ECEN 615 Problem Set #1 Fall 2020

Due 9/3/2020

1. Use Newton-Raphson to find one solution to the polynomial equation f(x) = 7, where y = 0 and
   $f(x) = x^4 + 3x^3 - 15x^2 - 19x + 30$. Start with x(0) = 0 and continue until (6.2.2) is satisfied with $\varepsilon = 0.001$

Aug 27th

ECEN 615_TA

HW01

Given:

(A.1) $f(x) = x^4 + 3x^3 - 15x^2 - 19x + 30$

$y = 0$; $f(x) = 7$; $x_0 = 0$

$\Rightarrow x^4 + 3x^3 - 15x^2 - 19x + 30 = 7$

$\therefore y = x^4 + 3x^3 - 15x^2 - 19x + 23 = 0$ — (1)

$\cancel{y(x) = x^4 + 3x^3 - 15x^2 - 19x + 23}$

$y' = 4x^3 + 9x^2 - 30x - 19$ — (2)

$$x_1 = x_0 - \frac{y(x_0)}{y'(x_0)}$$

where

$y(x_0) = y(0) = (0)^4 + 3(0)^3 - 15(0)^2 - 19(0) + 23 = 23$

$y'(x_0) = y'(0) = 4(0)^3 + 9(0)^2 - 30(0) - 19 = -19$

$\Rightarrow x_1 = 0 - \frac{23}{-19} = 1.2105$

2nd iteration:

$y(x_1) = y(1.2105) = (1.2105)^4 + 3(1.2105)^3 - 15(1.2105)^2 - 19(1.2105) + 23 = -14.5107$

$y'(x_1) = y'(1.2105) = 4(1.2105)^3 + 9(1.2105)^2 - 30(1.2105) - 19 = -35.0322$

$$x_2 = x_1 - \frac{y(x_1)}{y'(x_1)}$$

$$= 1.2105 - \frac{(-14.5107)}{-35.0322} = 1.2105 - 0.4142$$

$$= 0.7963$$

3rd iteration:

$$y(x_2) = y(0.7963) = 0.2758$$

$$y'(x_2) = y'(0.7963) = -35.1624$$

$$x_3 = x_2 - \frac{y(x_2)}{y'(x_2)} = 0.7963 - \frac{0.2758}{-35.1624}$$

$$= 0.8041$$

4th iteration:

$$y(x_3) = y(0.8041) = 0.0012$$

$$y'(x_3) = y'(0.8041) = -35.2242$$

$$x_4 = x_3 - \frac{y(x_3)}{y'(x_3)} = 0.8041 - \frac{0.0012}{-35.2242}$$

$$\simeq 0.8041$$

$$\varepsilon_a = |x_4 - x_3) = 0 < 0.001$$

$$\therefore \boxed{x = 0.8041}$$

2. The following nonlinear equations contain terms that are often found in the power flow equations:

$f_1(\mathbf{x}) = 12\, x_1 \sin x_2 + 1.5 = 0$

$f_2(\mathbf{x}) = 12\, (x_1)^2 - 12\, x_1 \cos x_2 + 0.75 = 0$

Start with an initial guess of $x_1(0) = 1$ and $x_2(0) = 0$ radians, and a stopping criteria of $\varepsilon = 10^{-4}$.

(A.2) Given:

$f_1(x) = 12 x_1 \sin x_2 + 1.5 = 0$

$f_2(x) = 12 (x_1)^2 - 12 x_1 \cos x_2 + 0.75 = 0$

$x_1(0) = 1$ & $x_2(0) = 0$ radians

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{new} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{old} - \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1^{old}} & \frac{\partial f_1}{\partial x_2^{old}} \\ \frac{\partial f_2}{\partial x_1^{old}} & \frac{\partial f_2}{\partial x_2^{old}} \end{bmatrix}^{-1}}_{[J(x)]^{-1}} \times \begin{bmatrix} f_1(x^{old}) \\ f_2(x^{old}) \end{bmatrix}$$

$$\frac{\partial f_1}{\partial x_1} = 12 \sin x_2 \qquad \frac{\partial f_1}{\partial x_2} = 12 x_1 \cos x_2$$

$$\frac{\partial f_2}{\partial x_1} = 24 x_1 - 12 \cos x_2 \qquad \frac{\partial f_2}{\partial x_2} = 12 x_1 \sin x_2$$

