ECEN 615
Methods of Electric Power Systems Analysis

Lecture 20: Economic Dispatch, Optimal Power Flow

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Announcements

- Read Chapter 8
- Homework 5 today
- Homework 6 will be assigned next time, due on Nov 12
- The second exam will be in class on Nov 17
  - Distance learners will be able to take the exam from Nov 16 to Nov 18
- Associated with Homework 6 will be student presentations; these will be about 15 minutes during class on Nov 19 or Nov 24
  - Other times can be arranged for the distance learners

Unit Commitment: Quick Coverage (Chapter 4)

- Unit commitment is used to determine which generator units should be committed to meet the load
- The electric load varies substantially so there is almost always more generator capacity available than load
- Units have availability constraints
  - Minimum up time, time to start, cost to start
  - Minimum down time, time to shutdown, cost to shutdown
  - Ramp rates, minimum MW output
  - Scheduled and unscheduled outages
- System constraints including load, reserve, emissions, network

Solving Unit Commitment

- Unit commitment involves a potentially large number of integer and continuous variables
  - Not just the status of each unit, but also the amount of time it has been in a particular state (i.e., off or on)
- Solved for a set of discrete time periods, which at each time period there are lots of different potential states
- Solution approaches include
  - Dynamic programming
  - Lagrangian relaxation
  - Mixed Integer Programming (currently state-of-the-art)

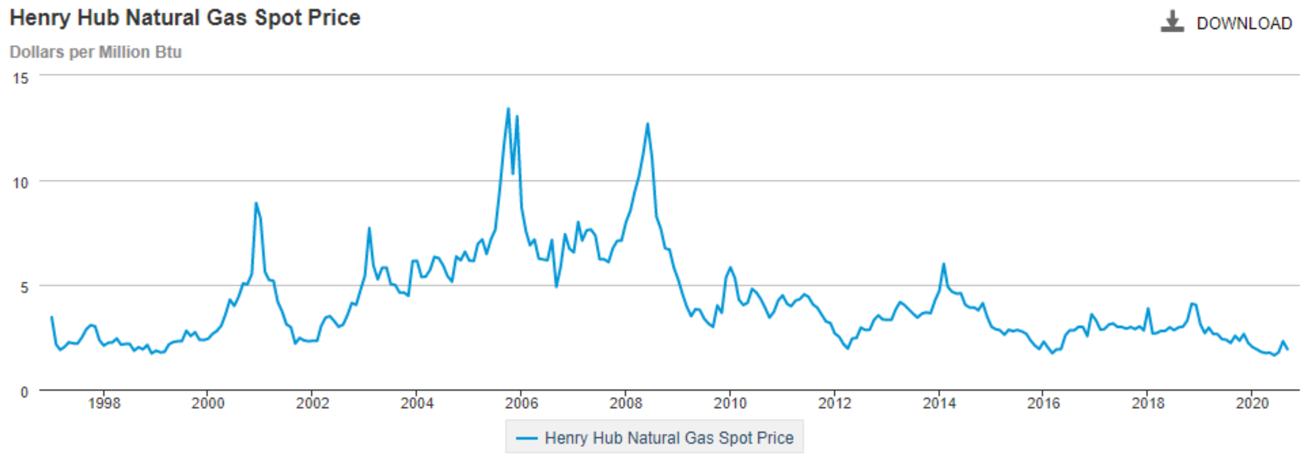
Longer Term Optimization: Quicker Coverage (Chapter 5)

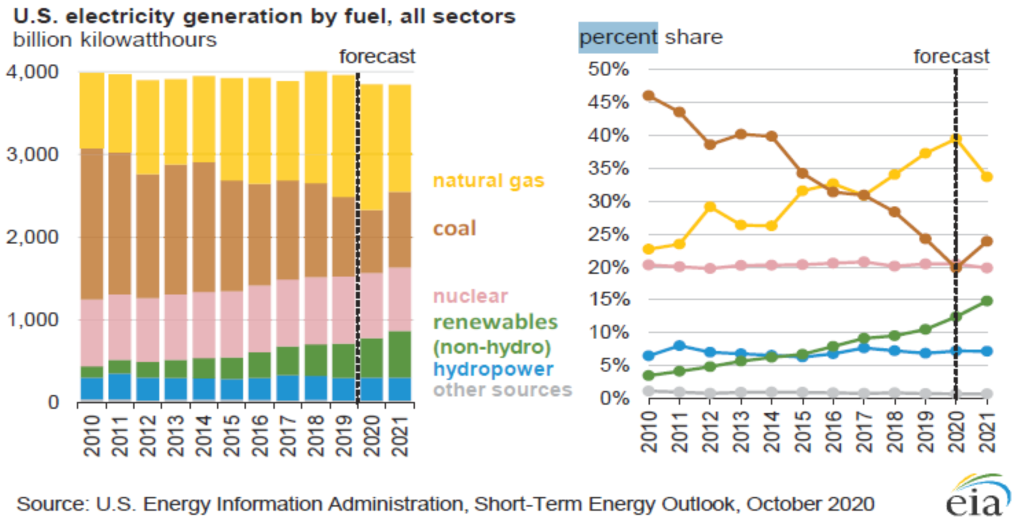
- Longer term optimization is a key consideration in hydro systems with significant reservoir storage
  - Use the water when it is the most valuable taking into account potentially many other constraints
- Generator maintenance scheduling
- Building generation often involves large upfront capital costs to create an asset that will last 20 to 40 years. Long-term contracts provide a way to share the risk
- Take-or-pay contracts obligate a purchaser to purchase so much of a product over a given time period

Power System Economic Dispatch

- Generators can have vastly different incremental operational costs
  - Some are essentially free or low cost (wind, solar, hydro, nuclear)
  - Because of the large amount of natural gas generation, electricity prices are very dependent on natural gas prices
- Economic dispatch is concerned with determining the best dispatch for generators without changing their commitment
- Economic dispatch is the foundation for the optimal power flow

Variation in Natural Gas Prices and Generation Sources





Source: www.eia.gov/dnav/ng/hist/rngwhhdm.htm

Power System Economic Dispatch

- Economic dispatch is formulated as a constrained minimization
  - The cost function is often total generation cost in an area
  - Single equality constraint is the real power balance equation
- Solved by setting up the Lagrangian (with P_D the load and P_L the losses, which are a function the generation)

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda\left(P_D + P_L(\mathbf{P}_G) - \sum_{i=1}^{m} P_{Gi}\right)$$

- A necessary condition for a minimum is that the gradient is zero. Without losses this occurs when all generators are dispatched at the same marginal cost (except when they hit a limit)

Power System Economic Dispatch

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi})$$

$$\frac{\partial \mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}) = 0$$

$$P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi} = 0$$

- If losses are neglected then there is a single marginal cost (lambda); if losses are included then each bus could have a different marginal cost

