ECEN 615
Methods of Electric Power Systems Analysis

Lecture 3:Per Unit, Ybus, Power Flow

Prof. Tom Overbye

Dept. of Electrical and Computer Engineering

Texas A&M University

overbye@tamu.edu

Announcements

- Exam 1 is moved to Oct 13 (Oct 10 and 11 are fall break)
- Homework 1 is assigned today. It is due on Thursday September 8
- ERCOT is having a Virtual College Day on September 16 at 930am (Central Time); details at ERCOT.COM/CAREERS
  - To register you can go to www.ercot.com/careers/edp

ERCOT Virtual College Day

To register you can go to
www.ercot.com/careers/edp

Brief History of Electric Power

- First real practical uses of electricity began with the telegraph (1860's) and then arc lighting in the 1870's
- Early 1880's – Edison introduced Pearl Street dc system in Manhattan supplying 59 customers
- 1884 – Sprague produces practical dc motor
- 1885 – invention of transformer
- Mid 1880's – Westinghouse/Tesla introduce rival ac system
- Late 1880's – Tesla invents ac induction motor
- 1893 – Three-phase transmission line at 2.3 kV

History, cont'd

- 1896 – ac lines deliver electricity from hydro generation at Niagara Falls to Buffalo, 20 miles away; also 30kV line in Germany
- Early 1900's – Private utilities supply all customers in area (city); recognized as a natural monopoly; states step in to begin regulation
- By 1920's – Large interstate holding companies control most electricity systems
- 1935 – Congress passes Public Utility Holding Company Act to establish national regulation, breaking up large interstate utilities (repealed 2005)
  - This gave rise to electric utilities that only operated in one state
- 1935/6 – Rural Electrification Act brought electricity to rural areas
- 1930's – Electric utilities established as vertical monopolies

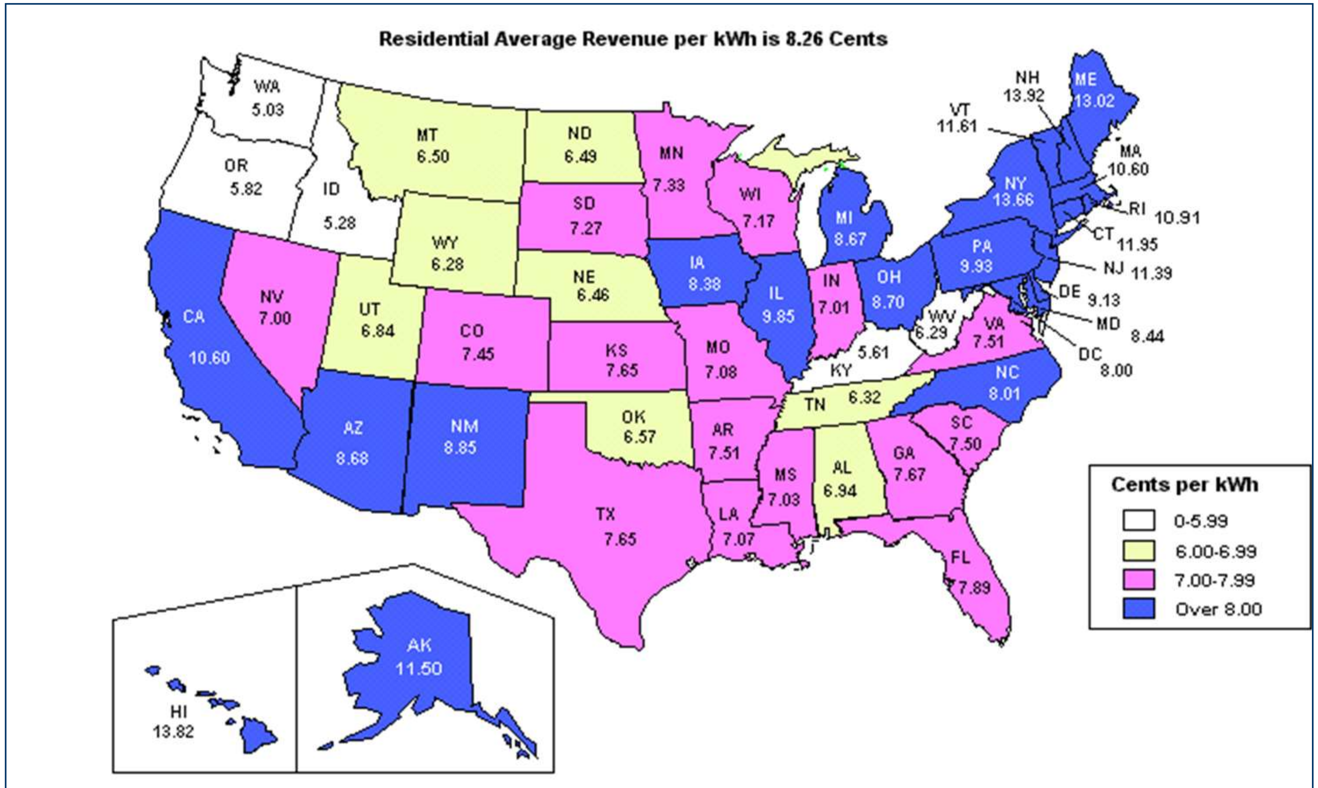
History, cont'd

- Frequency standardized in the 1930's
- During 1940's to 1960's utilities gradually interconnected their systems so by 1970 transmission lines crisscrossed North America, with voltages up to 765 kV
- 1970's brought inflation, increased fossil-fuel prices, calls for conservation and growing environmental concerns
- Increasing rates replaced decreasing ones
- As a result, U.S. Congress passed Public Utilities Regulator Policies Act (PURPA) in 1978, which mandated utilities must purchase power from independent generators located in their service territory (modified 2005)
- PURPA introduced some competition

