ECEN 667
Power System Stability

Lecture 20: SMIB Eigenvalues, Measurement-Based Modal Analysis

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Announcements

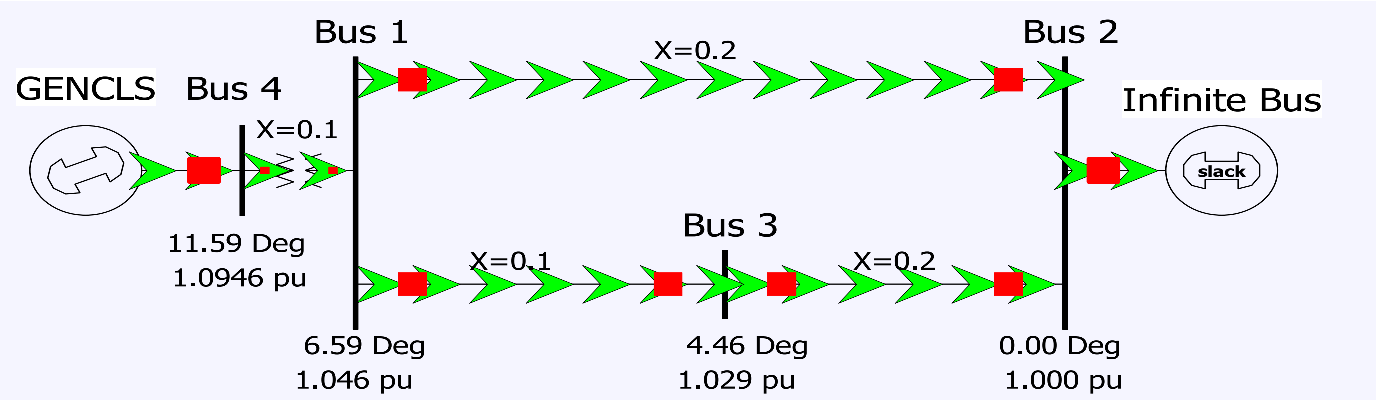
- Read Chapter 8
- Homework 6 is due on November 11
- There is a 2019 NERC document on oscillations at www.nerc.com/comm/PC/SMSResourcesDocuments/Interconnection_Oscillation_Analysis.pdf

Single Machine Infinite Bus

- A quite useful analysis technique is to consider the small signal stability associated with a single generator connected to the rest of the system through an equivalent transmission line
- Driving point impedance looking into the system is used to calculate the equivalent line's impedance
  - The Z_{ii} value can be calculated quite quickly using sparse vector methods
- Rest of the system is assumed to be an infinite bus with its voltage set to match the generator's real and reactive power injection and voltage

Small SMIB Example

- As a small example, consider the 4 bus system shown below, in which bus 2 really is an infinite bus



- To get the SMIB for bus 4, first calculate Z_{44}

$$Y_{bus} = j\begin{bmatrix} -25 & 0 & 10 & 10 \\ 0 & 1 & 0 & 0 \\ 10 & 0 & -15 & 0 \\ 10 & 0 & 0 & -13.33 \end{bmatrix} \rightarrow Z_{44} = j0.1269$$

Z_{44} is Z_{th} in parallel with $jX'_{d,4}$ (which is j0.3) so Z_{th} is j0.22

Small SMIB Example

- The infinite bus voltage is then calculated so as to match the bus i terminal voltage and current

$$\bar{V}_{\text{inf}} = \bar{V}_i - Z_i \bar{I}_i$$

$$\text{where } \left(\frac{P_i + jQ_i}{\bar{V}_i} \right)^* = \bar{I}_i$$

While this was demonstrated on an extremely small system for clarity, the approach works the same for any size system

- In the example we have

$$\left(\frac{P_4 + jQ_4}{\bar{V}_4} \right)^* = \left(\frac{1 + j0.572}{1.072 + j0.220} \right)^* = 1 - j0.328$$

$$\bar{V}_{\text{inf}} = (1.072 + j0.220) - (j0.22)(1 - j0.328)$$

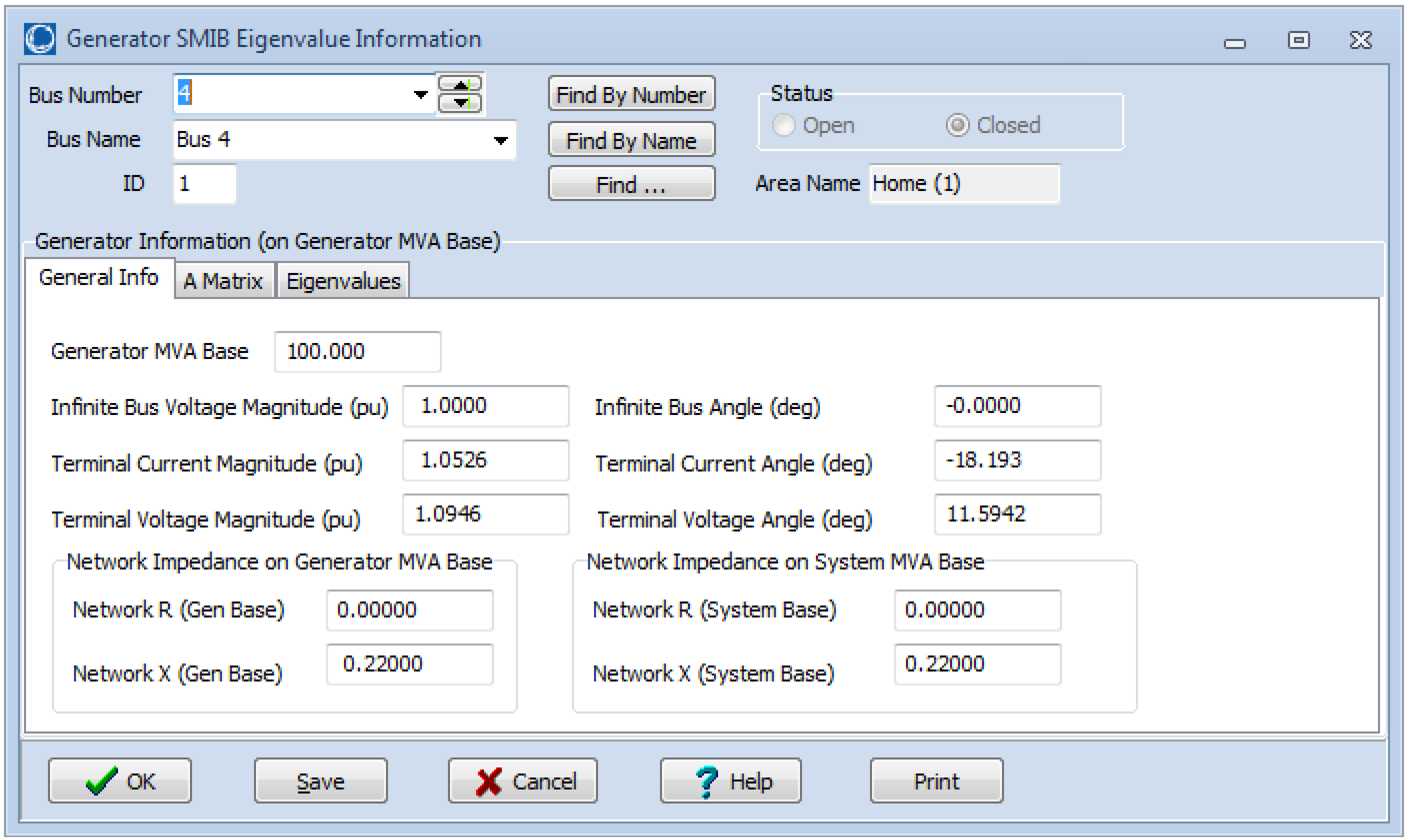
$$\bar{V}_{\text{inf}} = 1.0$$

Calculating the A Matrix

- The SMIB model **A** matrix can then be calculated either analytically or numerically
  - The equivalent line's impedance can be embedded in the generator model so the infinite bus looks like the "terminal"
- This matrix is calculated in PowerWorld by selecting Transient Stability, SMIB Eigenvalues
  - Select Run SMIB to perform an SMIB analysis for all the generators in a case
  - Right click on a generator on the SMIB form and select Show SMIB to see the Generator SMIB Eigenvalue Dialog
  - These two bus equivalent networks can also be saved, which can be quite useful for understanding the behavior of individual generators