Economic Dispatch Penalty Factors

Solving each equation for λ we get

$$\frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}} = 0$$

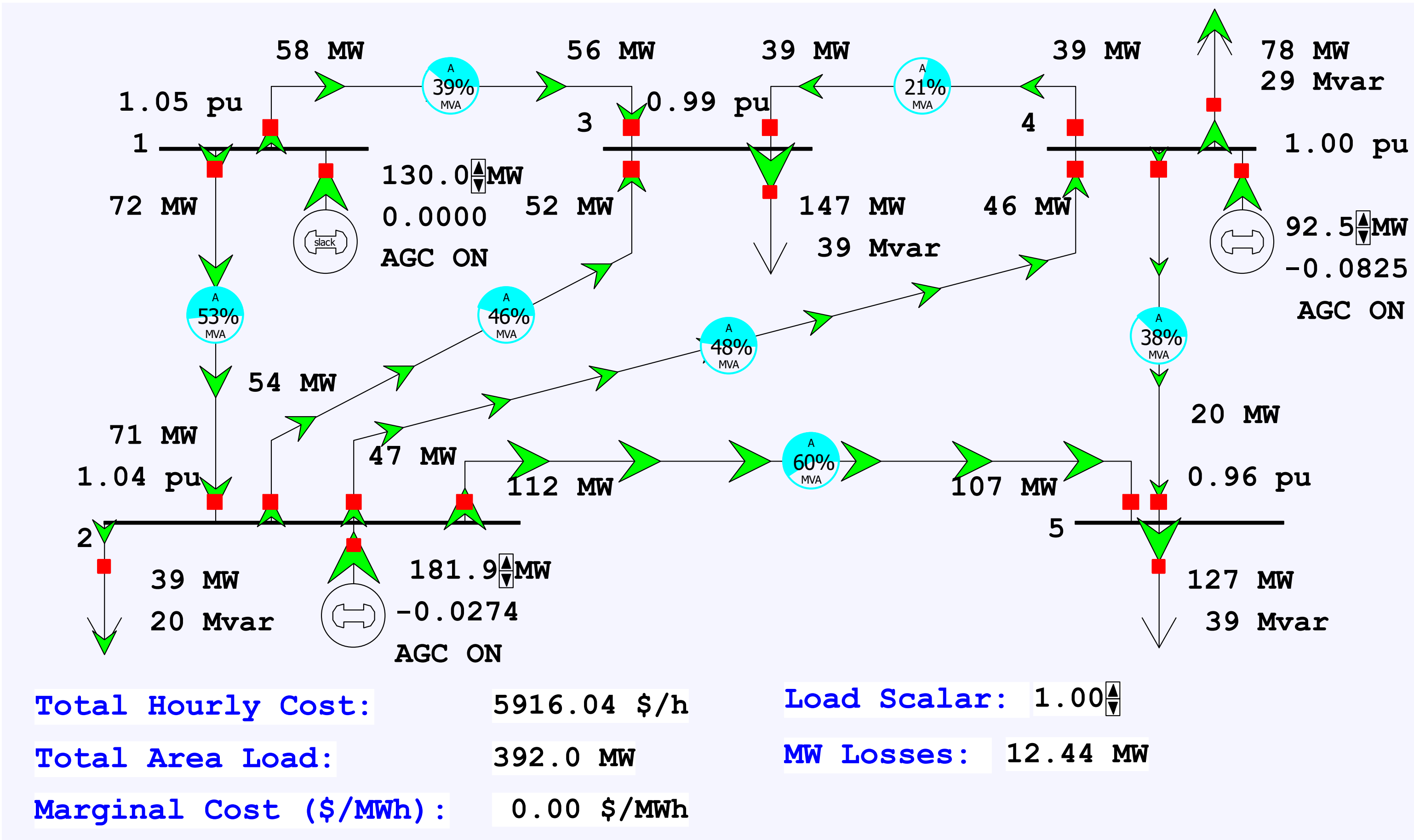
$$\lambda = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)} \frac{dC_i(P_{Gi})}{dP_{Gi}}$$

Define the penalty factor L_i for the i^{th} generator

$$L_i = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$$

The penalty factor at the slack bus is always unity!

Economic Dispatch Example



Case is GOS_Example6_22; use **Power Flow Solution Options, Advanced Options** to set Penalty Factors

Optimal Power Flow (OPF)

- OPF functionally combines the power flow with economic dispatch
- SCOPF adds in contingency analysis
- Goal of OPF and SCOPF is to minimize a cost function, such as operating cost, taking into account realistic equality and inequality constraints
- Equality constraints
  - bus real and reactive power balance
  - generator voltage setpoints
  - area MW interchange

OPF, cont.

- Inequality constraints
  - transmission line/transformer/interface flow limits
  - generator MW limits
  - generator reactive power capability curves
  - bus voltage magnitudes (not yet implemented in Simulator OPF)
- Available Controls
  - generator MW outputs
  - transformer taps and phase angles
  - reactive power controls

Two Example OPF Solution Methods

- Non-linear approach using Newton's method
  - handles marginal losses well, but is relatively slow and has problems determining binding constraints
  - Generation costs (and other costs) represented by quadratic or cubic functions
- Linear Programming
  - fast and efficient in determining binding constraints, but can have difficulty with marginal losses.
  - used in PowerWorld Simulator
  - generation costs (and other costs) represented by piecewise linear functions
- Both can be implemented using an ac or dc power flow

OPF and SCOPF Current Status

- OPF (really SCOPF) is currently an area of active research, with ARPA-E having an SCOPF competition (see gocompetition.energy.gov)
- A 2016 National Academies Press report, titled "Analytic Research Founds for the Next-Generation Electric Grid," recommended improved AC OPF models
  - I would recommend reading this report; it provides good background on power systems include OPF
  - It is available for free at www.nap.edu/catalog/21919/analytic-research-foundations-for-the-next-generation-electric-grid

OPF and SCOPF History

- A nice OPF history from Dec 2012 is provided by the below link, and briefly summarized here
- Prior to digital computers economic dispatch was solved by hand and the power flow with network analyzers
- Digital power flow developed in late 50's to early 60's
- First OPF formulations in the 1960's
  - J. Carpienterm, "Contribution e l'étude do Dispatching Economique," Bulletin Society Francaise Electriciens, 1962
  - H.W. Dommel, W.F. Tinney, "Optimal power flow solutions," *IEEE Trans. Power App. and Sys*tems, Oct. 1968
    - "Only a small extension of the power flow program is required"

www.ferc.gov/industries/electric/indus-act/market-planning/opf-papers/acopf-1-history-formulation-testing.pdf
(by M Cain, R. O'Neill, A. Castillo)

OPF and SCOPF History

- A linear programming (LP) approach was presented by Stott and Hobson in 1978
  - B. Stott, E. Hobson, “Power System Security Control Calculations using Linear Programming,” (Parts 1 and 2) *IEEE Trans. Power App and Syst*., Sept/Oct 1978
- Optimal Power Flow By Newton’s Method
  - D.I. Sun, B. Ashley, B. Brewer, B.A. Hughes, and W.F. Tinney, "Optimal Power Flow by Newton Approach", *IEEE Trans. Power App and Syst*., October 1984
- Follow-up LP OPF paper in 1990
  - O. Alsac, J. Bright, M. Prais, B. Stott, “Further Developments in LP-based Optimal Power Flow,” *IEEE Trans. Power Systems*, August 1990