History, cont'd

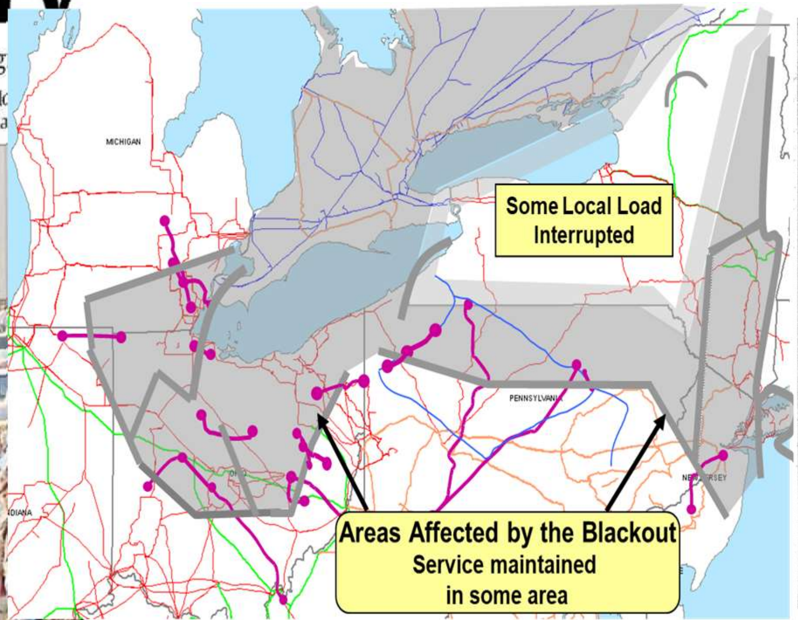
- Major opening of industry to competition occurred as a result of National Energy Policy Act of 1992
- This act mandated that utilities provide "nondiscriminatory" access to the high voltage transmission
- Goal was to set up true competition in generation
- Result over the last few years has been a dramatic restructuring of electric utility industry (for better or worse!)
- Energy Bill 2005 repealed PUHCA; modified PURPA
- Electric grid restructuring took place in the 1990's into the 2000's
  - Goal has been to reduce rates through the introduction of competition and consumer choice

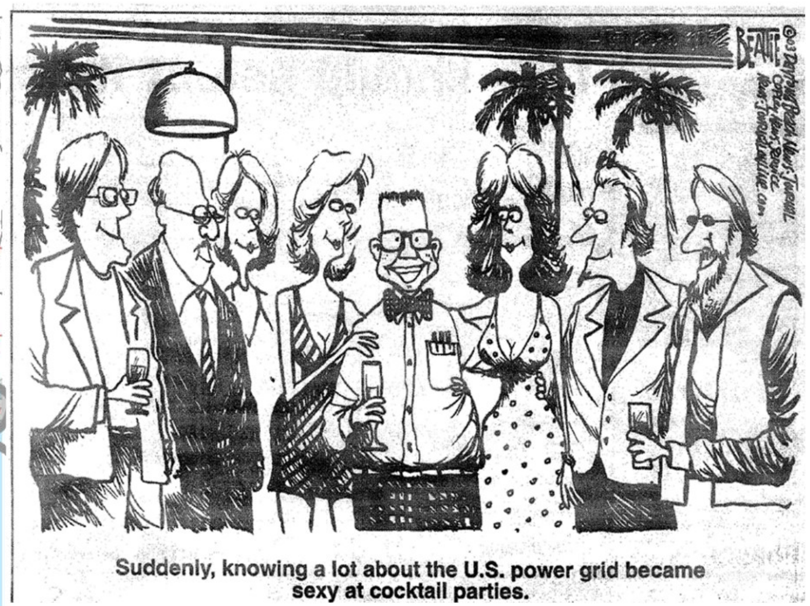
Historical State Variation in Electric Rates



KWh = Kilowatthour.
Note: The average revenue per kilowatthour of electricity sold is calculated by dividing revenue by sales.
Source: Energy information Administration, Form EIA-861, "Annual Electric Utility Report."

