# ECEN 667
# Power System Stability

## Lecture 15: PIDs, Governors, Transient Stability Solutions

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# Announcements

- Read Chapter 7

- Homework 5 is assigned today, due on Oct 31

# PID Controllers

- Governors and exciters often use proportional-integral-derivative (PID) controllers
  - Developed in 1890's for automatic ship steering by observing the behavior of experienced helmsman
- PIDs combine
  - Proportional gain, which produces an output value that is proportional to the current error
  - Integral gain, which produces an output value that varies with the integral of the error, eventually driving the error to zero
  - Derivative gain, which acts to predict the system behavior. This can enhance system stability, but it can be quite susceptible to noise

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# PID Controller Characteristics

- Four key characteristics of control response are 1) rise time, 2) overshoot, 3) settling time and 4) steady-state errors
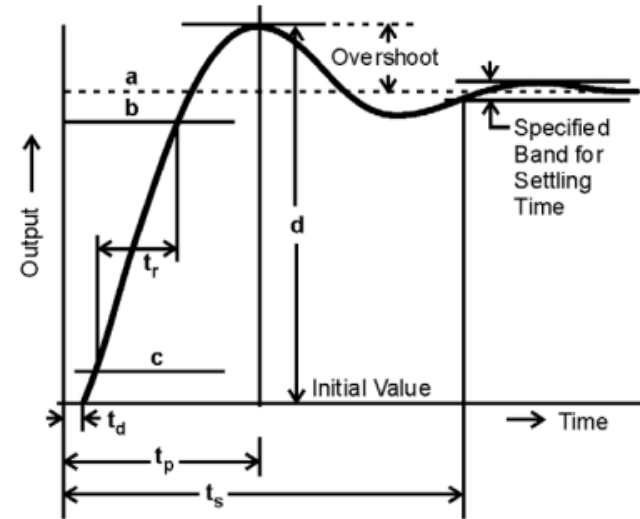


a - Steady-state value
b - 90% of steady-state value
c - 10% of steady-state value
d - peak value

$t_d$ - Delay time
$t_p$ - Time to reach peak value
$t_s$ - Settling time
$t_r$ - Rise time

**Figure F.1—Typical dynamic response of a turbine governing system to a step change**

| Increasing Gain | Rise Time | Overshoot | Setting Time | Steady-State Error |
|---|---|---|---|---|
| $K_P$ | Decreases | Increases | Little impact | Decreases |
| $K_I$ | Decreases | Increases | Increases | Zero |
| $K_D$ | Little impact | Decreases | Decreases | Little Impact |

Image source: Figure F.1, IEEE Std 1207-2011

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# PID Example: Car Cruise Control

- Say we wish to implement cruise control on a car by controlling the throttle position
  - Assume force is proportional to throttle position
  - Error is difference between actual speed and desired speed
- With just proportional control we would never achieve the desired speed because with zero error the throttle position would be at zero
- The integral term will make sure we stay at the desired point
- With derivative control we can improve control, but as noted it can be sensitive to noise

