# ECEN 667
# Power System Stability

## Lecture 25: Direct Methods, Modeling Wind and Solar

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# Announcements

- Read Chapter 9; also Homework 7 should be done before the second exam but need not be turned in
- As noted in the syllabus, the second exam is on Thursday Dec 2, 2021
  - On campus students will take it during class (80 minutes) whereas distance learning students should contact Wei.
  - The exam is comprehensive, but emphasizes the material since the first exam; it will be of similar form to the first exam
  - Two 8.5 by 11 inch hand written note sheets are allowed, front and back, as are calculators

# Stability Phenomena and Tools

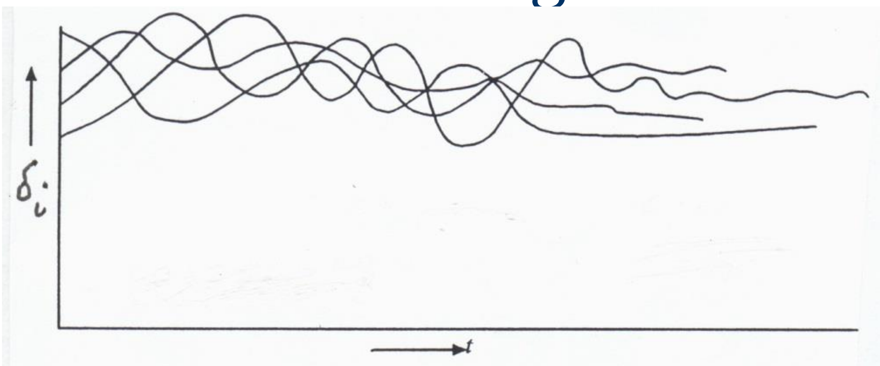
- Large Disturbance Stability (Non-linear Model)
- Small Disturbance Stability (Linear Model)
- Structural Stability (Non-linear Model)
  Loss of stability due to parameter variations.
- <u>Tools</u>
  - Simulation
  - Repetitive time-domain simulations are required to find critical parameter values, such as clearing time of circuit breakers.
  - Direct methods using Lyapunov-based theory (Also called Transient Energy Function (TEF) methods)
    - Can be useful for screening
  - Sensitivity based methods.

# Transient Energy Function (TEF) Techniques

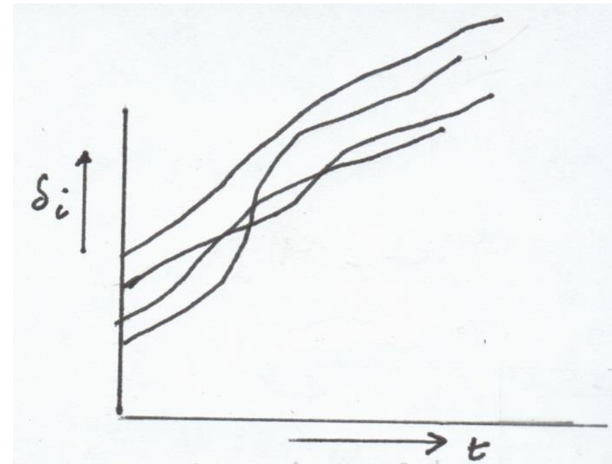
- No repeated simulations are involved.
- Limited somewhat by modeling complexity.
- Energy of the system used as Lyapunov function.
- Computing energy at the “controlling” unstable equilibrium point (CUEP) (critical energy).
- CUEP defines the mode of instability for a particular fault.
- Computing critical energy is <u>not</u> easy.

# Judging Stability / Instability

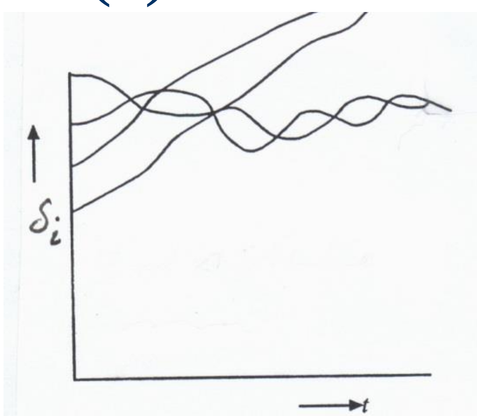
Monitor Rotor Angles



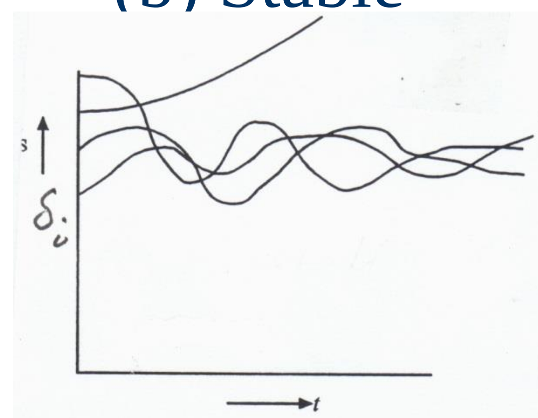

(a) Stable




(b) Stable




(c) Unstable




(d) Unstable

Stability is judged by Relative Rotor Angles.

# Mathematical Formulation

- A power system undergoing a disturbance (fault, etc), followed by clearing of the fault, has the following model

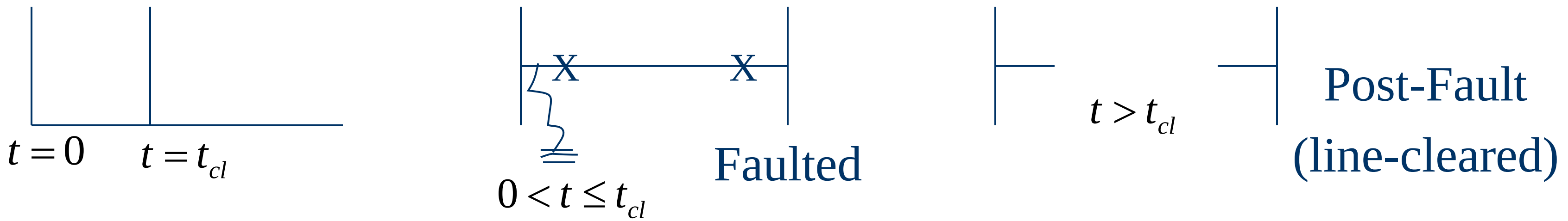
$$\dot{\mathbf{x}}(t) = \mathbf{f}^{\mathrm{I}}(\mathbf{x}(t)) \quad -\infty < t \le 0 \qquad (1)$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}^{\mathrm{F}}(\mathbf{x}(t)) \quad 0 < t \le t_{cl} \qquad (2)$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) \quad t_{cl} < t \le \infty \qquad (3)$$

  - (1) Prior to fault (Pre-fault)
  - (2) During fault (Fault-on or faulted)
  - (3) After the fault (Post-fault)

$T_{cl}$ is the clearing time

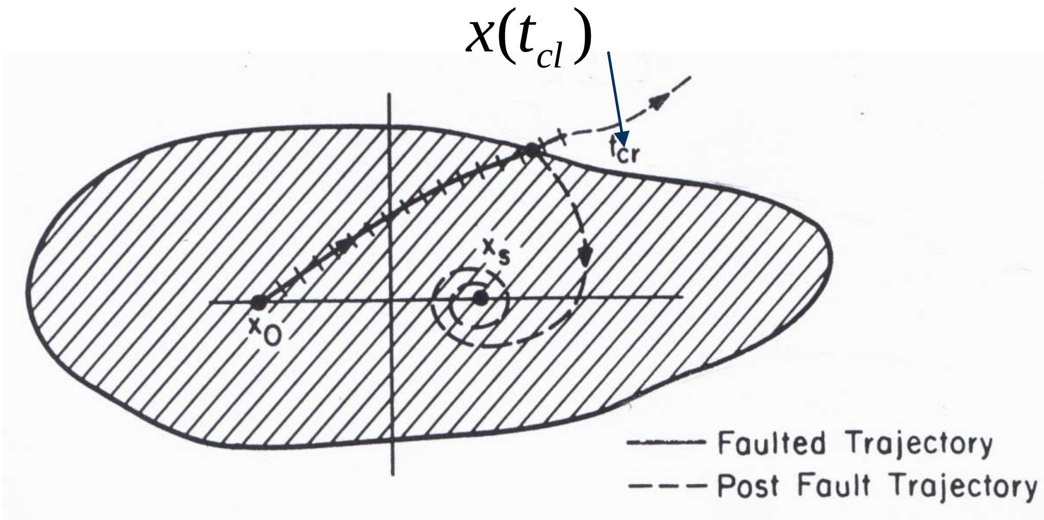


# Critical Clearing Time

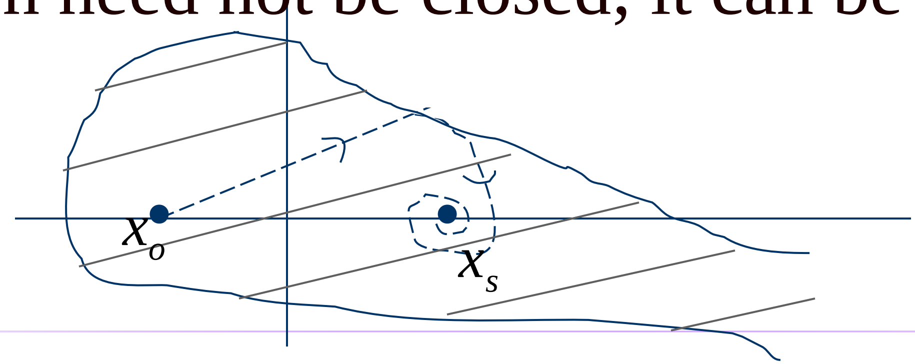
- Assume the post-fault system has a stable equilibrium point $\mathbf{x}_s$
- All possible values of $\mathbf{x}(t_{cl})$ for different clearing times provide the initial conditions for the post-fault system
  - Question is then will the trajectory of the post fault system, starting at $\mathbf{x}(t_{cl})$, converge to $\mathbf{x}_s$ as $t \rightarrow \infty$
- Largest value of $t_{cl}$ for which this is true is called the critical clearing time, $t_{cr}$
- The value of $t_{cr}$ is different for different faults

