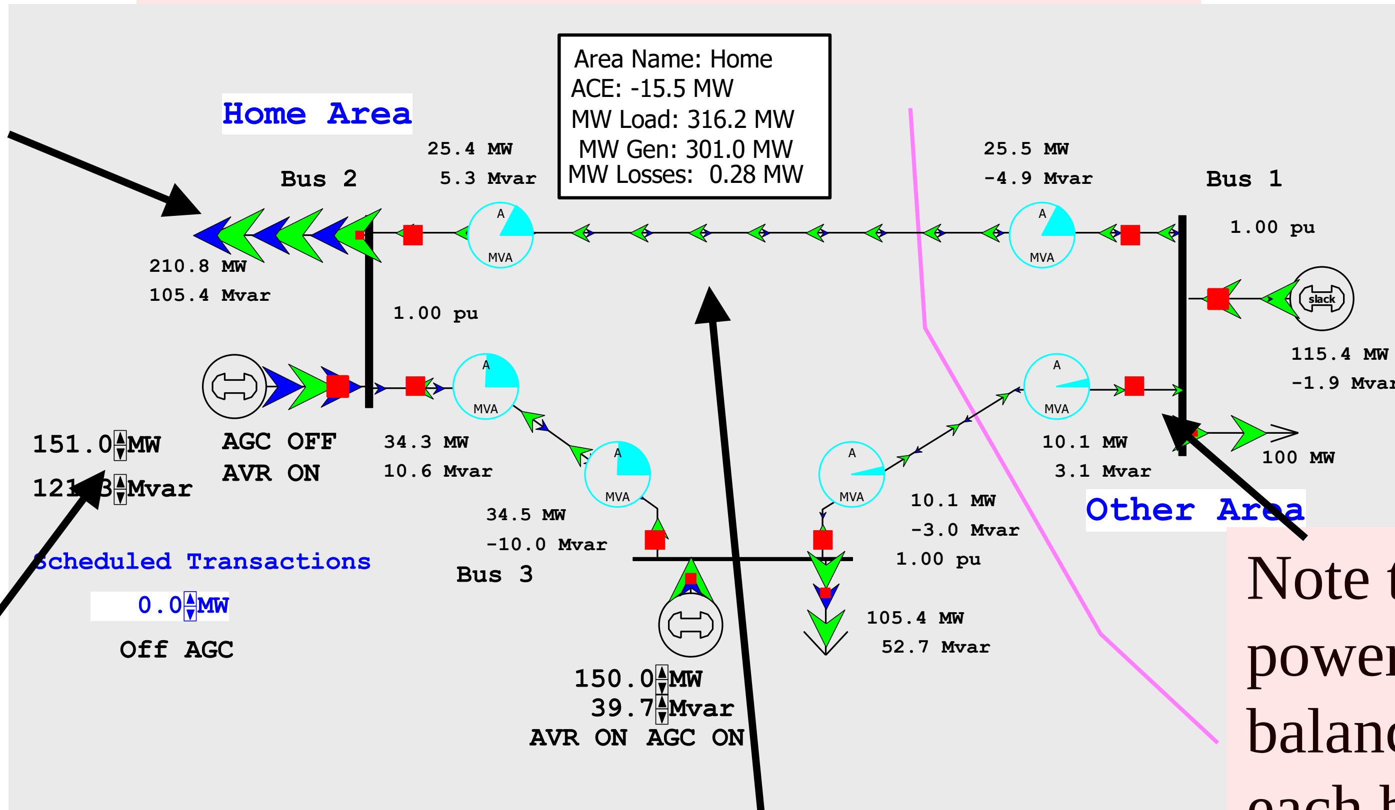


This case requires the commercial version of PowerWorld since the GOS Version is limited to 42 Buses