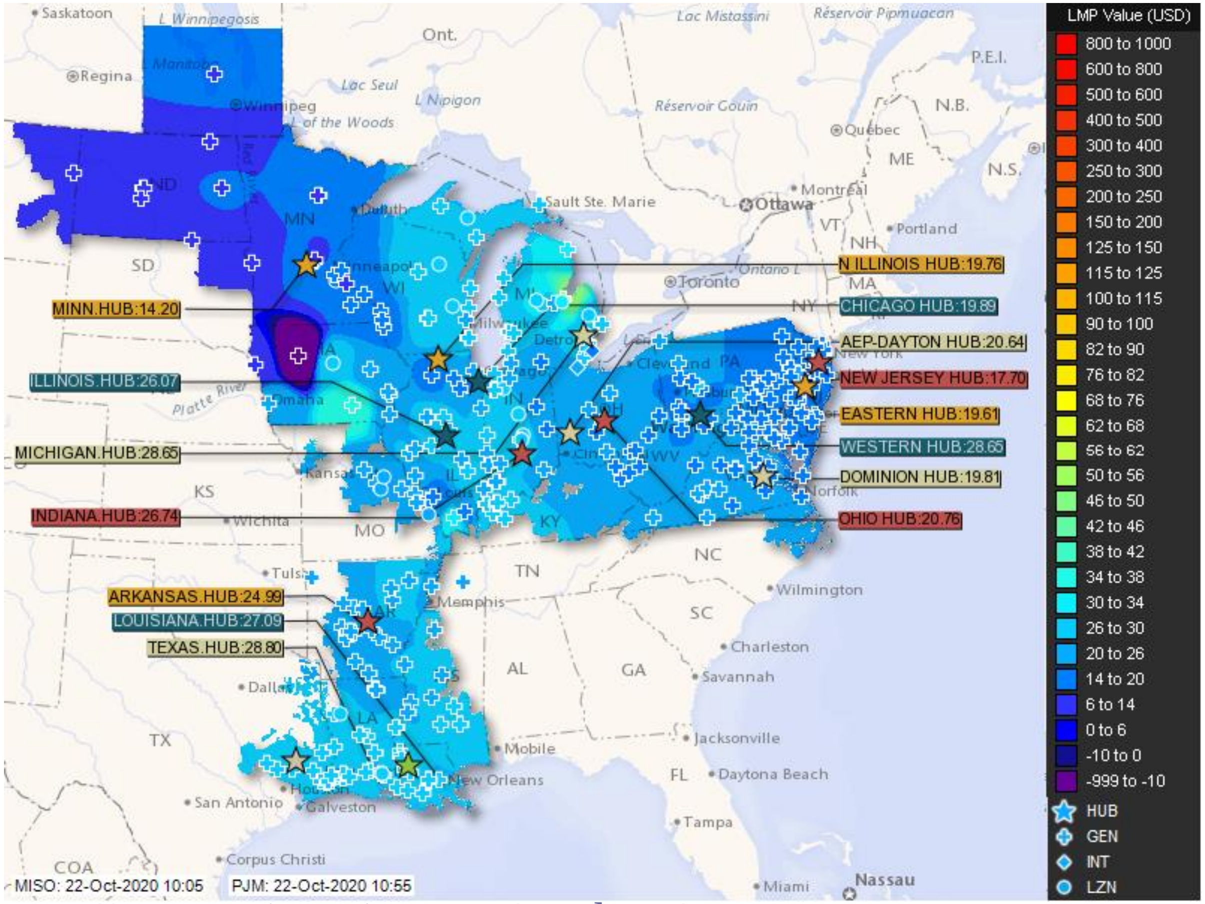
OPF and SCOPF History

- Critique of OPF Algorithms
  - W.F. Tinney, J.M. Bright, K.D. Demaree, B.A. Hughes, "Some Deficiencies in Optimal Power Flow," *IEEE Trans. Power Systems*, May 1988
- Hundreds of other papers on OPF
- Comparison of ac and dc optimal power flow methods
  - T.J. Overbye, X. Cheng, Y. San, "A Comparison of the AC and DC Power Flow Models for LMP Calculations," Proc. 37th Hawaii International Conf. on System Sciences, 2004

Key SCOPF Application: Locational Marginal Prices (LMPs)

- The locational marginal price (LMP) tells the cost of providing electricity to a given location (bus) in the system
- Concept introduced by Schweppe in 1985
  - F.C. Schweppe, M. Caramanis, R. Tabors, "Evaluation of Spot Price Based Electricity Rates," *IEEE Trans. Power App and Syst.*, July 1985
- LMPs are a direct result of an SCOPF, and are widely used in many electricity markets worldwide
  –

Example LMP Contour, 10/22/2020

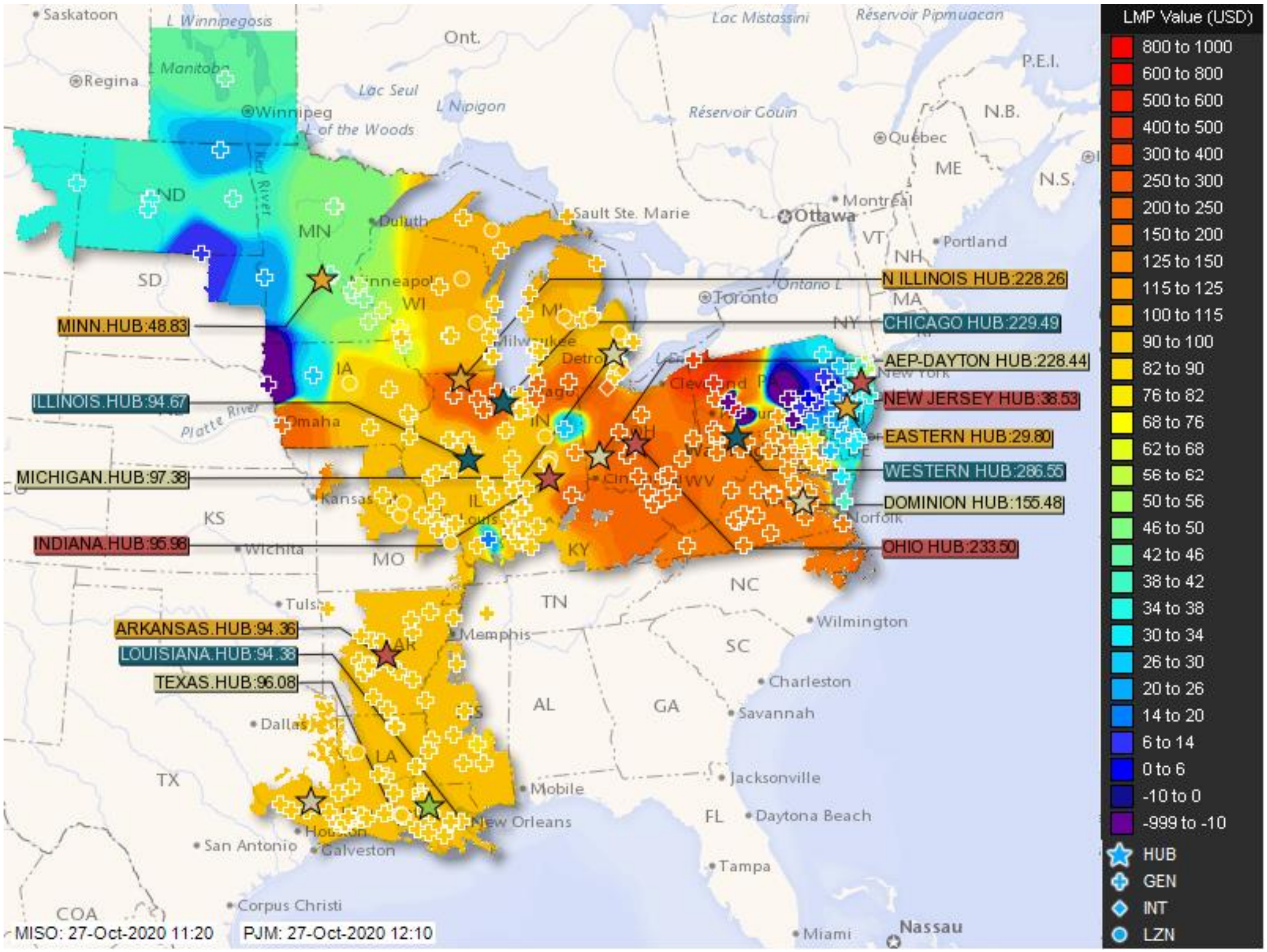


LMPs are now widely visualized using color contours; the first use of LMP color contours was presented in [1]

https://www.miso-pjm.com/markets/contour-map.aspx

[1] T.J. Overbye, R.P. Klump, J.D. Weber, "A Virtual Environment for Interactive Visualization of Power System Economic and Security Information," IEEE PES 1999 Summer Meeting, Edmonton, AB, Canada, July 1999

Example LMP Contour: 10/27/2020



Note the wide range in LMPs including some negative values!

This is just the real-time market; most electricity is not traded here.