# Region of Attraction (ROA)

All faulted trajectories cleared before they reach the boundary of the ROA will tend to $\mathbf{x}_s$ as $t \rightarrow \infty$ (stable)




The region need not be closed; it can be open:



# Methods to Compute RoA

- Had been a topic of intense research in power system literature since early 1960's.
- The stable equilibrium point (SEP) of the <u>post-fault</u> system, $\mathbf{x}_s$, is generally close to the pre-fault EP, $\mathbf{x}_0$
- Surrounding $\mathbf{x}_s$ there are a number of unstable equilibrium points (UEPs).
- The boundary of ROA is characterized via these UEPs

$$\mathbf{x}_{u,i}\ , i = 1, 2 \ldots$$

$$\mathbf{f}(\mathbf{x}) = 0 \quad i.e. \quad \mathbf{f}(\mathbf{x}_{u,i}) = 0 \quad i = 1, 2 \ldots$$

# Characterization of RoA

- Define a scalar energy function V(**x**) = sum of the kinetic and potential energy of the post-fault system.
- Compute $V(\mathbf{x}_{u,i})$ at each UEP, i=1,2,…
- Defined $V_{cr}$ as

$$V_{cr} = Min\, V(\mathbf{x})\Big|_{\mathbf{x}_{u,i}}$$

  - RoA is defined by $V(\mathbf{x}) < V_{cr}$
  - But this can be an extremely conservative result.
- Alternative method: Depending on the fault, identify the critical UEP, $\mathbf{x}_{u,cr}$, towards which the faulted trajectory is headed; then $V(\mathbf{x}) < V(\mathbf{x}_{u,cr})$ is a good estimate of the ROA.

# Lyapunov's Method

- Defining the function V(**x**) is a key challenge
- Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad \mathbf{f}(\mathbf{x}_s) = \mathbf{0}$$

- Lyapunov's method: If there exists a scalar function V(**x**) such that

1) $V(\mathbf{x}_s) = 0$

2) $V(\mathbf{x}) > 0$ for all $\mathbf{x}$ around $\mathbf{x}_s$

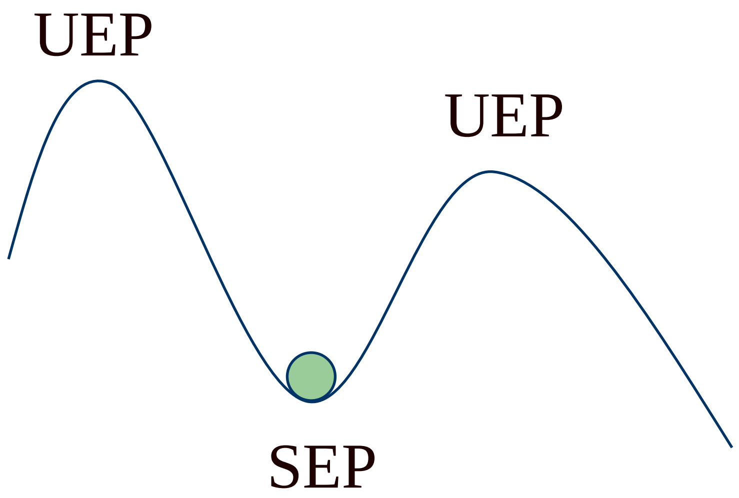
3) $\dot{V}(\mathbf{x}) \leq 0$ for all $\mathbf{x}$ around $\mathbf{x}_s$

Then $\mathbf{x}_s$ is stable in the sense of Lyapunov

EP $\mathbf{x}_s$ is asympotically stable if $\dot{V}(\mathbf{x}) < 0$ for $\mathbf{x} \neq \mathbf{x}_s$ around $\mathbf{x}_s$

# Ball in Well Analogy

- The classic Lyapunov example is the stability of a ball in a well (valley) in which the Lyapunov function is the ball's total energy (kinetic and potential)




- For power systems, defining a true Lyapunov function often requires using restrictive models

# Power System Example

- Consider the classical generator model using an internal node representation (load buses have been equivalenced)

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_i - \sum_{\substack{j=1 \\ j\neq i}}^{m} (C_{ij}\sin\delta_{ij} + D_{ij}\cos\delta_{ij})$$

$C_{ij}$ are the susceptance terms, $D_{ij}$ the conductance terms

$$P_i = T_{Mi} - E_i^2 G_{ii}$$

*Functionally*

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_i - P_{ei}(\delta_1, \ldots, \delta_m) \qquad i = 1, \ldots, m$$

$$\dot{\delta}_i = \omega_i - \omega_s$$

$$\dot{\omega}_i = \frac{1}{M_i}\left(P_i - P_{ei}(\delta_i, \ldots, \delta_m) - D_i(\omega_i - \omega_s)\right)$$

# Constructing the Transient Energy Function (TEF)

- The reference frame matters. Either relative rotor angle formulation, or COI reference frame.
  - COI is preferable since we measure angles with respect to the “mean motion” of the system.
- TEF for conservative system (i.e., zero damping)

$\delta_o = \frac{1}{M_T}\sum_{i=1}^{m} M_i \delta_i$ With center of speed as $\omega_o = \frac{1}{M_T}\sum_{i=1}^{m} M_i \omega_i$

where $M_T = \sum_{i=1}^{m} M_i$. We then transform the variables to the

COI variables as $\theta_i = \delta_i - \tilde{\delta}_o$, $\omega_i = \omega_i - \omega_o$.

It is easy to verify $\dot{\theta}_i = \dot{\delta}_i - \dot{\delta}_o = \omega_i - \tilde{\omega}_o \triangleq \omega_i$

# TEF

- We consider the general case in which all $M_i$'s are finite. We have two sets of differential equations:

$$M_i \frac{d\tilde{\omega}_i}{dt} = f_i^F(\theta) \quad 0 < t \le t_{cl} \qquad (Faulted)$$

$$\frac{d\theta_i}{dt} = \tilde{\omega}_i, \quad i = 1, 2, \ldots, m$$

And

$$M_i \frac{d\tilde{\omega}_i}{dt} = f_i \ (\theta) \quad t > t_{cl} \qquad (Post\ fault)$$

$$\frac{d\theta_i}{dt} = \tilde{\omega}_i, \quad i = 1, 2, \ldots, m$$

- Let the post fault system has a SEP at $\boldsymbol{\theta} = \boldsymbol{\theta}^s, \tilde{\boldsymbol{\omega}} = \mathbf{0}$
- This SEP is found by solving

$$\mathbf{f}_i(\boldsymbol{\theta}) = \mathbf{0}, \ i = 1, .., m$$

# TEF

- Steps for computing the critical clearing time are:
  - Construct a Lyapunov (energy) function for the post-fault system.
  - Find the critical value of the Lyapunov function (critical energy) for a given fault
  - Integrate the faulted equations until the energy is equal to the critical energy; this instant of time is called the critical clearing time
- Idea is once the fault is cleared the energy can only decrease, hence the critical clearing time is determined directly
- Methods differ as to how to implement steps 2 and 3.

# Potential Energy Boundary Surface

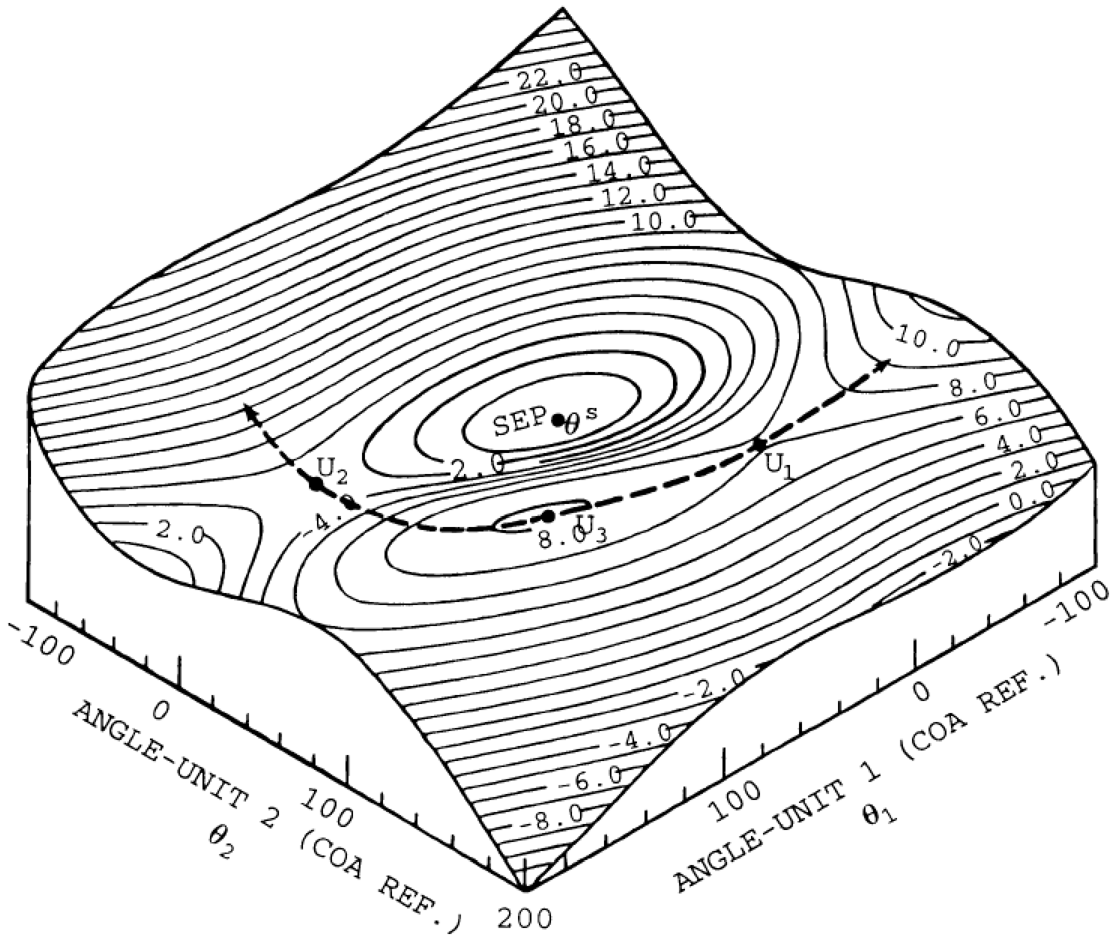



Figure 9.10: The potential energy boundary surface (reproduced from [97]