Three Bus PowerWorld Simulator Case

PowerWorld Case Name: **B3Slow**

Load with green arrows indicating amount of MW flow



Note the power balance at each bus

Used to control output of generator

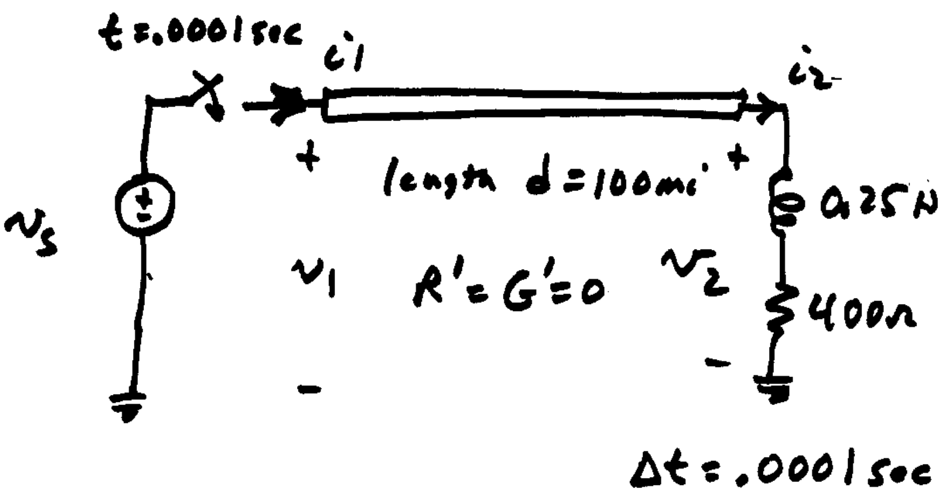
Direction of green arrow is used to indicate direction of real power (MW) flow; the blue arrows show the reactive power

Basic Power Control

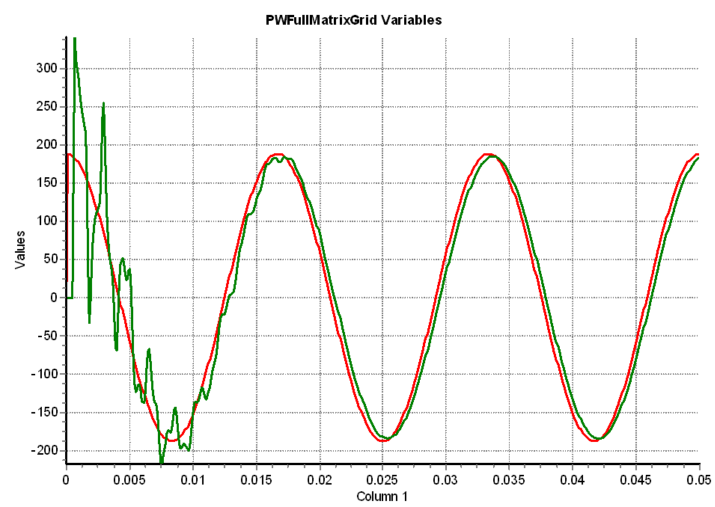
- Opening a circuit breaker causes the power flow to instantaneously (nearly) change.
- No other way to directly control power flow in a transmission line.
- By changing generation we can indirectly change this flow.
- Power flow in transmission line is limited by heating considerations
- Losses ($I^2 R$) can heat up the line, causing it to sag.

Modeling Consideration – Change is Not Really Instantaneous!

- The change isn't really instantaneous because of propagation delays, which are near the speed of light; there also wave reflection issues
  - This is covered in ECEN 667



