ECEN 667
Power System Stability

Lecture 15: Time-Domain Simulation Solutions (Transient Stability)

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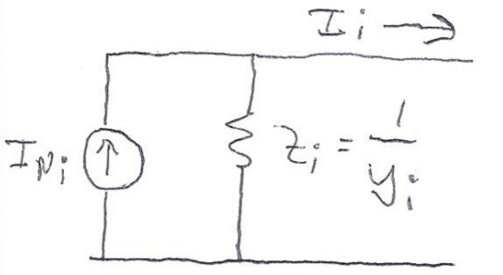
Announcements

- Read Chapter 7
- Homework 5 is due on Oct 28.
- A classic paper in this area is B. Stott, “Power System Dynamic Response Calculations,” Proc. IEEE February 1979, pp. 219-241
- We’ll cover the equal area criteria in Chapter 9
- IEEE Spectrum did have a nice biographical article on Charlie Concordia in 1999 (when he won the IEEE Medal of Honor at age 91)
  - He joined GE in 1926; his best contribution (he noted) was, “to increase the understanding of the dynamics of power systems”

Subtransient Models

- The Norton current injection approach is what is commonly used with subtransient models in industry
- If subtransient saliency is neglected (as is the case with GENROU and GENSAL in which $X''_d = X''_q$) then the current injection is

$$I_{Nd} + jI_{Nq} = \frac{E''_d + jE''_q}{R_s + jX''} = \frac{\left(-\psi''_q + j\psi''_d\right)\omega}{R_s + jX''}$$



  – Subtransient saliency can be handled with this approach, but it is more involved (see Arrillaga, *Computer Analysis of Power Systems*, section 6.6.3)

Subtransient Models

- Note, the values here are on the dq reference frame
- We can now extend the approach introduced for the classical machine model to subtransient models
- Initialization is as before, which gives the δ's and other state values
- Each time step is as before, except we use the δ's for each generator to transfer values between the network reference frame and each machine's dq reference frame
  - The currents provide the coupling

Two Bus Example with Two GENROU Machine Models

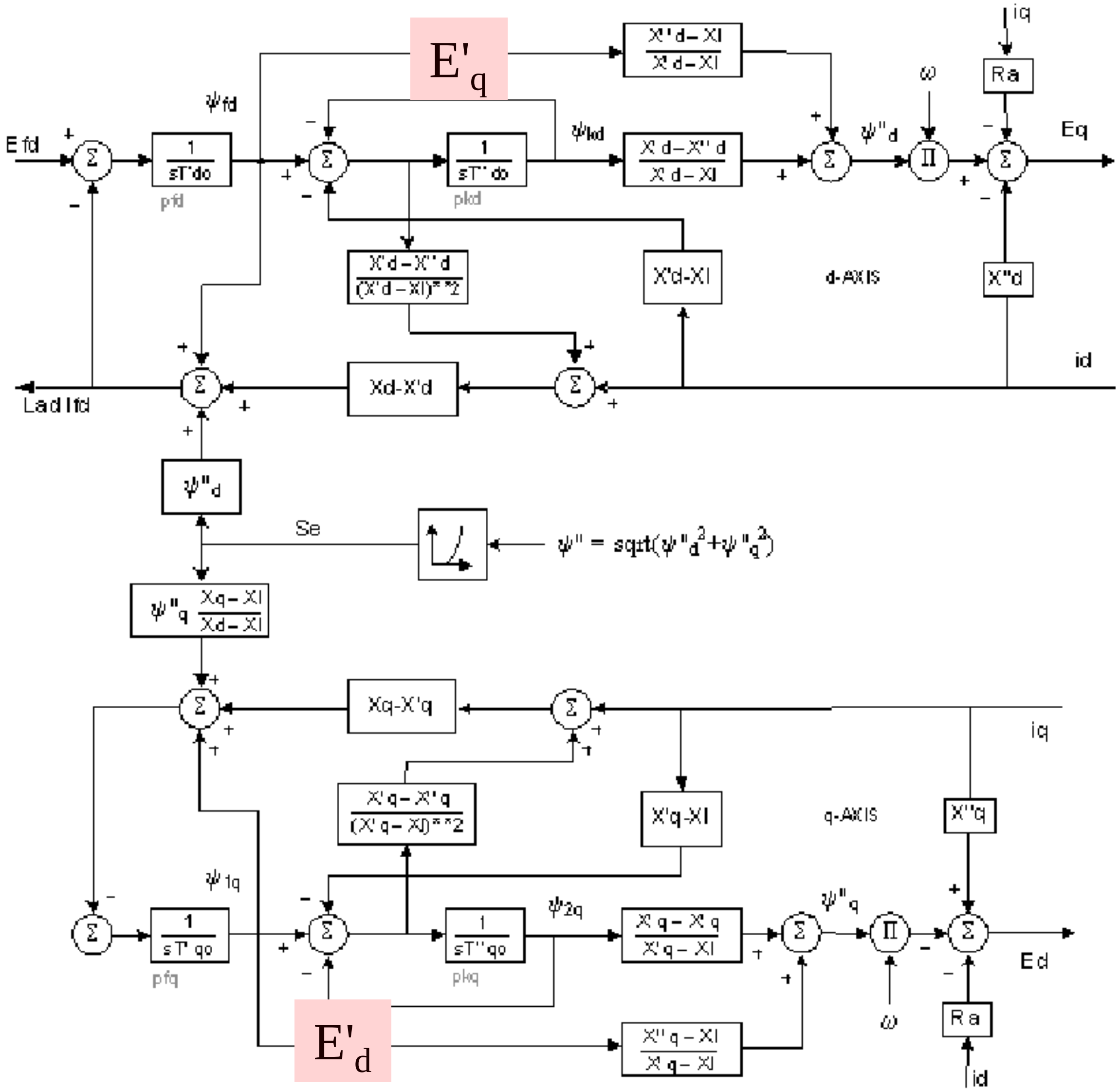
- Use the same system as before, except with we'll model both generators using GENROUs
  - For simplicity we'll make both generators identical except set H_1=3, H_2=6; other values are X_d=2.1, X_q=0.5, X'_d=0.2, X'_q=0.5, X''_q=X''_d=0.18, X_l=0.15, T'_{do} = 7.0, T'_{qo}=0.75, T''_{do}=0.035, T''_{qo}=0.05; no saturation
  - With no saturation the value of the δ's are determined (as per the earlier lectures) by solving

$$\left|E\right|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$

  - Hence for generator 1

$$\left|E_1\right|\angle\delta_1 = 1.0946\angle 11.59° + \left(j0.5\right)\left(1.052\angle -18.2°\right) = 1.431\angle 30.2°$$

GENROU Block Diagram



Two Bus Example with Two GENROU Machine Models

- Using the early approach the initial state vector is

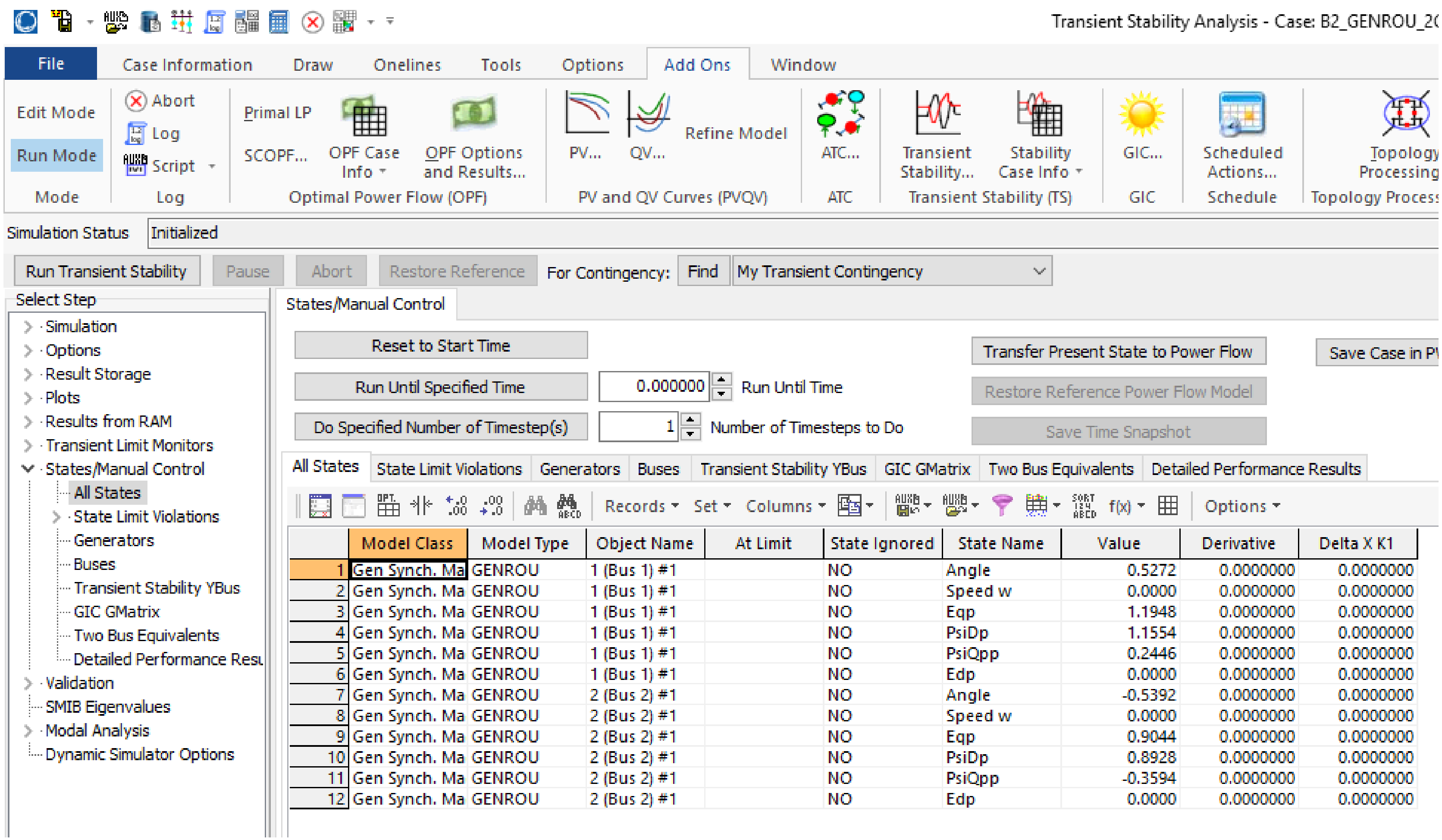
$$\mathbf{x}(0) = \begin{bmatrix} \delta_1 \\ \Delta\omega_1 \\ E'_{q1} \\ \psi_{1d1} \\ \psi_{2q1} \\ E'_{d1} \\ \delta_2 \\ \Delta\omega_2 \\ E'_{q2} \\ \psi_{1d2} \\ \psi_{2q2} \\ E'_{d2} \end{bmatrix} = \begin{bmatrix} 0.5273 \\ 0.0 \\ 1.1948 \\ 1.1554 \\ 0.2446 \\ 0 \\ -0.5392 \\ 0 \\ 0.9044 \\ 0.8928 \\ -0.3594 \\ 0 \end{bmatrix}$$