Example: Bus 4 SMIB Dialog

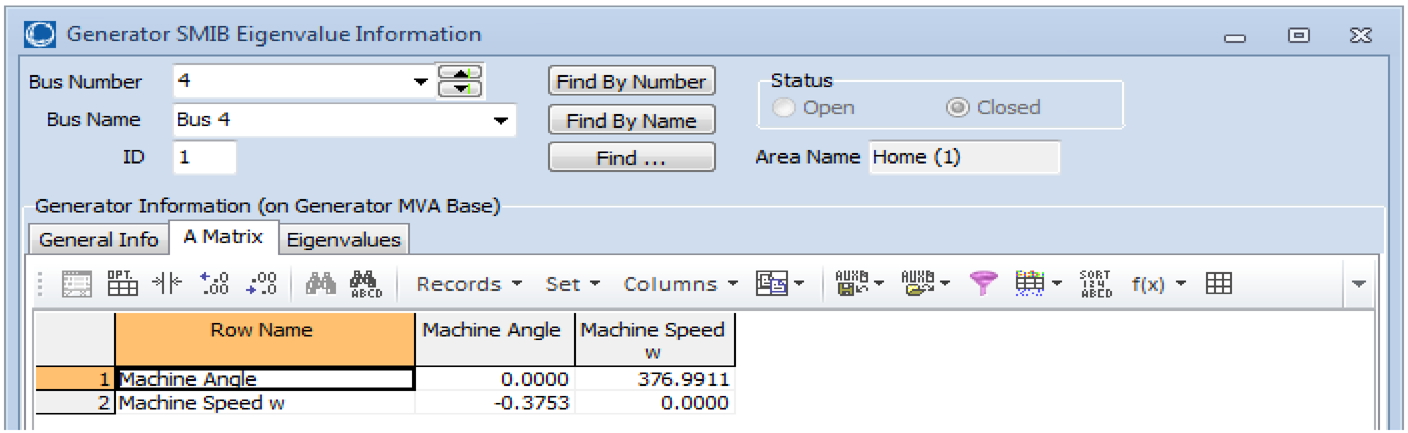
- On the SMIB dialog, the General Information tab shows information about the two bus equivalent



PowerWorld case **B4_SMIB**

Example: Bus 4 SMIB Dialog

- On the SMIB dialog, the **A** Matrix tab shows the $\mathbf{A}_{sys}$ matrix for the SMIB generator



- In this example A_{21} is showing

$$\frac{\partial \Delta\omega_{4,pu}}{\partial \delta_4} = \frac{1}{2H_4}\left(\frac{-\partial P_{E,4}}{\partial \delta_4}\right) = -\left(\frac{1}{6}\right)\left(\left(\frac{-1}{0.3+0.22}\right)\left(-1.2812\cos\left(23.94^\circ\right)\right)\right)$$

$$= -0.3753$$

Example: Bus 4 with GENROU

- The eigenvalues can be calculated for any set of generator models
- This example replaces the bus 4 generator classical machine with a GENROU model
  - There are now six eigenvalues, with the dominate response coming from the electro-mechanical mode with a frequency of 1.84 Hz, and damping of 6.9%

Generator Information (on Generator MVA Base)

General Info | A Matrix | Eigenvalues

Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)
1	-21.2472	0.0000	21.2472	1.0000	0.0000		3.3816
2	-0.8040	11.5563	11.5842	0.0694	1.8392	0.5437	1.8437
3	-0.8040	-11.5563	11.5842	0.0694	-1.8392	-0.5437	1.8437
4	-14.2256	0.0000	14.2256	1.0000	0.0000		2.2641
5	-3.7087	0.0000	3.7087	1.0000	0.0000		0.5903
6	-0.4248	0.0000	0.4248	1.0000	0.0000		0.0676

PowerWorld case **B4_GENROU_Sat_SMIB**

Example: Bus 4 with GENROU Model and Exciter

- Adding an relatively slow EXST1 exciter adds additional states (with K_A=200, T_A=0.2)
  - As the initial reactive power output of the generator is decreased, the system becomes unstable (below example is with a generator reactive power output of 0 Mvar)

General Info | A Matrix | Eigenvalues

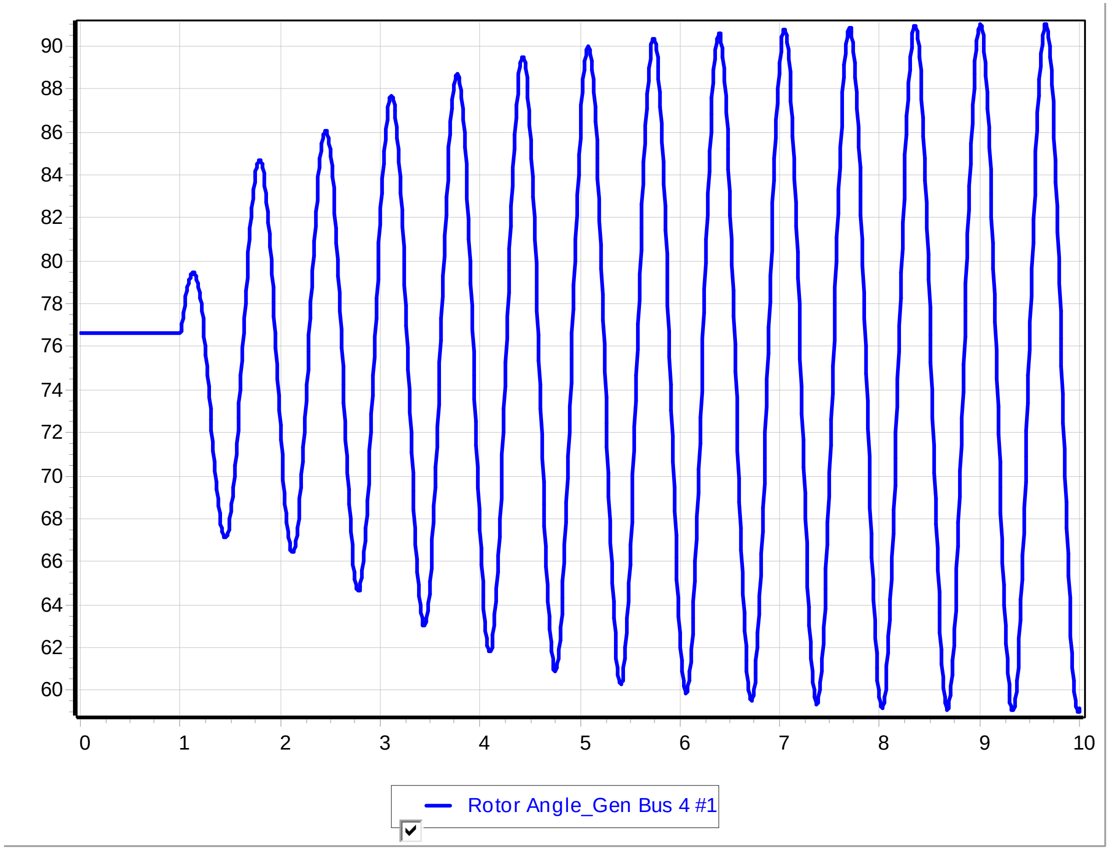
Records Set Columns Options

	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Ma
1	0.2704	-9.5336	9.5374	-0.0283	-1.5173	-0.6591	1.5179	
2	0.2704	9.5336	9.5374	-0.0283	1.5173	0.6591	1.5179	
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	
4	-3.0137	0.0000	3.0137	1.0000	0.0000		0.4796	
5	-3.6849	-6.4281	7.4094	0.4973	-1.0231	-0.9775	1.1792	
6	-3.6849	6.4281	7.4094	0.4973	1.0231	0.9775	1.1792	
7	-14.4234	0.0000	14.4234	1.0000	0.0000		2.2956	
8	-21.6978	0.0000	21.6978	1.0000	0.0000		3.4533	

PowerWorld case **B4_GENROU_Sat_SMIB_QZero**

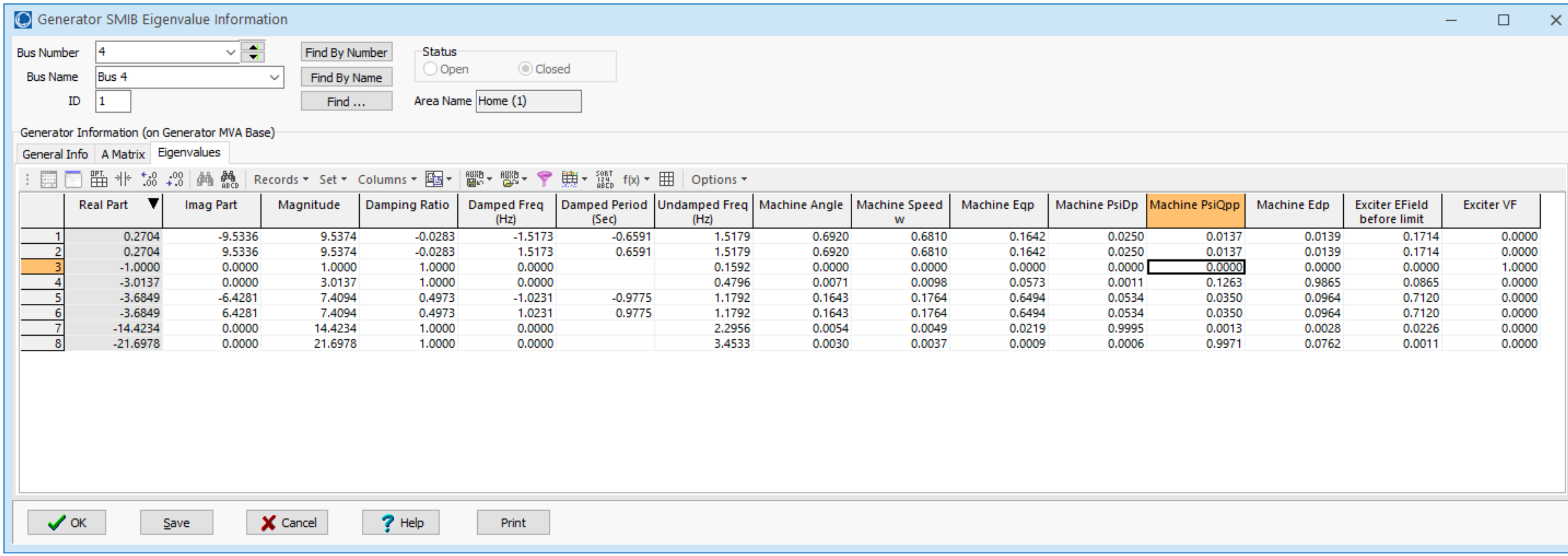
Example: Bus 4 with GENROU Model and Exciter