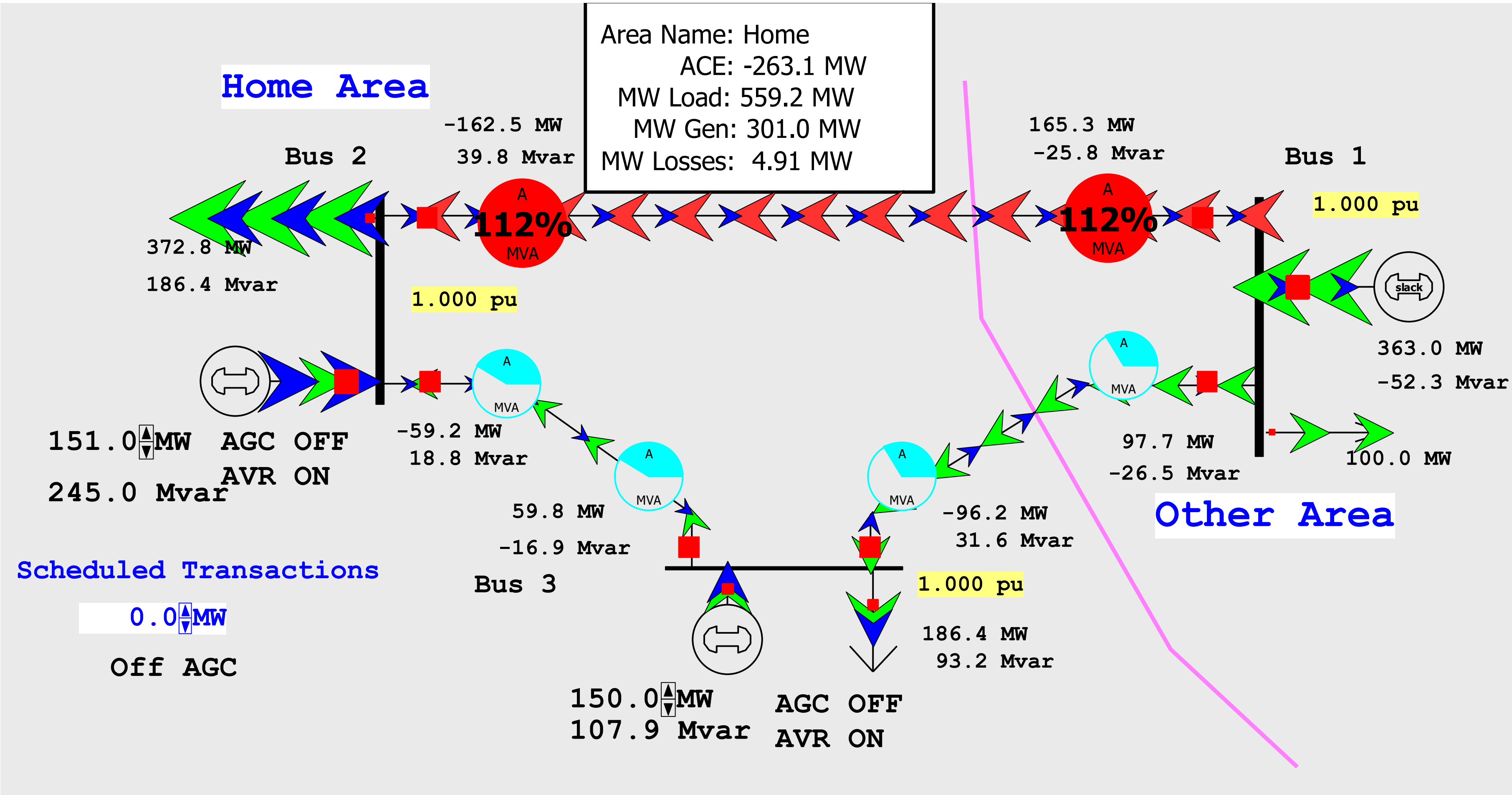
Red is the v_s end, green the v_2 end



Transmission Line Limits

- Power flow in transmission line is limited by heating considerations.
- Losses ($I^2 R$) can heat up the line, causing it to sag.
- Each line has a limit; many utilities use winter/summer limits.