# HYG3

- The HYG3 models has a PID or a double derivative
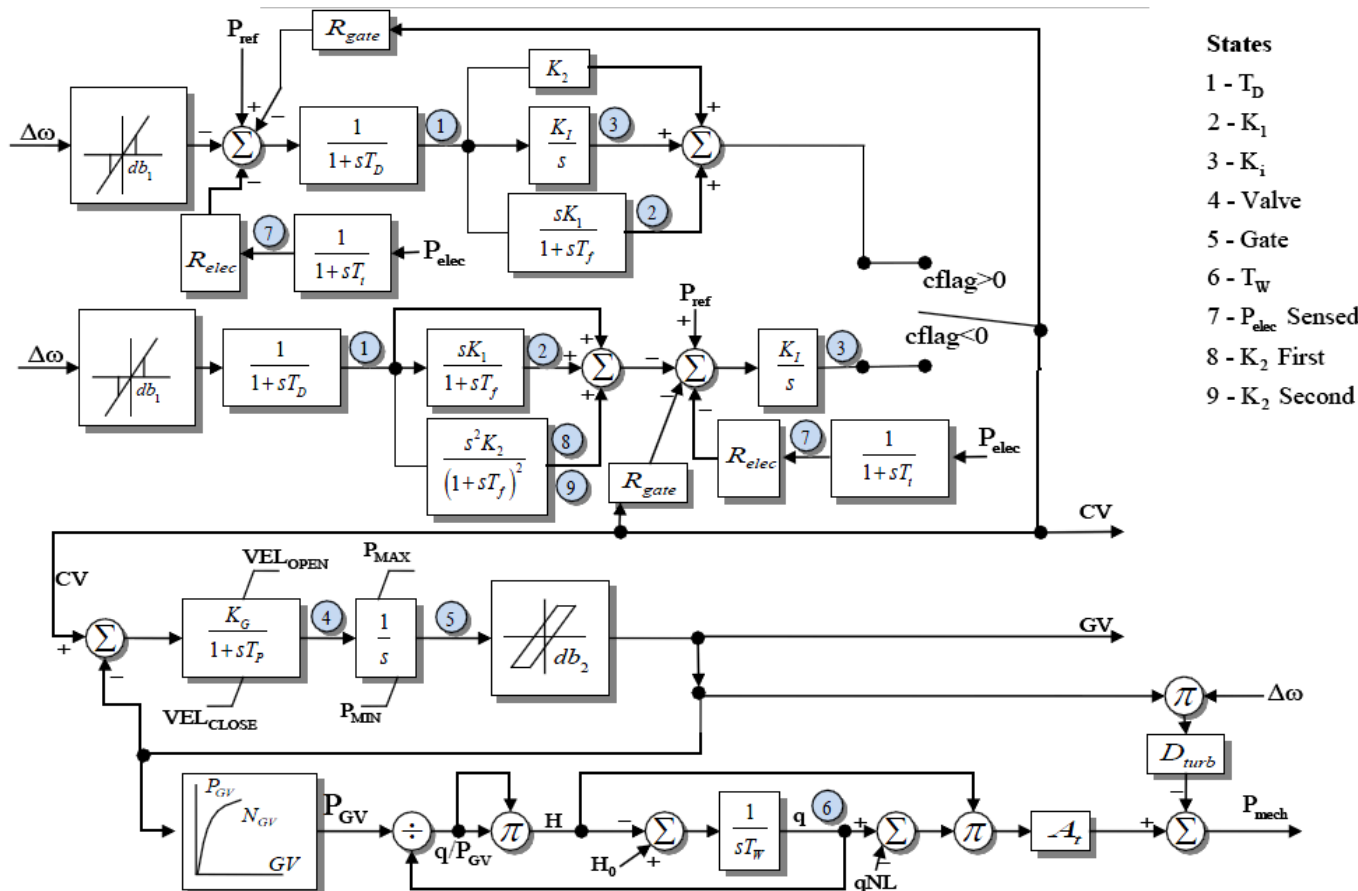


**States**
1 - $T_D$
2 - $K_1$
3 - $K_i$
4 - Valve
5 - Gate
6 - $T_W$
7 - $P_{elec}$ Sensed
8 - $K_2$ First
9 - $K_2$ Second