Note that this is a salient pole machine with $X'_q = X_q$; hence E'_d will always be zero

The initial currents in the dq reference frame are $I_{d1}=0.7872$, $I_{q1}=0.6988$, $I_{d2}=0.2314$, $I_{q2}=-1.0269$

Initial values of $\psi''_{q1} = -0.2236$, and $\psi''_{d1} = 1.179$

PowerWorld GENROU Initial States



	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value	Derivative	Delta X K1
1	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Angle	0.5272	0.0000000	0.0000000
2	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Speed w	0.0000	0.0000000	0.0000000
3	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Eqp	1.1948	0.0000000	0.0000000
4	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	PsiDp	1.1554	0.0000000	0.0000000
5	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.2446	0.0000000	0.0000000
6	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Edp	0.0000	0.0000000	0.0000000
7	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Angle	-0.5392	0.0000000	0.0000000
8	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Speed w	0.0000	0.0000000	0.0000000
9	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Eqp	0.9044	0.0000000	0.0000000
10	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8928	0.0000000	0.0000000
11	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.3594	0.0000000	0.0000000
12	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Edp	0.0000	0.0000000	0.0000000

Solving with Euler's

- We'll again solve with Euler's, except with Δt set now to 0.01 seconds (because now we have a subtransient model with faster dynamics)
  - We'll also clear the fault at t=0.05 seconds
- For the more accurate subtransient models the swing equation is written in terms of the torques

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\frac{d\omega_i}{dt} = \frac{2H_i}{\omega_s}\frac{d\Delta\omega_i}{dt} = T_{Mi} - T_{Ei} - D_i\left(\Delta\omega_i\right)$$

with $T_{Ei} = \psi''_{d,i} i_{qi} - \psi''_{q,i} i_{di}$

Other equations are solved based upon the block diagram

Norton Equivalent Current Injections

- The initial Norton equivalent current injections on the dq base for each machine are

$$I_{Nd1} + jI_{Nq1} = \frac{\left(-\psi''_{q1} + j\psi''_{d1}\right)\omega_1}{jX''_1} = \frac{\left(-0.2236 + j1.179\right)(1.0)}{j0.18}$$

$$= 6.55 + j1.242$$

$$I_{ND1} + jI_{NQ1} = 2.222 - j6.286$$

$$I_{Nd2} + jI_{Nq2} = 4.999 + j1.826$$

$$I_{ND2} + jI_{NQ2} = -1 - j5.227$$

Recall the dq values are on the machine's reference frame and the DQ values are on the system reference frame

Moving between DQ and dq

- Recall

$$\begin{bmatrix} I_{di} \\ I_{qi} \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_{Di} \\ I_{Qi} \end{bmatrix}$$

- And

$$\begin{bmatrix} I_{Di} \\ I_{Qi} \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_{di} \\ I_{qi} \end{bmatrix}$$

The currents provide the key coupling between the two reference frames

Bus Admittance Matrix

- The bus admittance matrix is as from before for the classical models, except the diagonal elements are augmented using

$$Y_i = \frac{1}{R_{s,i} + jX''_{d,i}}$$

$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.18} & 0 \\ 0 & \frac{1}{j0.18} \end{bmatrix} = \begin{bmatrix} -j10.101 & j4.545 \\ j4.545 & -j10.101 \end{bmatrix}$$

Algebraic Solution Verification

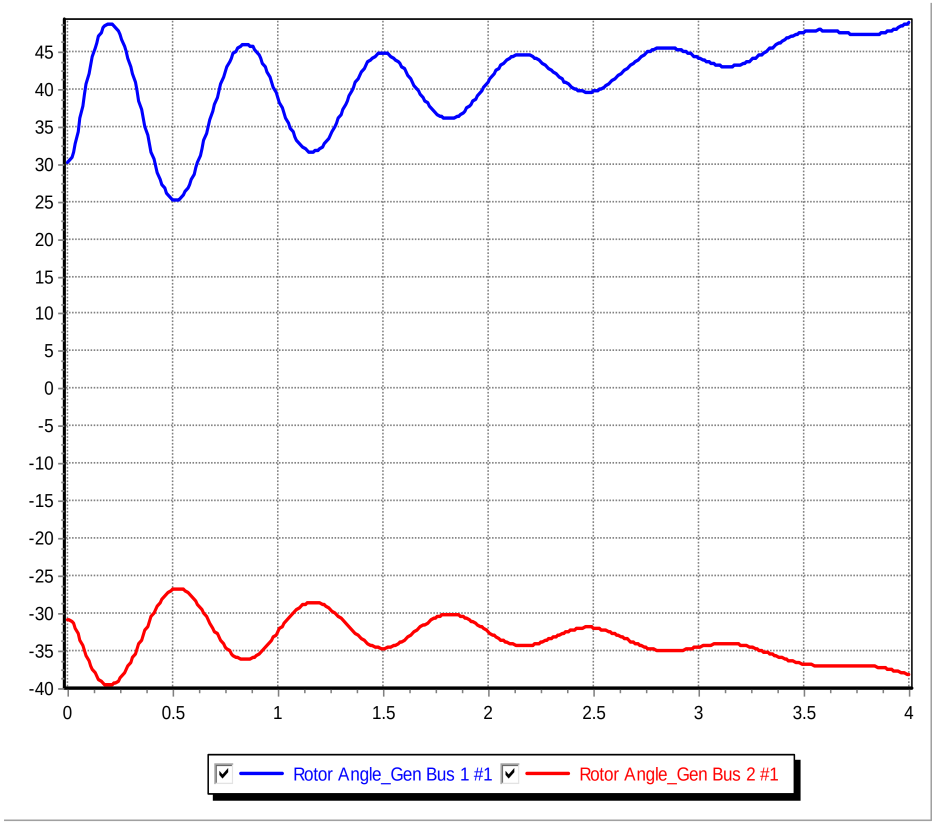
- To check the values solve (in the network reference frame)

$$\mathbf{V} = \begin{bmatrix} -j10.101 & j4.545 \\ j4.545 & -j10.101 \end{bmatrix}^{-1} \begin{bmatrix} 2.222 - j6.286 \\ -1 - j5.227 \end{bmatrix}$$

$$= \begin{bmatrix} 1.072 + j0.22 \\ 1.0 \end{bmatrix}$$

Results

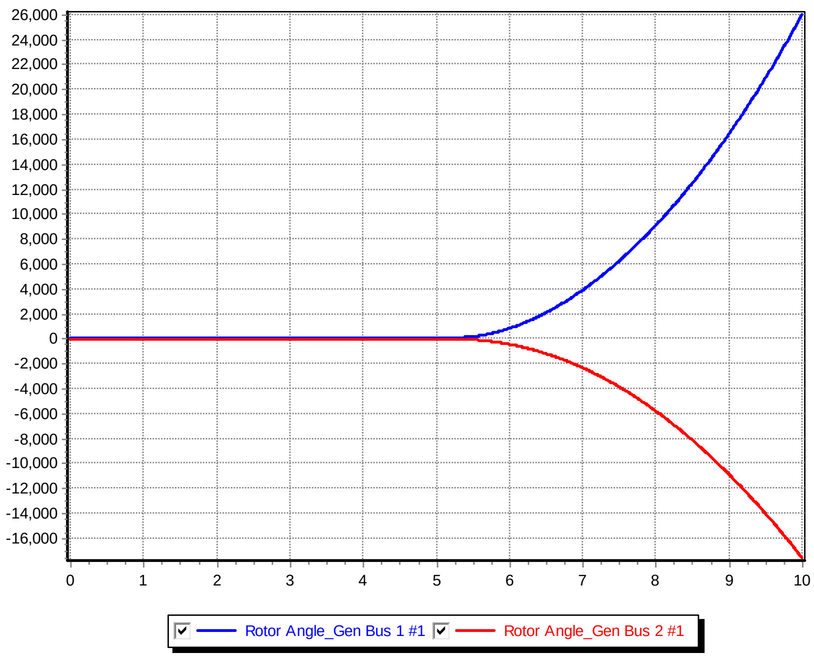
- The below graph shows the results for four seconds of simulation, using Euler's with Δt=0.01 seconds



