A.1) Taking $\theta_1 = 0$ [$\because$ DC flow]

$$M_{12} = \frac{1}{x_{12}}(\theta_1 - \theta_2) = -10\theta_2 = 0.62$$

$$M_{21} = \frac{1}{x_{21}}(\theta_2 - \theta_1) = 10\theta_2 = -0.52$$

$$\Rightarrow H = \begin{bmatrix} -10 \\ 10 \end{bmatrix} \; ; \quad z^{meas} = \begin{bmatrix} 0.62 \\ -0.52 \end{bmatrix} ;$$

$$R = \begin{bmatrix} \left(\frac{1}{100}\right)^2 & \\ & \left(\frac{5}{100}\right)^2 \end{bmatrix}$$

σ has to be also in pu.

$$x^{est} = [\theta_2^{est}] = -0.0616$$

$$\therefore |M_{21}| = |M_{12}| = 10\theta_2 = 0.616 \text{ pu}$$

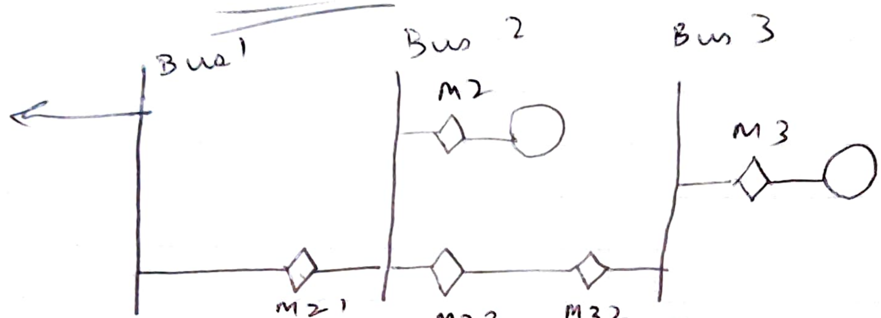
$$\Rightarrow 61.6 \text{ MW}$$

- Unequal life span incremental analysis (Ch 5)

HW05

(2) (a)

Base = 100 MVA



$\theta_3 = 0$

- $X_{12} = 0.1$ pu , $X_{23} = 0.25$ pu
- $M_{21} = 148$ MW, $M_{23} = -135$ MW, $M_{32} = 98$ MW; $M_2 = 49$ MW, $M_3 = 105$ MW
- $\sigma = 0.01$ pu for all devices

$$M_{21} = \frac{1}{X_{21}}(\theta_2 - \theta_1) = \frac{1}{0.1}(\theta_2 - \theta_1) = 10\theta_2 - 10\theta_1 = 1.48$$

$$M_{23} = \frac{1}{X_{23}}(\theta_2 - \theta_3) = \frac{1}{0.25}(\theta_2 - \theta_3) = 4\theta_2 - 4\theta_3 = -1.35$$

$$M_{32} = \frac{1}{X_{32}}(\theta_3 - \theta_2) = 4\theta_3 - 4\theta_2 = 0.98$$

Since it's given, $\theta_3 = 0°$

$$H = \begin{bmatrix} -10 & 10 \\ 0 & 4 \\ 0 & -4 \end{bmatrix} ; \quad R = \begin{bmatrix} (10^{-2})^2 & & \\ & (10^{-2})^2 & \\ & & (10^{-2})^2 \end{bmatrix}$$

$$Z_{meas} = \begin{bmatrix} 1.48 \\ -1.35 \\ 0.98 \end{bmatrix}$$

$$X^{est} = [H^T R^{-1} H]^{-1} H^T R^{-1} Z^{meas}$$

$$X^{est} = \begin{bmatrix} \theta_1^{est} \\ \theta_2^{est} \end{bmatrix} = \begin{bmatrix} -0.4392 \\ -0.2913 \end{bmatrix}$$

(b)

$$J(\theta_1, \theta_2) = \frac{[1.48 - [10\theta_2 - 10\theta_1]]^2}{(10^{-2})^2} + \frac{[-1.35 - [4\theta_2 - 4\theta_3]]^2}{(10^{-2})^2} + \frac{[0.98 - [4\theta_3 - 4\theta_2]]^2}{(10^{-2})^2}$$

$$= \frac{1}{(10^{-2})^2} [10^{-6} + 0.0342 + 0.0343]$$

$$= 685.0004$$

c) $M_{21} = \frac{1}{x_{21}}(\theta_2 - \theta_1) = \frac{1}{0.1}(-0.2913 + 0.4392)$

$= 1.479$

- $M_{23} = \frac{1}{x_{23}}(\theta_2 - \theta_3) = \frac{1}{0.25}(-0.2913)$

$= -1.1652$

- $M_{32} = \frac{1}{x_{32}}(\theta_3 - \theta_2) = 1.1652$

- $M_3 = M_{32} = 1.1652$

- $M_2 = |M_{21}| - |M_{23}|$ [using ~~KVL~~ Kirchhoff Current Law]

$= 0.3138$

d) Yes, there is error.

Because,

~~A~~ If we compare the given and the calculated measurements, they are different.

On comparison, M_{23} & M_{32} are different for the given & calculated, so they are more apt to ~~b~~ be in error.

A.3) Answers may vary based on order of selection.

My selection order was :
1) A[3,1]
2) A[2,1]
3) A[3,2]

<u>QR</u> :

$$A = (G_1 G_2 G_3)(G_3^T G_2^T G_1^T A)$$

$$= \begin{bmatrix} 0.3810 & 0.8401 & 0.3862 \\ 0.8890 & -0.4476 & 0.0966 \\ 0.2540 & 0.3065 & -0.9173 \end{bmatrix} \times \begin{bmatrix} 7.8740 & 9.3980 \\ 0 & 5.2613 \\ 0 & 0 \end{bmatrix}$$

A.4)

$$y = a + bx + cx^2$$

$$y = \begin{bmatrix} 1 & x & x^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ 1 & x_4 & x_4^2 \\ 1 & x_5 & x_5^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 6 \\ 12 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$a = -7.8$
$b = 11.3429$
$c = -1.8571$ //

A.5) $Y_{bus} =$

		e (1 2 3)	s (4 5 6 7)
e	1, 2, 3	Y_{ee}	Y_{es}
s	4, 5, 6, 7	Y_{se}	Y_{ss}

$$Y_{eq} = Y_{ss} - Y_{se} * (Y_{ee})^{-1} * Y_{es}$$