Figure from course textbook

# TEF

- Integrating the equations between the post-fault SEP and the current state gives

$$V(\theta,\omega) = \frac{1}{2}\sum_{i=1}^{m} M_i\omega_i^2 - \sum_{i=1}^{m}\int_{\theta_i^s}^{\theta_i} f_i(\theta)d\theta_i$$

$$= \frac{1}{2}\sum_{i=1}^{m} M_i\omega_i^2 - \sum_{i=1}^{m} P_i(\theta_i - \theta_i^s) - \sum_{i=1}^{m-1}\sum_{j=i+1}^{m}[C_{ij}(\cos\theta_{ij} - \cos\theta_{ij}^s)$$

$$-\int_{\theta_i^s+\theta_j^s}^{\theta_i+\theta_j} D_{ij}\cos\theta_{ij}\,d(\theta_i - \theta_j)]$$

$$= V_{KE}(\omega) + V_{PE}(\theta)$$

$C_{ij}$ are the susceptance terms, $D_{ij}$ the conductance terms; the conductance term is path dependent

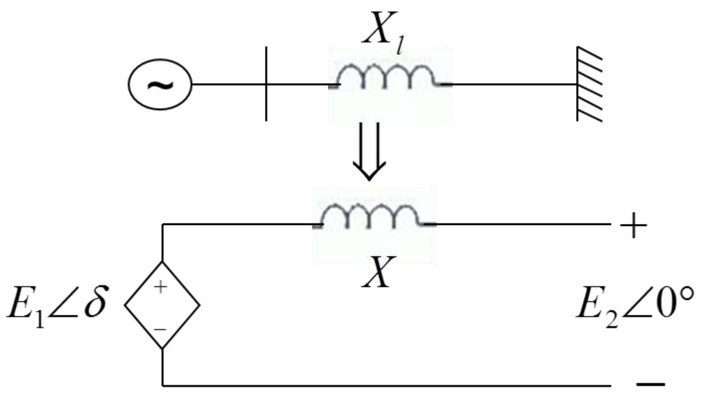
# TEF

- $V(\boldsymbol{\theta}, \tilde{\boldsymbol{\omega}})$ contains path dependent terms.
- Cannot claim that $V(\boldsymbol{\theta}, \tilde{\boldsymbol{\omega}})$ is p.d.
- If conductance terms are ignored then it can be shown to be a Lyapunov function
- Methods to compute the UEPS are
  - Potential Energy Boundary Surface (PEBS) method.
  - Boundary Controlling Unstable (BCU) equilibrium point method.
  - Other methods (Hybrid, Second-kick etc)

(a) and (b) are the most important ones.

# Equal Area Criterion and TEF

- For an SMIB system with classical generators this reduces to the equal area criteria
  - TEF is for the post-fault system
  - Change notation from $T_m$ to $P_m$




$$M\frac{d^2\delta}{dt^2} = P_m - P_e^{\max}\sin\delta \qquad (1)$$

$$P_e^{\max} = \frac{E_1 E_2}{X^I}\sin\delta \qquad (2)$$

$$X = X^F \quad (Faulted)$$

$$X = X^I \quad (Post-fault)$$

$$P_e = \frac{E_1 E_2}{X^F}\sin\delta \quad (Faulted)$$

$$P_e = \frac{E_1 E_2}{X^I}\sin\delta \quad (Post-fault)$$

# TEF for SMIB System

$$M\frac{d^2\delta}{dt^2} = P_m - P_e^{\max}\sin\delta \qquad (1)$$

The right hand side of (1) can be written as $-\dfrac{\partial V_{PE}}{\partial \delta}$ ,where

$$V_{PE}(\delta) = -P_m\delta - P_e^{\max}\cos\delta \qquad (2)$$

Multiplying (1) by $\frac{d\delta}{dt}$ , re-write

$$\frac{d}{dt}\left[\frac{M}{2}\left(\frac{d\delta}{dt}\right)^2 + V_{PE}(\delta)\right] = 0 \quad \text{since} \quad \frac{d\delta}{dt} = \omega$$

i.e $\dfrac{d}{dt}\left[\dfrac{1}{2}M\omega^2 + V_{PE}(\delta)\right] = 0$

i.e $\dfrac{d}{dt}[V(\delta,\omega)] = 0$

Hence, the energy function is

$$V(\delta,\omega) = \frac{1}{2}M\omega^2 + V_{PE}(\delta)$$

# TEF for SMIB System (contd)

- The equilibrium point is given by

$$0 = P_m - P_e^{\max} \sin\delta \qquad (1)$$

$$\delta^s = \sin^{-1}\left(\frac{P_m}{P_e^{\max}}\right) \qquad (2)$$

- This is the stable e.p.
- Can be verified by linearizing.
- Eigenvalues on jω axis. (Marginally Stable)
- With slight damping eigenvalues are in L.H.P.
- TEF is still constructed for undamped system.

# TEF for SMIB System

- The energy function is

$$V(\delta,\omega) = V_{KE} + V_{PE}(\delta) = \tfrac{1}{2}M\omega^2 - P_m\delta - P_e^{\max}\cos\delta$$

- There are two UEP: $\delta^{u1} = \pi-\delta^s$ and $\delta^{u2} = -\pi-\delta^s$
- A change in coordinates sets $V_{PE}=0$ for $\delta=\delta^s$

$$V_{PE}(\delta,\delta^s) = -P_m(\delta-\delta^s) - P_e^{\max}(\cos\delta - \cos\delta^s)$$

- With this, the energy function is

$$V(\delta,\omega) = \tfrac{1}{2}M\omega^2 - P_m(\delta-\delta^s) - P_e^{\max}(\cos\delta - \cos\delta^s)$$