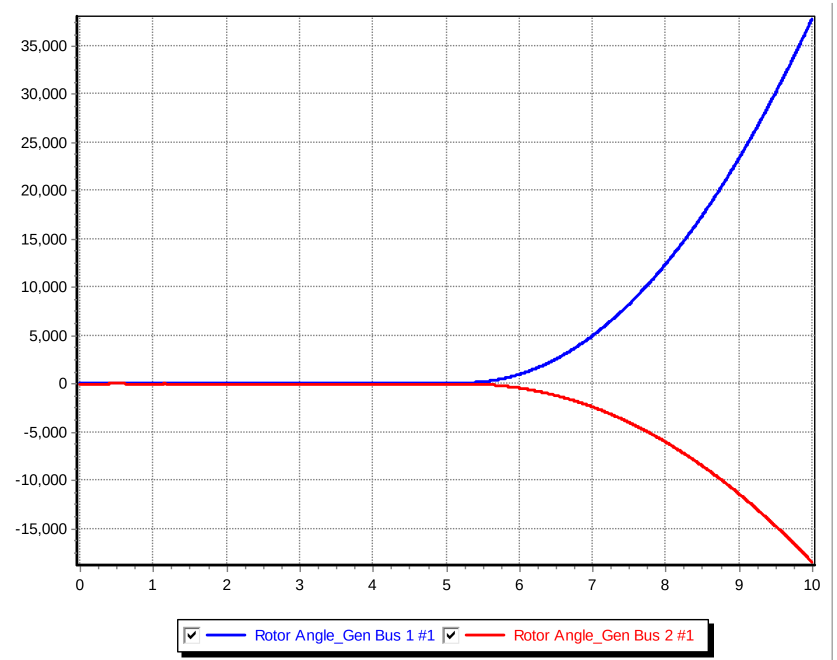
PowerWorld case is **B2_GENROU_2GEN_EULER**

Results for Longer Time

- Simulating out 10 seconds indicates an unstable solution, both using Euler's and RK2 with $\Delta t=0.005$, so it is really unstable!



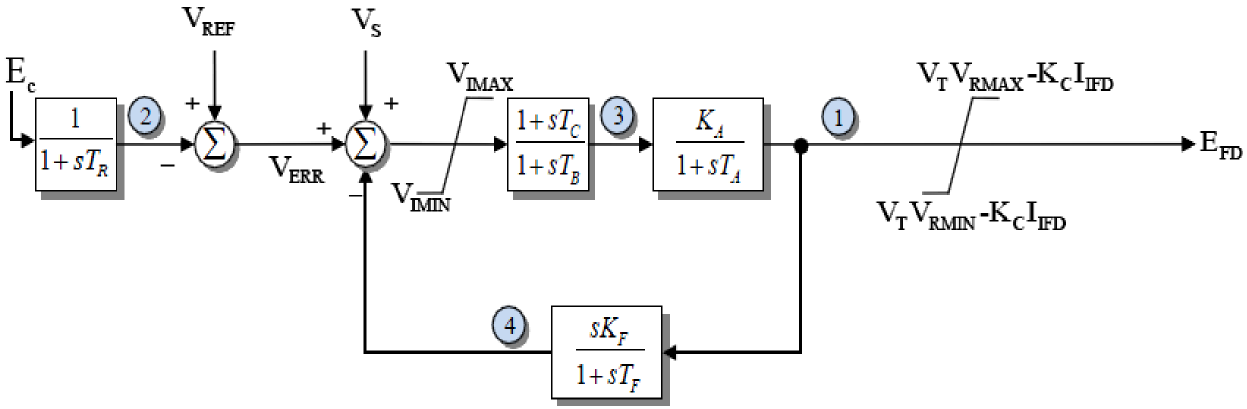
Euler's with $\Delta t=0.01$



RK2 with $\Delta t=0.005$

Adding More Models

- In this situation the case is unstable because we have not modeled exciters
- To each generator add an EXST1 with T_R=0, T_C=T_B=0, K_f=0, K_A=100, T_A=0.1



  - This just adds one differential equation per generator

$$\frac{dE_{FD}}{dt} = \frac{1}{T_A}\left(K_A\left(V_{REF} - |V_t|\right) - E_{FD}\right)$$

Two Bus, Two Gen With Exciters

- Below are the initial values for this case from PowerWorld

All States | State Limit Violations | Generators | Buses | Transient Stability YBus | GIC GMatrix | Two Bus Equivalents

Records ▾ Set ▾ Columns ▾ f(x) ▾

	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value
1	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Angle	0.5273
2	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Speed w	0.0000
3	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Eqp	1.1948
4	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiDp	1.1554
5	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.2446
6	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Edp	0.0000
7	Gen Exciter	EXST1	1 (Bus 1) #1		NO	EField before lim	2.6904
8	Gen Exciter	EXST1	1 (Bus 1) #1		YES	Sensed Vt	1.0946
9	Gen Exciter	EXST1	1 (Bus 1) #1		YES	VLL	0.0269
10	Gen Exciter	EXST1	1 (Bus 1) #1		NO	VF	0.0000
11	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Angle	-0.5392
12	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Speed w	0.0000
13	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Eqp	0.9044
14	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8928
15	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.3594
16	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Edp	0.0000
17	Gen Exciter	EXST1	2 (Bus 2) #1		NO	EField before lim	1.3441
18	Gen Exciter	EXST1	2 (Bus 2) #1		YES	Sensed Vt	1.0000
19	Gen Exciter	EXST1	2 (Bus 2) #1		YES	VLL	0.0134
20	Gen Exciter	EXST1	2 (Bus 2) #1		NO	VF	0.0000

Because of the zero values the other differential equations for the exciters are included but treated as ignored

Case is **B2_GENROU_2GEN_EXCITER**

Viewing the States

- PowerWorld allows one to single-step through a solution, showing the **f**(**x**) and the $\mathbf{K}_1$ values
  - This is mostly used for education or model debugging

All States | State Limit Violations | Generators | Buses | Transient Stability YBus | GIC GMatrix | Two Bus Equivalents

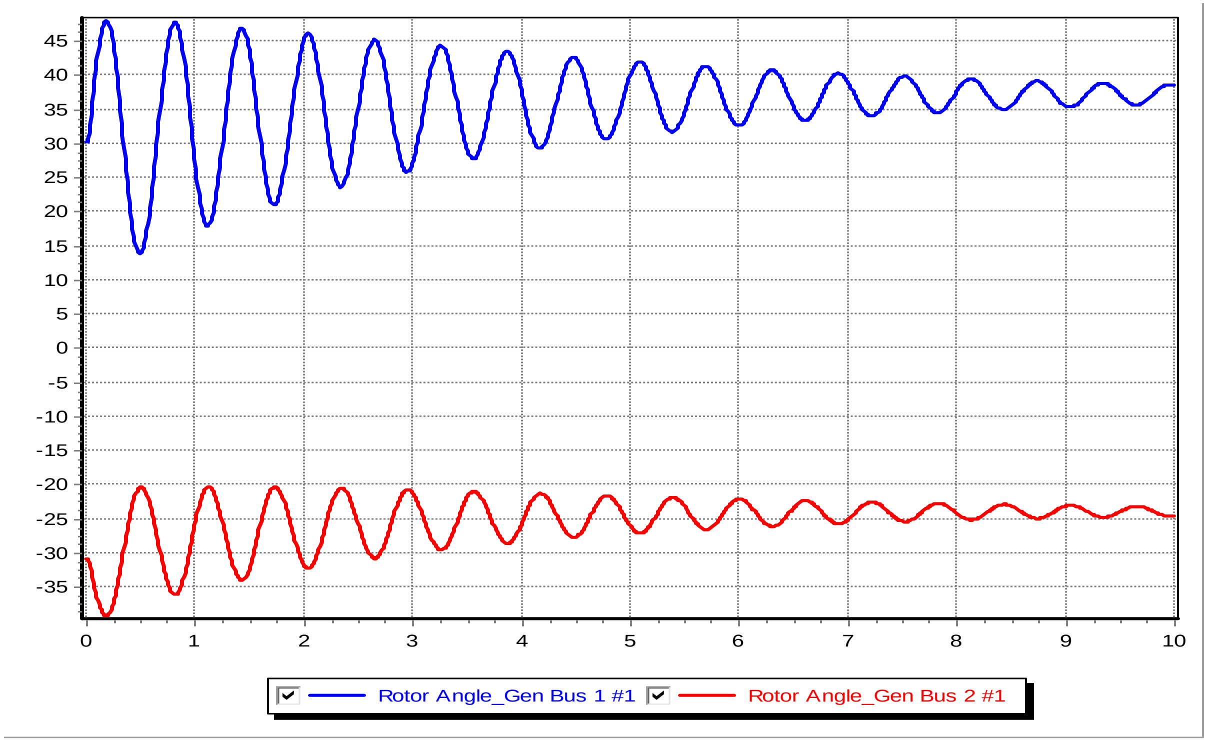
Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value	Derivative	Delta X K1
1	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Angle	0.5288	0.6283185	0.0015708
2	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Speed w	0.0017	0.1666667	0.0016667
3	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Eqp	1.1813	-1.4246850	-0.0135115
4	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiDp	1.0788	-6.1374236	-0.0766226
5	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.1276	-7.0939033	-0.1170377
6	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Edp	0.0000	0.0000000	0.0000000
7	Gen Exciter	EXST1	1 (Bus 1) #1		NO	EField before lim	3.4214	65.7861970	0.7309577
8	Gen Exciter	EXST1	1 (Bus 1) #1		YES	Sensed Vt	0.0000	0.0000000	0.0000000
9	Gen Exciter	EXST1	1 (Bus 1) #1		YES	VLL	0.1000	0.0000000	0.0000000
10	Gen Exciter	EXST1	1 (Bus 1) #1		NO	VF	0.0000	0.0000000	0.0000000
11	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Angle	-0.5400	-0.2896794	-0.0007854
12	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Speed w	-0.0008	-0.0833331	-0.0007684
13	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Eqp	0.9010	-0.2497156	-0.0033918
14	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8661	-2.1684713	-0.0267221
15	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.2480	8.9252864	0.1113928
16	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Edp	0.0000	0.0000000	0.0000000
17	Gen Exciter	EXST1	2 (Bus 2) #1		NO	EField before lim	2.2097	77.9031593	0.8655907
18	Gen Exciter	EXST1	2 (Bus 2) #1		YES	Sensed Vt	0.5032	0.0000000	0.0000000
19	Gen Exciter	EXST1	2 (Bus 2) #1		YES	VLL	0.1000	0.0000000	0.0000000
20	Gen Exciter	EXST1	2 (Bus 2) #1		NO	VF	0.0000	0.0000000	0.0000000