August 14th, 2003 Blackout

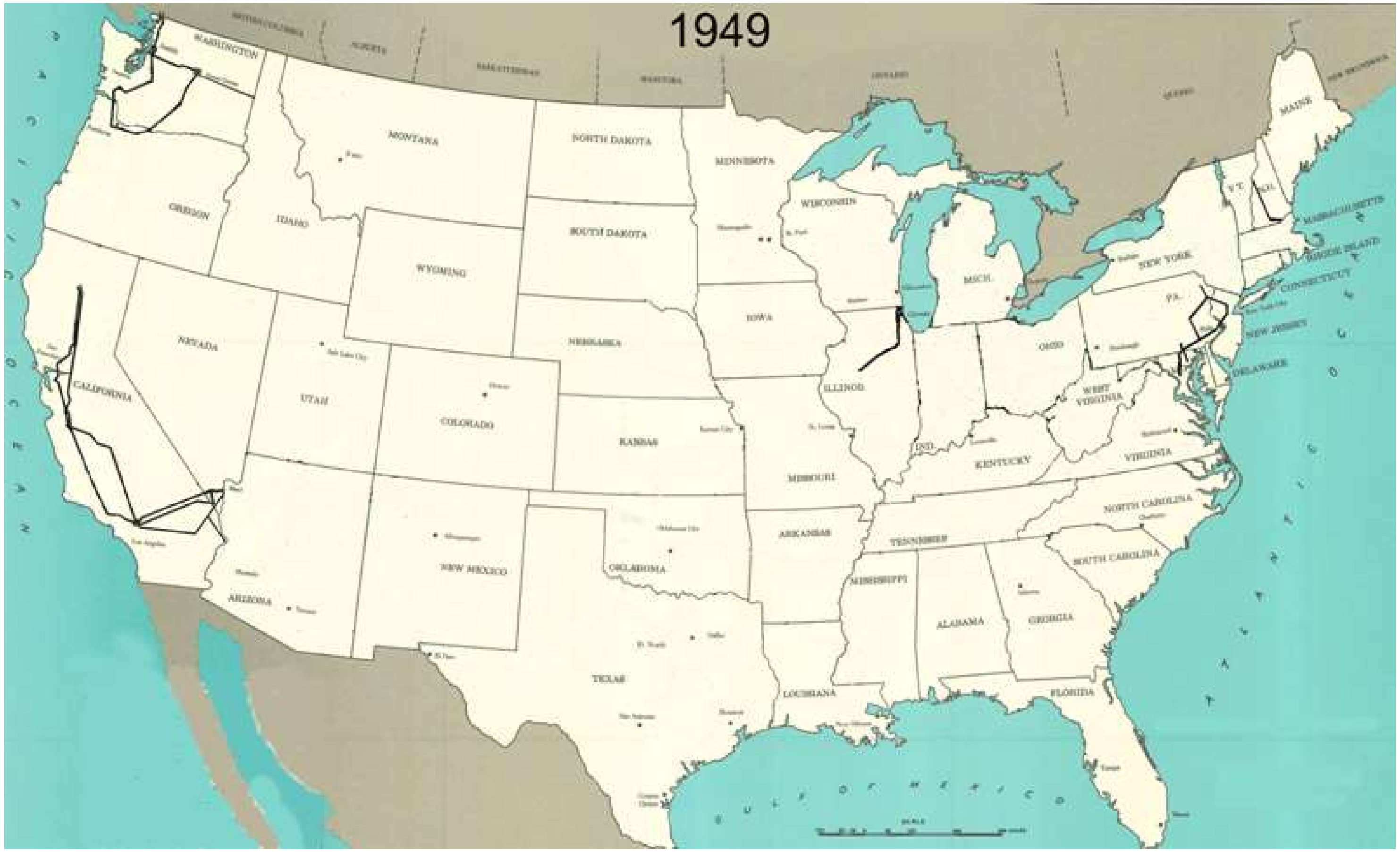




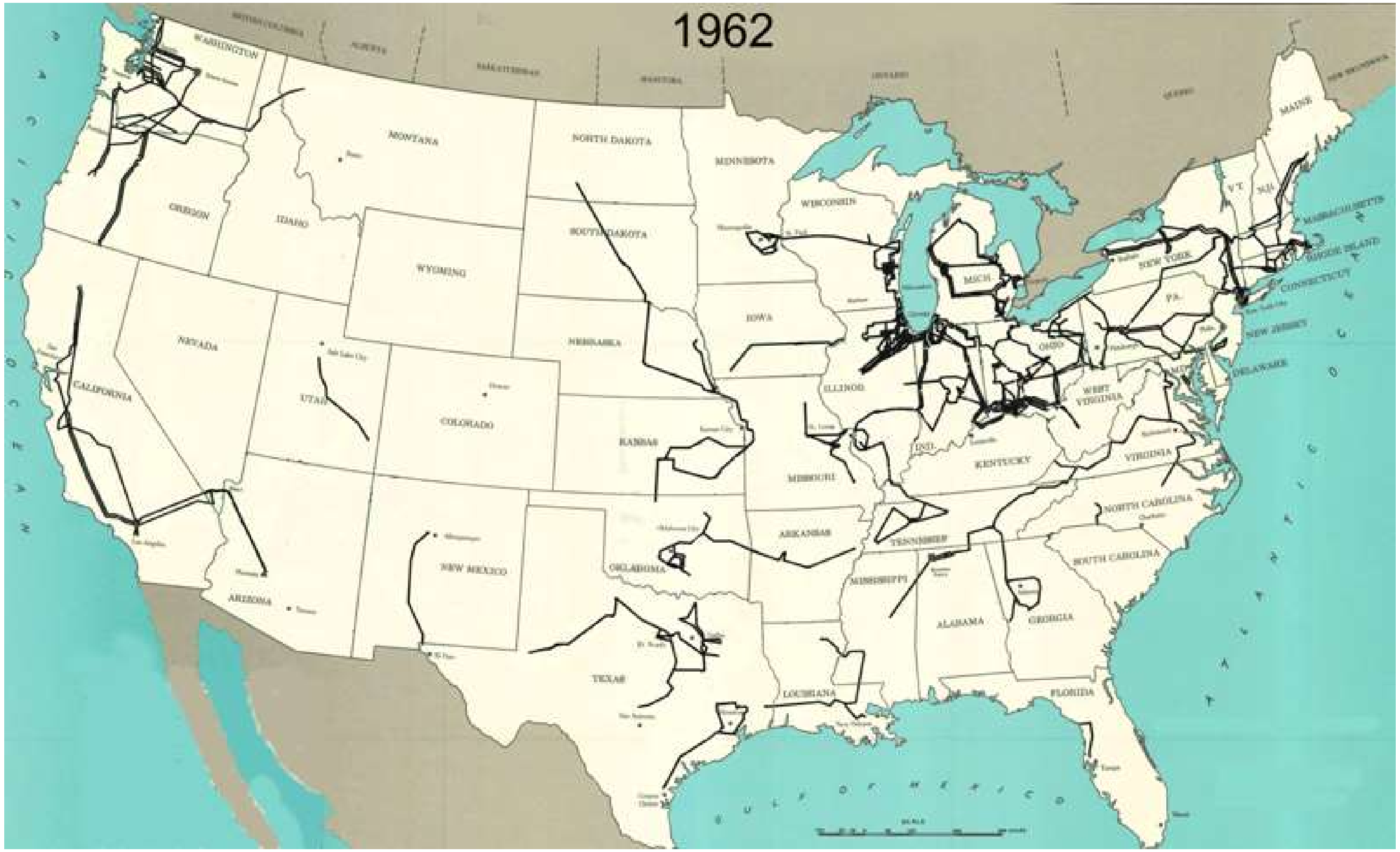
Above image from energy.gov, August 14, 2003 Blackout Final Report

I will talk about this event and the 2021 Texas one later in the semester

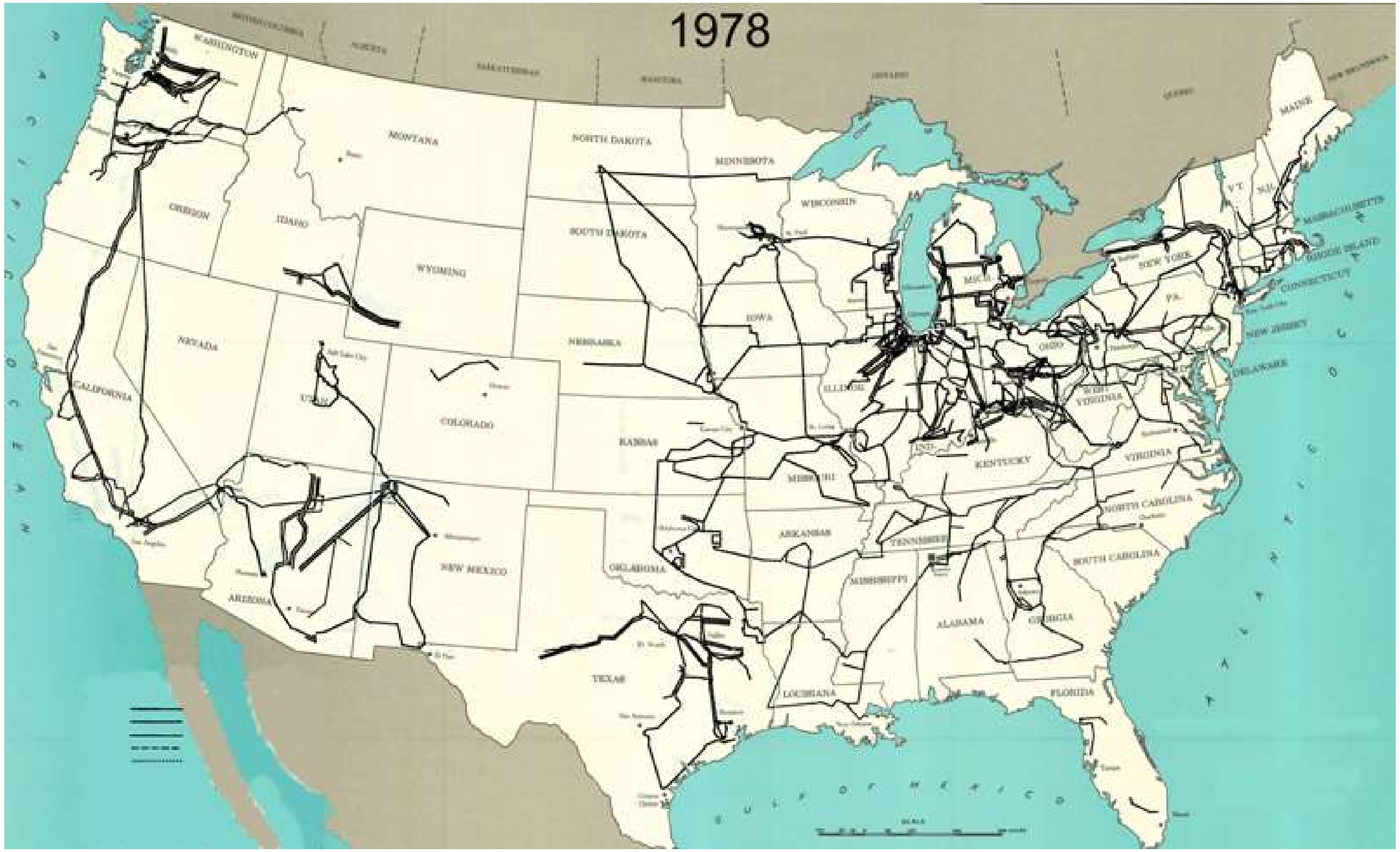
345 kV+ Transmission Growth at a Glance (From Jay Caspary)