$$= V_{KE} + V_{PE}(\delta,\delta^s)$$

- The kinetic energy term is $V_{KE} = \tfrac{1}{2}M\omega^2$

# Equal-Area Criterion

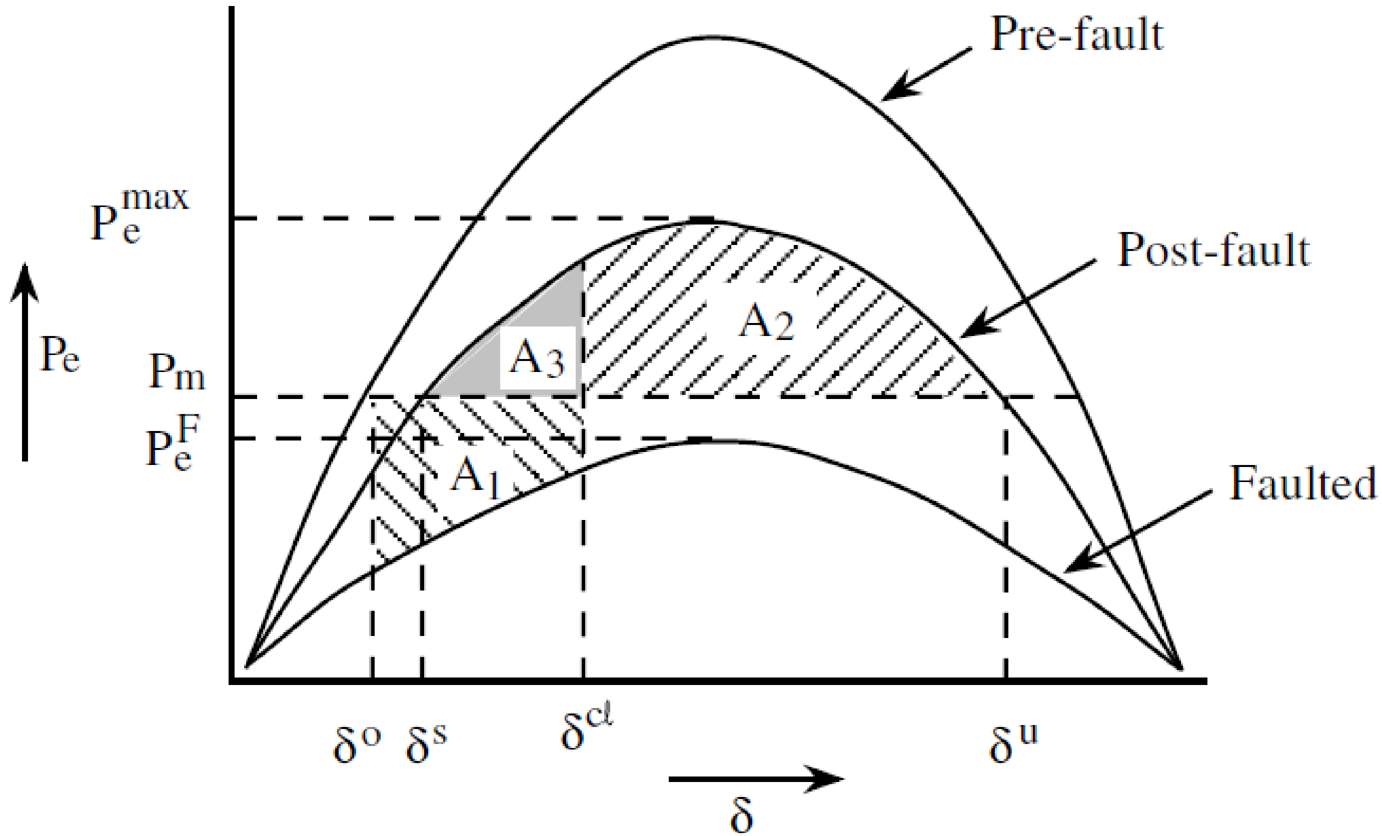



Figure 9.9: Equal-area criterion for the SMIB case

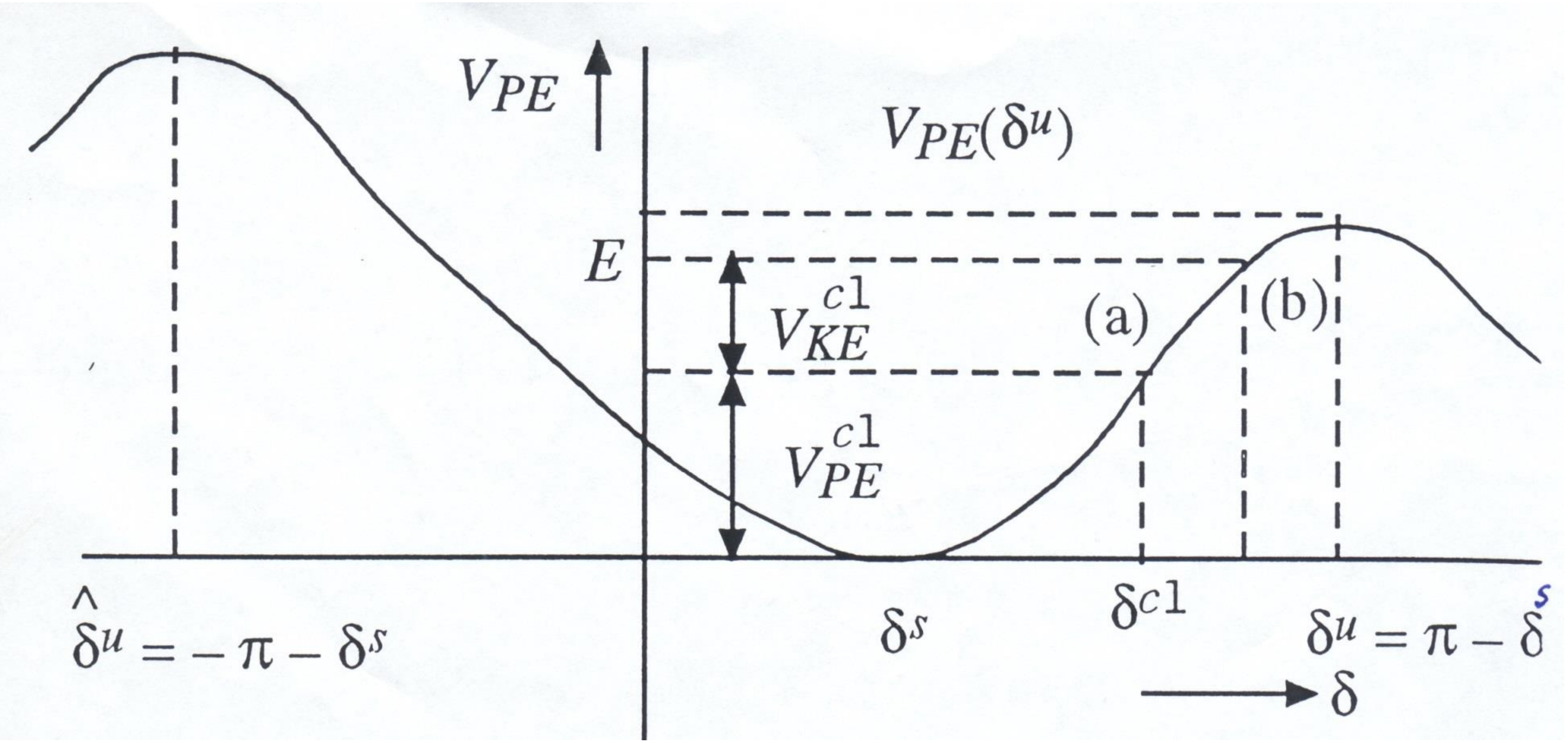
During the fault $A_1$ is the gain in the kinetic energy and $A_3$ the gain in potential energy

Figure from course textbook

# Energy Function for SMIB System

- $V(\delta,\omega)$ is equal to a constant E, which is the sum of the kinetic and potential energies.
- It remains constant once the fault is cleared since the system is conservative (with no damping)
- $V(\delta,\omega)$ evaluated at $t=t_{cl}$ from the fault trajectory represents the total energy *E* present in the system at $t=t_{cl}$
- This energy must be absorbed by the system once the fault is cleared if the system is to be stable.
- The kinetic energy is always positive, and is the difference between *E* and $V_{PE}(\delta, \delta^s)$

# Potential Energy Well for SMIB System




- Potential energy "well" or P.E. curve

- How is $E$ computed?

# Structure Preserving Energy Function

- If we retain the power flow equations

$$
\begin{aligned}
\dot{\theta}_i &= \omega_i - \omega_s && (9.69) \\
M_i \dot{\omega}_i &= T_{Mi} - \sum_{j=1}^{n+m} V_i V_j B_{ij} \sin(\theta_i - \theta_j) \\
& \quad i = n+1, \ldots, n+m && (9.70) \\
P_{Li}(V_i) &= \sum_{j=1}^{n+m} V_i V_j B_{ij} \sin(\theta_i - \theta_j) \;\; i = 1, \ldots, n && (9.71) \\
Q_L(V_i) &= -\sum_{j=1}^{n+m} V_i V_j B_{ij} \cos(\theta_i - \theta_j) \;\; i = 1, \ldots, n. && (9.72)
\end{aligned}
$$

# Structure Preserving Energy Function

- Then we can get the following energy function

$$V(\tilde{\omega}, \tilde{\theta}, V) \quad = \quad V_{KE}(\tilde{\omega}) + V_{P1}(\tilde{\theta}, V) + V_{P2}(\tilde{\theta}) \tag{9.73}$$

where

$$V_{KE}(\tilde{\omega}) \quad = \quad \frac{1}{2}\sum_{i=1}^{m} M_i \tilde{\omega}_i^2$$

$$\begin{aligned}
V_{p1}(\tilde{\theta}, V) \quad &= \quad -\sum_{i=n+1}^{n+m} T_{Mi}(\tilde{\theta}_i - \tilde{\theta}_i^s) + \sum_{i=1}^{n} \int_{V_i^s}^{V_i} \frac{Q_{Li}(V_i)}{V_i} dV_i && (9.74) \\
&\quad\ \frac{1}{2}\sum_{1}^{n} B_{ii}(V_i^2 - (V_i^s)^2) && (9.75) \\
&\quad - \sum_{i=1}^{n+m-1} \sum_{j=i+1}^{n+m} B_{ij}(V_i V_j \cos\tilde{\theta}_{ij} - V_i^s V_j^s \cos\tilde{\theta}_{ij}^s) && (9.76) \\
V_{p2}(\tilde{\theta}) \quad &= \quad -\sum_{1}^{n} P_{Li}(\tilde{\theta}_i - \tilde{\theta}_i^s). && (9.77)
\end{aligned}$$

# Energy Functions for a Large System

- Need an energy function that at least approximates the actual system dynamics
  - This can be quite challenging!
- In general there are many UEPs; need to determine the UEPs for closely associated with the faulted system trajectory (known as the controlling UEP)
- Energy of the controlling UEP can then be used to determine the critical clearly time (i.e., when the fault-on energy is equal to that of the controlling UEP)
- For on-line transient stability, technique can be used for fast screening

# Renewable Resource Modeling

- With the advent of more renewable generation in power systems worldwide it is important to have correct models
- Hydro systems have already been covered
- Solar thermal and geothermal are modeled similar to existing stream generation, so they are not covered here
- Coverage will focus on transient stability level models for wind and solar PV for integrated system studies
  - More detailed EMTP-level models may be needed for individual plant issues, like subsynchronous resonance
  - Models are evolving, with a desire by many to have as generic as possible models