Derivatives shown are evaluated at the end of the time step

Two Bus Results with Exciters

- Below graph shows the angles with Δt=0.01 and a fault clearing at t=0.05 using Euler's
  – With the addition of the exciters case is now stable



Load Models Introduced

- The simplest approach for modeling the loads is to treat them as constant impedances, embedding them in the bus admittance matrix
  - Only impact the $\mathbf{Y}_{\text{bus}}$ diagonals
- The admittances are set based upon their power flow values, scaled by the inverse of the square of the power flow bus voltage

$$\bar{S}_{load,i} = \bar{V}_i \bar{I}^*_{load,i} = \left|\bar{V}_i\right|^2 \left(G_{load,i} - jB_{load,i}\right)$$

$$G_{load,i} - jB_{load,i} = \frac{\bar{S}_{load,i}}{\left|\bar{V}_i\right|^2}$$

Note the positive sign comes from the sign convention on $\bar{\mathrm{I}}_{\mathrm{load,i}}$

In PowerWorld the default load model is specified on **Transient Stability, Options, Power System Model** page

Example 7.4 Case (WSCC 9 Bus)

- PowerWorld Case **Example_7_4** duplicates the example 7.4 case from the book, with the exception of using different generator models

	Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6	Bus 7	Bus 8	Bus 9
1	Bus1	0.000 - j42.361			-0.000 + j17.361					
2	Bus 2		0.000 - j27.111					-0.000 + j16.000		
3	Bus 3			0.000 - j23.732						-0.000 + j17.065
4	Bus 4	-0.000 + j17.361			3.307 - j39.309	-1.365 + j11.604	-1.942 + j10.511			
5	Bus 5				-1.365 + j11.604	3.814 - j17.843		-1.188 + j5.975		
6	Bus 6				-1.942 + j10.511		4.102 - j16.133			-1.282 + j5.588
7	Bus 7		-0.000 + j16.000			-1.188 + j5.975		2.805 - j35.446	-1.617 + j13.698	
8	Bus 8							-1.617 + j13.698	3.741 - j23.642	-1.155 + j9.784
9	Bus 9			-0.000 + j17.065			-1.282 + j5.588		-1.155 + j9.784	2.437 - j32.154

Bus 5 Example: Without the load $Y_{55} \; = \; 2.553 - j17.339$

$$\bar{S}_{load,5} = 1.25 + j0.5 \text{ and } \left|\bar{\mathrm{V}}_5\right| = 0.996$$

$$\mathbf{Y}_{55} = 2.553 - j17.579 + \frac{(1.25 - j0.5)}{|0.996|^2} = 3.813 - j17.843$$

Nonlinear Network Equations

- With constant impedance loads the network equations can usually be written with **I** independent of **V**, then they can be solved directly (as we've been doing)

$$\mathbf{V} = \mathbf{Y}^{-1}\,\mathbf{I}(\mathbf{x})$$

- In general this is not the case, with constant power loads one common example. Hence in general a nonlinear solution with Newton's method is used
- We'll generalize the dependence on the algebraic variables, replacing **V** by **y** since they may include other values beyond just the bus voltages

Nonlinear Network Equations

- Just like in the power flow, the complex equations are rewritten, here as a real current and a reactive current
  YV – I(x,y) = 0
- The values for bus i are

$$g_{Di}(\mathbf{x},\mathbf{y}) = \sum_{k=1}^{n}\left(G_{ik}V_{Dk} - B_{ik}V_{QK}\right) - I_{NDi} = 0$$

$$g_{Qi}(\mathbf{x},\mathbf{y}) = \sum_{k=1}^{n}\left(G_{ik}V_{Qk} + B_{ik}V_{DK}\right) - I_{NQi} = 0$$

This is a rectangular formulation; we also could have written the equations in polar form

- For each bus we add two new variables and two new equations
- If an infinite bus is modeled then its variables and equations are omitted since its voltage is fixed

Nonlinear Network Equations

- The network variables and equations are then

$$\mathbf{y} = \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ \vdots \\ V_{Dn} \\ V_{Qn} \end{bmatrix} \qquad \mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{bmatrix} \sum_{k=1}^{n} \left(G_{1k} V_{Dk} - B_{1k} V_{QK} \right) - I_{ND1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{ik} V_{Qk} + B_{ik} V_{DK} \right) - I_{NQ1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{2k} V_{Dk} - B_{2k} V_{QK} \right) - I_{ND2}(\mathbf{x},\mathbf{y}) = 0 \\ \vdots \\ \sum_{k=1}^{n} \left(G_{nk} V_{Dk} - B_{nk} V_{QK} \right) - I_{NDn}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{nk} V_{Qk} + B_{nk} V_{DK} \right) - I_{NQn}(\mathbf{x},\mathbf{y}) = 0 \end{bmatrix}$$

Nonlinear Network Equation Newton Solution

The network equations are solved using a similar procedure to that of the Netwon-Raphson power flow

$$\begin{aligned}
&\text{Set } v = 0\text{; make an initial guess of } \mathbf{y},\ \mathbf{y}^{(v)} \\
&\text{While } \left\| \mathbf{g}(\mathbf{y}^{(v)}) \right\| > \varepsilon \text{ Do} \\
&\qquad \mathbf{y}^{(v+1)} = \mathbf{y}^{(v)} - \mathbf{J}(\mathbf{y}^{(v)})^{-1}\mathbf{g}(\mathbf{y}^{(v)}) \\
&\qquad v = v+1 \\
&\text{End While}
\end{aligned}$$

Network Equation Jacobian Matrix

- The most computationally intensive part of the algorithm is determining and factoring the Jacobian matrix, **J**(**y**)

$$\mathbf{J}(\mathbf{y}) = \begin{bmatrix} \frac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \frac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \frac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \\ \frac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \frac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \frac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \frac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \frac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \end{bmatrix}$$

Network Jacobian Matrix

- The Jacobian matrix can be stored and computed using a 2 by 2 block matrix structure
- The portion of the 2 by 2 entries just from the $\mathbf{Y}_{\text{bus}}$ are

$$\begin{bmatrix} \dfrac{\partial \hat{g}_{Di}(\mathbf{x},\mathbf{y})}{\partial V_{Dj}} & \dfrac{\partial \hat{g}_{Di}(\mathbf{x},\mathbf{y})}{\partial V_{Qj}} \\ \dfrac{\partial \hat{g}_{Qi}(\mathbf{x},\mathbf{y})}{\partial V_{Dj}} & \dfrac{\partial \hat{g}_{Qi}(\mathbf{x},\mathbf{y})}{\partial V_{Qj}} \end{bmatrix} = \begin{bmatrix} G_{ij} & -B_{ij} \\ B_{ij} & G_{ij} \end{bmatrix}$$

The "hat" was added to the g functions to indicate it is just the portion from the $\mathbf{Y}_{\text{bus}}$

- The major source of the current vector voltage sensitivity comes from non-constant impedance loads; also dc transmission lines

Example: Constant Current and Constant Power Load

- As an example, assume the load at bus k is represented with a ZIP model

$$P_{Load,k} = P_{BaseLoad,k}\left(P_{z,k}\left|\bar{V}_k^2\right| + P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(Q_{z,k}\left|\bar{V}_k^2\right| + Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right)$$

The base load values are set from the power flow

- Constant impedance could be in the $\mathbf{Y}_{\text{bus}}$

$$\hat{P}_{Load,k} = P_{BaseLoad,k}\left(P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right) = \left(P_{BL,i,k}\left|\bar{V}_k\right| + P_{BL,p,k}\right)$$

$$\hat{Q}_{Load,k} = Q_{BaseLoad,k}\left(Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right) = \left(Q_{BL,i,k}\left|\bar{V}_k\right| + Q_{BL,p,k}\right)$$

- Usually solved in per unit on network MVA base

Example: Constant Current and Constant Power Load

- The current is then

$$\bar{I}_{Load,k} = I_{D,Load,k} + jI_{Q,Load,k} = \left(\frac{\hat{P}_{Load,k} + j\hat{Q}_{Load,k}}{\bar{V}_k} \right)^*$$

$$= \left(\frac{\left(P_{BL,i,k}\sqrt{V_{DK}^2 + V_{QK}^2} + P_{BL,p,k} \right) - j\left(Q_{BL,i,k}\sqrt{V_{DK}^2 + V_{QK}^2} + Q_{BL,p,k} \right)}{V_{Dk} - jV_{Qk}} \right)$$

- Multiply the numerator and denominator by $V_{DK}+jV_{QK}$ to write as the real current and the reactive current

Example: Constant Current and Constant Power Load

$$I_{D,Load,k} = \frac{V_{Dk}P_{BL,p,k} + V_{QK}Q_{BL,p,k}}{V_{DK}^2 + V_{QK}^2} + \frac{V_{Dk}P_{BL,i,k} + V_{QK}Q_{BL,i,k}}{\sqrt{V_{DK}^2 + V_{QK}^2}}$$

$$I_{Q,Load,k} = \frac{V_{Qk}P_{BL,p,k} - V_{DK}Q_{BL,p,k}}{V_{DK}^2 + V_{QK}^2} + \frac{V_{Qk}P_{BL,i,k} - V_{DK}Q_{BL,i,k}}{\sqrt{V_{DK}^2 + V_{QK}^2}}$$