1st iteration :

- $x_1^0 = 1 \quad , \quad x_2^0 = 0$

- $\frac{\partial f_1}{\partial x_1} = 0 \qquad \frac{\partial f_1}{\partial x_2} = 12$

$\frac{\partial f_2}{\partial x_1} = 24 - 12 = 12 \qquad \frac{\partial f_2}{\partial x_2} = 0$

$$J = \begin{bmatrix} 0 & 12 \\ 12 & 0 \end{bmatrix}$$

$$\Rightarrow J^{-1} = \begin{bmatrix} 0 & 0.0833 \\ 0.0833 & 0 \end{bmatrix}$$

- $f_1(x^{old}) = 12(1) \sin(0) + 1.5 = 1.5$

$f_2(x^{old}) = 12(1)^2 - 12(1) \cos(0) + 0.75 = 0.75$

$$\begin{bmatrix} x_1^1 \\ x_2^1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 & 0.0833 \\ 0.0833 & 0 \end{bmatrix}}^{2\times 2} \overbrace{\begin{bmatrix} 1.5 \\ 0.75 \end{bmatrix}}^{2\times 1} = \overbrace{\begin{bmatrix} 0.9375 \\ -0.1250 \end{bmatrix}}^{2\times 1}$$

2nd iteration:

$$\frac{\partial f_1}{\partial x_1} = 12\sin(-0.1250) = -1.4961 \qquad \frac{\partial f_1}{\partial x_2} = 11.1622$$

$$\frac{\partial f_2}{\partial x_1} = 10.5936 \qquad \frac{\partial f_2}{\partial x_2} = -1.4026$$

$$\Rightarrow J^{-1} = \begin{bmatrix} 0.0121 & 0.0961 \\ 0.0912 & 0.0129 \end{bmatrix}$$

$$f_1(x^1) = 12(0.9375)\sin(-0.1250) + 1.5 = -1.4026$$

$$f_2(x^1) = 12(0.9375)^2 - 12(0.9375)\cos(-0.1250) + 0.75$$
$$= 0.1347$$

$$\begin{bmatrix} x_1^2 \\ x_2^2 \end{bmatrix} = \begin{bmatrix} 0.9375 \\ -0.1250 \end{bmatrix} - \begin{bmatrix} 0.0121 & 0.0961 \\ 0.0912 & 0.0129 \end{bmatrix} \begin{bmatrix} -1.4026 \\ 0.1347 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9415 \\ 0.0012 \end{bmatrix}$$

3rd iteration:

$$\frac{\partial f_1}{\partial x_1} = 12\sin(0.0012) = 0.0144 \qquad \frac{\partial f_1}{\partial x_2} = 12(0.9415)\cos(0.0012) = 11.2980$$

$$\frac{\partial f_2}{\partial x_1} = 24(0.9415) - 12\cos(0.0012) = 10.5960 \qquad \frac{\partial f_2}{\partial x_2} = 12(0.9415)\sin(0.0012) = 0.0136$$

$$\Rightarrow J^{-1} = \begin{bmatrix} -0.0001 & 0.0944 \\ 0.0885 & -0.0001 \end{bmatrix}$$

$$f_1(x^2) = 12(0.9415)\sin(0.0012) + 1.5 = 0.0136$$

$$f_2(x^2) = 12(0.9415)^2 - 12(0.9415)\cos(0.0012) + 0.75$$
$$= 0.0891$$

$$\begin{bmatrix} x_1^3 \\ x_2^3 \end{bmatrix} = \begin{bmatrix} 0.9415 \\ 0.0012 \end{bmatrix} - \begin{bmatrix} -0.0001 & 0.0944 \\ 0.0885 & -0.0001 \end{bmatrix} \begin{bmatrix} 0.0136 \\ 0.0891 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9331 \\ 0 \end{bmatrix}$$