Looks more complicated than it is since depending on cflag only one of the upper paths is used
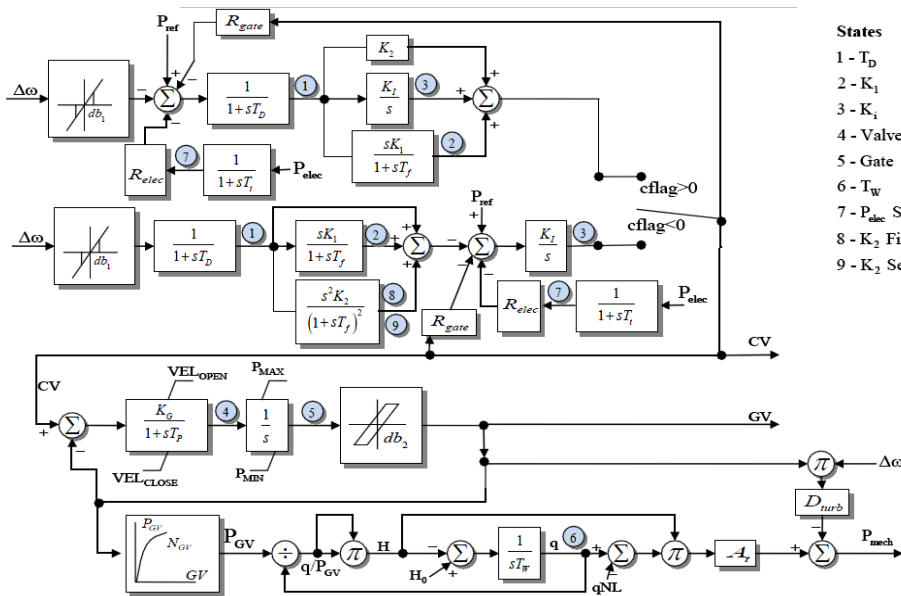
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# Tuning PID Controllers

- Tuning PID controllers can be difficult, and there is no single best method
  - Conceptually simple since there are just three parameters, but there can be conflicting objectives (rise time, overshoot, setting time, error)
- One common approach is the Ziegler-Nichols method
  - First set $K_I$ and $K_D$ to zero, and increase $K_P$ until the response to a unit step starts to oscillate (marginally stable); define this value as $K_u$ and the oscillation period at $T_u$
  - For a P controller set $K_p = 0.5K_u$
  - For a PI set $K_P = 0.45\ K_u$ and $K_I = 1.2*\ K_p/T_u$
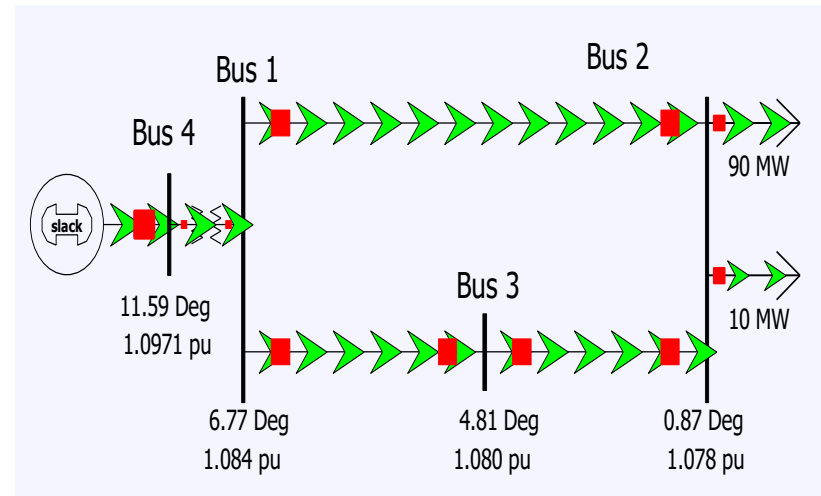  - For a PID set $K_P=0.6\ K_u$, $K_I=2*\ K_p/T_u$, $K_D=K_pT_u/8$