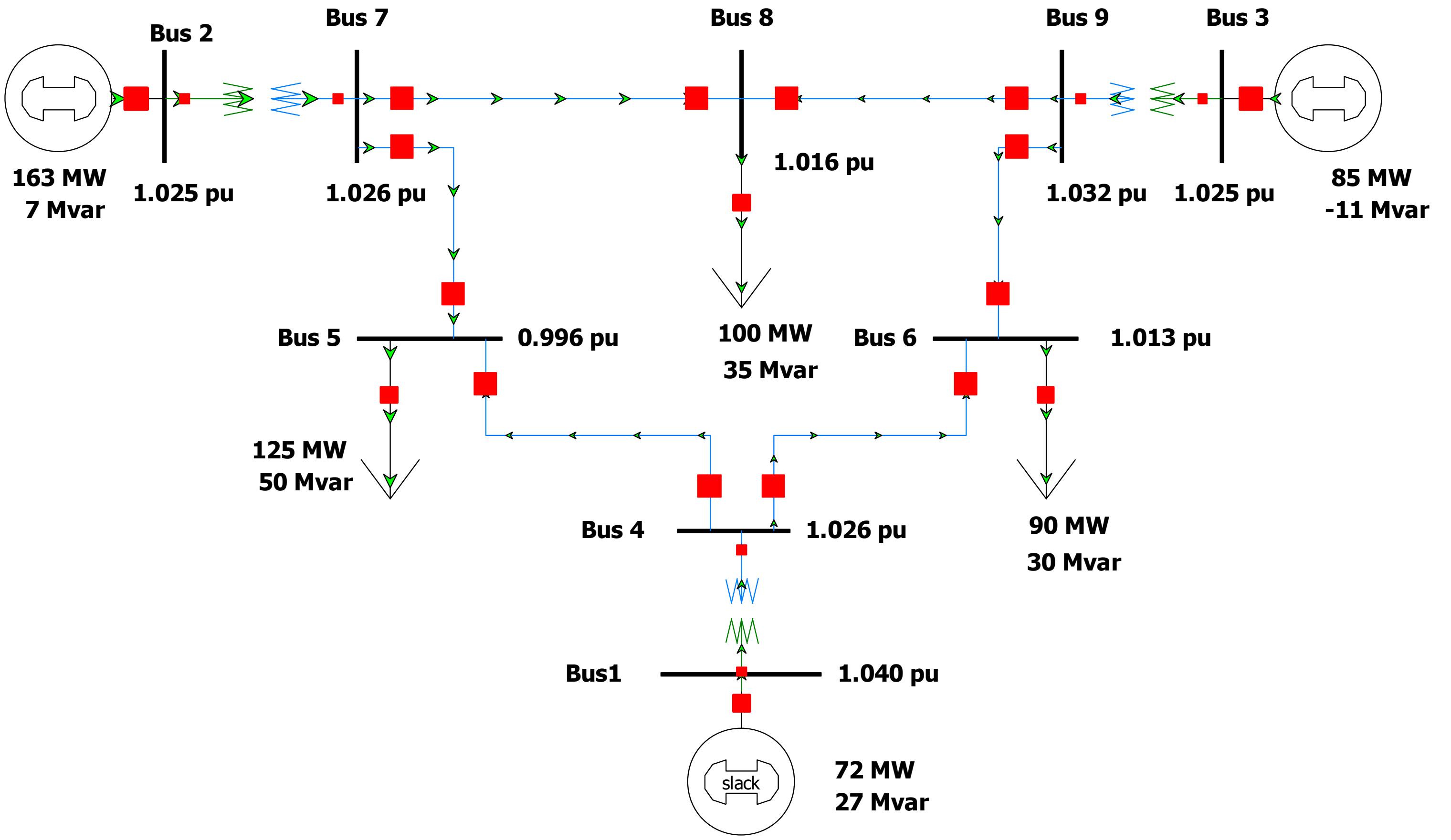
- The Jacobian entries are then found by differentiating with respect to V_{DK} and V_{QK}
  - Only affect the 2 by 2 block diagonal values
- Usually constant current and constant power models are replaced by a constant impedance model if the voltage goes too low, like during a fault

Example: 7.4 ZIP Case

- Example 7.4 is modified so the loads are represented by a model with 30% constant power, 30% constant current and 40% constant impedance
  - In PowerWorld load models can be entered in a number of different ways; a tedious but simple approach is to specify a model for each individual load
    - Right click on the load symbol to display the Load Options dialog, select Stability, and select WSCC to enter a ZIP model, in which p1&q1 are the normalized about of constant impedance load, p2&q2 the amount of constant current load, and p3&q3 the amount of constant power load

Case is **Example_7_4_ZIP**

Example 7.4 ZIP One-line



Example 7.4 ZIP Bus 8 Load Values

- As an example the values for bus 8 are given (per unit, 100 MVA base)

$$1.00 = P_{BaseLoad,8}\left(0.4\times 1.016^2 + 0.3\times 1.016 + 0.3\right)$$

$$\rightarrow P_{BaseLoad,8} = 0.983$$

$$0.35 = Q_{BaseLoad,8}\left(0.4\times 1.016^2 + 0.3\times 1.016 + 0.3\right)$$

$$\rightarrow Q_{BaseLoad,8} = 0.344$$

$$I_{D,Load,8} + jI_{Q,Load,8} = \left(\frac{1 + j0.35}{1.0158 + j0.0129}\right)^* = 0.9887 - j0.332$$

Example: 7.4 ZIP Case Jacobian

- For this case the 2 by 2 block between buses 8 and 7 is

$$\begin{bmatrix} -1.155 & 9.784 \\ -9.784 & -1.155 \end{bmatrix}$$

This is referencing slides 6 and 9

- And between 8 and 9 is

$$\begin{bmatrix} -1.617 & 13.698 \\ -13.698 & -1.617 \end{bmatrix}$$

These entries are easily checked with the $\mathbf{Y}_{bus}$

- The 2 by 2 block for the bus 8 diagonal is

$$\begin{bmatrix} 2.876 & -23.352 \\ 23.632 & 3.745 \end{bmatrix}$$

The check here is left for the student

Additional Comments

- When coding Jacobian values, a good way to check that the entries are correct is to make sure that for a small perturbation about the solution the Newton's method has quadratic convergence
- When running the simulation the Jacobian is actually seldom rebuilt and refactored
  - If the Jacobian is not too bad it will still converge
- To converge Newton's method needs a good initial guess, which is usually the last time step solution
  - Convergence can be an issue following large system disturbances, such as a fault

Explicit Method Long-Term Solutions

- The explicit method can be used for long-term solutions
  - For example in PowerWorld DS we've done solutions of large systems for many hours
- Numerical errors do not tend to build-up because of the need to satisfy the algebraic equations
- However, sometimes models have default parameter values that cause unexpected behavior when run over longer periods of time (such as default trips after 99 seconds below 0.1 Hz).
- Some models have slow unstable modes

Simultaneous Implicit

- The other major solution approach is the simultaneous implicit in which the algebraic and differential equations are solved simultaneously
- This method has the advantage of being numerically stable

Simultaneous Implicit

- Recalling an initial lecture, we covered two common implicit integration approaches for solving $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
  - Backward Euler $\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t \mathbf{f}\left(\mathbf{x}(t+\Delta t)\right)$

    For a linear system we have

    $$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\, \mathbf{A}\right]^{-1} \mathbf{x}(t)$$

  - Trapezoidal $\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \dfrac{\Delta t}{2}\left[\mathbf{f}\left(\mathbf{x}(t)\right) + \mathbf{f}\left(\mathbf{x}(t+\Delta t)\right)\right]$

    For a linear system we have

    $$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\, \mathbf{A}\right]^{-1} \left[I + \frac{\Delta t}{2}\mathbf{A}\right] \mathbf{x}(t)$$

- We'll just consider trapezoidal, but for nonlinear cases

Nonlinear Trapezoidal

- We can use Newton's method to solve $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ with the trapezoidal

$$-\mathbf{x}(t+\Delta t)+\mathbf{x}(t)+\frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)\right)+\mathbf{f}\left(\mathbf{x}(t)\right)\right)=\mathbf{0}$$

- We are solving for **x**(t+Δt); **x**(t) is known
- The Jacobian matrix is

$$\mathbf{J}\left(\mathbf{x}(t+\Delta t)\right)=\frac{\Delta t}{2}\begin{bmatrix}\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n}\\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n}\end{bmatrix}-\mathbf{I}$$

Right now we are just considering the differential equations; we'll introduce the algebraic equations shortly

The –**I** comes from differentiating -**x**(t+Δt)

Nonlinear Trapezoidal using Newton's Method

- The full solution would be at each time step
  - Set the initial guess for $\mathbf{x}(t+\Delta t)$ as $\mathbf{x}(t)$, and initialize the iteration counter k = 0
  - Determine the mismatch at each iteration k as

$$\mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) \triangleq -\mathbf{x}(t+\Delta t)^{(k)} + \mathbf{x}(t) + \frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) + \mathbf{f}\left(\mathbf{x}(t)\right)\right)$$