4th iteration:

$$\frac{\partial f_1}{\partial x_1} = 0 \qquad \frac{\partial f_1}{\partial x_2} = 12(0.9331)\cos(0) = 11.1972$$

$$\frac{\partial f_2}{\partial x_1} = 24(0.9331) - 12 = 10.3944 \qquad \frac{\partial f_2}{\partial x_2} = 0$$

$$\Rightarrow J^{-1} = \begin{bmatrix} 0 & 0.0962 \\ 0.0893 & 0 \end{bmatrix}$$

$$f_1(x^3) = 12(0.9331)(0) + 1.5 = 1.5$$

$$f_2(x^3) = 12(0.9331)^2 - 12(0.9331)(1) + 0.75 = 0.0009$$

$$\begin{bmatrix} x_1^4 \\ x_2^4 \end{bmatrix} = \begin{bmatrix} 0.9331 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0.0562 \\ 0.0893 & 0 \end{bmatrix} \begin{bmatrix} 1.5 \\ 0.0009 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9331 \\ -0.1340 \end{bmatrix}$$

<u>5^{th} iteration</u>

$$\frac{\partial f_1}{\partial x_1} = 12\sin(-0.1340) = -1.6032 \quad ; \quad \frac{\partial f_1}{\partial x_2} = 12 x_1 \cos x_2 = 11.0968$$

$$\frac{\partial f_2}{\partial x_1} = 24(0.9331) - 12\cos(0.1340) = 10.5020 \quad ; \quad \frac{\partial f_2}{\partial x_2} = 12 x_1 \sin x_2 = -1.4959$$

$$\Rightarrow J^{-1} = \begin{bmatrix} 0.0131 & 0.0972 \\ 0.0920 & 0.0140 \end{bmatrix}$$

$$f_1(x^4) = 12(0.9331)\sin(-0.1340) + 1.5 = 0.0041$$

$$f_2(x^4) = 12(0.9331)^2 - 12(0.9331)\cos(-0.1340) + 0.75 = 0.1013$$

$$\begin{bmatrix} x_1^5 \\ x_2^5 \end{bmatrix} = \begin{bmatrix} 0.9331 \\ -0.1340 \end{bmatrix} - \begin{bmatrix} 0.0131 & 0.0972 \\ 0.0920 & 0.0140 \end{bmatrix} \begin{bmatrix} 0.0041 \\ 0.1013 \end{bmatrix}$$

$$\begin{pmatrix} x_1^5 \\ x_2^5 \end{pmatrix} = \begin{bmatrix} 0.9331 \\ -0.1340 \end{bmatrix} - \begin{bmatrix} 0.0099 \\ 0.0018 \end{bmatrix} = \begin{bmatrix} 0.9232 \\ -0.1358 \end{bmatrix}$$

6^{th} iteration :

$$\frac{\partial f_1}{\partial x_1} = 12\sin(-0.1358) = 1.6246 \qquad \frac{\partial f_1}{\partial x_2} = 12(0.9232)\cos(0.1358) = 10.9764$$

$$\frac{\partial f_2}{\partial x_1} = 24(0.9232) - 12\cos(0.1358) = 10.2673 \qquad \frac{\partial f_2}{\partial x_2} = 12(0.9232)\sin(-0.1358) = 1.4998$$

$$\Rightarrow J^{-1} = \begin{bmatrix} -0.0136 & 0.0995 \\ 0.0931 & -0.0147 \end{bmatrix}$$

$$f_1(x^5) = 12(0.9232)\sin(-0.1358) + 1.5 = 0.0002$$

$$f_2(x^5) = 12(0.9232)^2 - 12(0.9232)\cos(0.1358) + 0.\cancel{75} = 0.0012$$

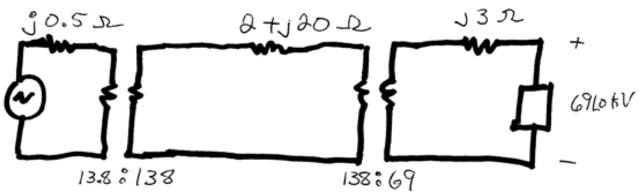
$$\begin{pmatrix} x_1^6 \\ x_2^6 \end{pmatrix} = \begin{bmatrix} 0.9232 \\ -0.1358 \end{bmatrix} - \begin{bmatrix} -0.0136 & 0.0995 \\ 0.0931 & -0.0147 \end{bmatrix} \begin{bmatrix} 0.0002 \\ 0.0012 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9232 \\ -0.1358 \end{bmatrix} - \begin{bmatrix} 0.0001 \\ 0 \end{bmatrix} //$$