OPF Problem Formulation

- The OPF is usually formulated as a minimization with equality and inequality constraints

$$\text{Minimize } F(\mathbf{x},\mathbf{u})$$

$$\mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0}$$

$$\mathbf{h}_{\min} \leq \mathbf{h}(\mathbf{x},\mathbf{u}) \leq \mathbf{h}_{\max}$$

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}$$

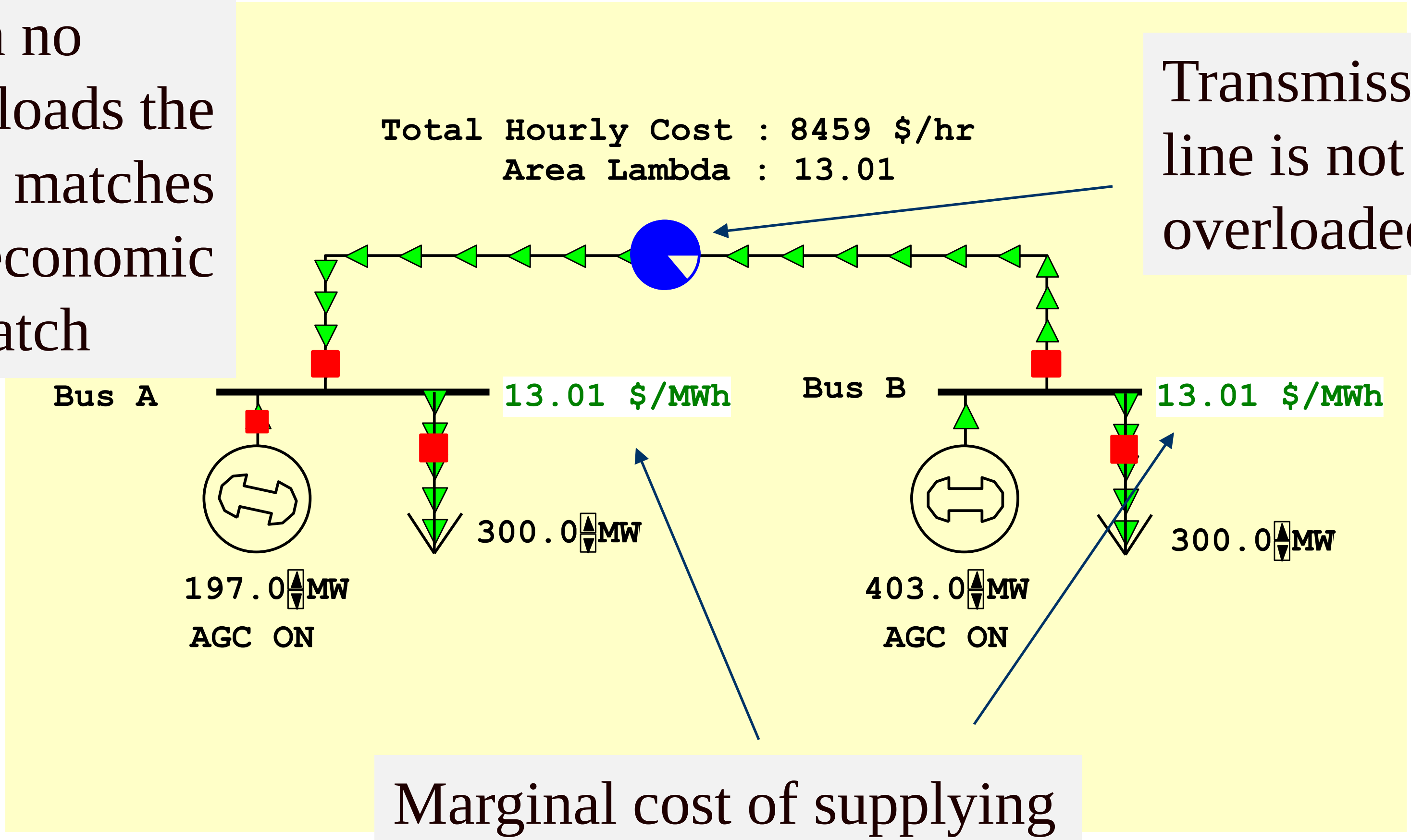
where **x** is a vector of dependent variables (such as the bus voltage magnitudes and angles), **u** is a vector of the control variables, F(**x**,**u**) is the scalar objective function, **g** is a set of equality constraints (e.g., the power balance equations) and **h** is a set of inequality constraints (such as line flows)

LP OPF Solution Method

- Solution iterates between
  - solving a full ac or dc power flow solution
    - enforces real/reactive power balance at each bus
    - enforces generator reactive limits
    - system controls are assumed fixed
    - takes into account non-linearities
  - solving a primal LP
    - changes system controls to enforce linearized constraints while minimizing cost

Two Bus with Unconstrained Line

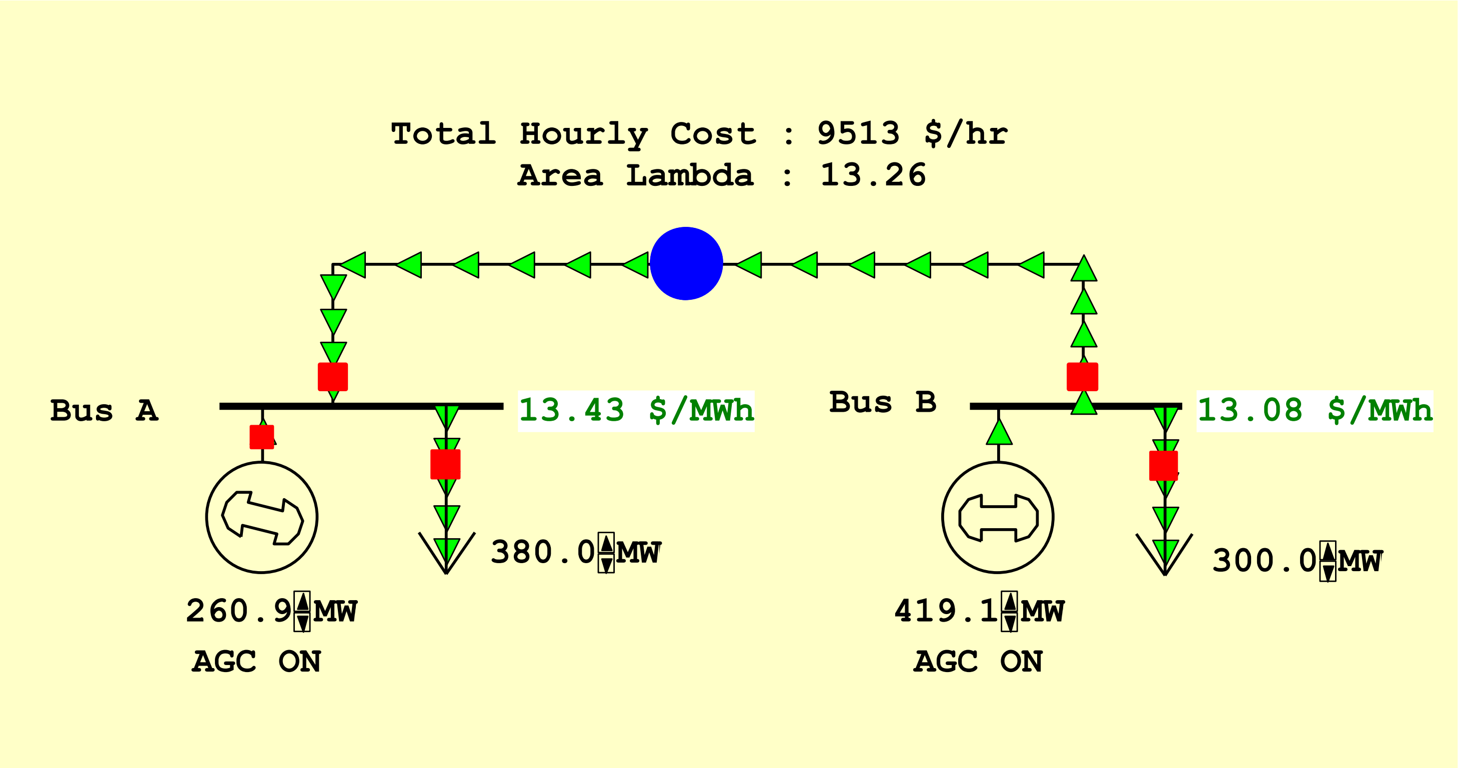
With no overloads the OPF matches the economic dispatch