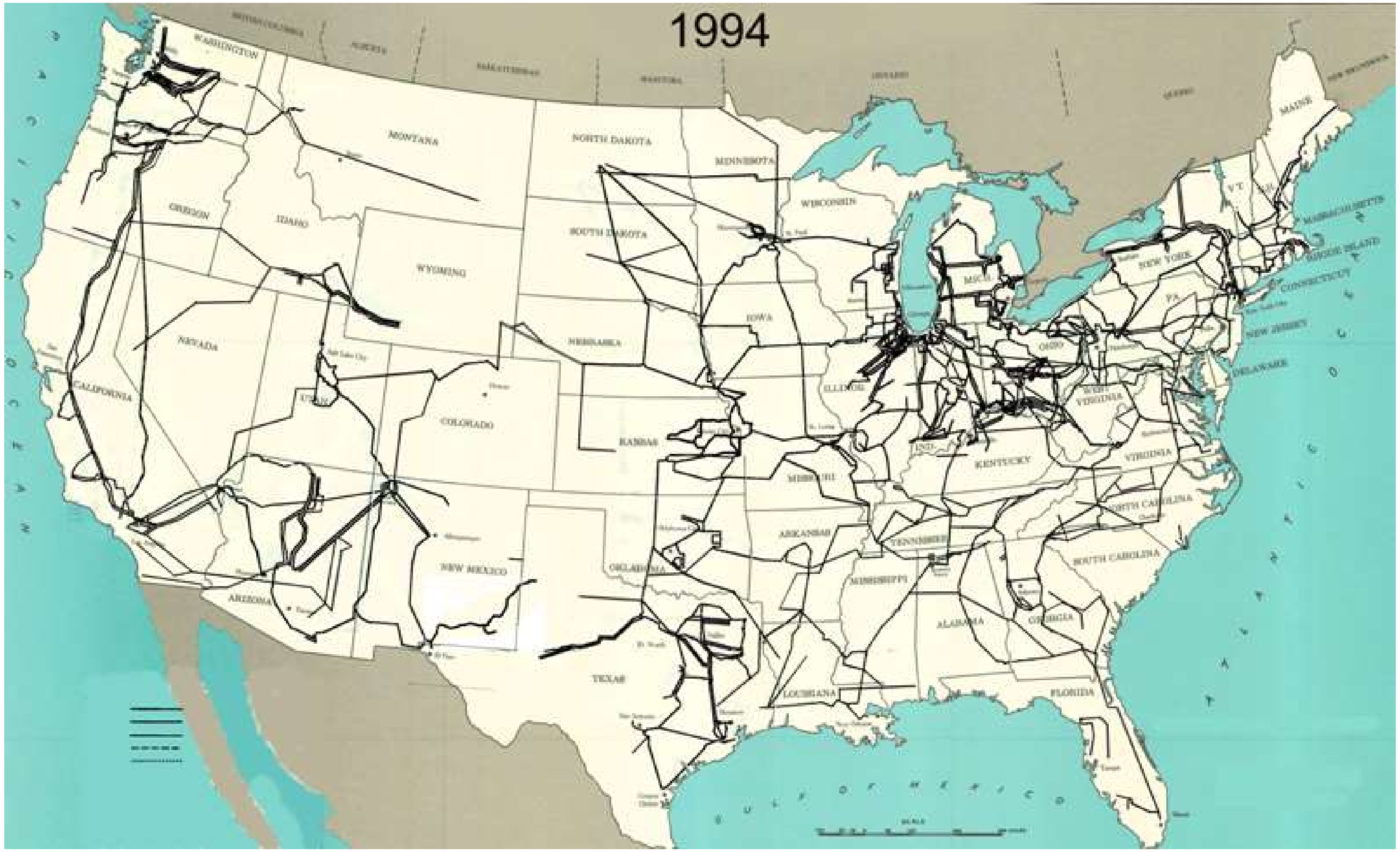
345 kV+ Transmission Growth at a Glance (From Jay Caspary)



345 kV+ Transmission Growth at a Glance (From Jay Caspary)