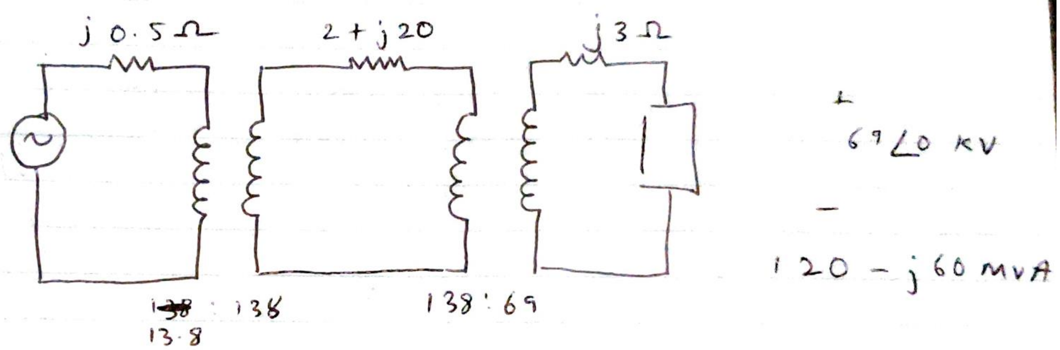
Since $| x^6 - x^5 | < \underset{(\varepsilon)}{10^{-4}}$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1^6 \\ x_2^6 \end{pmatrix} = \begin{bmatrix} 0.9231 \\ -0.1358 \end{bmatrix}$$

3. Assume the below diagram models a balanced three-phase system in which a 120- + j60 MVA load (total for all three phases) is supplied at 69 kV (line-to-line). First, redraw the network using a per unit representation with a 100 MVA base, and a 69 kV voltage base for the load. How much real and reactive power is being supplied by the generator (source) on the left?



A.3



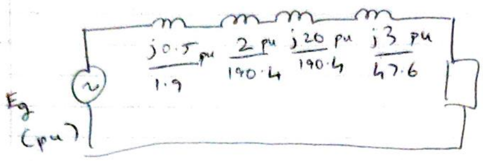
$MVA_B = 100\,MVA$; $kV_B = 69\,kV$

$$Z_B^{left} = \frac{(13.8 \times 10^3)^2}{100\,MVA} = 1.9\,\Omega$$

$$Z_B^{middle} = \frac{(138 \times 10^3)^2}{100\,MVA} = 190.4\,\Omega$$

$$Z_B^{right} = \frac{(69 \times 10^3)^2}{100\,MVA} = 47.6\,\Omega$$

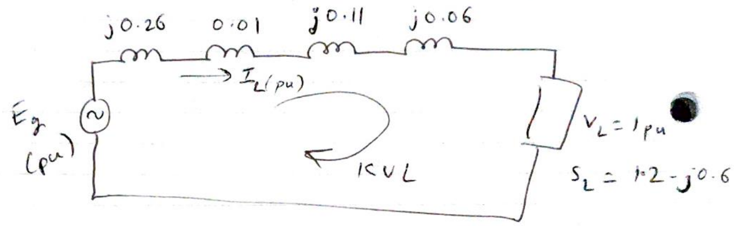
Creating pu representation :



$\frac{120 - j60 \text{ MVA}}{100 \text{ MVA}_B}$

$\frac{69 \angle 0 \text{ KV}}{69 \text{ KV}_B}$

$\equiv$



$$I_L = \left(\frac{S_L}{V_L}\right)^* = \left(\frac{1.2 - j0.6}{1}\right)^* = 1.2 + j0.6 \text{ pu}$$