# Changing Sources of Generation

- In the US and worldwide the sources of electricity are rapidly changing

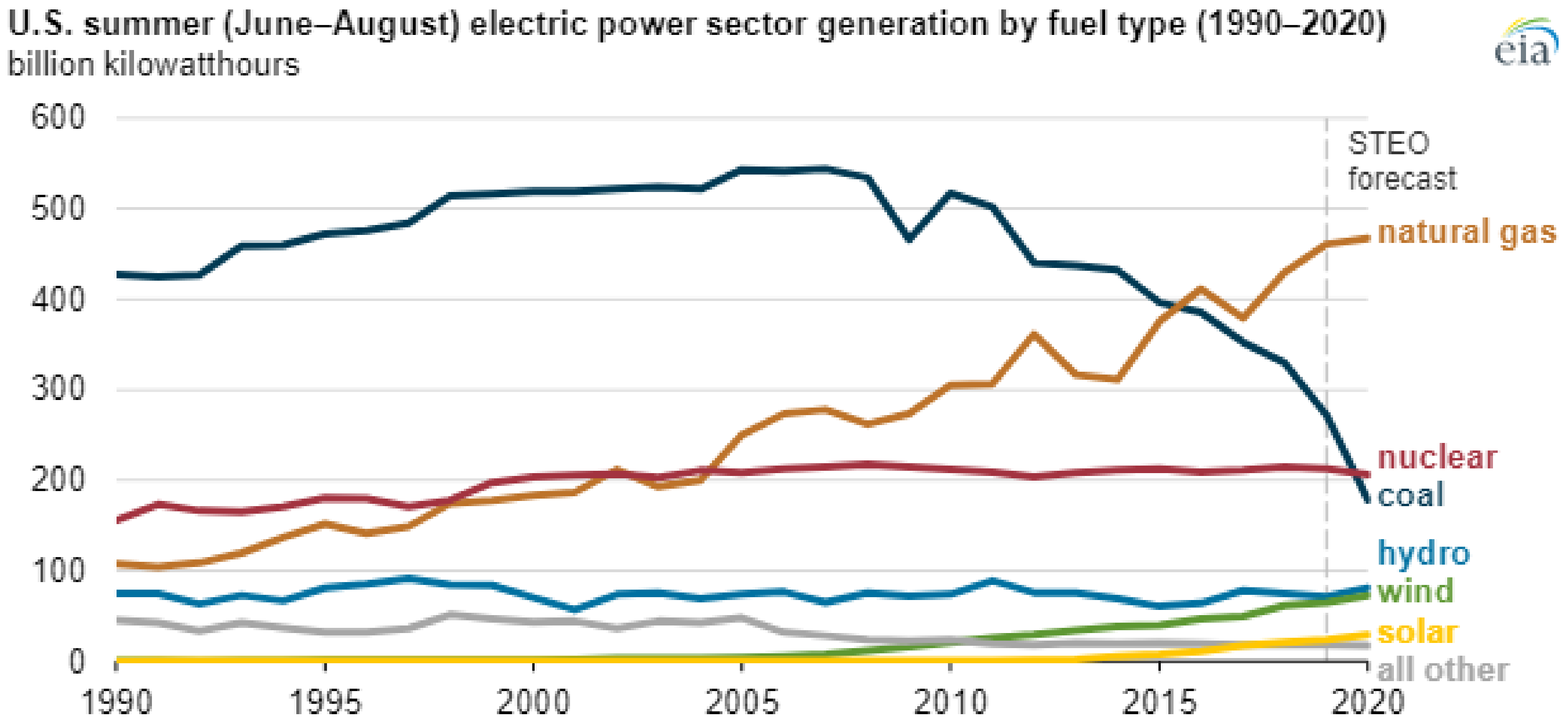
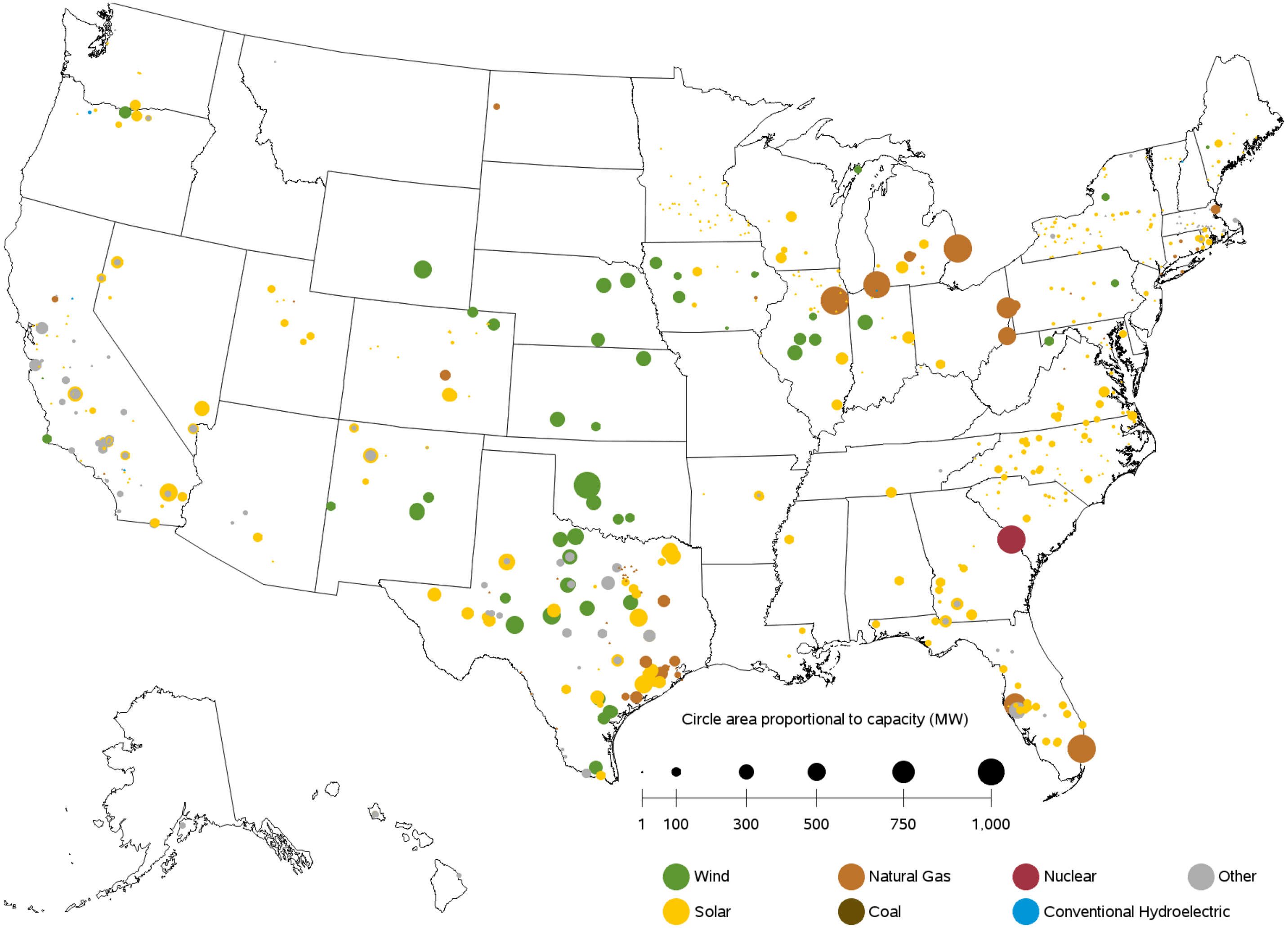



Image Source: www.eia.gov/todayinenergy/detail.php?id=44055

# Natural Gas Prices


Source: fred.stlouisfed.org/series/DHHNGSP

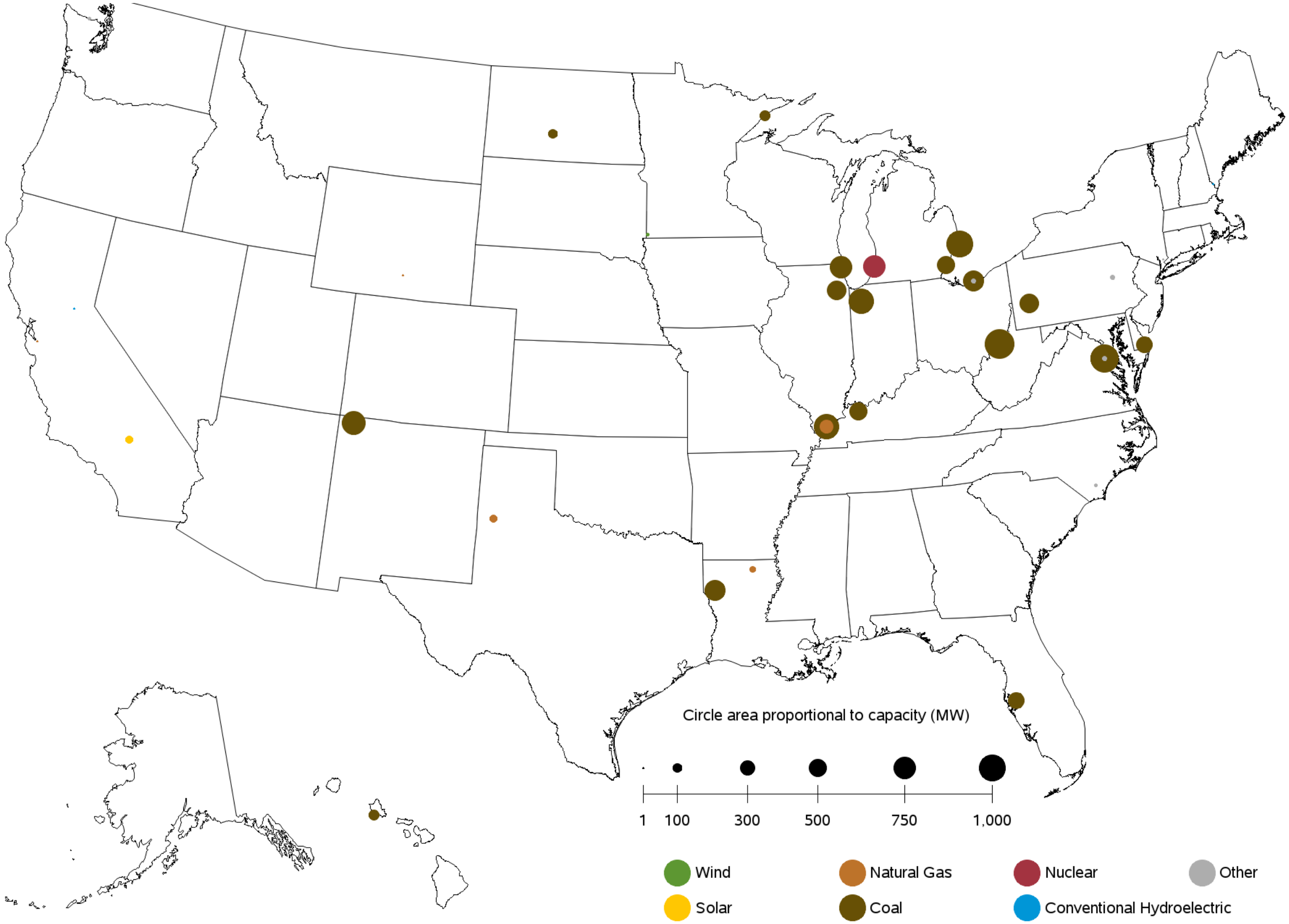
# Planned New Utility-Scale Generation, Oct 2021 to Sept 2022




In the US the new nuclear is Vogtle 3 and 4 (1100 MW each); about 50 are under construction worldwide