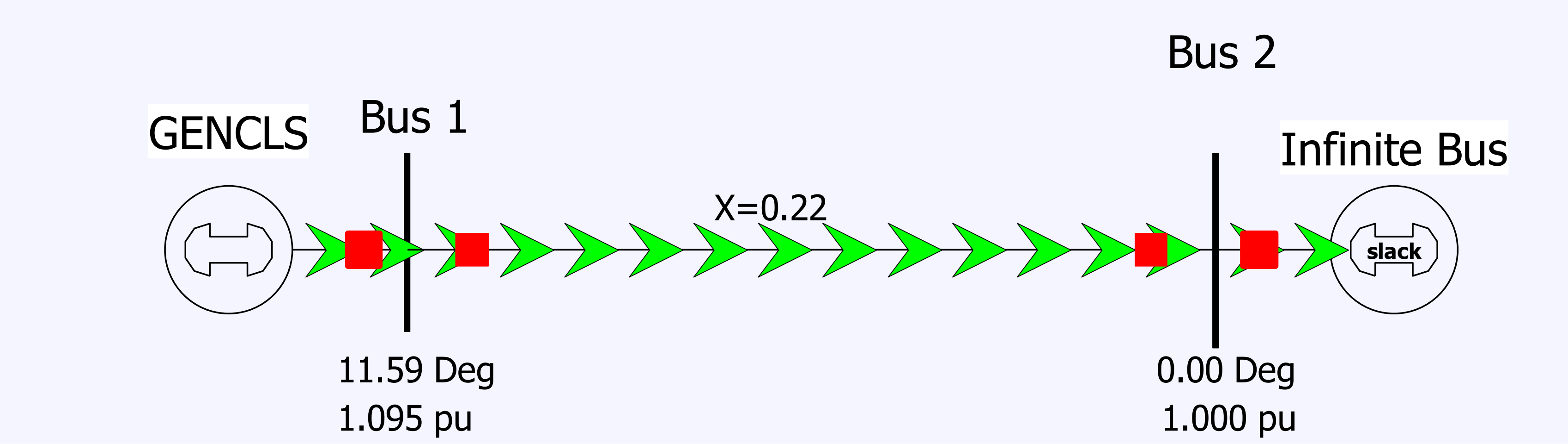
  - Determine the Jacobian matrix
  - Solve

$$\mathbf{x}(t+\Delta t)^{(k+1)} = \mathbf{x}(t+\Delta t)^{(k)} - \left[\mathbf{J}(\mathbf{x}(t+\Delta t)^{(k)}\right]^{-1} \mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right)$$

  - Iterate until done

Infinite Bus GENCLS Example

- Use the previous two bus system with gen 4 again modeled with a classical model with X_d'=0.3, H=3 and D=0



In this example X_{th} = (0.22 + 0.3), with the internal voltage $\bar{E}'_1 = 1.281\angle 23.95°$ giving E'_1=1.281 and δ_1= 23.95°

Infinite Bus GENCLS Implicit Solution

- Assume a solid three phase fault is applied at the bus 1 generator terminal, reducing P_{E1} to zero during the fault, and then the fault is self-cleared at time T^{clear}, resulting in the post-fault system being identical to the pre-fault system
  - During the fault-on time the equations reduce to

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times 3}\left(1-0\right)$$

That is, with a solid fault on the terminal of the generator, during the fault $P_{E1} = 0$

Infinite Bus GENCLS Implicit Solution

- The initial conditions are

$$\mathbf{x}(0) = \begin{bmatrix} \delta(0) \\ \omega_{pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

- Let $\Delta t = 0.02$ seconds
- During the fault the Jacobian is

$$\mathbf{J}\left(\mathbf{x}(t+\Delta t)\right) = \frac{0.02}{2}\begin{bmatrix} 0 & \omega_s \\ 0 & 0 \end{bmatrix} - \mathbf{I} = \begin{bmatrix} -1 & 3.77 \\ 0 & -1 \end{bmatrix}$$

- Set the initial guess for **x**(0.02) as **x**(0), and

$$\mathbf{f}\left(\mathbf{x}(0)\right) = \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- Then calculate the initial mismatch

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(0)}\right) \square -\mathbf{x}(0.02)^{(0)} + \mathbf{x}(0) + \frac{0.02}{2}\left(\mathbf{f}\left(\mathbf{x}(0.02)^{(0)}\right) + \mathbf{f}\left(\mathbf{x}(0)\right)\right)$$

- With $\mathbf{x}(0.02)^{(0)} = \mathbf{x}(0)$ this becomes

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(0)}\right) = -\begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{0.02}{2}\left(\begin{bmatrix} 0 \\ 0.167 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.167 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0.00334 \end{bmatrix}$$

- Then

$$\mathbf{x}(0.02)^{(1)} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 & 3.77 \\ 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0.00334 \end{bmatrix} = \begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- Repeating for the next iteration

$$\mathbf{f}\left(\mathbf{x}\left(0.02\right)^{(1)}\right)=\begin{bmatrix}1.259\\0.1667\end{bmatrix}$$

$$\mathbf{h}\left(\mathbf{x}(0.02)^{(1)}\right)=-\begin{bmatrix}0.4306\\0.00334\end{bmatrix}+\begin{bmatrix}0.418\\0\end{bmatrix}+\frac{0.02}{2}\left(\begin{bmatrix}1.259\\0.167\end{bmatrix}+\begin{bmatrix}0\\0.167\end{bmatrix}\right)$$

$$=\begin{bmatrix}0.0\\0.0\end{bmatrix}$$

- Hence we have converged with $\mathbf{x}(0.02)=\begin{bmatrix}0.4306\\0.00334\end{bmatrix}$

Infinite Bus GENCLS Implicit Solution

- Iteration continues until t = T^{clear}, assumed to be 0.1 seconds in this example

$$\mathbf{x}(0.10) = \begin{bmatrix} 0.7321 \\ 0.0167 \end{bmatrix}$$

- At this point, when the fault is self-cleared, the equations change, requiring a re-evaluation of $\mathbf{f}(\mathbf{x}(T^{clear}))$

$$\frac{d\delta}{dt} = \Delta\omega_{pu}\omega_s$$

$$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{6}\left(1 - \frac{1.281}{0.52}\sin\delta\right)$$

$$\mathbf{f}\left(\mathbf{x}\left(0.1^{+}\right)\right) = \begin{bmatrix} 6.30 \\ -0.1078 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution

- With the change in **f**(**x**) the Jacobian also changes

$$\mathbf{J}\left(\mathbf{x}(0.12^{(0)})\right) = \frac{0.02}{2}\begin{bmatrix} 0 & \omega_s \\ -0.305 & 0 \end{bmatrix} - \mathbf{I} = \begin{bmatrix} -1 & 3.77 \\ -0.00305 & -1 \end{bmatrix}$$

- Iteration for **x**(0.12) is as before, except using the new function and the new Jacobian

$$\mathbf{h}\left(\mathbf{x}(0.12)^{(0)}\right) \triangleq -\mathbf{x}(0.12)^{(0)} + \mathbf{x}(0.01) + \frac{0.02}{2}\left(\mathbf{f}\left(\mathbf{x}(0.12)^{(0)}\right) + \mathbf{f}\left(\mathbf{x}(0.10^{+})\right)\right)$$

$$\mathbf{x}(0.12)^{(1)} = \begin{bmatrix} 0.7321 \\ 0.0167 \end{bmatrix} - \begin{bmatrix} -1 & 3.77 \\ -0.00305 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0.1257 \\ -0.00216 \end{bmatrix} = \begin{bmatrix} 0.848 \\ 0.0142 \end{bmatrix}$$

This also converges quickly, with one or two iterations

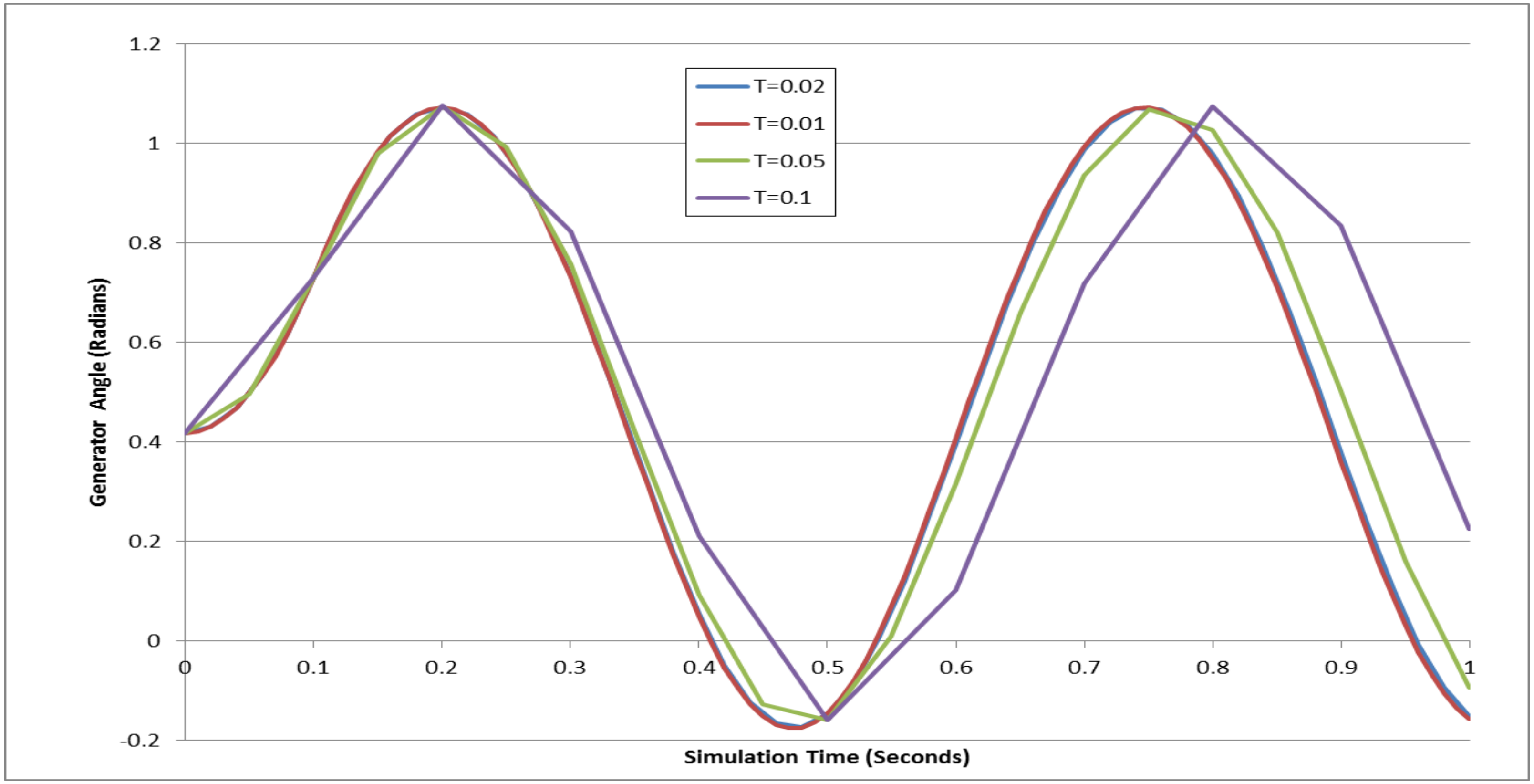
Computational Considerations

- As presented for a large system most of the computation is associated with updating and factoring the Jacobian. But the Jacobian actually changes little and hence seldom needs to be rebuilt/factored
- Rather than using **x**(t) as the initial guess for **x**(t+Δt), prediction can be used when previous values are available

$$\mathbf{x}(t+\Delta t)^{(0)} = \mathbf{x}(t) + \left(\mathbf{x}(t) - \mathbf{x}(t-\Delta t)\right)$$