# Tuning PID Controller Example

- Assume four bus case with infinite bus replaced by load, and gen 4 has a HYG3 governor with cflag > 0; tune $K_P$, $K_I$ and $K_D$ for full load to respond to a 10% drop in load ($K_2$, $K_I$, $K_1$ in the model; assume $T_f$=0.1)
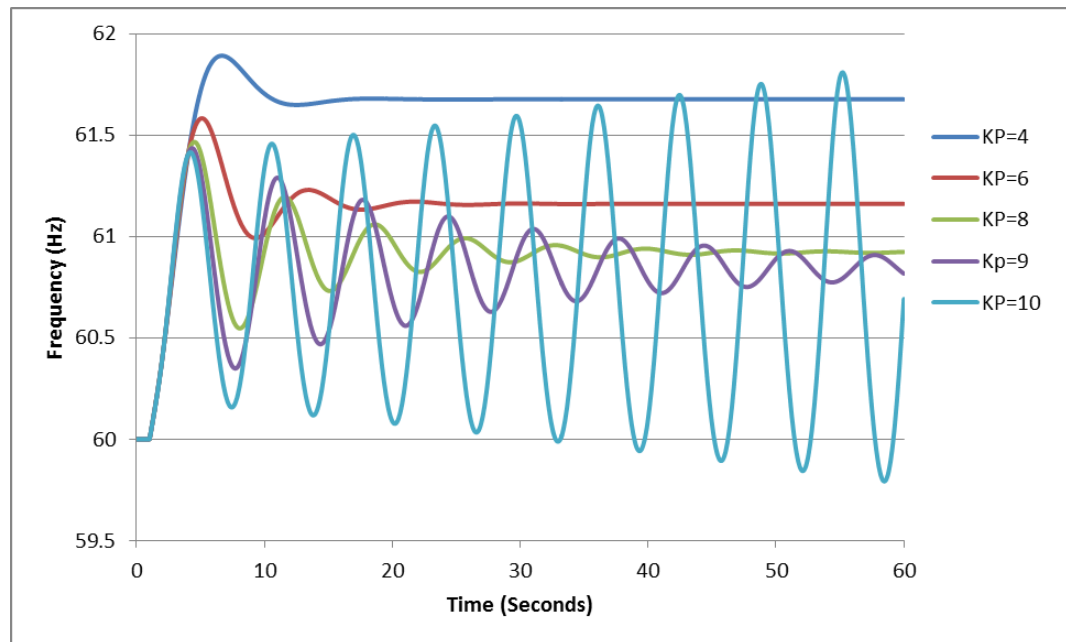