- The below image shows the system response to a brief bus 4 self-clearing fault



Example: Bus 4 with GENROU Model and Exciter

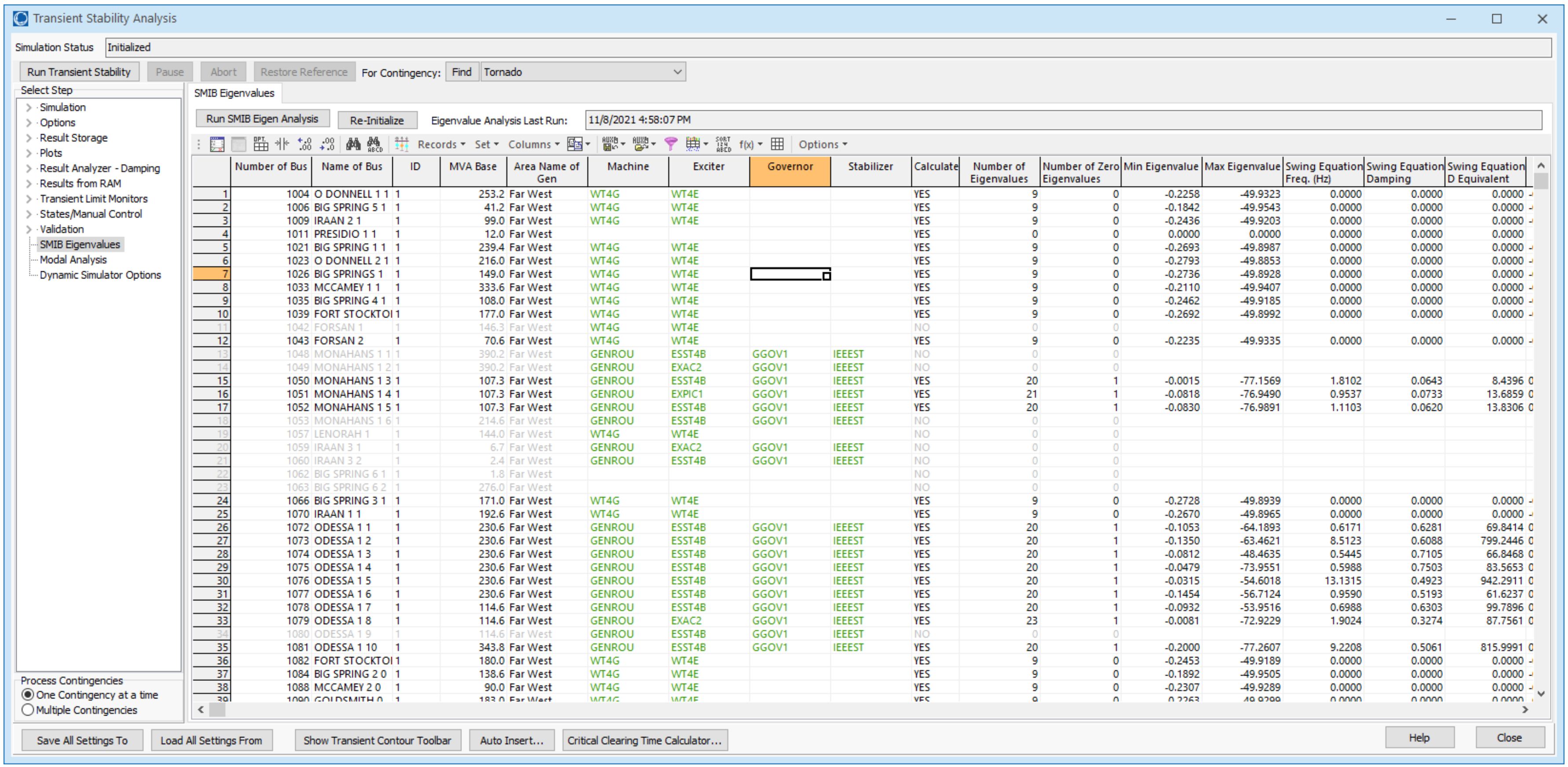
- The remainder of the Eigenvalues page shows the participation factors for the various states in the modes



	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Speed w	Machine Eqp	Machine PsiDp	Machine PsiQpp	Machine Edp	Exciter EField before limit	Exciter VF
1	0.2704	-9.5336	9.5374	-0.0283	-1.5173	-0.6591	1.5179	0.6920	0.6810	0.1642	0.0250	0.0137	0.0139	0.1714	0.0000
2	0.2704	9.5336	9.5374	-0.0283	1.5173	0.6591	1.5179	0.6920	0.6810	0.1642	0.0250	0.0137	0.0139	0.1714	0.0000
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
4	-3.0137	0.0000	3.0137	1.0000	0.0000		0.4796	0.0071	0.0098	0.0573	0.0011	0.1263	0.9865	0.0865	0.0000
5	-3.6849	-6.4281	7.4094	0.4973	-1.0231	-0.9775	1.1792	0.1643	0.1764	0.6494	0.0534	0.0350	0.0964	0.7120	0.0000
6	-3.6849	6.4281	7.4094	0.4973	1.0231	0.9775	1.1792	0.1643	0.1764	0.6494	0.0534	0.0350	0.0964	0.7120	0.0000
7	-14.4234	0.0000	14.4234	1.0000	0.0000		2.2956	0.0054	0.0049	0.0219	0.9995	0.0013	0.0028	0.0226	0.0000
8	-21.6978	0.0000	21.6978	1.0000	0.0000		3.4533	0.0030	0.0037	0.0009	0.0006	0.9971	0.0762	0.0011	0.0000

SMIB Eigenvalues for TSGC_2000 Case

- All the SMIB eigenvalues can be calculated quickly even for relatively large grids