Source: www.eia.gov/electricity/monthly/, November 2021

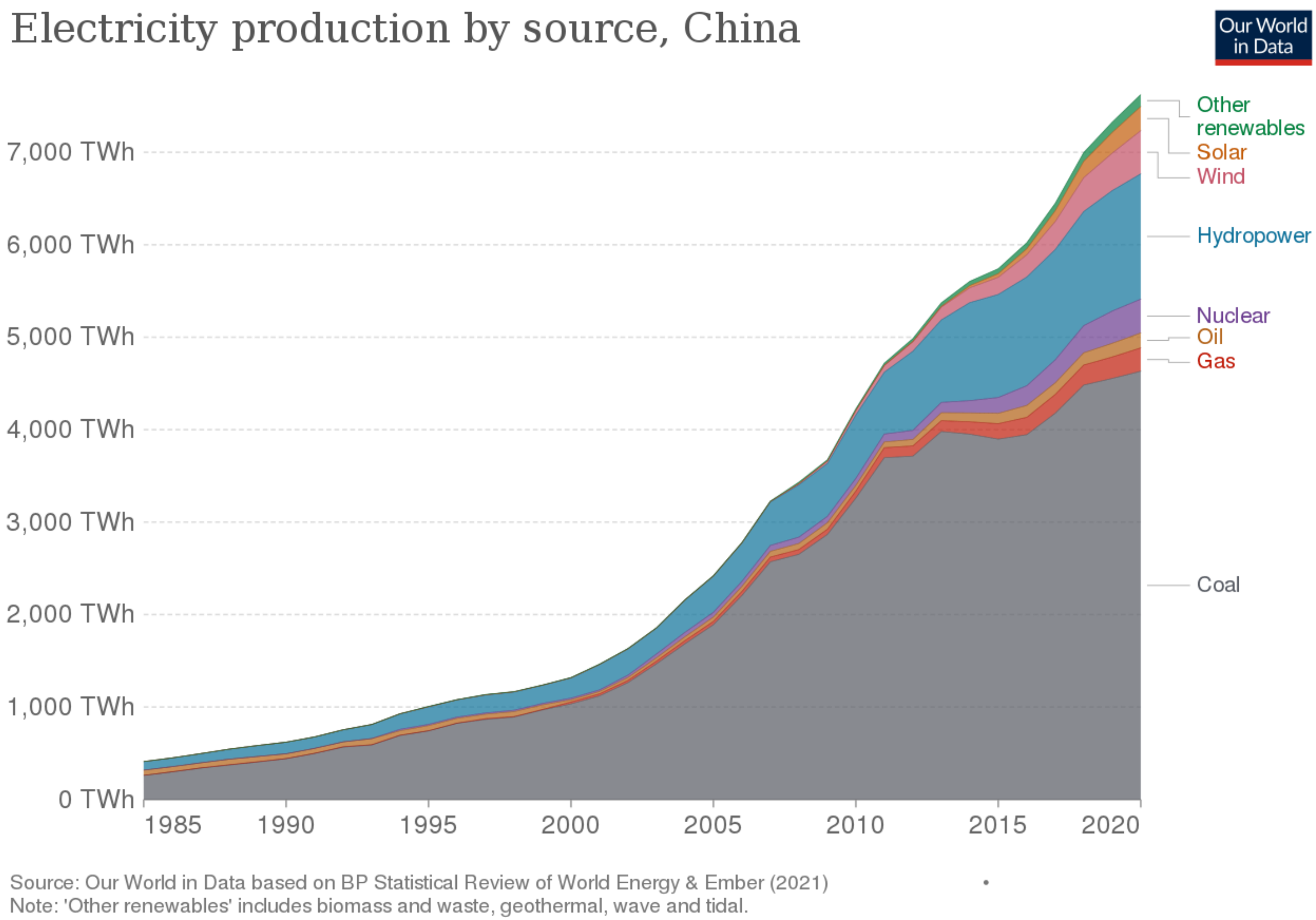
# Planned Utility-Scale Generation Retirements, Oct 2021 to Sept 2022



# Most Recently There Has Been a Surge in US Coal Generation

- The amount of US generation from coal has been rapidly decreasing, but in 2021 it has risen about 25% from 2020 levels
  - The rolling 12 months between 2020 and 2021 has gone from 788 to 919 billion kWh (in 2011 it was about 1700)
  - This is mostly due to the increasing natural gas prices, and it expected to be short lived because of coal plant retirements

# Electricity Production in China

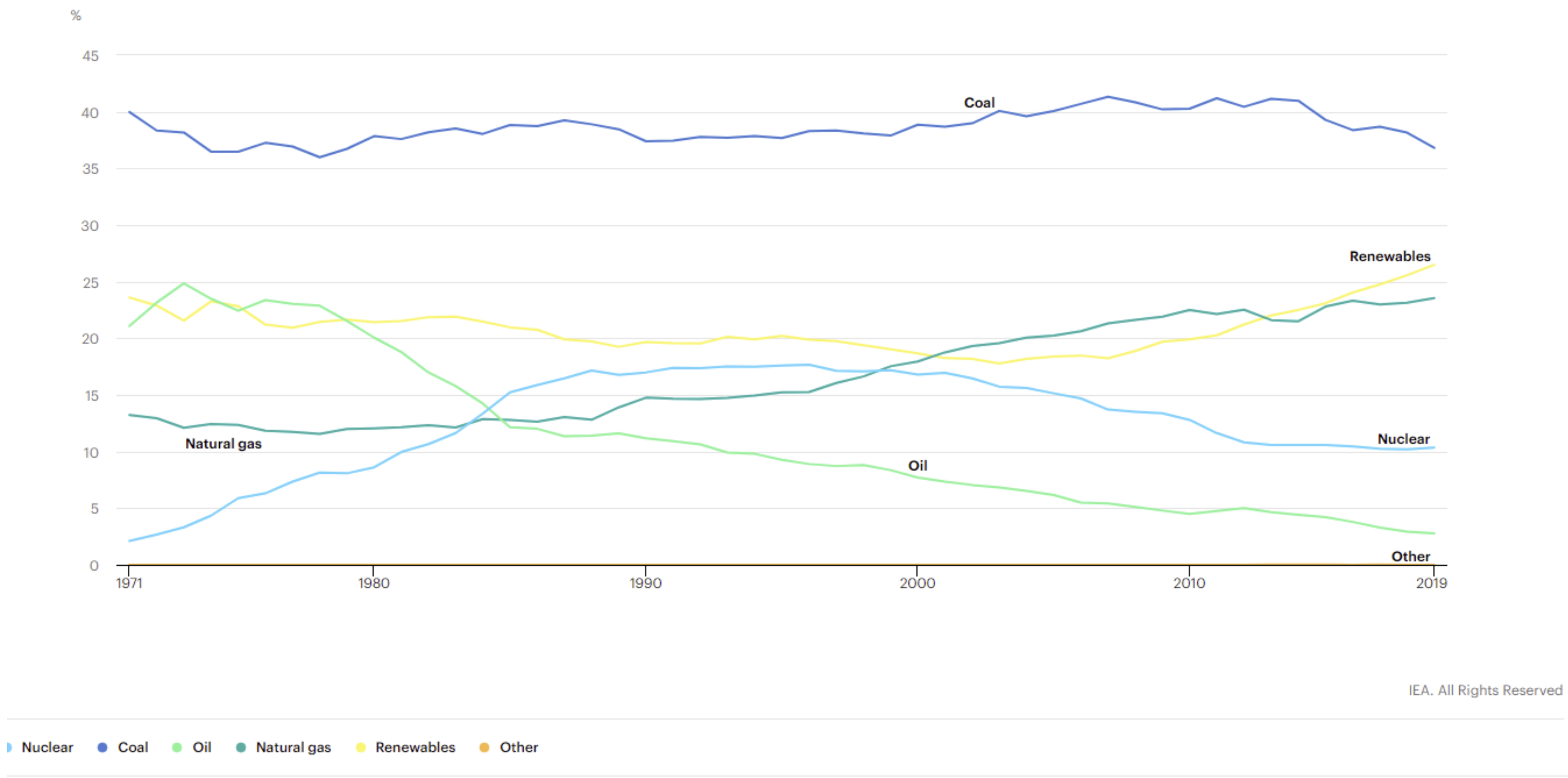



TWh are the same as billion kWh; for reference the US total in 2021 is about 4000 TWh

Source: en.wikipedia.org/wiki/Electricity_sector_in_China#/media/File:China-electricity-prod-source-stacked.svg

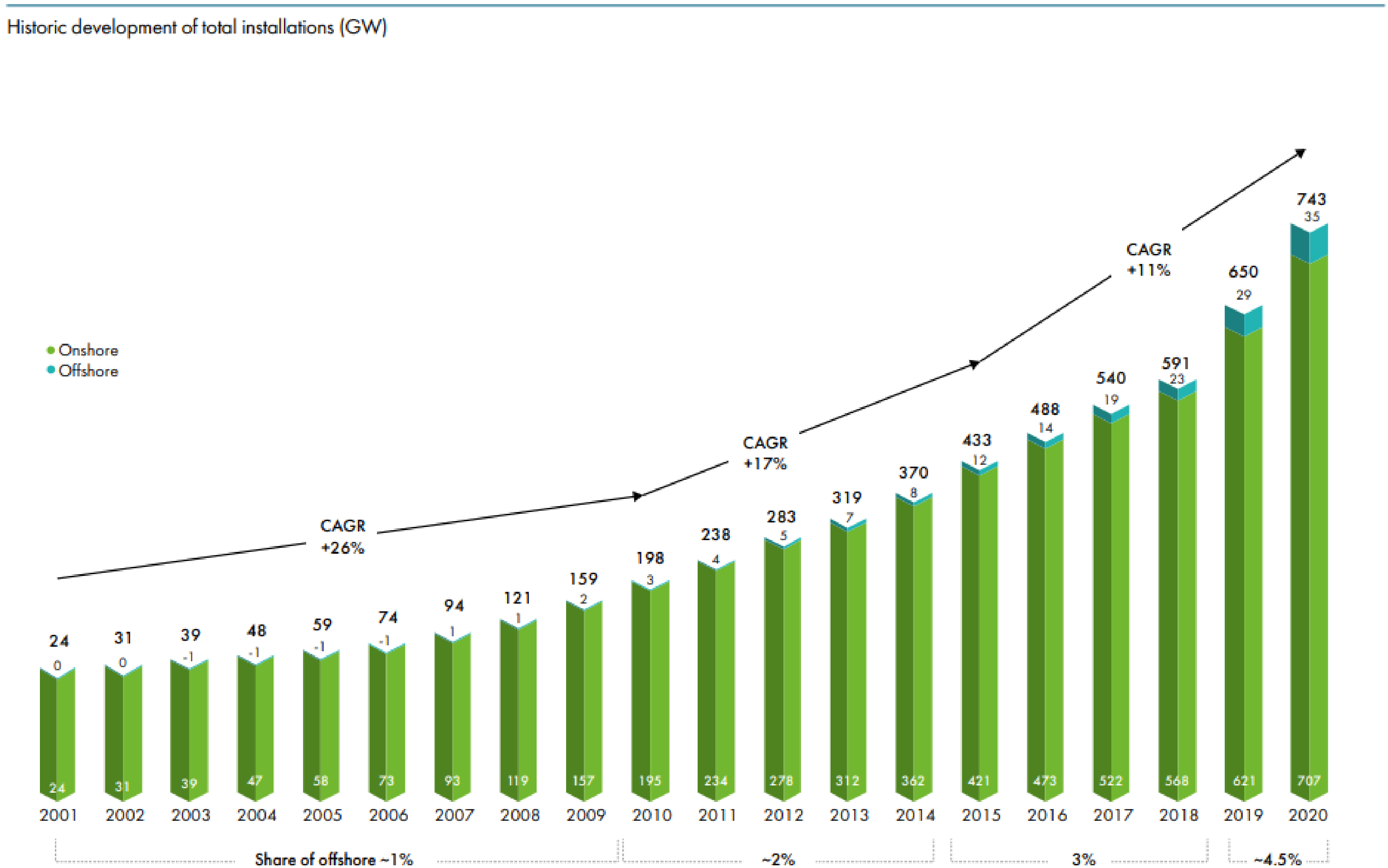
# World Electricity Generation (Energy)