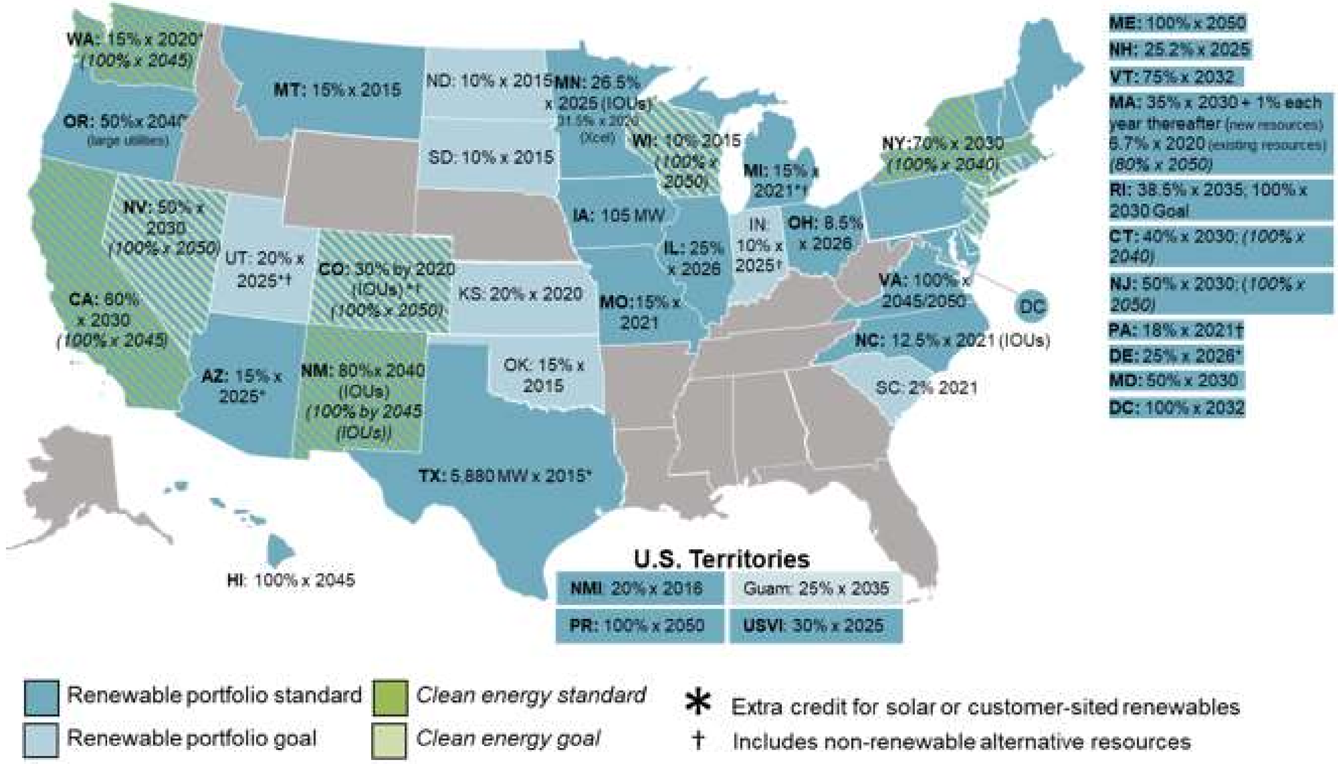
345 kV+ Transmission Growth at a Glance (From Jay Caspary)



The Smart Grid

- The term “Smart Grid” dates officially to the 2007 “Energy Independence and Security Act”, Title 13 (“Smart Grid”)
  - Use of digital information and control techniques
  - Dynamic grid optimization with cyber-security
  - Deployment of distributed resources including
  - Customer participation and smart appliances
  - Integration of storage including PHEVs
  - Development of interoperability standards

Renewable Portfolio Standards (September 2020)



TX is now 10 GW by 2025 which we've met several times over; CA is 60% by 2030 and 100% by 2045

Image source: http://www.dsireusa.org/

See also www.ncsl.org/research/energy/renewable-portfolio-standards.aspx

Interconnected Power System Basic Characteristics

- Three – phase AC systems:
  - generation and transmission equipment is usually three phase
  - industrial loads are three phase
  - residential and commercial loads are single phase and distributed equally among the phases; consequently, a balanced three – phase system results
- Synchronous machines have traditionally generated electricity, though this is rapidly changing with the addition of more inverter-connected generation (such as solar and most wind)
- Interconnection transmits power over a wider region with subsystems operating at different voltage levels

Quick Review of Per Unit (From Undergrad Class)

- A key problem in analyzing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer impedances to the different sides of the transformers
- This problem is avoided by a normalization of all variables.
- This normalization is known as per unit analysis.

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Per Unit Conversion Procedure, Single-Phase

1. Pick a 1ϕ VA base for the entire system, S_B
2. Pick a voltage base for each different voltage level, V_B. Voltage bases are related by transformer turns ratios. Voltages are line to neutral.
3. Calculate the impedance base, $Z_B = (V_B)^2/S_B$
4. Calculate the current base, $I_B = V_B/Z_B$
5. Convert actual values to per unit

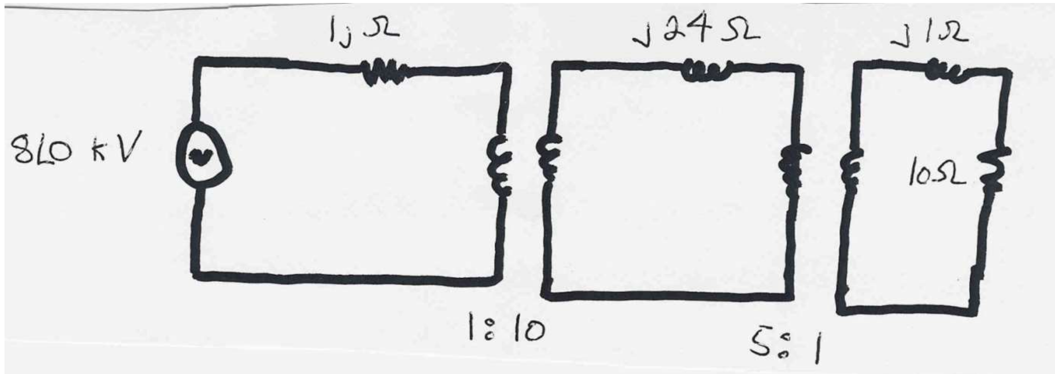
Note, per unit conversion on affects magnitudes, not the angles. Also, per unit quantities no longer have units (i.e., a voltage is 1.0 p.u., not 1 p.u. volts)