Overloaded Transmission Line

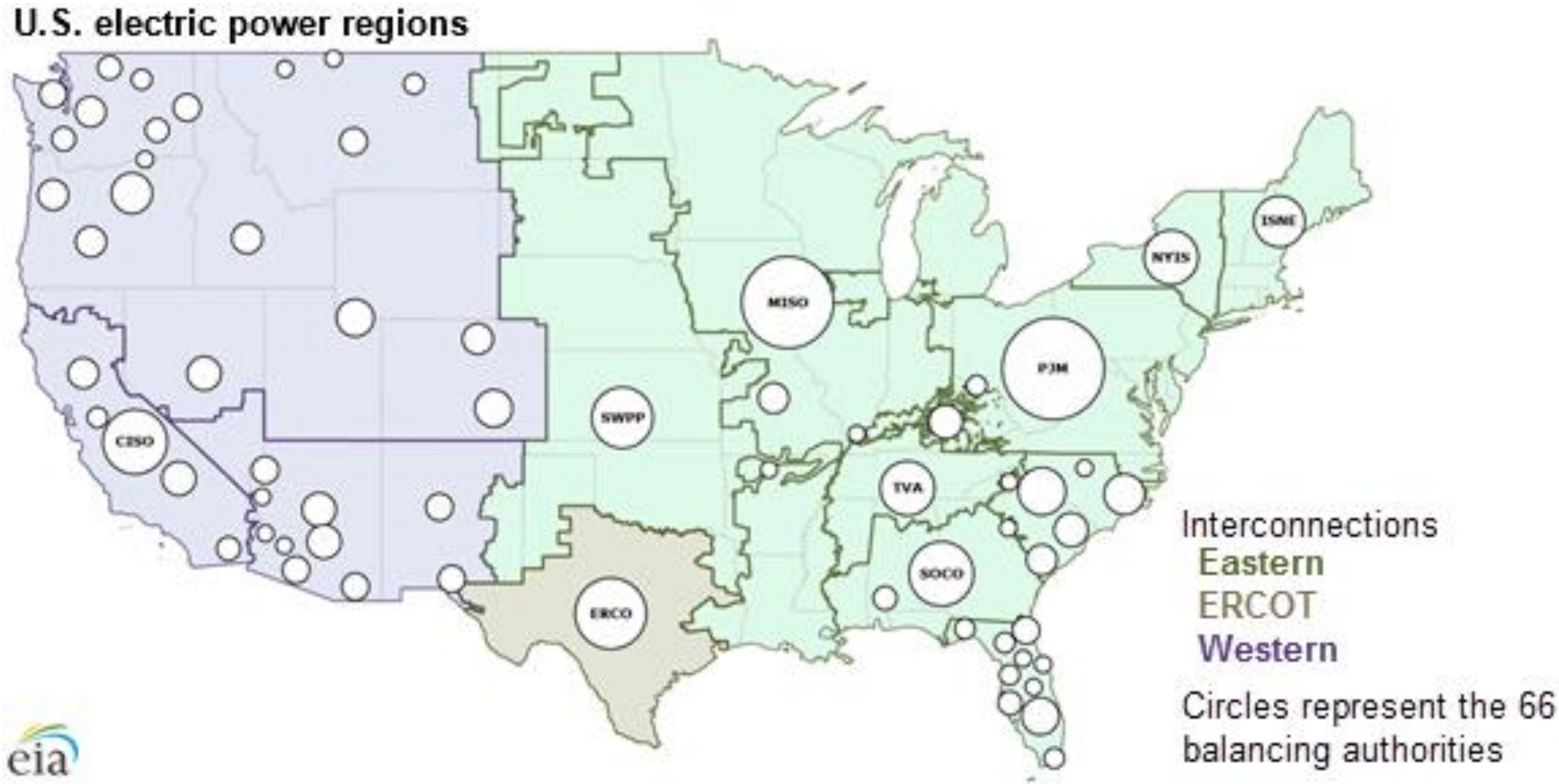


Interconnected Operation Balancing Authority (BA) Areas

- North American Eastern and Western grids are divided into balancing authority areas (BA)
  - Often just called an area
- Transmission lines that join two areas are known as tie-lines.
- The net power out of an area is the sum of the flow on its tie-lines.
- The flow out of an area is equal to