Transmission line is not overloaded

Marginal cost of supplying power to each bus (locational marginal costs)

Two Bus with Constrained Line

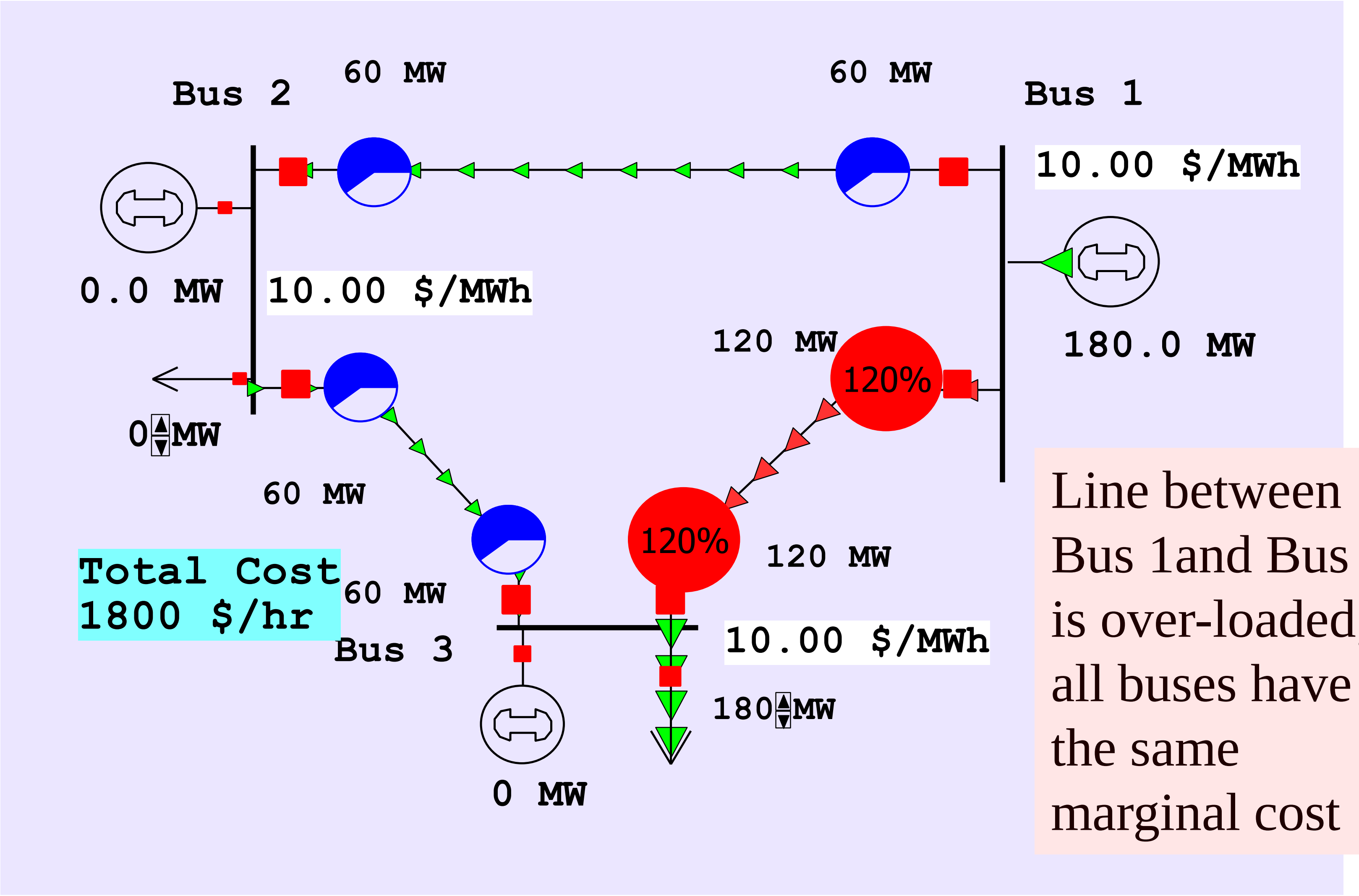


With the line loaded to its limit, additional load at Bus A must be supplied locally, causing the marginal costs to diverge.

Three Bus (B3) Example

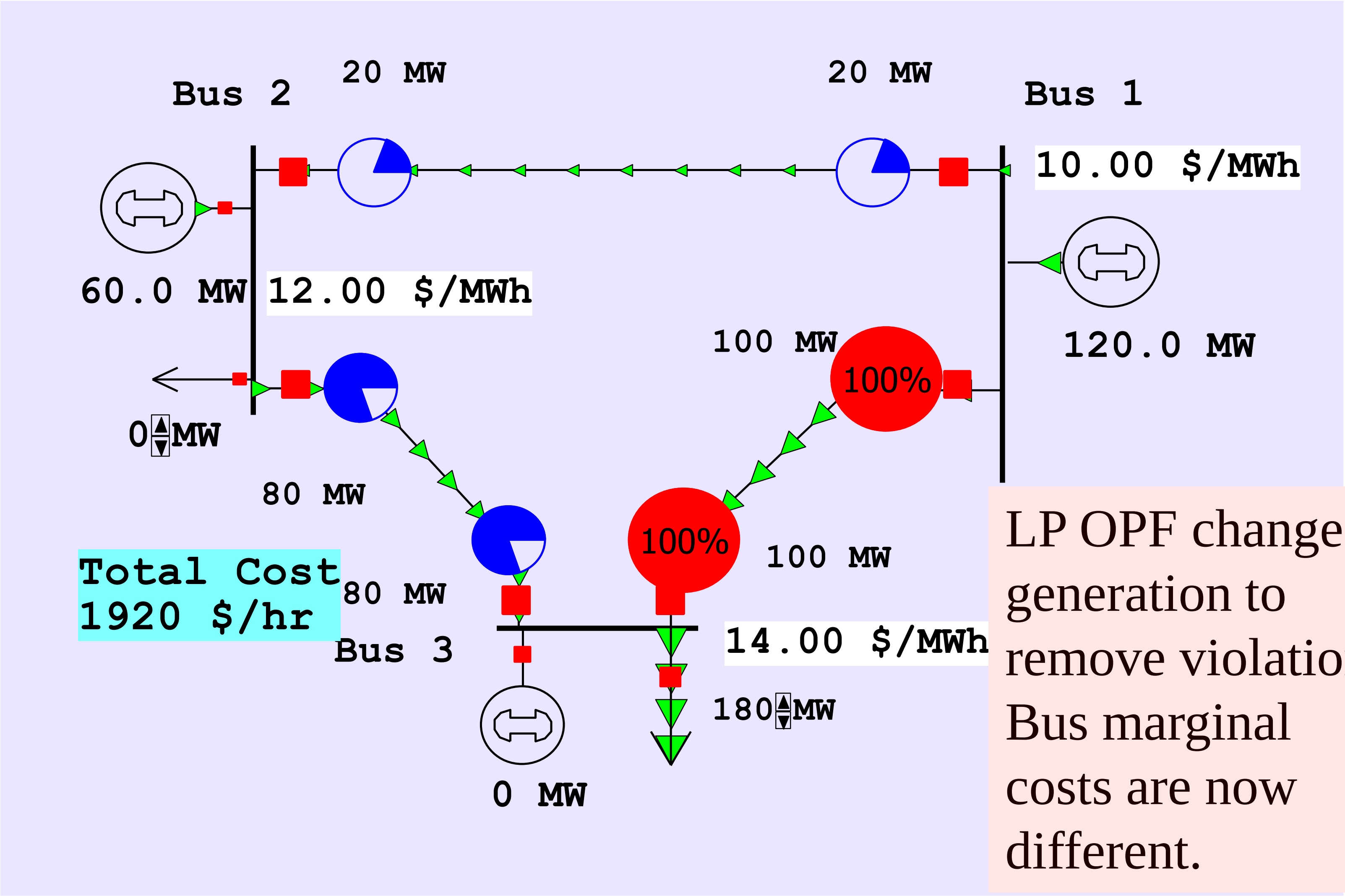
- Consider a three bus case (Bus 1 is system slack), with all buses connected through 0.1 pu reactance lines, each with a 100 MVA limit
- Let the generator marginal costs be
  - Bus 1: 10 $ / MWhr; Range = 0 to 400 MW
  - Bus 2: 12 $ / MWhr; Range = 0 to 400 MW
  - Bus 3: 20 $ / MWhr; Range = 0 to 400 MW
- Assume a single 180 MW load at bus 2