	Number of Bus	Name of Bus	ID	MVA Base	Area Name of Gen	Machine	Exciter	Governor	Stabilizer	Calculate	Number of Eigenvalues	Number of Zero Eigenvalues	Min Eigenvalue	Max Eigenvalue	Swing Equation Freq. (Hz)	Swing Equation Damping	Swing Equation D Equivalent
1	1004	O DONNELL 1 1	1	253.2	Far West	WT4G	WT4E			YES	9	0	-0.2258	-49.9323	0.0000	0.0000	0.0000
2	1006	BIG SPRING 5 1	1	41.2	Far West	WT4G	WT4E			YES	9	0	-0.1842	-49.9543	0.0000	0.0000	0.0000
3	1009	IRAAN 2 1	1	99.0	Far West	WT4G	WT4E			YES	9	0	-0.2436	-49.9203	0.0000	0.0000	0.0000
4	1011	PRESIDIO 1 1	1	12.0	Far West					YES	0	0	0.0000	0.0000	0.0000	0.0000	0.0000
5	1021	BIG SPRING 1 1	1	239.4	Far West	WT4G	WT4E			YES	9	0	-0.2693	-49.8987	0.0000	0.0000	0.0000
6	1023	O DONNELL 2 1	1	216.0	Far West	WT4G	WT4E			YES	9	0	-0.2793	-49.8853	0.0000	0.0000	0.0000
7	1026	BIG SPRINGS 1	1	149.0	Far West	WT4G	WT4E			YES	9	0	-0.2736	-49.8928	0.0000	0.0000	0.0000
8	1033	MCCAMEY 1 1	1	333.6	Far West	WT4G	WT4E			YES	9	0	-0.2110	-49.9407	0.0000	0.0000	0.0000
9	1035	BIG SPRING 4 1	1	108.0	Far West	WT4G	WT4E			YES	9	0	-0.2462	-49.9185	0.0000	0.0000	0.0000
10	1039	FORT STOCKTOI	1	177.0	Far West	WT4G	WT4E			YES	9	0	-0.2692	-49.8992	0.0000	0.0000	0.0000
11	1042	FORSAN 1	1	146.3	Far West	WT4G	WT4E			NO	0	0					
12	1043	FORSAN 2	1	70.6	Far West	WT4G	WT4E			YES	9	0	-0.2235	-49.9335	0.0000	0.0000	0.0000
13	1048	MONAHANS 1 1	1	390.2	Far West	GENROU	ESST4B	GGOV1	IEEEST	NO	0	0					
14	1049	MONAHANS 1 2	1	390.2	Far West	GENROU	EXAC2	GGOV1	IEEEST	NO	0	0					
15	1050	MONAHANS 1 3	1	107.3	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0015	-77.1569	1.8102	0.0643	8.4396
16	1051	MONAHANS 1 4	1	107.3	Far West	GENROU	EXPIC1	GGOV1	IEEEST	YES	21	1	-0.0818	-76.9490	0.9537	0.0733	13.6859
17	1052	MONAHANS 1 5	1	107.3	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0830	-76.9891	1.1103	0.0620	13.8306
18	1053	MONAHANS 1 6	1	214.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	NO	0	0					
19	1057	LENORAH 1	1	144.0	Far West	WT4G	WT4E			NO	0	0					
20	1059	IRAAN 3 1	1	6.7	Far West	GENROU	EXAC2	GGOV1	IEEEST	NO	0	0					
21	1060	IRAAN 3 2	1	2.4	Far West	GENROU	ESST4B	GGOV1	IEEEST	NO	0	0					
22	1062	BIG SPRING 6 1	1	1.8	Far West					NO	0	0					
23	1063	BIG SPRING 6 2	1	276.0	Far West					NO	0	0					
24	1066	BIG SPRING 3 1	1	171.0	Far West	WT4G	WT4E			YES	9	0	-0.2728	-49.8939	0.0000	0.0000	0.0000
25	1070	IRAAN 1 1	1	192.6	Far West	WT4G	WT4E			YES	9	0	-0.2670	-49.8965	0.0000	0.0000	0.0000
26	1072	ODESSA 1 1	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.1053	-64.1893	0.6171	0.6281	69.8414
27	1073	ODESSA 1 2	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.1350	-63.4621	8.5123	0.6088	799.2446
28	1074	ODESSA 1 3	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0812	-48.4635	0.5445	0.7105	66.8468
29	1075	ODESSA 1 4	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0479	-73.9551	0.5988	0.7503	83.5653
30	1076	ODESSA 1 5	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0315	-54.6018	13.1315	0.4923	942.2911
31	1077	ODESSA 1 6	1	230.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.1454	-56.7124	0.9590	0.5193	61.6237
32	1078	ODESSA 1 7	1	114.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0932	-53.9516	0.6988	0.6303	99.7896
33	1079	ODESSA 1 8	1	114.6	Far West	GENROU	EXAC2	GGOV1	IEEEST	YES	23	1	-0.0081	-72.9229	1.9024	0.3274	87.7561
34	1080	ODESSA 1 9	1	114.6	Far West	GENROU	ESST4B	GGOV1	IEEEST	NO	0	0					
35	1081	ODESSA 1 10	1	343.8	Far West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.2000	-77.2607	9.2208	0.5061	815.9991
36	1082	FORT STOCKTOI	1	180.0	Far West	WT4G	WT4E			YES	9	0	-0.2453	-49.9189	0.0000	0.0000	0.0000
37	1084	BIG SPRING 2 0	1	138.6	Far West	WT4G	WT4E			YES	9	0	-0.1892	-49.9505	0.0000	0.0000	0.0000
38	1088	MCCAMEY 2 0	1	90.0	Far West	WT4G	WT4E			YES	9	0	-0.2307	-49.9289	0.0000	0.0000	0.0000
39	1090	GOLDSMITH 0	1	183.0	Far West	WT4G	WT4E			YES	9	0	[illegible]	[illegible]	0.0000	0.0000	0.0000

Saving a Two Bus Equivalent

- PowerWorld makes it easy to save a two bus equivalent from the **SMIB Eigenvalues** page
  - Right-click and select **Save Two Bus Equivalent**
- As the name implies, the two bus equivalent is the generator connected to an infinite bus through its driving point impedance
- Two bus equivalents provide a convenient way to track down at least some causes of instability issues

Small Signal Analysis and Measurement-Based Modal Analysis

- Small signal analysis has been used for decades to determine power system frequency response
  - It is a model-based approach that considers the properties of a power system, linearized about an operating point
- Measurement-based modal analysis determines the observed dynamic properties of a system
  - Input can either be measurements from devices (such as PMUs) or dynamic simulation results
  - The same approach can be used regardless of the measurement source
- Focus in this section is on the measurement-based approach

Ring-down Modal Analysis

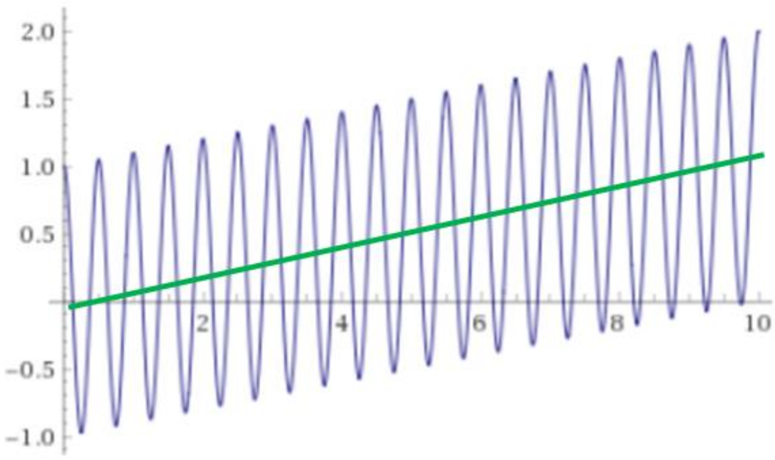
- Ring-down analysis seeks to determine the frequency and damping of key power system modes following some disturbance
- There are several different techniques, with the Prony approach the oldest (from 1795); introduced into power in 1990 by Hauer, Demeure and Scharf
- Regardless of technique, the goal is to represent the response of a sampled signal as a set of exponentially damped sinusoidals (modes)

$$y(t) = \sum_{i=1}^{q} A_i e^{\sigma_i t} \cos\left(\omega_i t + \phi_i\right) \qquad \text{Damping } (\%) = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \omega_i^2}} \times 100$$

Where We Are Going: Extracting the Modes from Signals

- The goal is to gain information about the electric grid by extracting modal information from its signals
  - The frequency and damping of the modes is key
- The premise is we'll be able to reproduce a complex signal, over a period of time, as a set a of sinusoidal modes
  - We'll also allow for linear detrending

$$0.1t + \cos(2\pi 2t)$$

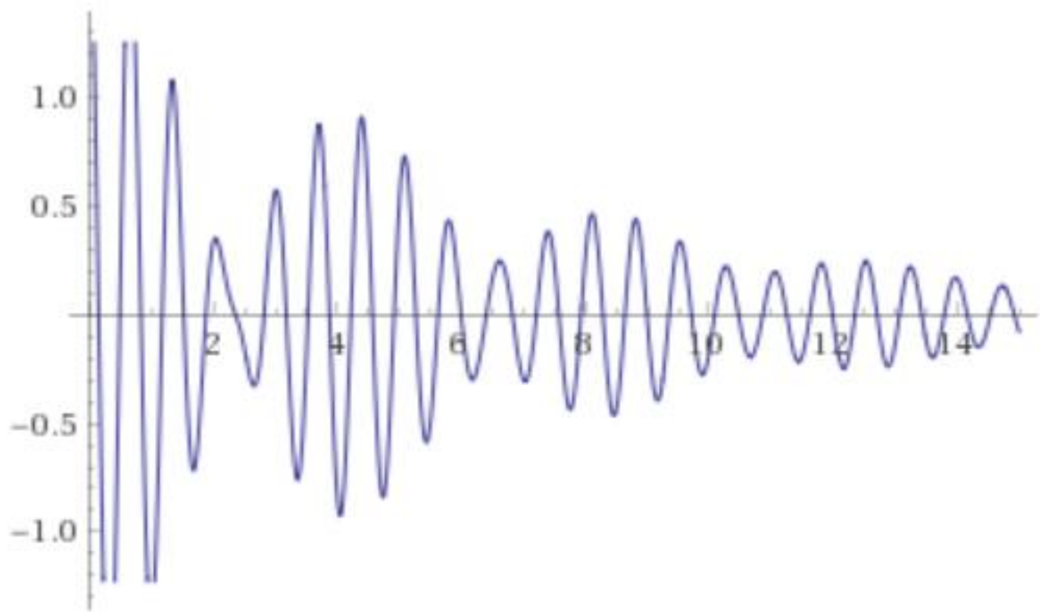


Example: The Summation of two damped exponentials

- This example was created by going from the modes to a signal
- We'll be going in the opposite direction (i.e., from a measured signal to the modes)

plot	$e^{-0.25\,x}\cos(10\,x)+e^{-0.125\,x}\cos\left(8.5\,x+\frac{\pi}{8}\right)$

Plot:

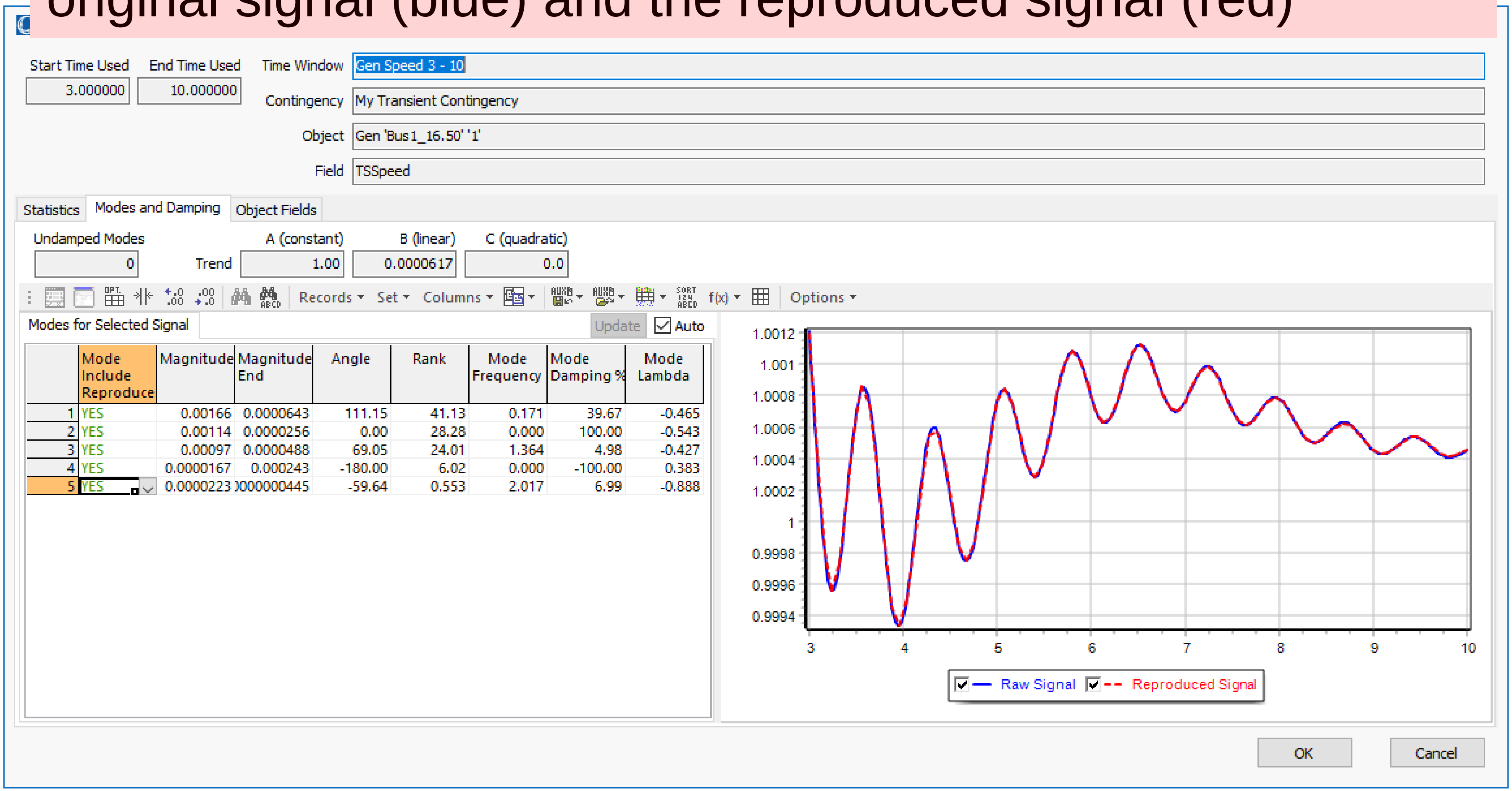


Some Reasonable Expectations

- **Verifiable** to show how well the modes match the original signal(s)
  - We'll show this
- **Flexible** to handle between one and many signals
  - We'll go up to simultaneously considering 40,000 signals
- **Fast**
  - What is presented will be, with a discussion of the computational scaling
- **Easy to use**
  - This is software implementation specific; results shown here were done using the modal analysis tool integrated into PowerWorld Simulator (version 22)

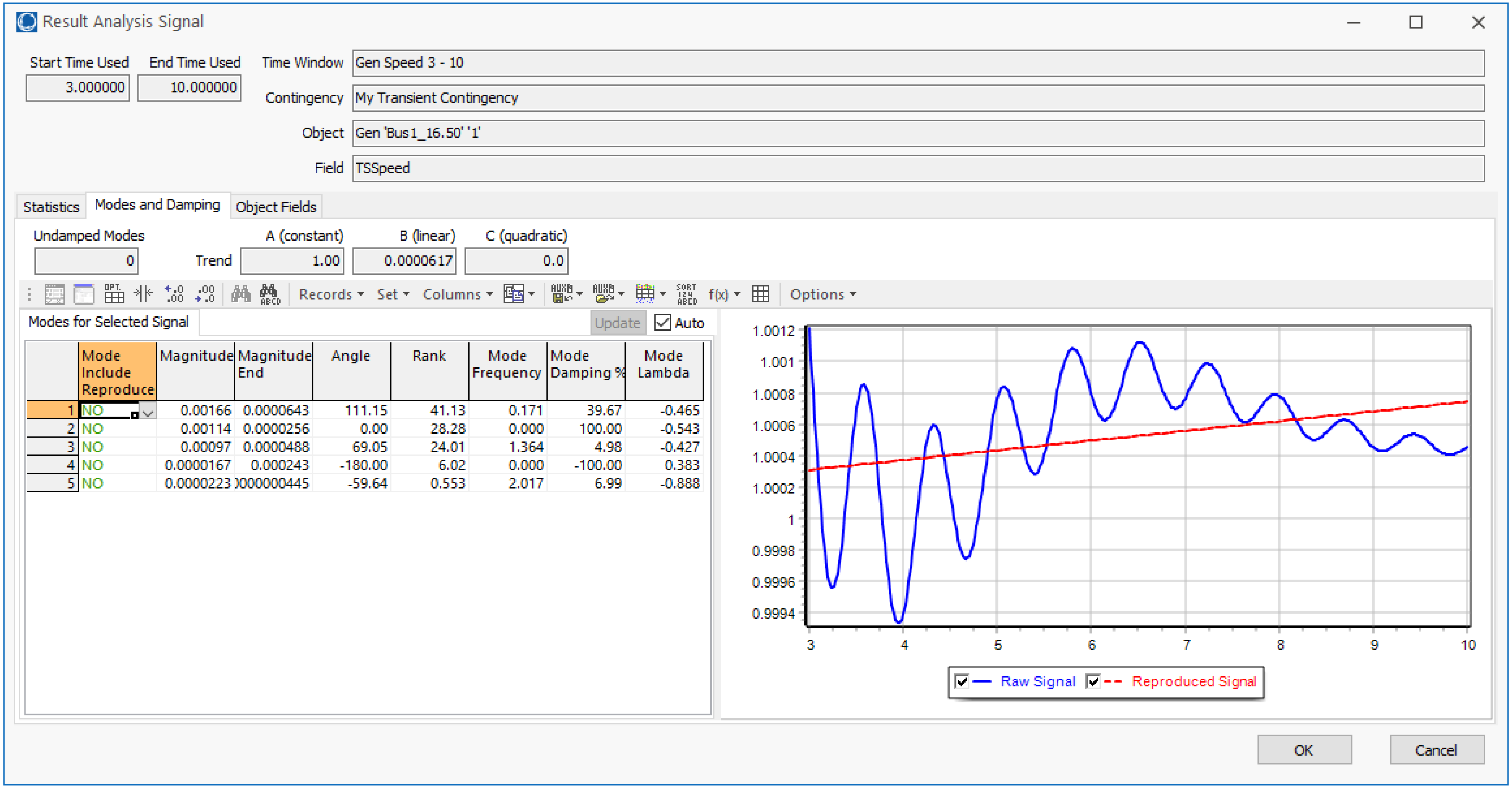
Example: One Signal

This could be any signal; image shows the result of the original signal (blue) and the reproduced signal (red)



	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	YES	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	YES	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888

Verification: Linear Trend Line Only



	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	NO	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	NO	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	NO	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	NO	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888

Verification: Linear Trend Line + One Mode

Result Analysis Signal

Start Time Used	End Time Used
3.000000	10.000000

Time Window: Gen Speed 3 - 10

Contingency: My Transient Contingency

Object: Gen 'Bus1_16.50' '1'

Field: TSSpeed

Statistics | Modes and Damping | Object Fields

Undamped Modes		A (constant)	B (linear)	C (quadratic)
0	Trend	1.00	0.0000617	0.0

Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

Modes for Selected Signal   Update   Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	NO	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
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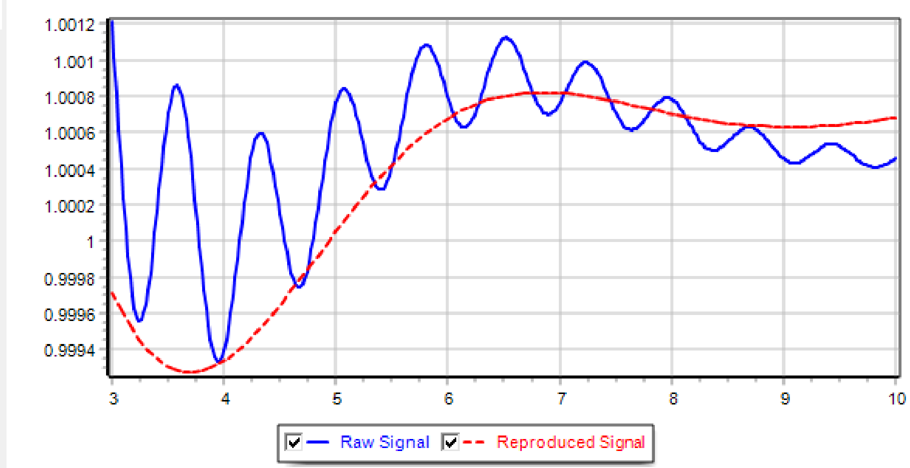