Per Unit Solution Procedure

1. Convert to per unit (p.u.) (many problems are already in per unit)
2. Solve
3. Convert back to actual as necessary

Per Unit Example

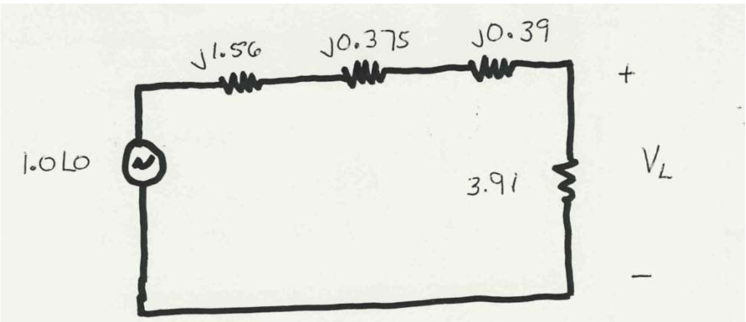
- Solve for the current, load voltage and load power in the circuit shown below using per unit analysis with an S_B of 100 MVA, and voltage bases of 8 kV, 80 kV and 16 kV



$$Z_B^{Left} = \frac{8kV^2}{100MVA} = 0.64\Omega$$

$$Z_B^{Middle} = \frac{80kV^2}{100MVA} = 64\Omega$$

$$Z_B^{Right} = \frac{16kV^2}{100MVA} = 2.56\Omega$$



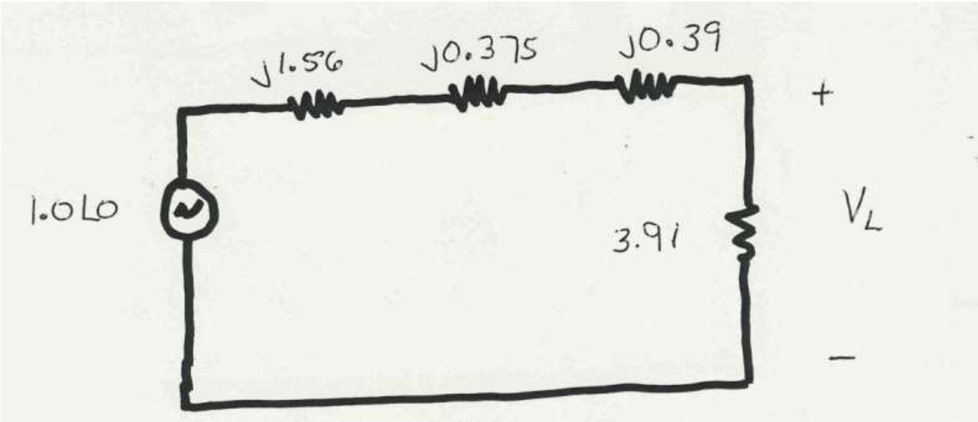
Same circuit, with values expressed in per unit.

Per Unit Example, cont'd

$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ \text{ p.u. (not amps)}$$

$$V_L = 1.0\angle 0^\circ - 0.22\angle -30.8^\circ \times 2.327\angle 90^\circ$$

$$= 0.859\angle -30.8^\circ \text{ p.u.}$$



$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

Per Unit Example, cont'd

- To convert back to actual values just multiply the per unit values by their per unit base

$$V_{\mathrm{L}}^{\mathrm{Actual}} = 0.859\angle -30.8° \times 16 \text{ kV} = 13.7\angle -30.8° \text{ kV}$$

$$S_{\mathrm{L}}^{\mathrm{Actual}} = 0.189\angle 0° \times 100 \text{ MVA} = 18.9\angle 0° \text{ MVA}$$

$$S_{\mathrm{G}}^{\mathrm{Actual}} = 0.22\angle 30.8° \times 100 \text{ MVA} = 22.0\angle 30.8° \text{ MVA}$$

$$\mathrm{I}_{\mathrm{B}}^{\mathrm{Middle}} = \frac{100 \text{ MVA}}{80 \text{ kV}} = 1250 \text{ Amps}$$

$$\mathrm{I}_{\mathrm{Middle}}^{\mathrm{Actual}} = 0.22\angle -30.8° \times 1250 \text{ Amps} = 275\angle -30.8° \text{ A}$$

Procedure for Balanced Three-Phase Circuits

Procedure is very similar to 1ϕ except we use a 3ϕ VA base, and use line to line voltage bases

1. Pick a 3ϕ VA base for the entire system,
2. Pick a voltage base for each different voltage level, V_B. Voltages are line to line.
3. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\,V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

Three-Phase Per Unit, cont'd

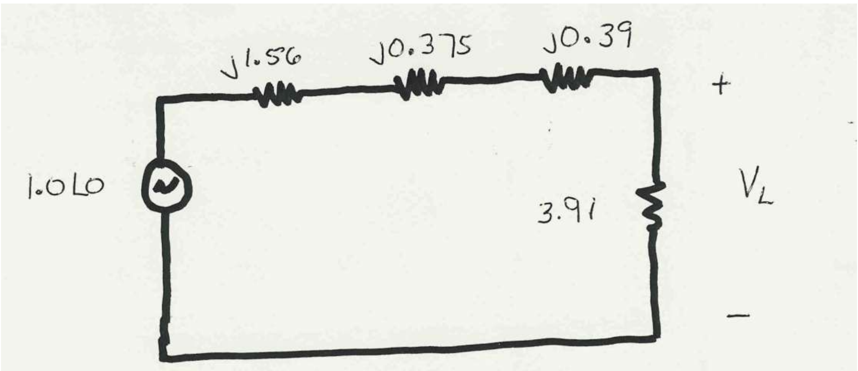
4. Calculate the current base, I_B

$$\mathrm{I}_{\mathrm{B}}^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\, V_{B,LL}} = \frac{3\, S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\, V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = \mathrm{I}_{\mathrm{B}}^{1\phi}$$

5. Convert actual values to per unit

Three-Phase Per Unit Example

Solve for the current, load voltage and load power in the previous circuit, assuming a 3ϕ power base of **300 MVA**, and line to line voltage bases of 13.8 kV, 138 kV and 27.6 kV (square root of 3 larger than the 1ϕ example voltages). Also assume the generator is Y-connected so its line to line voltage is 13.8 kV.



Convert to per unit as before. Note the system is exactly the same!

Three-Phase Per Unit Example, cont.

$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ \text{ p.u. (not amps)}$$

$$\mathrm{V_L} = 1.0\angle 0^\circ - 0.22\angle -30.8^\circ \times 2.327\angle 90^\circ$$

$$= 0.859\angle -30.8^\circ \text{ p.u.}$$

Again, analysis is exactly the same!