B3 with Line Limits NOT Enforced



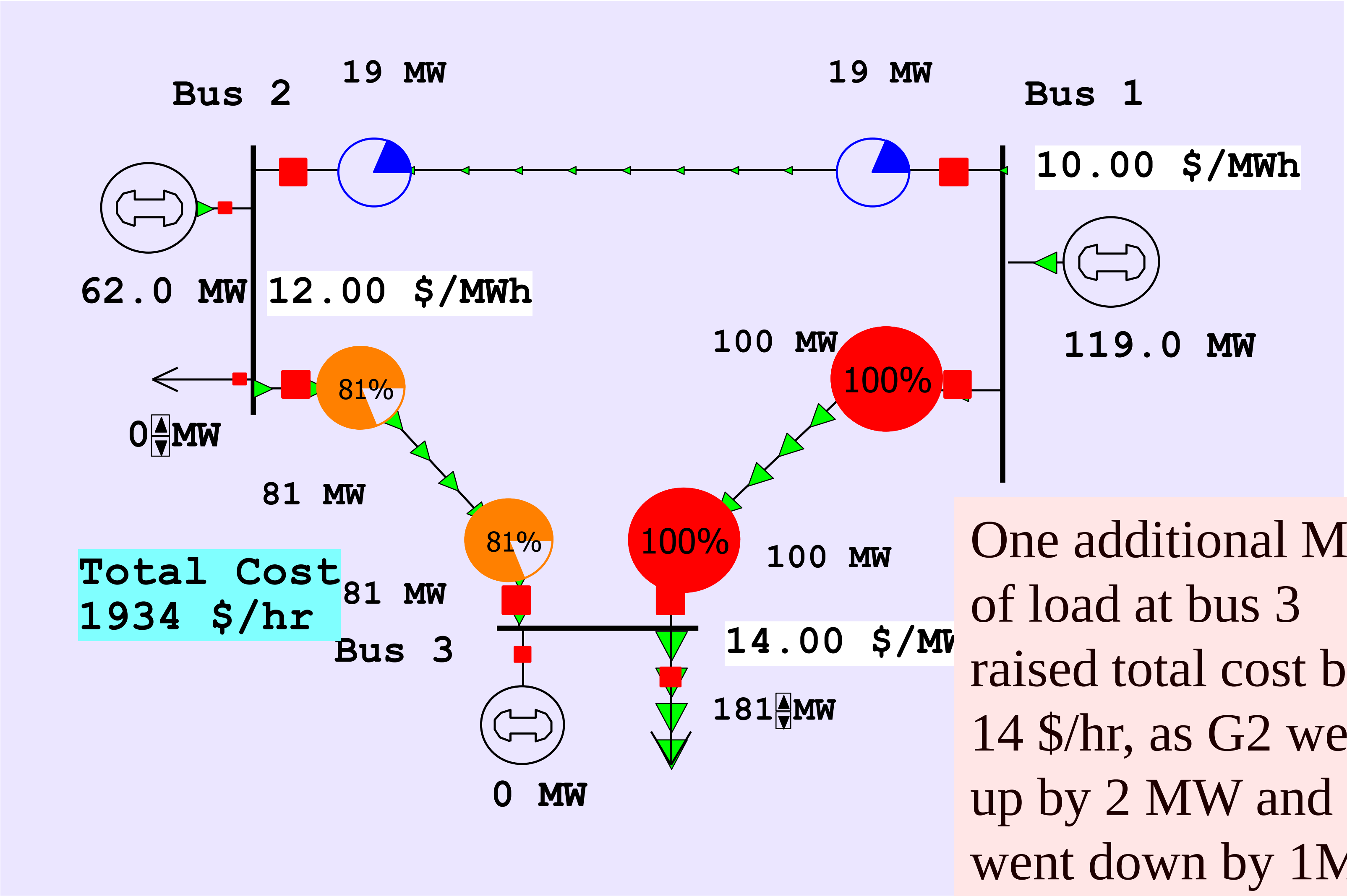
Line between Bus 1and Bus 3 is over-loaded; all buses have the same marginal cost

B3 with Line Limits Enforced



LP OPF changes generation to remove violation. Bus marginal costs are now different.

Verify Bus 3 Marginal Cost



One additional MW of load at bus 3 raised total cost by 14 $/hr, as G2 went up by 2 MW and G1 went down by 1MW