OK   Cancel

Verification:
Linear Trend Line + Two Modes

Result Analysis Signal

Start Time Used: 3.000000
End Time Used: 10.000000
Time Window: Gen Speed 3 - 10
Contingency: My Transient Contingency
Object: Gen 'Bus1_16.50' '1'
Field: TSSpeed

Statistics | Modes and Damping | Object Fields

Undamped Modes: 0
Trend: A (constant) 1.00 | B (linear) 0.0000617 | C (quadratic) 0.0

Records Set Columns f(x) Options

Modes for Selected Signal | Update | Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
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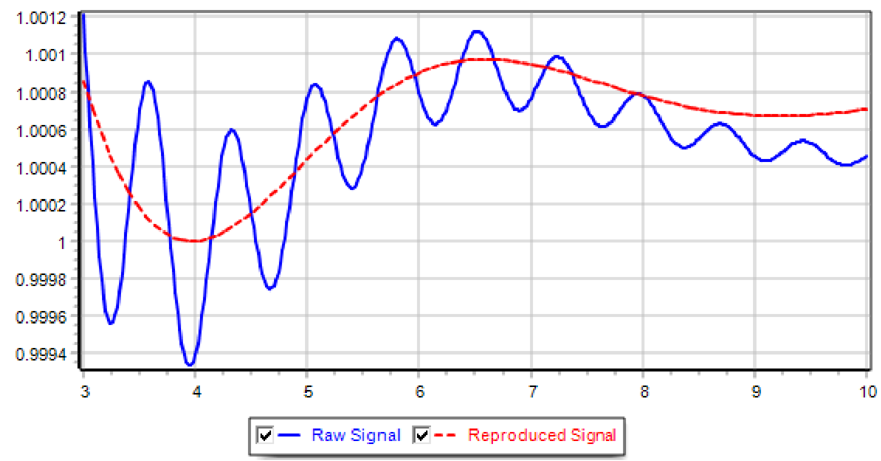


OK | Cancel

Verification: Linear Trend Line + Three Modes

Result Analysis Signal

Start Time Used	End Time Used
3.000000	10.000000

Time Window: Gen Speed 3 - 10

Contingency: My Transient Contingency

Object: Gen 'Bus1_16.50' '1'

Field: TSSpeed

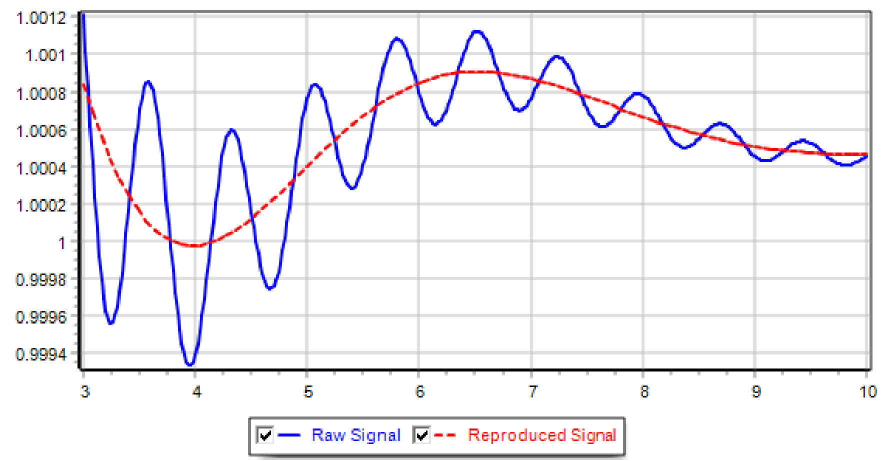
Statistics | Modes and Damping | Object Fields

Undamped Modes		A (constant)	B (linear)	C (quadratic)
0	Trend	1.00	0.0000617	0.0

Records Set Columns f(x) Options

Modes for Selected Signal — Update — Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	NO	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888



OK Cancel

Verification:
Linear Trend Line + Four Modes

Result Analysis Signal

Start Time Used	End Time Used
3.000000	10.000000

Time Window: Gen Speed 3 - 10

Contingency: My Transient Contingency

Object: Gen 'Bus1_16.50' '1'

Field: TSSpeed

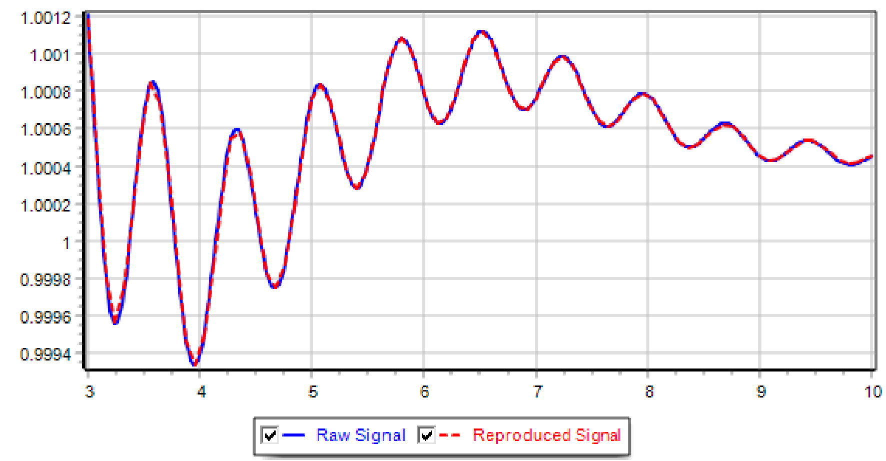
Statistics | Modes and Damping | Object Fields

Undamped Modes		A (constant)	B (linear)	C (quadratic)
0	Trend	1.00	0.0000617	0.0

Records Set Columns f(x) Options

Modes for Selected Signal | Update | Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	YES	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888



OK | Cancel

Verification: Linear Trend Line + Five Modes

Result Analysis Signal

Start Time Used	End Time Used
3.000000	10.000000

Time Window: Gen Speed 3 - 10

Contingency: My Transient Contingency

Object: Gen 'Bus1_16.50' '1'

Field: TSSpeed

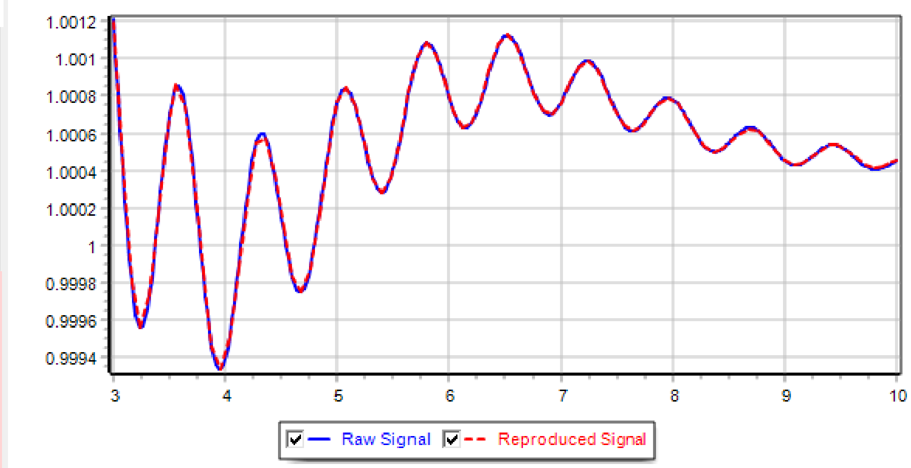
Statistics | Modes and Damping | Object Fields

Undamped Modes		A (constant)	B (linear)	C (quadratic)
0	Trend	1.00	0.0000617	0.0

Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

Modes for Selected Signal — Update — Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	YES	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	YES	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888