In 2019 the capacity is 4200 GW fossil fuel,  2500 GW renewables (1140 hydro, 650 wind, 584 solar), 369 GW nuclear



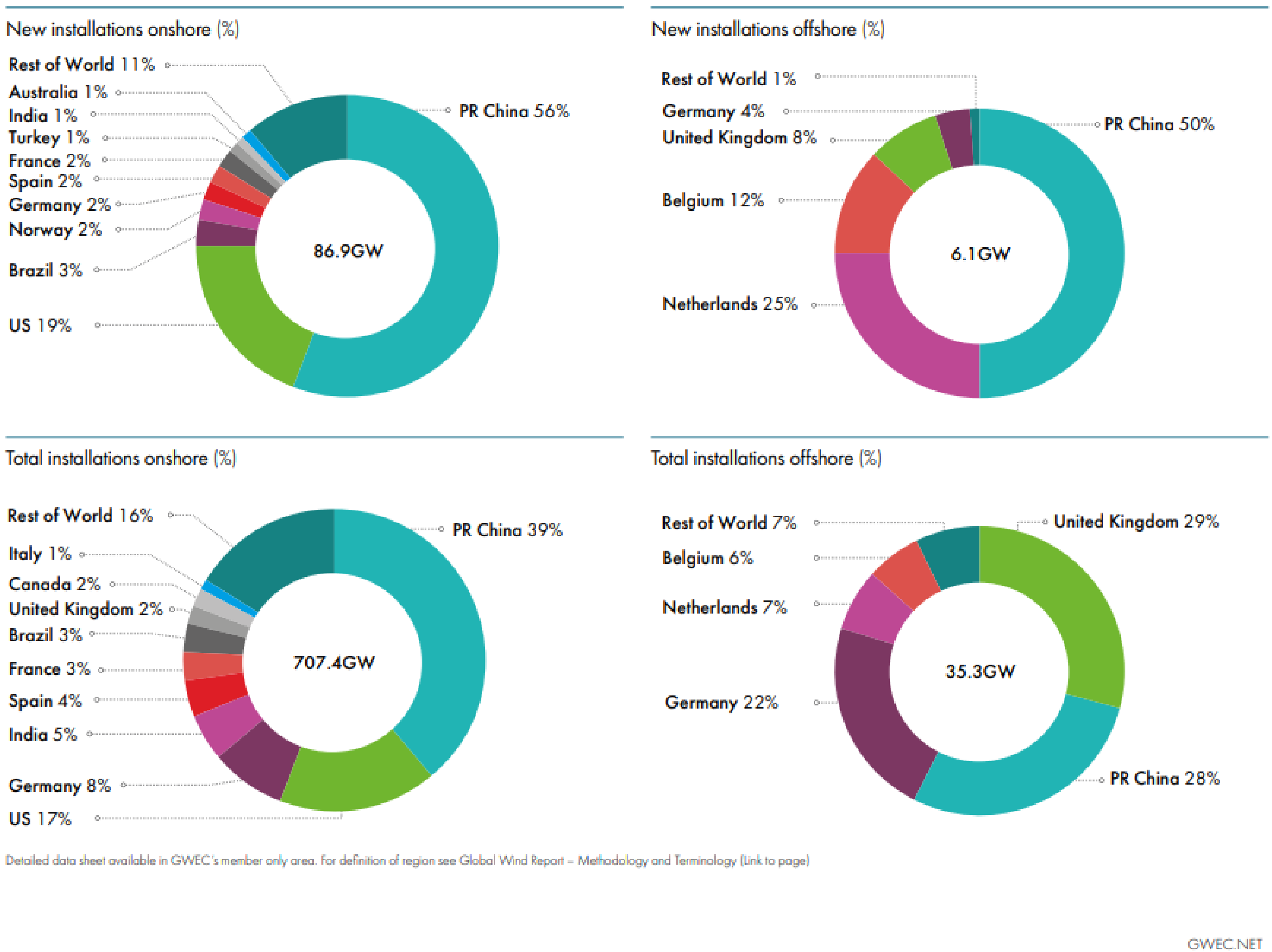

Graph source: www.iea.org/data-and-statistics/charts/world-electricity-generation-mix-by-fuel-1971-2019