Two Bus System Results

- The below graph shows the generator angle for varying values of Δt; recall the implicit method is numerically stable



Adding the Algebraic Constraints

- Since the classical model can be formulated with all the values on the network reference frame, initially we just need to add the network equations
- We'll again formulate the network equations using the form

$$\mathbf{I}(\mathbf{x},\mathbf{y}) = \mathbf{Y}\,\mathbf{V} \quad \text{or} \quad \mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x},\mathbf{y}) = \mathbf{0}$$

- As before the complex equations will be expressed using two real equations, with voltages and currents expressed in rectangular coordinates

Adding the Algebraic Constraints

- The network equations are as before

$$\mathbf{y} = \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ \vdots \\ V_{Dn} \\ V_{Qn} \end{bmatrix} \qquad \mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{bmatrix} \sum_{k=1}^{n}\left(G_{1k}V_{Dk} - B_{1k}V_{QK}\right) - I_{ND1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n}\left(G_{ik}V_{Qk} + B_{ik}V_{DK}\right) - I_{NQ1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n}\left(G_{2k}V_{Dk} - B_{2k}V_{QK}\right) - I_{ND2}(\mathbf{x},\mathbf{y}) = 0 \\ \vdots \\ \sum_{k=1}^{n}\left(G_{nk}V_{Dk} - B_{nk}V_{QK}\right) - I_{NDn}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n}\left(G_{nk}V_{Qk} + B_{nk}V_{DK}\right) - I_{NQn}(\mathbf{x},\mathbf{y}) = 0 \end{bmatrix}$$

Coupling of x and y with the Classical Model

- In the simultaneous implicit method **x** and **y** are determined simultaneously; hence in the Jacobian we need to determine the dependence of the network equations on **x**, and the state equations on **y**
- With the classical model the Norton current depends on **x** as

$$\bar{I}_{Ni} = \frac{E_i' \angle \delta_i}{R_{s,i} + jX'_{d,i}}, \quad G_i + jB_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$

$$\bar{I}_{Ni} = I_{DNi} + jI_{QNi} = E_i' \left(\cos\delta_i + j\sin\delta_i\right)\left(G_i + jB_i\right)$$

$$E_{Di} + jE_{Qi} = E_i' \left(\cos\delta_i + j\sin\delta_i\right)$$

$$I_{DNi} = E_{Di} G_i - E_{Qi} B_i$$

$$I_{QNi} = E_{Di} B_i + E_{Qi} G_i$$

Recall with the classical model E_i' is constant

Coupling of x and y with the Classical Model

- In the state equations the coupling with **y** is recognized by noting

$$\mathrm{P}_{Ei} = E_{Di} I_{Di} + E_{Qi} I_{Qi}$$

$$I_{Di} + jI_{Qi} = \left(\left(E_{Di} - V_{Di}\right) + j\left(E_{Qi} - V_{Qi}\right)\right)\left(G_i + jB_i\right)$$

$$I_{Di} = \left(E_{Di} - V_{Di}\right)G_i - \left(E_{Qi} - V_{Qi}\right)B_i$$

$$I_{Qi} = \left(E_{Di} - V_{Di}\right)B_i + \left(E_{Qi} - V_{Qi}\right)G_i$$

These are the algebraic equations

$$\mathrm{P}_{Ei} = E_{Di}\left(\left(E_{Di} - V_{Di}\right)G_i - \left(E_{Qi} - V_{Qi}\right)B_i\right) + E_{Qi}\left(\left(E_{Di} - V_{Di}\right)B_i + \left(E_{Qi} - V_{Qi}\right)G_i\right)$$

$$\mathrm{P}_{Ei} = \left(E_{Di}^2 - E_{Di}V_{Di}\right)G_i + \left(E_{Qi}^2 - E_{Qi}V_{Qi}\right)G_i + \left(E_{Di}V_{Qi} - E_{Qi}V_{Di}\right)B_i$$

Hence we have P_{Ei} written in terms of the voltages (**y**)

Variables and Mismatch Equations

- In solving the Newton algorithm the variables now include **x** and **y** (recalling that here **y** is just the vector of the real and imaginary bus voltages
- The mismatch equations now include the state integration equations

$$\mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)}\right) =$$

$$-\mathbf{x}(t+\Delta t)^{(k)} + \mathbf{x}(t) + \frac{\Delta t}{2}\left(\mathbf{f}\left(\mathbf{x}(t+\Delta t)^{(k)}, \mathbf{y}(t+\Delta t)^{(k)}\right) + \mathbf{f}\left(\mathbf{x}(t), \mathbf{y}(t)\right)\right)$$

- And the algebraic equations

$$\mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)}, \mathbf{y}(t+\Delta t)^{(k)}\right)$$

Jacobian Matrix

- Since the **h**(**x**,**y**) and **g**(**x**,**y**) are coupled, the Jacobian is

$$J\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)$$

$$=\begin{bmatrix} \dfrac{\partial \mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{x}} & \dfrac{\partial \mathbf{h}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{y}} \\ \dfrac{\partial \mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{x}} & \dfrac{\partial \mathbf{g}\left(\mathbf{x}(t+\Delta t)^{(k)},\mathbf{y}(t+\Delta t)^{(k)}\right)}{\partial \mathbf{y}} \end{bmatrix}$$

  – With the classical model the coupling is the Norton current at bus i depends on δ_i (i.e., **x**) and the electrical power (P_{Ei}) in the swing equation depends on V_{Di} and V_{Qi} (i.e., **y**)

Jacobian Matrix Entries

- The dependence of the Norton current injections on δ is

$$I_{DNi} = E_i' \cos \delta_i G_i - E_i' \sin \delta_i B_i$$

$$I_{QNi} = E_i' \cos \delta_i B_i + E_i' \sin \delta_i G_i$$

$$\frac{\partial I_{DNi}}{\partial \delta_i} = -E_i' \sin \delta_i G_i - E_i' \cos \delta_i B_i$$

$$\frac{\partial I_{QNi}}{\partial \delta_i} = -E_i' \sin \delta_i B_i + E_i' \cos \delta_i G_i$$

  - In the Jacobian the sign is flipped because we defined

$$\mathbf{g}(\mathbf{x}, \mathbf{y}) = \mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x}, \mathbf{y})$$

Jacobian Matrix Entries

- The dependence of the swing equation on the generator terminal voltage is

$$\dot{\delta}_i = \Delta\omega_{i.pu}\omega_s$$

$$\Delta\dot{\omega}_{i,pu} = \frac{1}{2H_i}\left(P_{Mi} - P_{Ei} - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

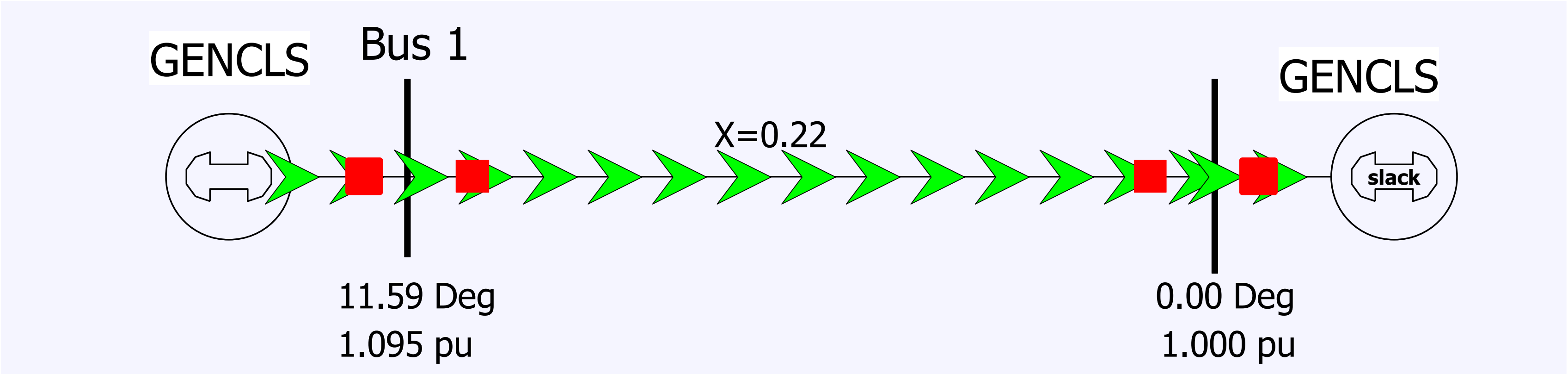
$$\mathrm{P}_{Ei} = \left(E_{Di}^2 - E_{Di}V_{Di}\right)G_i + \left(E_{Qi}^2 - E_{Qi}V_{Qi}\right)G_i + \left(E_{Di}V_{Qi} - E_{Qi}V_{Di}\right)B_i$$

$$\frac{\partial\Delta\dot{\omega}_{i,pu}}{\partial V_{Di}} = \frac{1}{2H_i}\left(E_{Di}G_i + E_{Qi}B_i\right)$$

$$\frac{\partial\Delta\dot{\omega}_{i,pu}}{\partial V_{Qi}} = \frac{1}{2H_i}\left(E_{Qi}G_i - E_{Di}B_i\right)$$