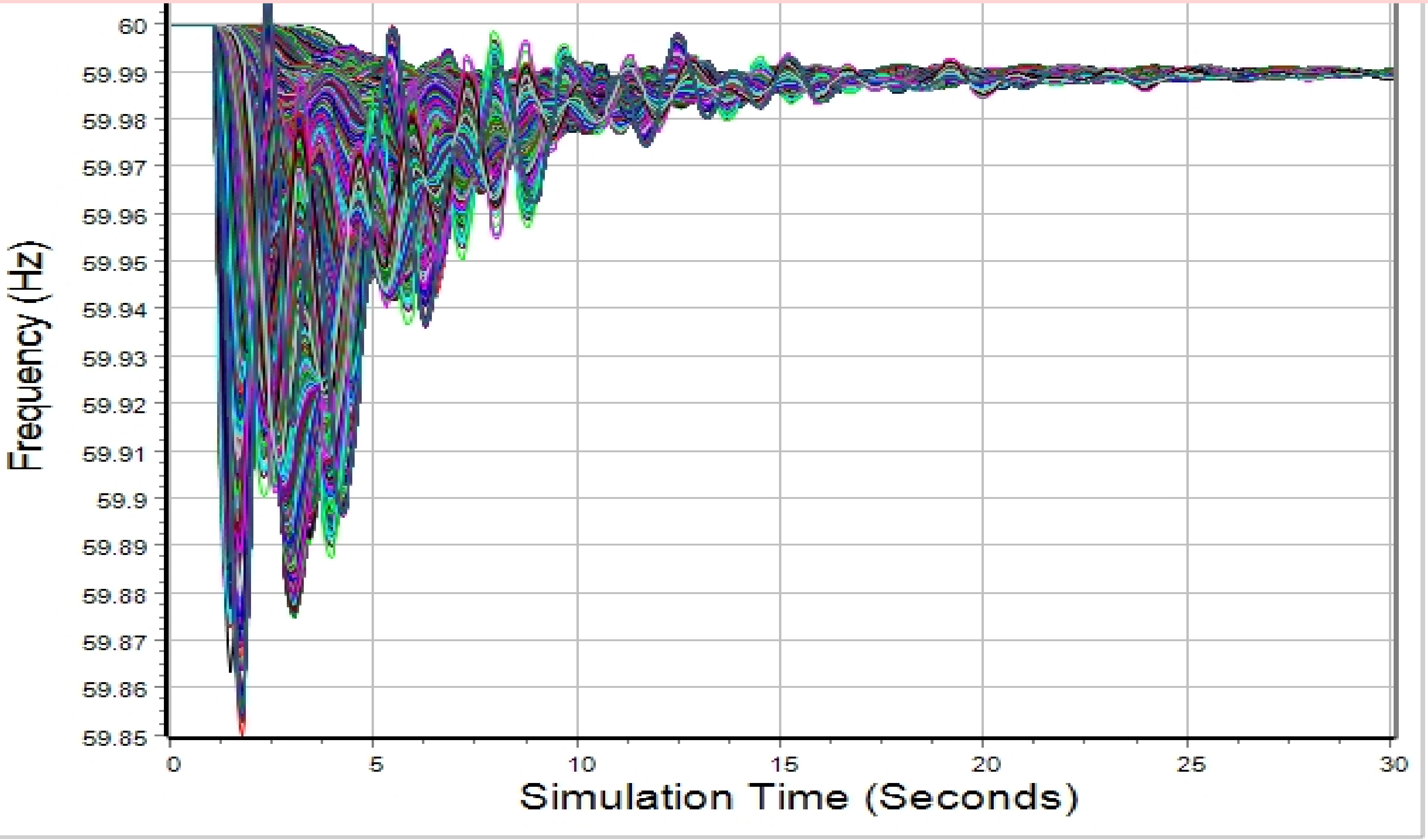


OK Cancel

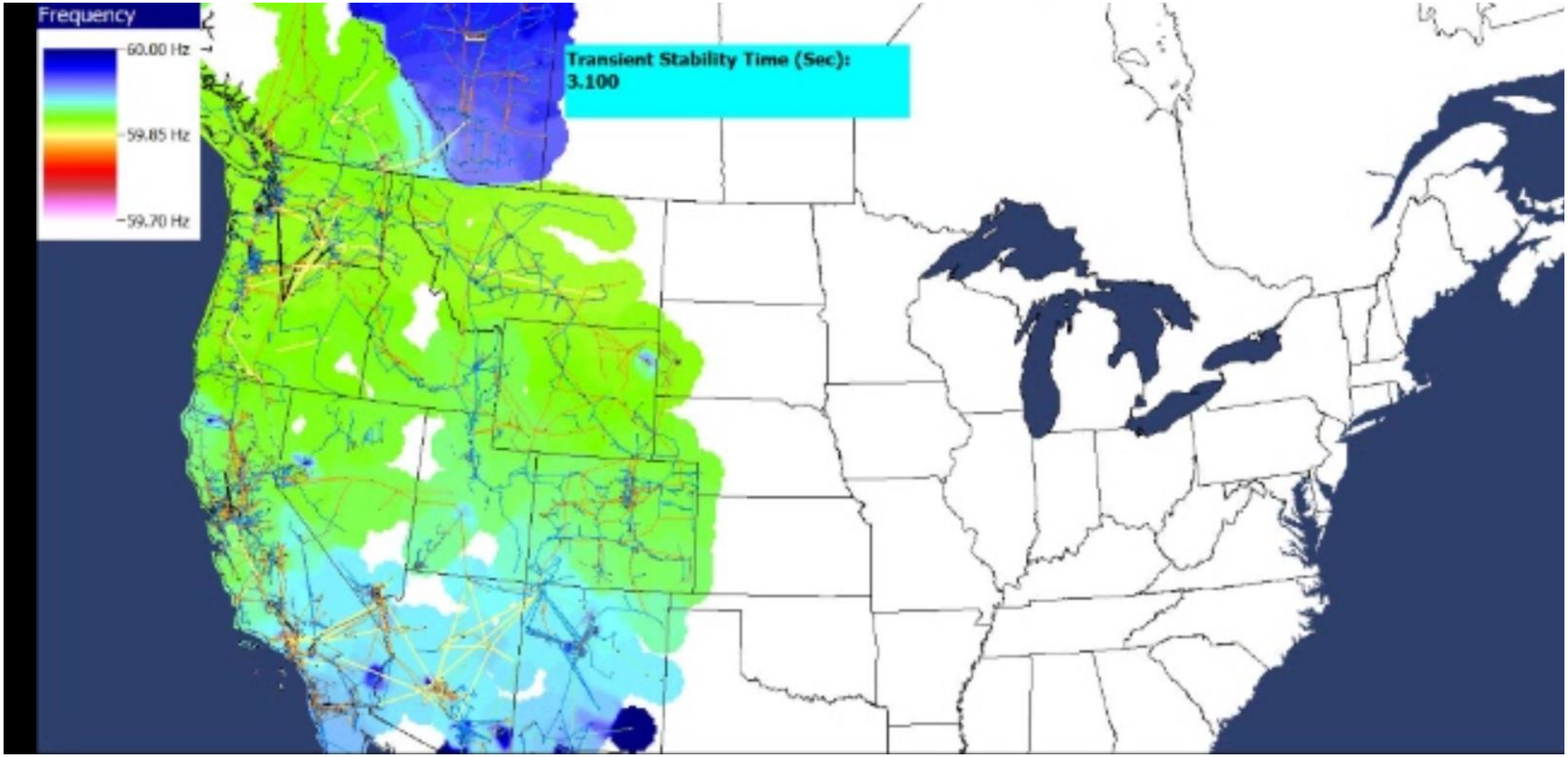
It is hard to tell a difference on this one, illustrating that modes manifest differently in different signals

A Larger Example We'll Finish With

Applying the developed techniques to the response of all 43,400 substation frequencies from an 110,000 bus electric grid(20 million plus values)



Spatial Visualization of Frequency



Measurement-Based Modal Analysis

- There are a number of different approaches
- The idea of all techniques is to approximate a signal, $y_{org}(t)$, by the sum of other, simpler signals (basis functions)
  - Basis functions are usually exponentials, with linear and quadratic functions used to detrend the signal
  - Properties of the original signal can be quantified from basis function properties
    - Examples are frequency and damping
  - Signal is considered over time with t=0 as the start
- Approaches sample the original signal $y_{org}(t)$

Measurement-Based Modal Analysis

- Vector **y** consists of m uniformly sampled points from $y_{org}(t)$ at a sampling value of ΔT, starting with t=0, with values y_j for j=1…m
  - Times are then $t_j = (j-1)\Delta T$
  - At each time point j, the approximation of y_j is

$$\hat{y}_j(\mathrm{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}$ is a vector with the real and imaginary eigenvalue components,

with $\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j}$ for α_i corresponding to a real eigenvalue, and

$\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j} \cos(\alpha_{\mathrm{i+1}} \mathrm{t}_j)$ and $\phi_{i+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \sin(\alpha_{\mathrm{i+1}} \mathrm{t}_j)$

for a complex eigenvector value

Measurement-Based Modal Analysis

- Error (residual) value at each point j is

$$r_j(t_j, \boldsymbol{\alpha}) = y_j - \hat{y}_j(t_j, \boldsymbol{\alpha})$$

- The closeness of the fit can be quantified using the Euclidean norm of the residuals

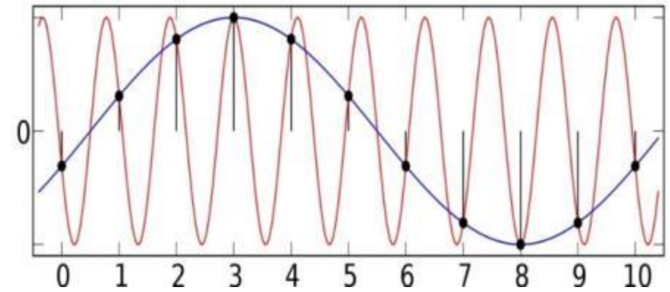
$$\frac{1}{2}\sum_{j=1}^{m}(y_j - \hat{y}_j(t_j, \boldsymbol{\alpha}))^2 = \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\alpha})\right\|_2^2$$