Case name: B4_PIDTuning

# Tuning PID Controller Example

- Based on testing, Ku is about 9.5 and Tu 6.4 seconds

- Using Ziegler-Nichols a good P value 4.75, is good PI values are $K_P = 4.3$ and $K_I = 0.8$, while good PID values are $K_P = 5.7$, $K_I = 1.78$, $K_D = 4.56$
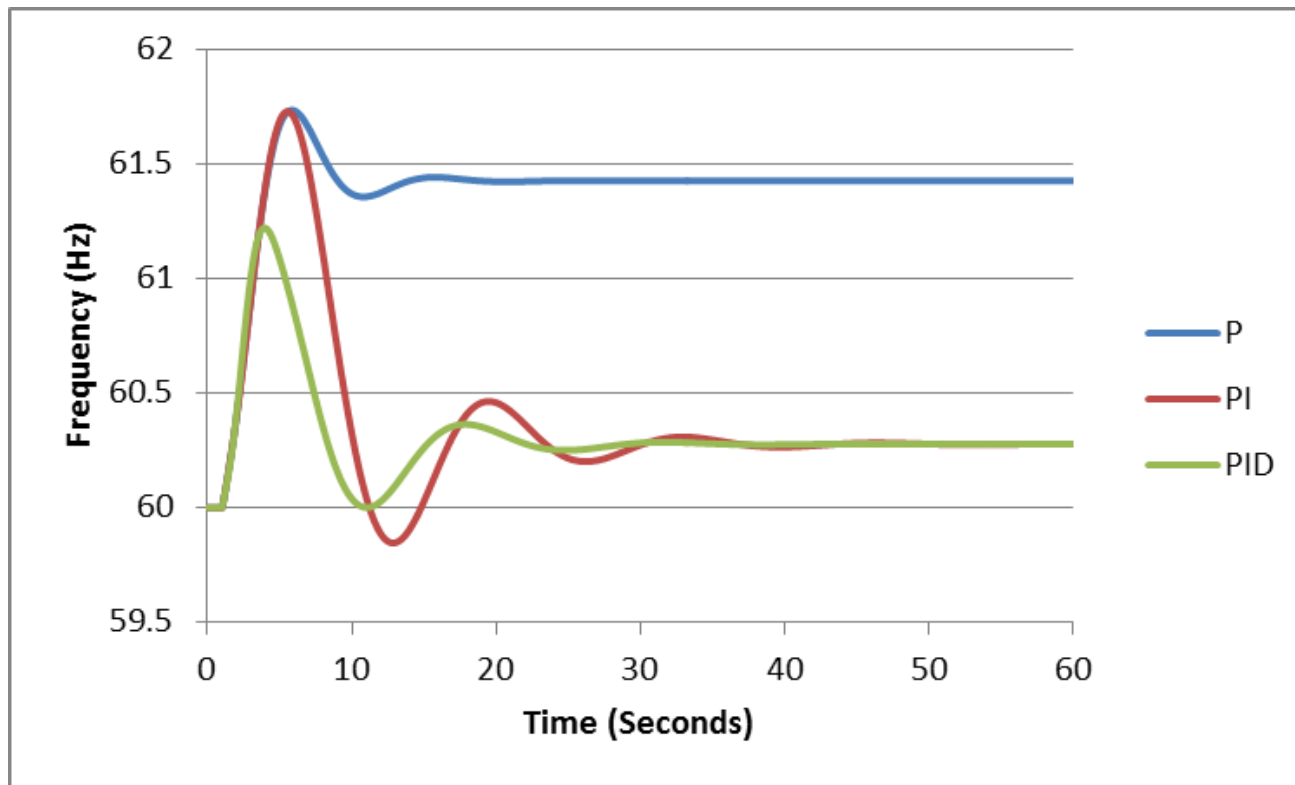


Further details on tuning are covered in IEEE Std. 1207-2011
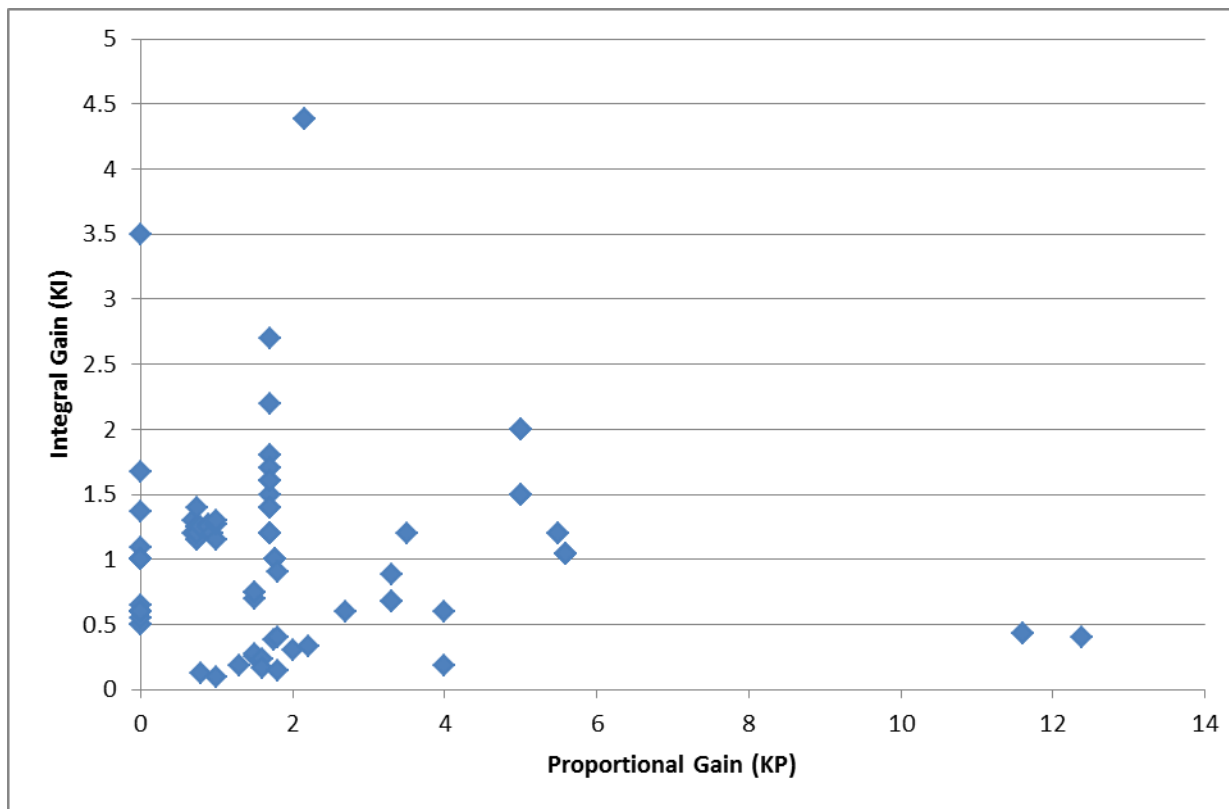
# Tuning PID Controller Example