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

Three-Phase Per Unit Example, cont'd

- Differences appear when we convert back to actual values

$$V_{\mathrm{L}}^{\mathrm{Actual}} = 0.859\angle -30.8° \times 27.6 \text{ kV} = 23.8\angle -30.8° \text{ kV}$$

$$S_{\mathrm{L}}^{\mathrm{Actual}} = 0.189\angle 0° \times 300 \text{ MVA} = 56.7\angle 0° \text{ MVA}$$

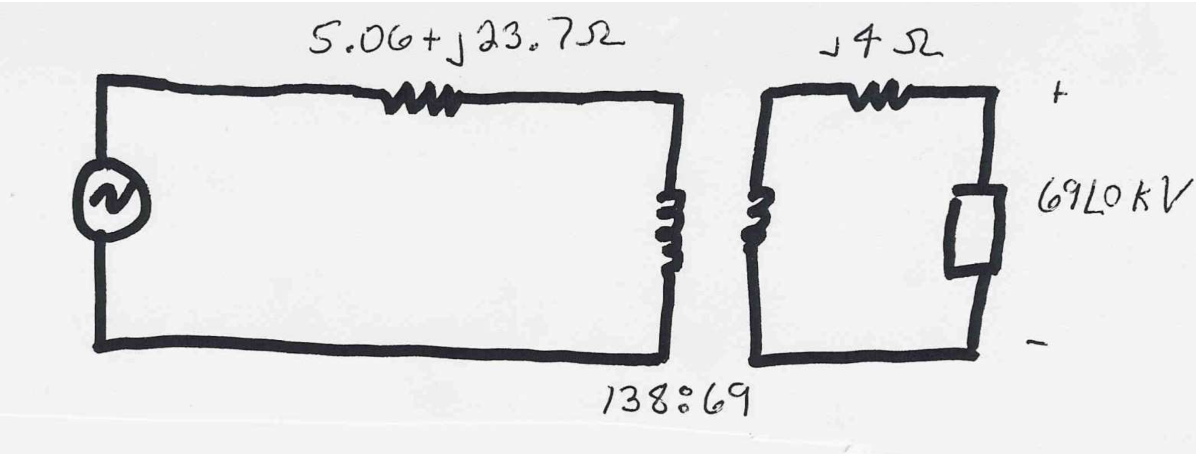
$$S_{\mathrm{G}}^{\mathrm{Actual}} = 0.22\angle 30.8° \times 300 \text{ MVA} = 66.0\angle 30.8° \text{ MVA}$$

$$\mathrm{I}_{\mathrm{B}}^{\mathrm{Middle}} = \frac{300 \text{ MVA}}{\sqrt{3}\, 138 \text{ kV}} = 1250 \text{ Amps} \quad \text{(same current!)}$$

$$\mathrm{I}_{\mathrm{Middle}}^{\mathrm{Actual}} = 0.22\angle -30.8° \times 1250 \text{ Amps} = 275\angle -30.8° \text{ A}$$

Three-Phase Per Unit Example 2

- Assume a 3ϕ load of 100+j50 MVA with V_{LL} of 69 kV is connected to a source through the below network:



What is the supply current and complex power?

Answer: I=467 amps, S = 103.3 + j76.0 MVA

Power Flow Analysis

- We now have the necessary models to start to develop the power system analysis tools
- The most common power system analysis tool is the power flow (also known sometimes as the load flow)
  - power flow determines how the power flows in a network
  - also used to determine all bus voltages and all currents
  - because of constant power models, power flow is a nonlinear analysis technique
  - power flow is a steady-state analysis tool

Linear versus Nonlinear Systems

A function **H** is linear if

$$\mathbf{H}(\alpha_1\mu_1 + \alpha_2\mu_2) = \alpha_1\mathbf{H}(\mu_1) + \alpha_2\mathbf{H}(\mu_2)$$

That is

1) the output is proportional to the input
2) the principle of superposition holds

Linear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}$

$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2) = c\mathbf{x}_1 + c\,\mathbf{x}_2$$

Nonlinear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}^2$

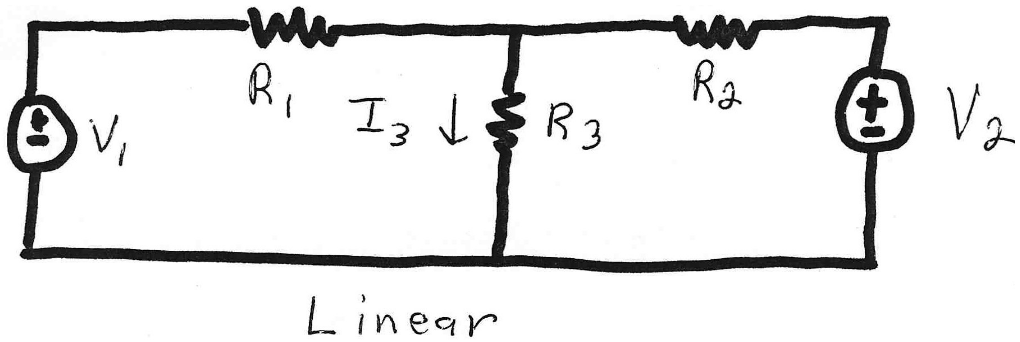
$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2)^2 \neq (c\mathbf{x}_1)^2 + (c\,\mathbf{x}_2)^2$$

Linear Power System Elements

Resistors, inductors, capacitors, independent voltage sources and current sources are linear circuit elements

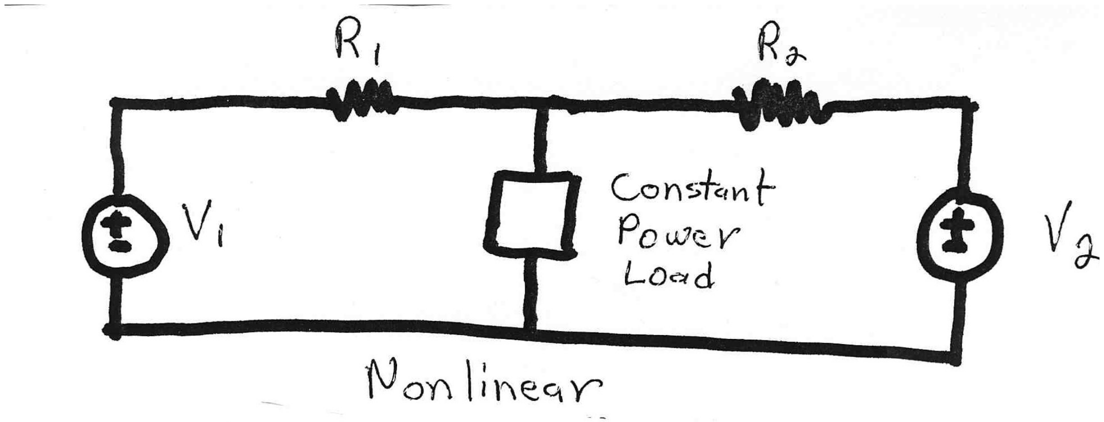
$$V = R I \qquad V = j\omega L\, I \qquad V = \frac{1}{j\omega C} I$$