Two Bus, Two Gen GENCLS Example

- We'll reconsider the two bus, two generator case from the previous lecture ; fault at Bus 1, cleared after 0.06 seconds
  - Initial conditions and $\mathbf{Y}_{bus}$ are as covered in Lecture 16



PowerWorld Case **B2_CLS_2Gen**

Two Bus, Two Gen GENCLS Example

- Initial terminal voltages are

$$V_{D1} + jV_{Q1} = 1.0726 + j0.22, \quad V_{D2} + jV_{Q2} = 1.0$$

$$\bar{E}_1 = 1.281\angle 23.95°, \quad \bar{E}_2 = 0.955\angle -12.08$$

$$\bar{I}_{N1} = \frac{1.1709 + j0.52}{j0.3} = 1.733 - j3.903$$

$$\bar{I}_{N2} = \frac{0.9343 - j0.2}{j0.2} = -1 - j4.6714$$

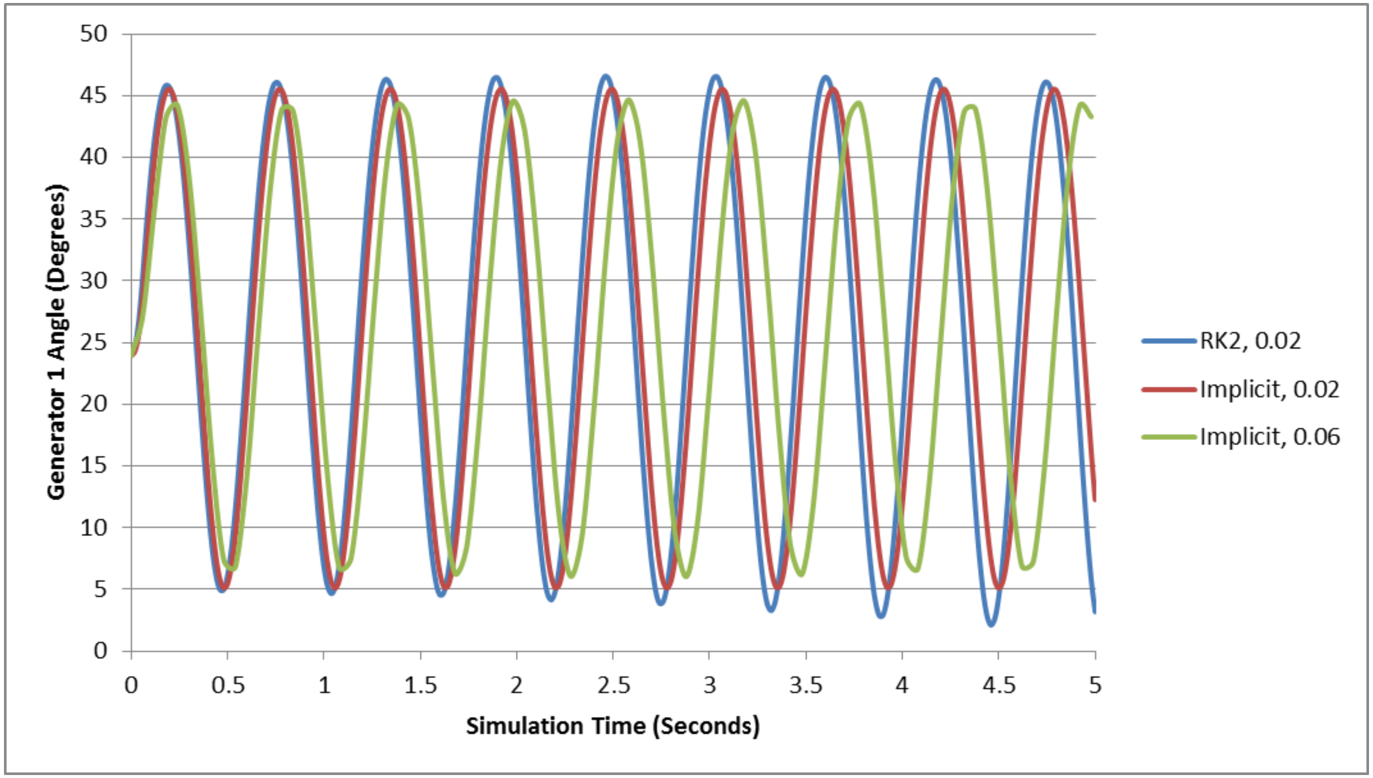
$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.333} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}$$

Two Bus, Two Gen Initial Jacobian

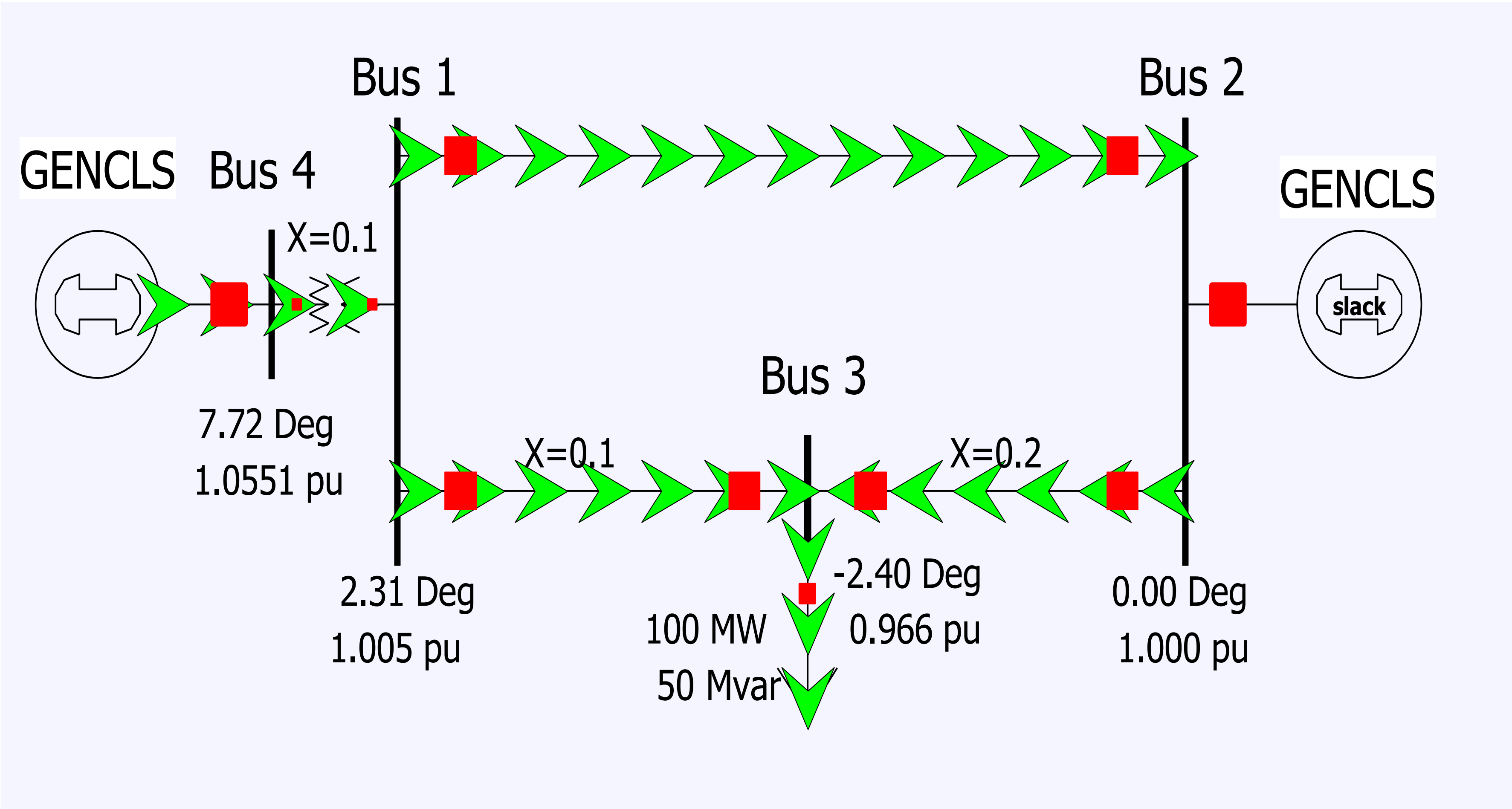
$$
\begin{array}{c|cccccccc}
 & \delta_1 & \Delta\omega_1 & \delta_2 & \Delta\omega_2 & V_{D1} & V_{Q1} & V_{D2} & V_{Q2} \\
\hline
\dot{\delta}_1 & -1 & 3.77 & 0 & 0 & 0 & 0 & 0 & 0 \\
\Delta\dot{\omega}_1 & -0.0076 & -1 & 0 & 0 & -0.0029 & 0.0065 & 0 & 0 \\
\dot{\delta}_2 & 0 & 0 & -1 & 3.77 & 0 & 0 & 0 & 0 \\
\Delta\dot{\omega}_2 & 0 & 0 & -0.0039 & -1 & 0 & 0 & 0.0008 & 0.0039 \\
I_{D1} & -3.90 & 0 & 0 & 0 & 0 & 7.879 & 0 & -4.545 \\
I_{Q1} & -1.73 & 0 & 0 & 0 & -7.879 & 0 & 4.545 & 0 \\
I_{D2} & 0 & 0 & -4.67 & 0 & 0 & -4.545 & 0 & 9.545 \\
I_{Q2} & 0 & 0 & 1.00 & 0 & 4.545 & 0 & -9.545 & 0
\end{array}
$$

Results Comparison

- The below graph compares the angle for the generator at bus 1 using Δt=0.02 between RK2 and the Implicit Trapezoidal; also Implicit with Δt=0.06



Four Bus Comparison



Four Bus Comparison

Fault at Bus 3 for 0.12 seconds; self-cleared