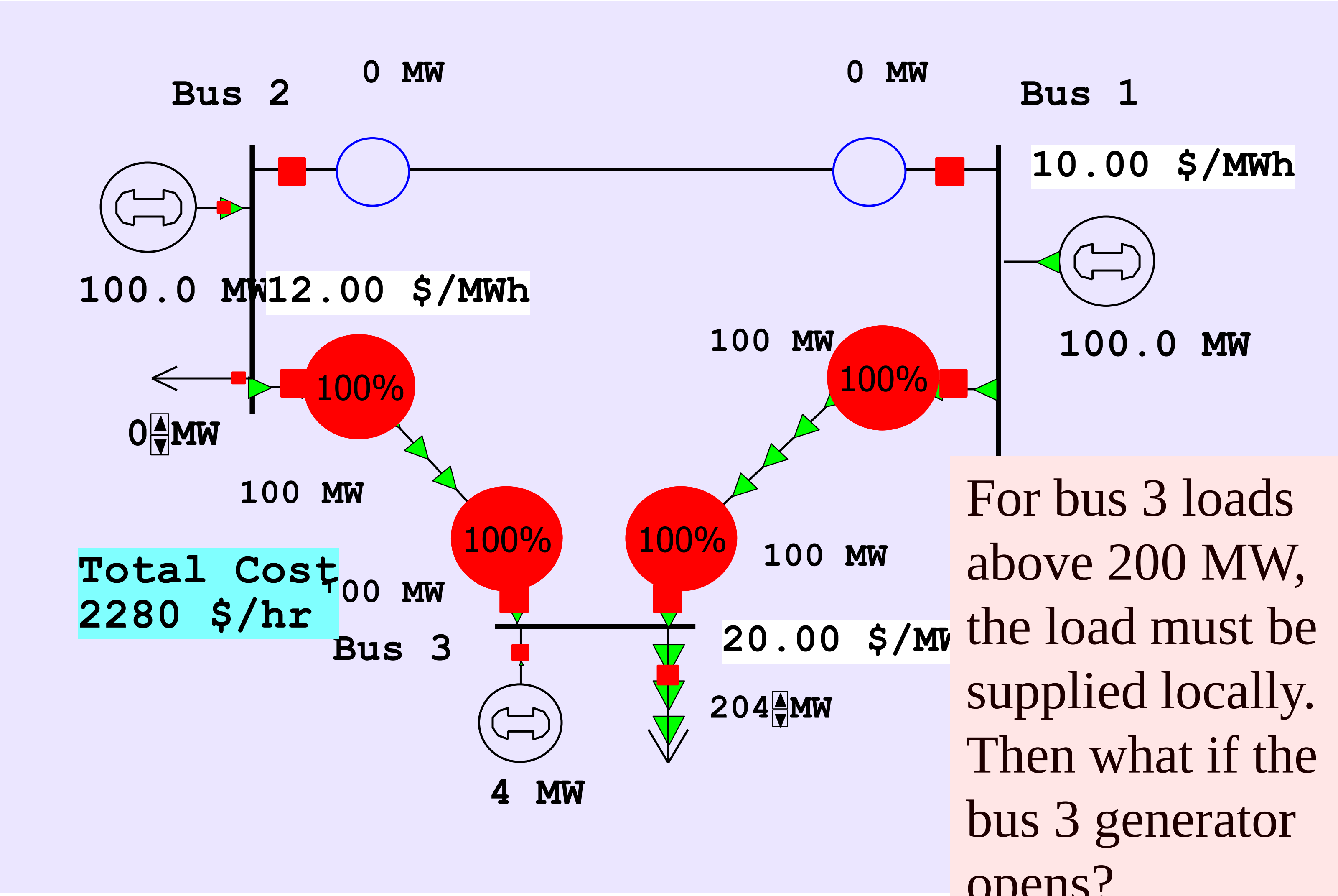
Why is bus 3 LMP = $14 /MWh

- All lines have equal impedance. Power flow in a simple network distributes inversely to impedance of path.
  - For bus 1 to supply 1 MW to bus 3, 2/3 MW would take direct path from 1 to 3, while 1/3 MW would “loop around” from 1 to 2 to 3.
  - Likewise, for bus 2 to supply 1 MW to bus 3, 2/3MW would go from 2 to 3, while 1/3 MW would go from 2 to 1to 3.

Why is bus 3 LMP \$ 14 / MWh, cont'd

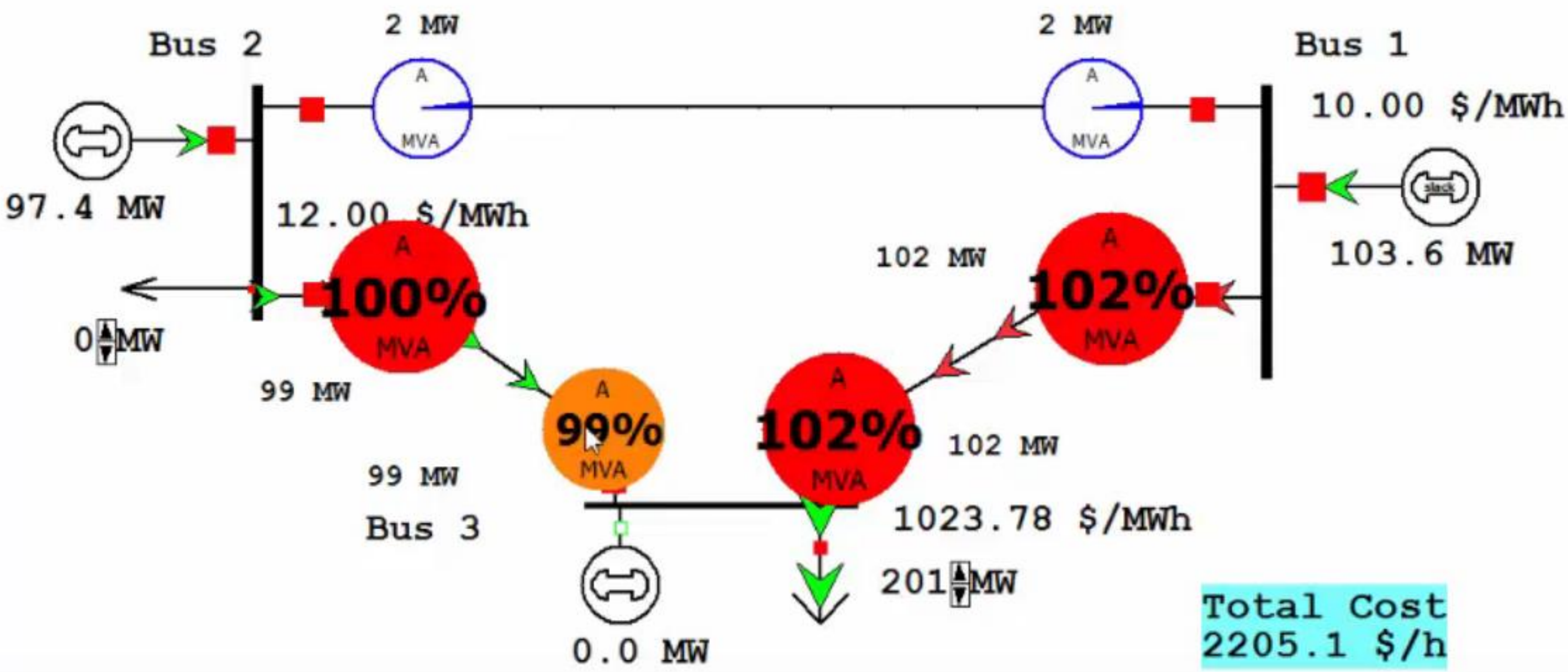
- With the line from 1 to 3 limited, no additional power flows are allowed on it.
- To supply 1 more MW to bus 3 we need
  - $\Delta P_{G1} + \Delta P_{G2} = 1$ MW
  - $2/3\ \Delta\ P_{G1} + 1/3\ \Delta\ P_{G2} = 0$; (no more flow on 1-3)
- Solving requires we up P_{G2} by 2 MW and drop P_{G1} by 1 MW -- a net increase of \$24 – \$10 = \$14.

Both lines into Bus 3 Congested



For bus 3 loads above 200 MW, the load must be supplied locally. Then what if the bus 3 generator opens?

Both lines into Bus 3 Congested



Quick Coverage of Linear Programming

- LP is probably the most widely used mathematical programming technique
- It is used to solve linear, constrained minimization (or maximization) problems in which the objective function and the constraints can be written as linear functions

Example Problem 1

- Assume that you operate a lumber mill which makes both construction-grade and finish-grade boards from the logs it receives. Suppose it takes 2 hours to rough-saw and 3 hours to plane each 1000 board feet of construction-grade boards. Finish-grade boards take 2 hours to rough-saw and 5 hours to plane for each 1000 board feet. Assume that the saw is available 8 hours per day, while the plane is available 15 hours per day. If the profit per 1000 board feet is \$100 for construction-grade and \$120 for finish-grade, how many board feet of each should you make per day to maximize your profit?

Problem 1 Setup

Let x_1=amount of cg, x_2= amount of fg

Maximize $\quad 100x_1 + 120x_2$

s.t. $\quad 2x_1 + 2x_2 \le 8$

$$3x_1 + 5x_2 \le 15$$

$$x_1, x_2 \ge 0$$

Notice that all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are seeking to determine the values of x_1 and x_2

Example Problem 2

- A nutritionist is planning a meal with 2 foods: A and B. Each ounce of A costs \$ 0.20, and has 2 units of fat, 1 of carbohydrate, and 4 of protein. Each ounce of B costs \$0.25, and has 3 units of fat, 3 of carbohydrate, and 3 of protein. Provide the least cost meal which has no more than 20 units of fat, but with at least 12 units of carbohydrates and 24 units of protein.