- Figure shows the Ziegler-Nichols for a P, PI and PID controls.  Note, this is for stand-alone, not interconnected operation

# Example $K_I$ and $K_P$ Values

- Figure shows example $K_I$ and $K_P$ values from an actual system case



About 60% of the models also had a derivative term with an average value of 2.8, and an average $T_D$ of 0.04 sec

# Non-windup Limits

- An important open question is whether the governor PI controllers should be modeled with non-windup limits
  - Currently models show no limit, but transient stability verification seems to indicate limits are being enforced
- This could be an issue if frequency goes low, causing governor PI to "windup" and then goes high (such as in an islanding situation)
  - How fast governor backs down depends on whether the limit winds up

# PI Non-windup Limits

- There is not a unique way to handle PI non-windup limits; the below shows two approaches from IEEE Std 425.5



$y > A$, then $x = A$ and $dz/dt = 0$
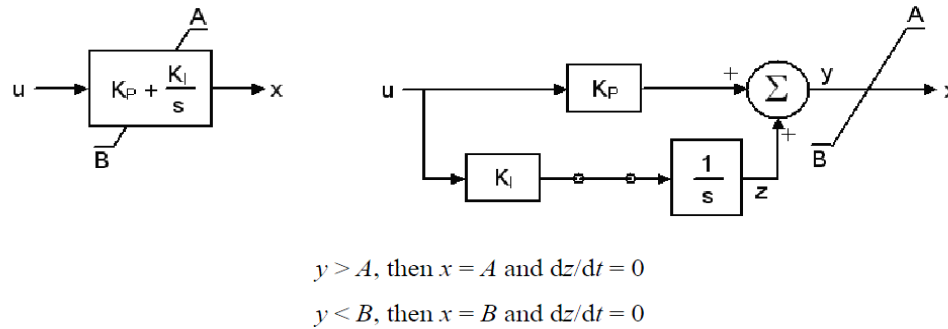
$y < B$, then $x = B$ and $dz/dt = 0$