Such systems may be analyzed by superposition



Nonlinear System Example

- Constant power loads and generator injections are nonlinear and hence systems with these elements can not be analyzed by superposition



Nonlinear problems can be very difficult to solve, and usually require an iterative approach

Nonlinear Systems May Have Multiple Solutions or No Solution

Example 1: $x^2 - 2 = 0$ has solutions $x = \pm 1.414\ldots$

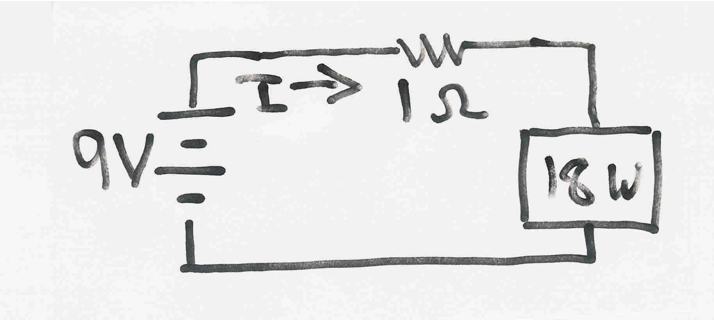
Example 2: $x^2 + 2 = 0$ has no real solution

$f(x) = x^2 - 2$ $\qquad\qquad$ $f(x) = x^2 + 2$

two solutions where $f(x) = 0$ $\qquad$ no solution $f(x) = 0$

Multiple Solution Example

- The dc system shown below has two solutions:



where the 18 watt load is a resistive load

What is the maximum P_{Load}?

The equation we're solving is

$$I^2 R_{Load} = \left(\frac{9 \text{ volts}}{1\Omega + R_{Load}}\right)^2 R_{Load} = 18 \text{ watts}$$

One solution is $R_{Load} = 2\Omega$

Other solution is $R_{Load} = 0.5\Omega$

Bus Admittance Matrix or Y_{bus}

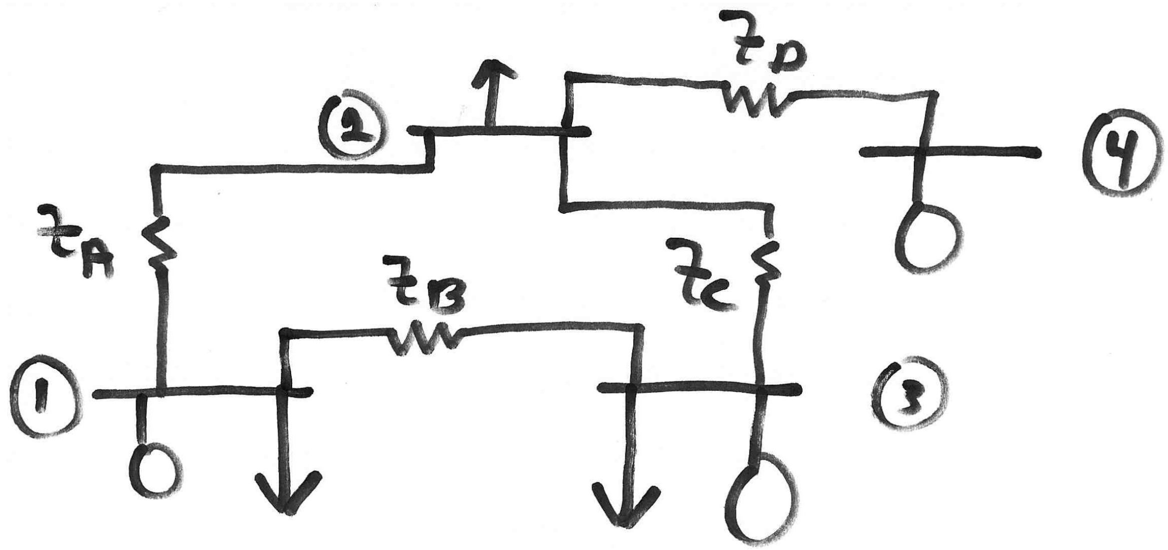
- First step in solving the power flow is to create what is known as the bus admittance matrix, often call the $\mathbf{Y}_{bus}$.
- The $\mathbf{Y}_{bus}$ gives the relationships between all the bus current injections, $\mathbf{I}$, and all the bus voltages, $\mathbf{V}$,

$$\mathbf{I} = \mathbf{Y}_{bus}\,\mathbf{V}$$

- The $\mathbf{Y}_{bus}$ is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances

Y_{bus} Example

Determine the bus admittance matrix for the network shown below, assuming the current injection at each bus i is $I_i = I_{Gi} - I_{Di}$ where I_{Gi} is the current injection into the bus from the generator and I_{Di} is the current flowing into the load



Y_{bus} Example, cont'd

By KCL at bus 1 we have

$$I_1 = I_{G1} - I_{D1}$$

$$I_1 = I_{12} + I_{13} = \frac{V_1 - V_2}{Z_A} + \frac{V_1 - V_3}{Z_B}$$

$$I_1 = (V_1 - V_2)Y_A + (V_1 - V_3)Y_B \qquad (\text{with } \mathrm{Y_j} = \frac{1}{Z_j})$$

$$= (Y_A + Y_B)V_1 - Y_A V_2 - Y_B V_3$$

Similarly

$$I_2 = I_{21} + I_{23} + I_{24}$$

$$= -Y_A V_1 + (Y_A + Y_C + Y_D)V_2 - Y_C V_3 - Y_D V_4$$

Y_{bus} Example, cont'd

We can get similar relationships for buses 3 and 4

The results can then be expressed in matrix form

$$\mathbf{I} = \mathbf{Y}_{bus}\mathbf{V}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_A & -Y_B & 0 \\ -Y_A & Y_A + Y_C + Y_D & -Y_C & -Y_D \\ -Y_B & -Y_C & Y_B + Y_C & 0 \\ 0 & -Y_D & 0 & Y_D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

For a system with n buses the Y_{bus} is an n by n symmetric matrix (i.e., one where $A_{ij} = A_{ji}$); however this will not be true in general when we consider phase shifting transformers

Y_{bus} General Form

- The diagonal terms, Y_{ii}, are the self admittance terms, equal to the sum of the admittances of all devices incident to bus i.
- The off-diagonal terms, Y_{ij}, are equal to the negative of the sum of the admittances joining the two buses.
- With large systems Y_{bus} is a sparse matrix (that is, most entries are zero)
- Shunt terms, such as with the π line model, only affect the diagonal terms.