Problem 2 Setup

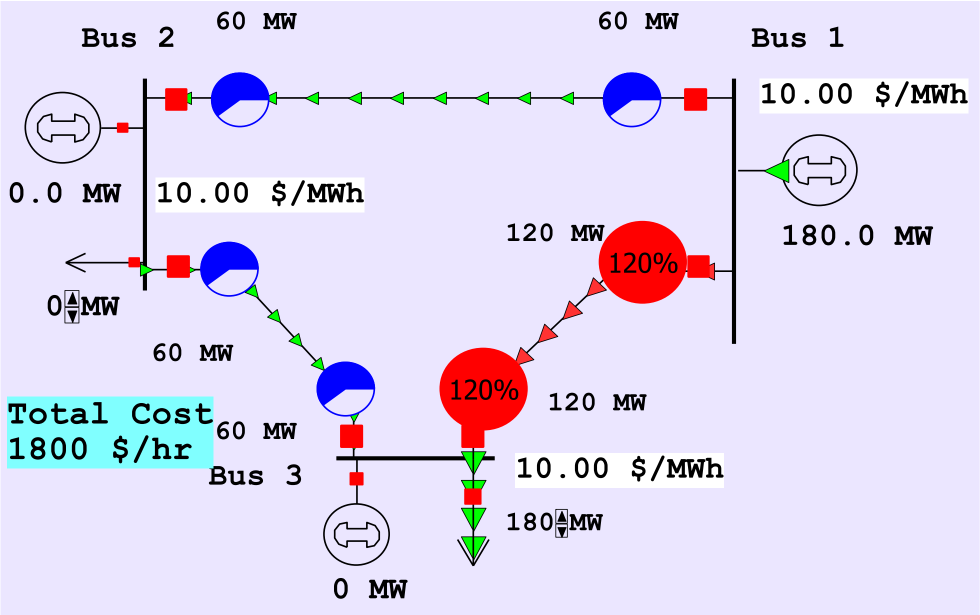
Let x_1=ounces of A, x_2= ounces of B

$$
\begin{aligned}
\text{Minimize} \quad & 0.20x_1 + 0.25x_2 \\
\text{s.t.} \quad & 2x_1 + 3x_2 \le 20 \\
& x_1 + 3x_2 \ge 12 \\
& 4x_1 + 3x_2 \ge 24
\end{aligned}
$$

Again all of the equations are linear, but
they are inequality, as opposed to equality, constraints;
we are again seeking to determine the values of x_1 and x_2;
notice there are also more constraints then solution variables

Three Bus Case Formulation

- For the earlier three bus system given the initial condition of an overloaded transmission line, minimize the cost of generation such that the change in generation is zero, and the flow on the line between buses 1 and 3 is not violating its limit



- Can be setup considering the change in generation, (ΔP_{G1}, ΔP_{G2}, ΔP_{G3})

Three Bus Case Problem Setup

Let $x_1 = \Delta P_{G1}$, $x_2 = \Delta P_{G2}$, $x_3 = \Delta P_{G3}$

Minimize $\quad 10x_1 + 12x_2 + 20x_3$

s.t. $\quad \frac{2}{3}x_1 + \frac{1}{3}x_2 \le -20$

Line flow constraint

$x_1 + x_2 + x_3 = 0$

Power balance constraint

enforcing limits on $x_1,\ x_2,\ x_3$

LP Standard Form

The standard form of the LP problem is

$$\begin{aligned} \text{Minimize} \quad & \mathbf{c}\,\mathbf{x} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

Maximum problems can be treated as minimizing the negative

where $\mathbf{x}$ = n-dimensional column vector

$\mathbf{c}$ = n-dimensional row vector

$\mathbf{b}$ = m-dimensional column vector

$\mathbf{A}$ = m×n matrix

For the LP problem usually n>> m

The previous examples were not in this form!

Replacing Inequality Constraints with Equality Constraints

- The LP standard form does not allow inequality constraints
- Inequality constraints can be replaced with equality constraints through the introduction of slack variables, each of which must be greater than or equal to zero

$$\ldots \le b_i \rightarrow \ldots + y_i = b_i \quad \text{with } y_i \ge 0$$
$$\ldots \ge b_i \rightarrow \ldots - y_i = b_i \quad \text{with } y_i \ge 0$$

- Slack variables have no cost associated with them; they merely tell how far a constraint is from being binding, which will occur when its slack variable is zero

Lumber Mill Example with Slack Variables

- Let the slack variables be x_3 and x_4, so

$$\text{Minimize} \quad -(100x_1 + 120x_2)$$

Minimize the negative

$$\text{s.t.} \quad 2x_1 + 2x_2 + x_3 = 8$$

$$3x_1 + 5x_2 + x_4 = 15$$

$$x_1, x_2, x_3, x_4 \geq 0$$

LP Definitions

A vector **x** is said to be basic if

1. $\mathbf{Ax} = \mathbf{b}$
2. At most m components of **x** are non-zero; these are called the basic variables; the rest are non basic variables; if there as less than m non-zeros then **x** is called degenerate

$\mathbf{A}_\mathrm{B}$ is called the basis matrix

Define $\mathbf{x} = \begin{bmatrix} \mathbf{x}_\mathrm{B} \\ \mathbf{x}_\mathrm{N} \end{bmatrix}$ (with $\mathbf{x}_\mathrm{B}$ basic) and $\mathbf{A} = \begin{bmatrix} \mathbf{A}_B & \mathbf{A}_N \end{bmatrix}$

With $\begin{bmatrix} \mathbf{A}_B & \mathbf{A}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_\mathrm{B} \\ \mathbf{x}_\mathrm{N} \end{bmatrix} = \mathbf{b}$ so $\mathbf{x}_\mathrm{B} = \mathbf{A}_B^{-1}\left(\mathbf{b} - \mathbf{A}_N \mathbf{x}_\mathrm{N}\right)$

Fundamental LP Theorem

- Given an LP in standard form with **A** of rank m then
  - If there is a feasible solution, there is a basic feasible solution
  - If there is an optimal, feasible solution, then there is an optimal, basic feasible solution
- Note, there could be a LARGE number of basic, feasible solutions
  - Simplex algorithm determines the optimal, basic feasible solution usually very quickly

LP Graphical Interpretation

- The LP constraints define a polyhedron in the solution space
  - This is a polytope if the polyhedron is bounded and nonempty
  - The basic, feasible solutions are vertices of this polyhedron
  - With the linear cost function the solution will be at one of vertices

APPENDIX 3B: LINEAR PROGRAMMING (LP) 11

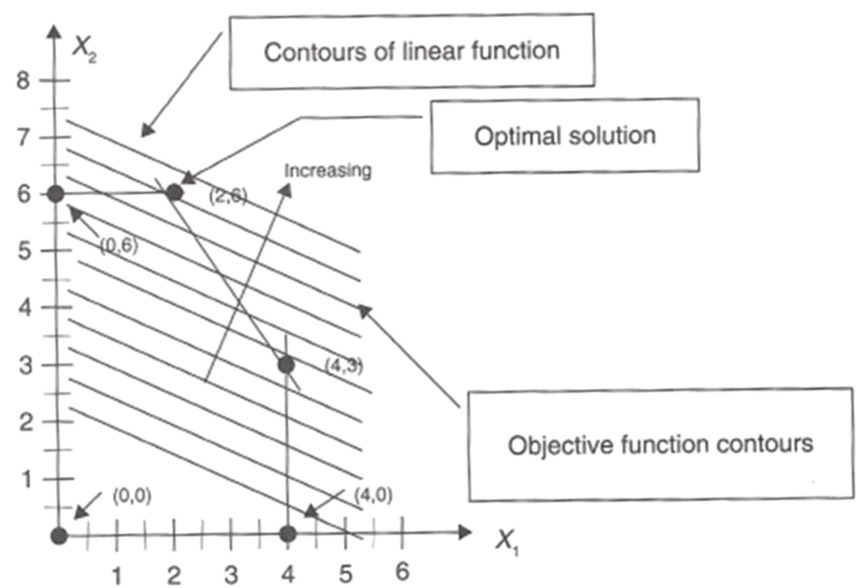


FIGURE 3.26 x_1, x_2 plane with cost contours and the optimal solution shown.

Image: Figure 3.26 from course text