Using KVL :

$$E_g(pu) = V_{L_{pu}} + I_{L_{pu}}(0.01 + j0.43)$$

$$= 1 + (1.2 + j0.6)(0.01 + j0.43)$$

$$= 0.7540 + 0.5220j$$

Power generated from source :

$$S_{g_{pu}} = E_{g_{pu}} I_{L_{pu}}^*$$

$$= (0.7540 + 0.5220j)(1.2 - j0.6)$$

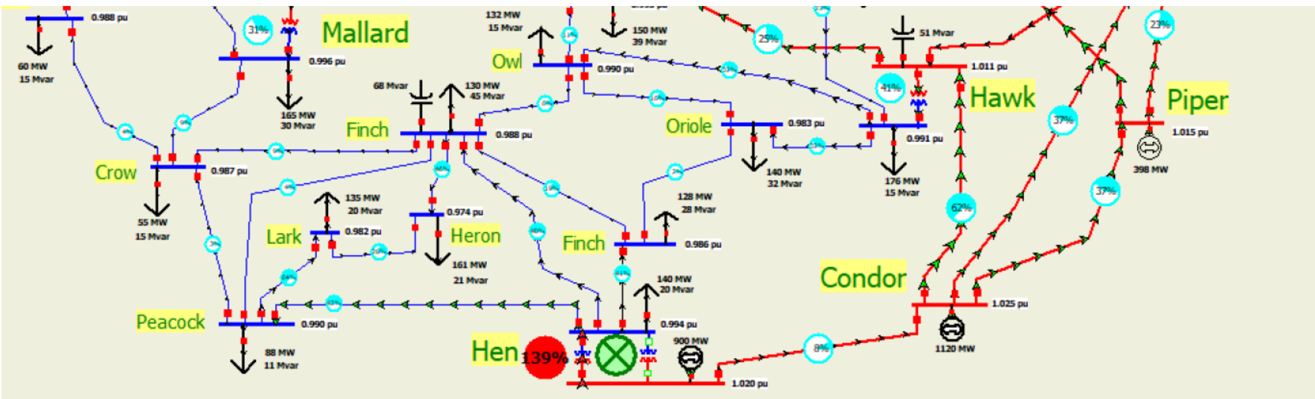
$$= 1.2180 + 0.1740j$$

$\therefore$ Real power generated = 121.8 MW

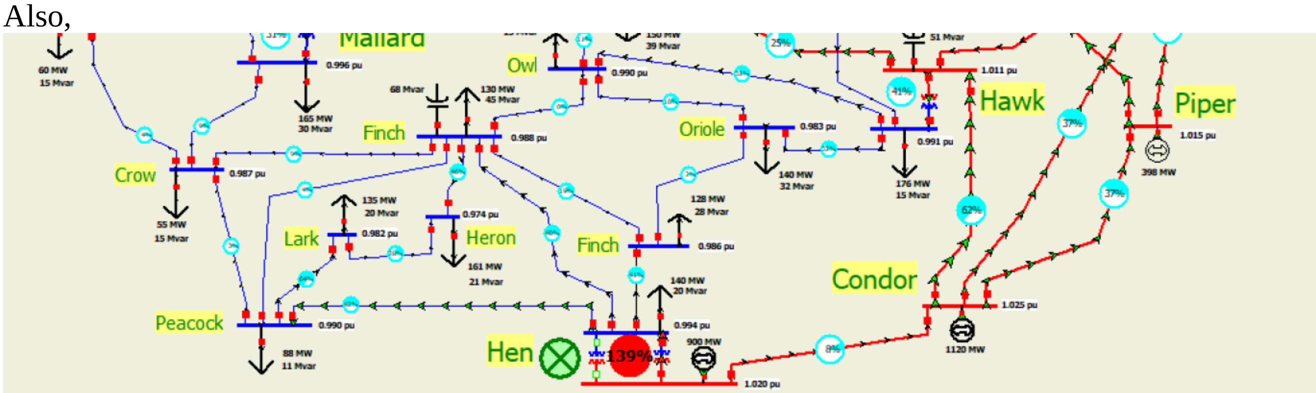
Reactive power generated = 17.4 MVAR

4. Using PowerWorld Simulator and the case ECEN_615_2020_HW1, give the bus numbers and circuit of two transmission lines or transformers that when individually opened cause at least one other transmission line or transformer to be overloaded.

Circuit #	From Bus# (Bus Name)	To Bus# (Bus Name)	Line/Transformer
1	35 (Hen345)	40 (Hen161)	Transformer
2	35 (Hen345)	40 (Hen161)	Transformer



Also,



5. Search the *IEEE Transactions on Power Systems* to find an important power flow paper that has not been mentioned in class (and doesn't have Overbye as an author). Write and turn in an approximately one page extended summary of the paper including explaining why you think it is an important paper. This should be a minimum of 750 words.