**Figure E.7—Non-windup proportional-integral block**



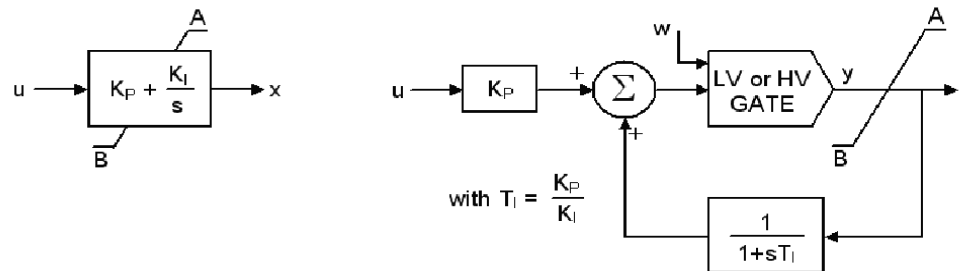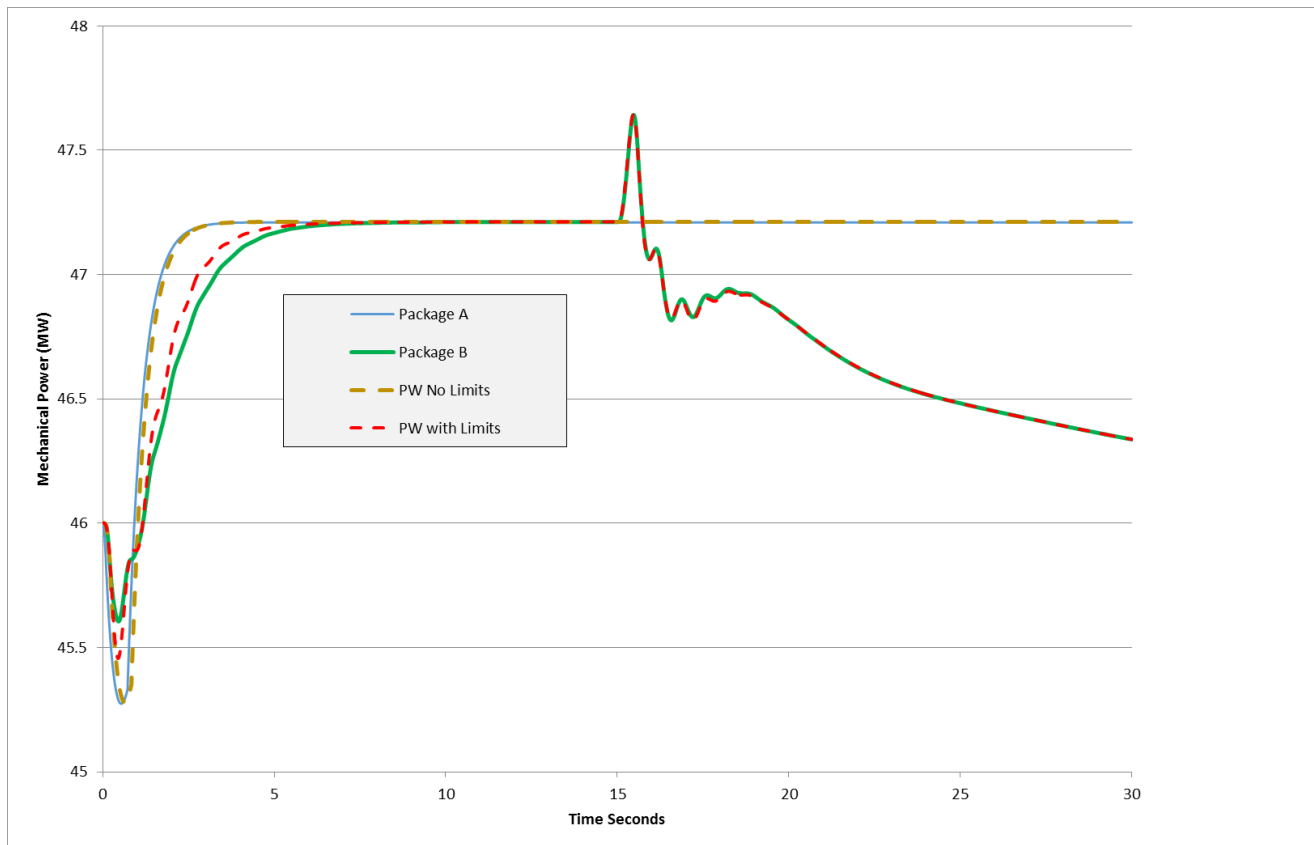**Figure E.8—Non-windup proportional-integral block**