  total gen - total load - total losses = tie-flow

US Balancing Authorities



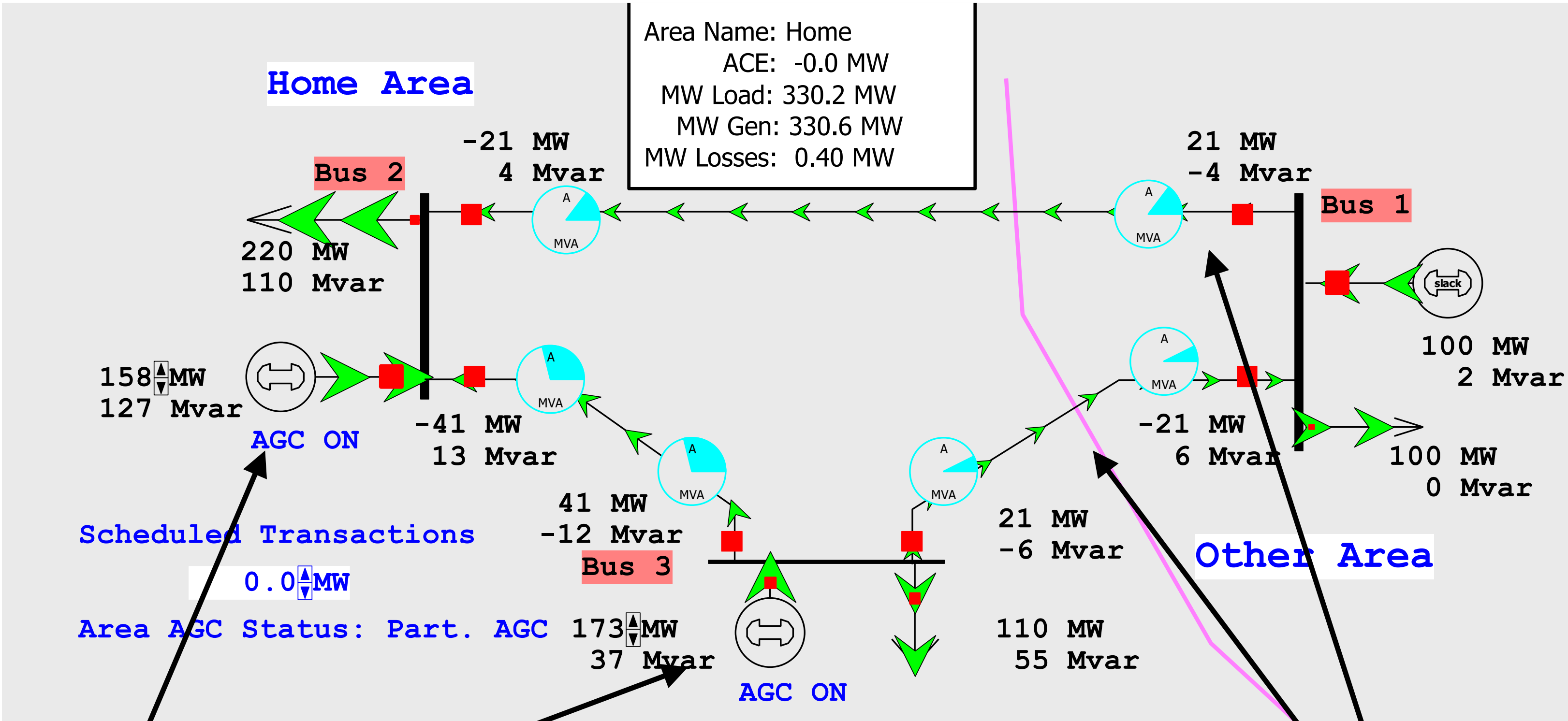
Area Control Error (ACE)

- The area control error is the difference between the actual flow out of an area, and the scheduled flow
  - ACE also includes a frequency component that we will probably consider later in the semester
- Ideally the ACE should always be zero
- Because the load is constantly changing, each utility (or ISO) must constantly change its generation to "chase" the ACE
- ACE was originally computed by utilities; increasingly it is computed by larger organizations such as ISOs

Automatic Generation Control

- Most utilities (ISOs) use automatic generation control (AGC) to automatically change their generation to keep their ACE close to zero.
- Usually the control center calculates ACE based upon tie-line flows; then the AGC module sends control signals out to the generators every couple seconds.

Three Bus Case on AGC



Generation is automatically changed to match change in load

Net tie flow is close to zero