- Hence we need to determine **α** and **b**

$$\hat{y}_j(\mathrm{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

Sampling Rate and Aliasing

- The Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency
  - For example, to see a 5 Hz frequency we need to sample the signal at a rate of at least 10 Hz
- Sampling shifts the frequency spectrum by 1/T (where T is the sample time), which causes frequency overlap
- This is known as aliasing, which can cause a high frequency signal to appear to be a lower frequency signal



  - Aliasing can be reduced by fast sampling and/or low pass filters

Image: upload.wikimedia.org/wikipedia/commons/thumb/2/28/AliasingSines.svg/2000px-AliasingSines.svg.png

One Solution Approach: The Matrix Pencil Method

- There are several algorithms for finding the modes. We'll use the Matrix Pencil Method
  - This is a newer technique for determining modes from noisy signals (from about 1990, introduced to power system problems in 2005); it is an alternative to the Prony Method
  - The Matrix Pencil Method is useful when there is signal noise
- Given m samples, with L=m/2, the first step is to form the Hankel Matrix, **Y** such that

This not a sparse matrix

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{L+1} \\ y_2 & y_3 & \cdots & y_{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L} & y_{m-L+1} & \cdots & y_m \end{bmatrix}$$

Reference: A. Singh and M. Crow, "The Matrix Pencil for Power System Modal Extraction," IEEE Transactions on Power Systems, vol. 20, no. 1, pp. 501-502, Institute of Electrical and Electronics Engineers (IEEE), Feb 2005.

Algorithm Details, cont.

- Then calculate **Y**'s singular values using an economy singular value decomposition (SVD)

$$\mathbf{Y} = \mathbf{U\Sigma V}^{\mathbf{T}}$$

- The ratio of each singular value is then compared to the largest singular value σ_c; retain the ones with a ratio > than a threshold
  - This determines the modal order, M
  - Assuming **V** is ordered by singular values (highest to lowest), let $\mathbf{V}_p$ be then matrix with the first M columns of **V**

The computational complexity increases with the cube of the number of measurements!

This threshold is a value that can be changed; decrease it to get more modes.

Aside: The Matrix Singular Value Decomposition (SVD)

- The SVD is a factorization of a matrix that generalizes the eigendecomposition to any m by n matrix to produce

$$\mathbf{Y} = \mathbf{U\Sigma V^T}$$

The original concept is more than 100 years old, but has found lots of recent applications

where $\mathbf{\Sigma}$ is a diagonal matrix of the singular values

- The singular values are non-negative, real numbers that can be used to indicate the major components of a matrix (the gist is they provide a way to decrease the rank of a matrix)

Aside: SVD Image Compression Example

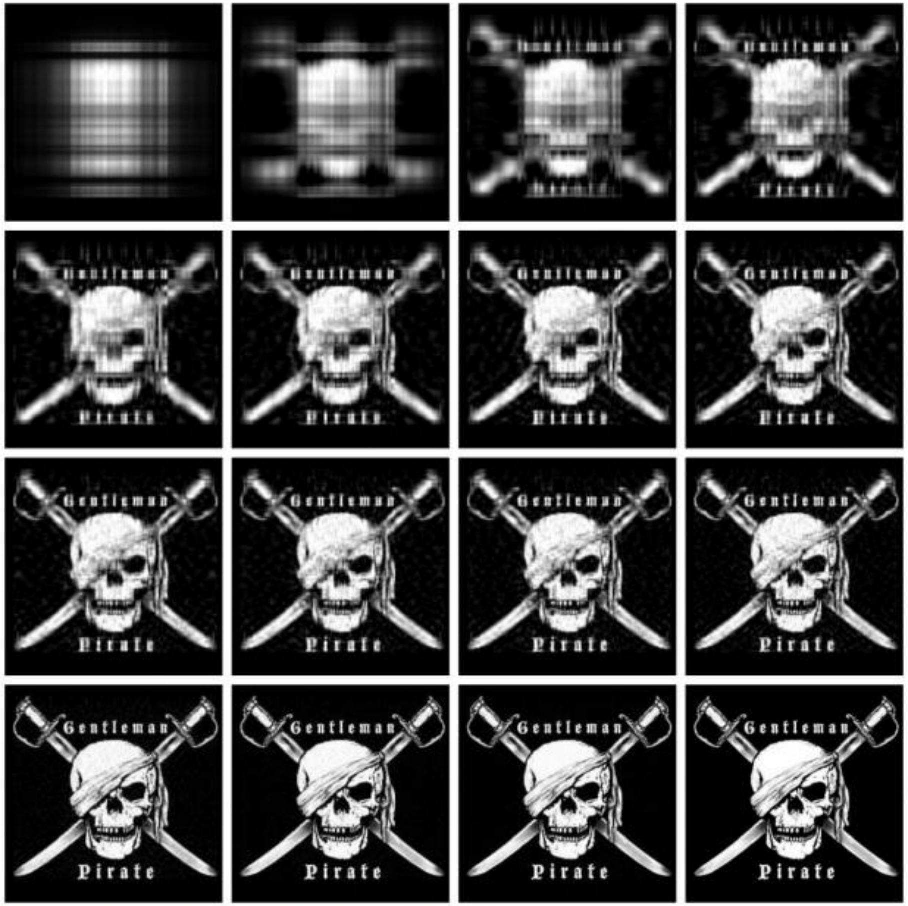


Figure 3.1: Image size 250x236 – modes used {{1,2,4,6},{8,10,12,14},{16,18,20,25},{50,75,100,original image}}

Images can be represented with matrices. When an SVD is applied and only the largest singular values are retained the image is compressed.

Image Source: www.math.utah.edu/~goller/F15_M2270/BradyMathews_SVDImage.pdf

Matrix Pencil Algorithm Details, cont.

- Then form the matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ such that
  - $\mathbf{V}_1$ is the matrix consisting of all but the last row of $\mathbf{V}_p$
  - $\mathbf{V}_2$ is the matrix consisting of all but the first row of $\mathbf{V}_p$
- Discrete-time poles are found as the generalized eigenvalues of the pair $(\mathbf{V}_2^T\mathbf{V}_1, \mathbf{V}_1^T\mathbf{V}_1) = (\mathbf{A},\mathbf{B})$
- These eigenvalues are the discrete-time poles, z_i with the modal eigenvalues then

If **B** is nonsingular (the situation here) then the generalized eigenvalues are the eigenvalues of $\mathbf{B}^{-1}\mathbf{A}$

$$\lambda_i = \frac{\ln(z_i)}{\Delta T}$$

The log of a complex number $z=r\angle\theta$ is $\ln(r) + j\theta$

Matrix Pencil Method with Many Signals

- The Matrix Pencil approach can be used with one signal or with multiple signals
- Multiple signals are handled by forming a $\mathbf{Y}_\text{k}$ matrix for each signal k using the measurements for that signal and then combining the matrices

$$\mathbf{Y}_k = \begin{bmatrix} y_{1,\text{k}} & y_{2,\text{k}} & \cdots & y_{L+1,\text{k}} \\ y_{2,\text{k}} & y_{3,k} & \cdots & y_{L+2,\text{k}} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L,\text{k}} & y_{m-L+1,\text{k}} & \cdots & y_{m,\text{k}} \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_N \end{bmatrix}$$

The required computation scales linearly with the number of signals

Matrix Pencil Method with Many Signals

- However, when dealing with many signals, usually the signals are somewhat correlated, so vary few of the signals are actually need to be included to determine the desired modes
- Ultimately we are finding

$$y_j(\mathrm{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

- The **α** is common to all the signals (i.e., the system modes) while the **b** vector is signal specific (i.e., how the modes manifest in that signal)

Quickly Determining the b Vectors

- A key insight is from an approach known as the Variable Projection Method (from Borden, 2013) that for any signal k

$$\mathbf{y}_k = \mathbf{\Phi}(\boldsymbol{\alpha})\mathbf{b}_k$$

And then the residual is minimized by selecting $\mathbf{b}_k = \mathbf{\Phi}(\boldsymbol{\alpha})^+ \mathbf{y}_k$

where $\mathbf{\Phi}(\boldsymbol{\alpha})$ is the m by n matrix with values

$\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ if α_i corresponds to a real eigenvalue,

and $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{i+1} t_j)$ and $\Phi_{ji+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{i+1} t_j)$

for a complex eigenvalue; $t_j = \left(j-1\right)\Delta T$

Finally, $\mathbf{\Phi}(\boldsymbol{\alpha})^+$ is the pseudoinverse of $\mathbf{\Phi}(\boldsymbol{\alpha})$

Where m is the number of measurements and n is the number of modes

A. Borden, B.C. Lesieutre, J. Gronquist, "Power System Modal Analysis Tool Developed for Industry Use," *Proc. 2013 North American Power Symposium*, Manhattan, KS, Sept. 2013