# Growth in Wind Worldwide




Source: Global Wind 2021 Report, Global Wind Energy Council

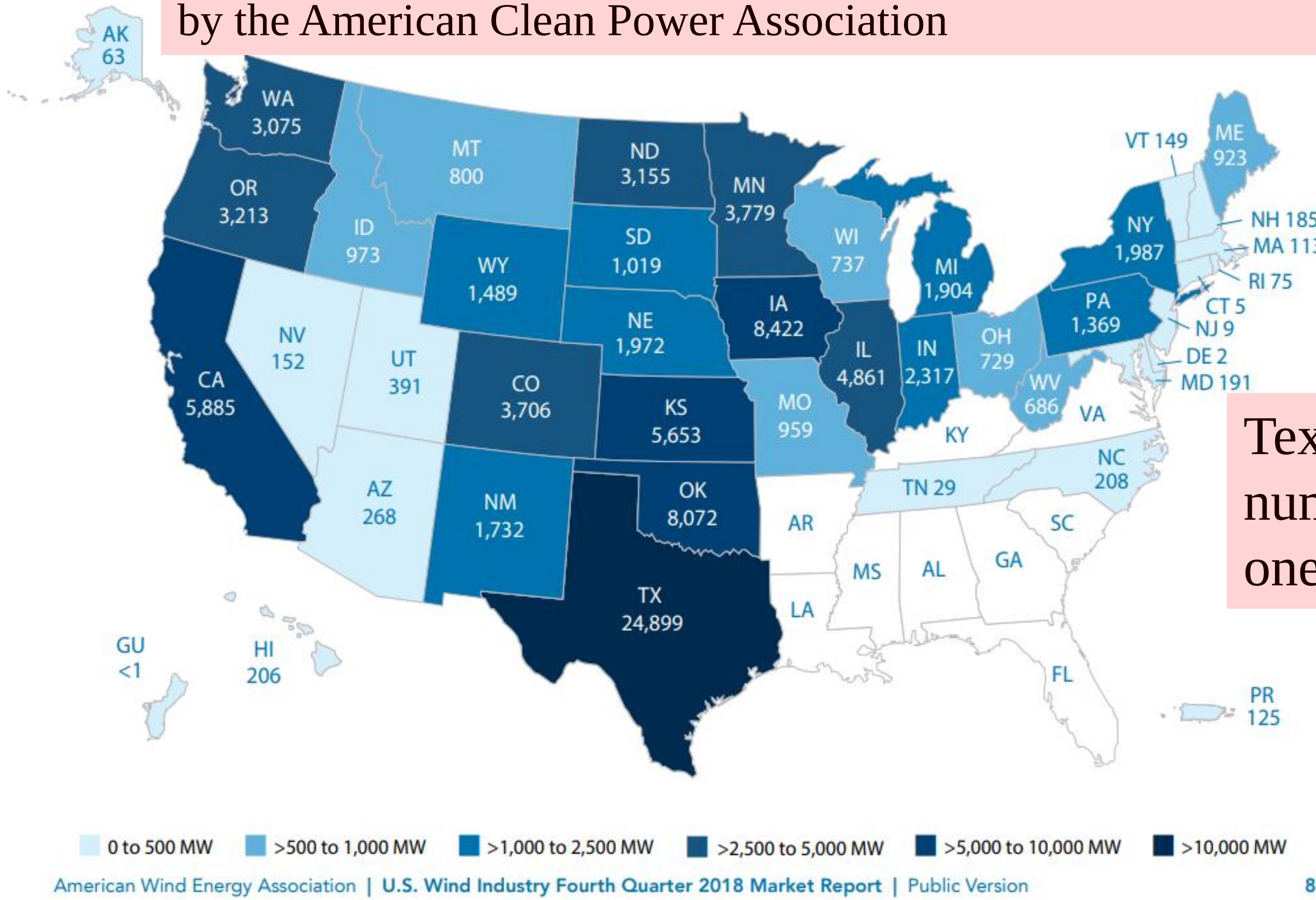
# Growth in Wind Worldwide, 2020




Source: Global Wind 2021 Report, Global Wind Energy Council

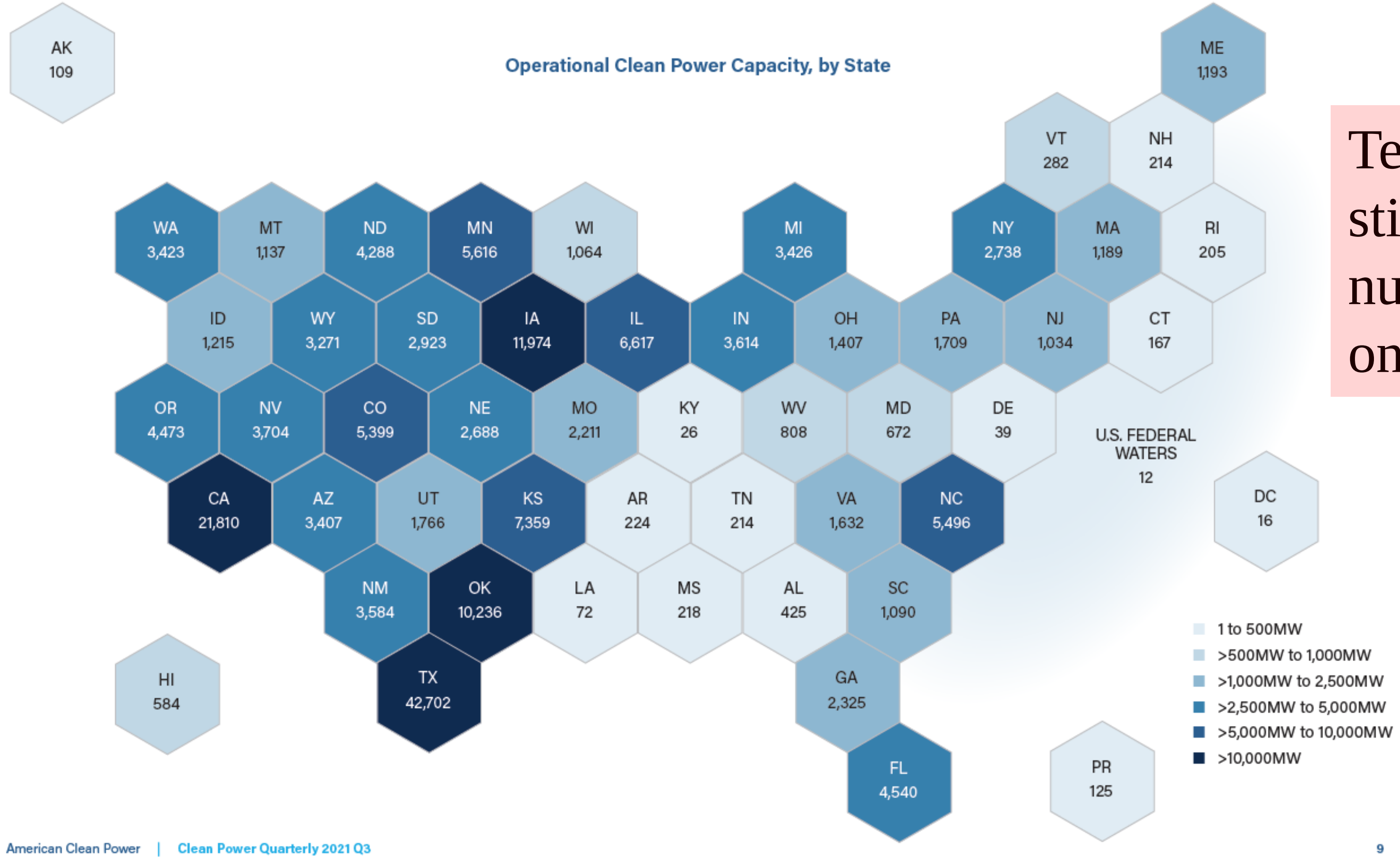
# 2018 Installed Wind Capacity by State

In January 2021 the American Wind Energy Association was succeeded by the American Clean Power Association




Texas is number one!

# 2021 Wind/Solar Capacity by State




Texas is still number one!

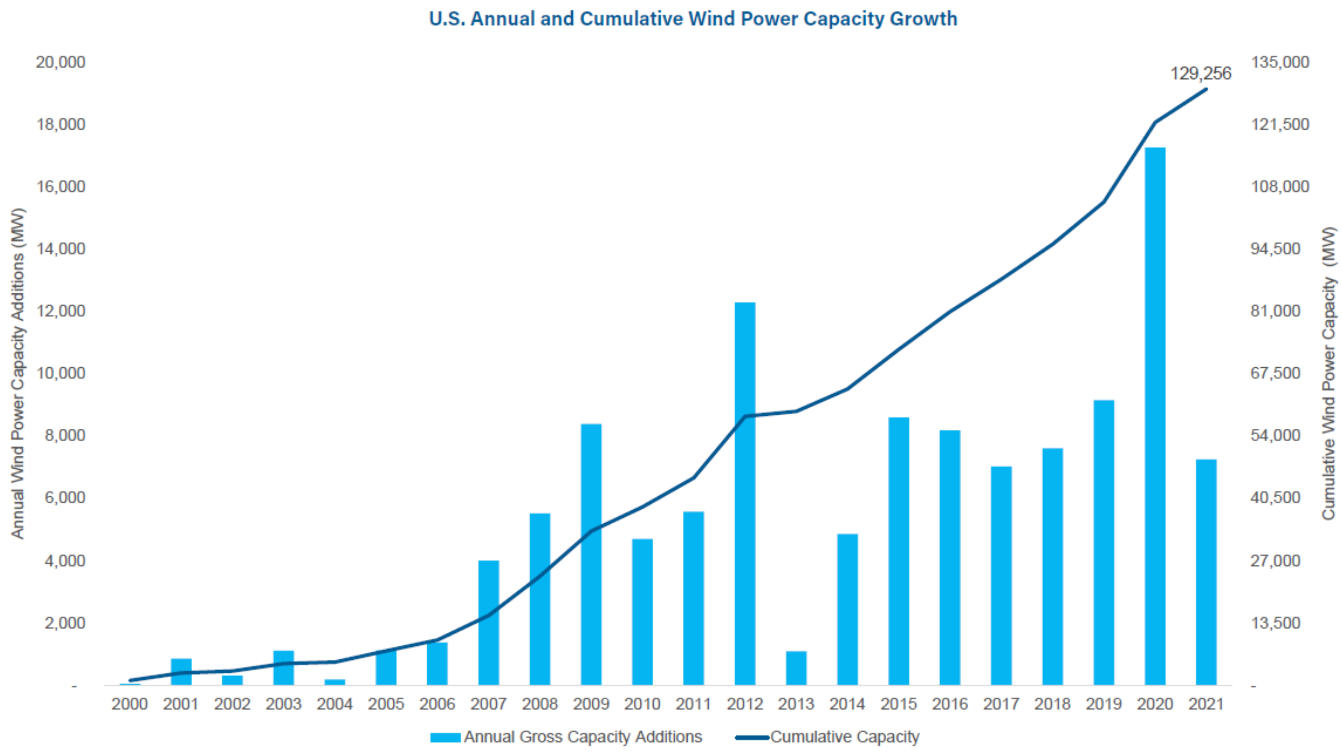
Source: American Clean Power Quarterly, 2021 Q3

# US Annual and Cumulative Wind Power Capacity Growth

LAND-BASED WIND ACTIVITY

## Over 1.5 GW of land-based wind online in Q3

- The wind industry installed 1,551 MW of new capacity. Total wind capacity installed in 2021 through September is now 7,248 MW.
- The volume of wind projects that came online in the third quarter is lower than pervious quarters this year, and lower than third quarter installations in recent years. This is due to projects originally planned to be online in the third quarter being pushed to a later date. Some developers cited supply chain issues as the reason for this delay.
- Ørsted's 367 MW Western Trail wind farm was the largest project to start commercial operation in the third quarter.
- Year-to-date the industry added 37 projects across 18 states totaling 7,248 MW, an increase of 15% compared to the first three quarters of 2020.
- The average size of wind projects installed in the third quarter was 129 MW.




Source: American Clean Power Quarterly, 2021 Q3

# US Wind Farm Locations

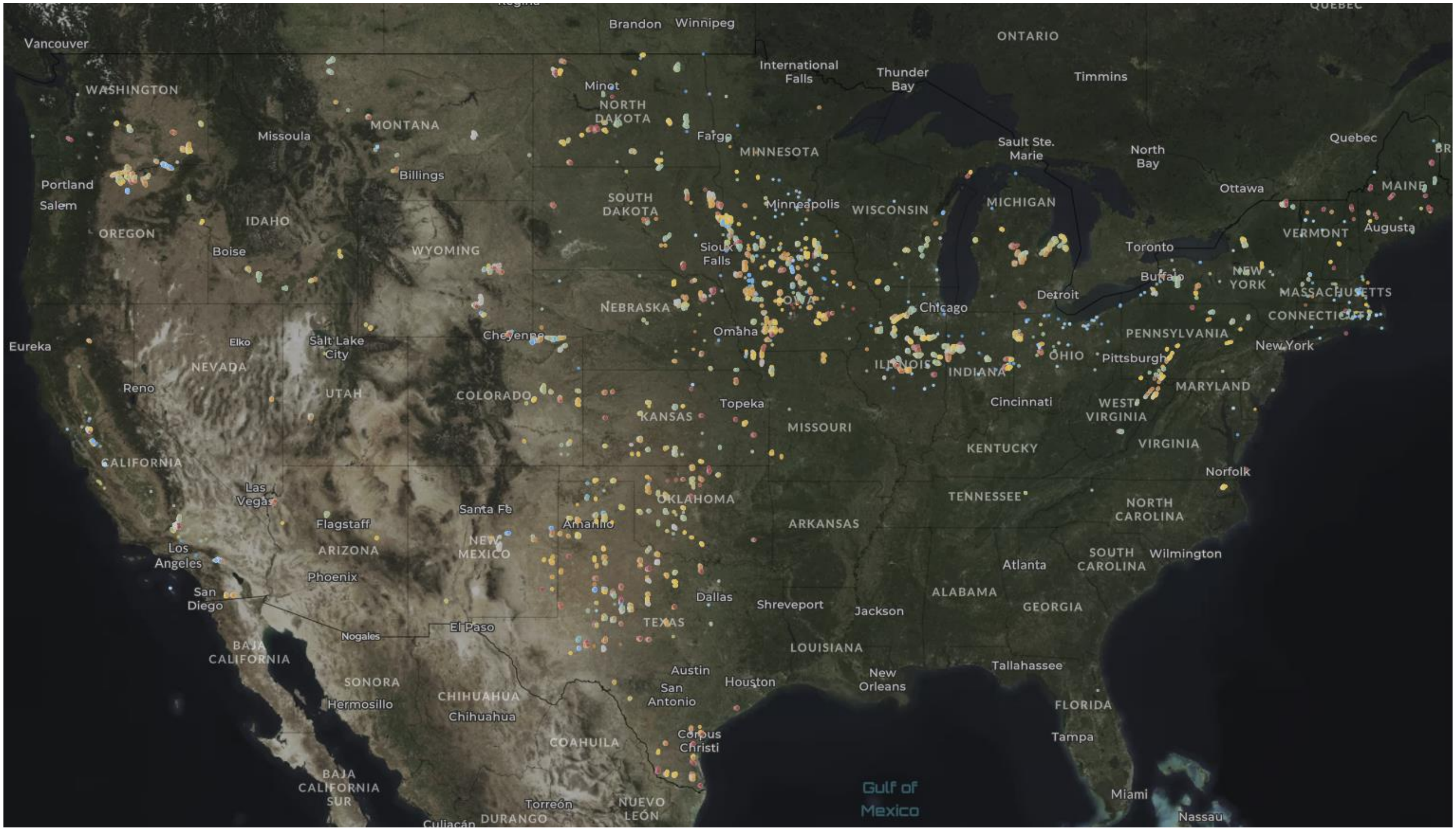



Image source: USGS at https://eerscmap.usgs.gov/uswtdb/