Another common approach is to cap the output and put a non-windup limit on the integratore

# PIDGOV Model Results

- Below graph compares the Pmech response for a two bus system for a frequency change, between three transient stability packages



Packages A and B both say they have no governor limits, but B seems to; PW can do either approach

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# PI Limit Problems with Actual Hydro Models

- UIUC research in 2014 found there were significant differences between hydro model implementations with respect to how PI limits were modeled

  – One package modeled limits but did not document them, another did not model them; limits were recommended at WECC MVWG in May 2014
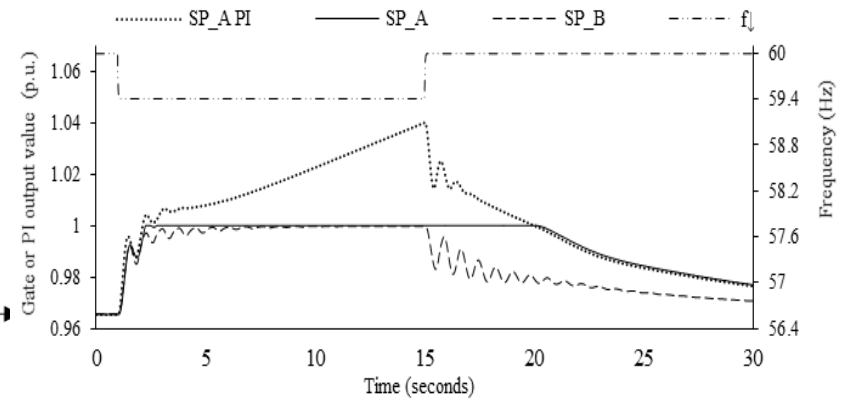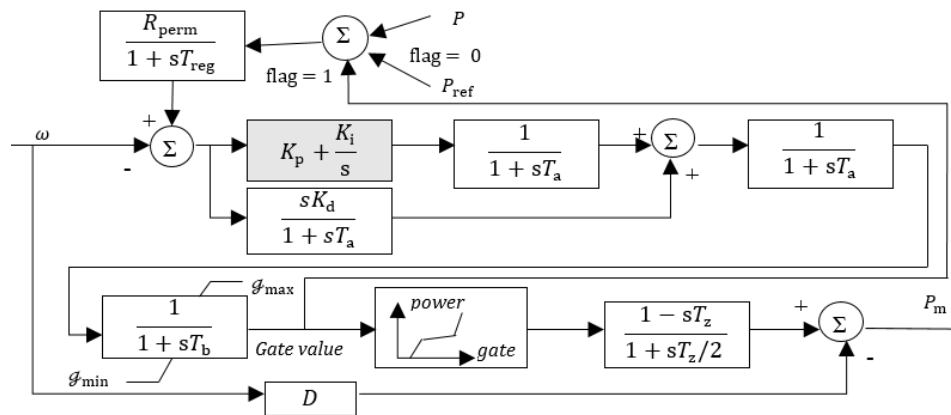
# GGOV1

- GGOV1 is a newer governor model introduced in early 2000's by WECC for modeling thermal plants
  - Existing models greatly under-estimated the frequency drop
  - GGOV1 is now the most common WECC governor, used with about 40% of the units

- A useful reference is L. Pereira, J. Undrill, D. Kosterev, D. Davies, and S. Patterson, "A New Thermal Governor Modeling Approach in the WECC," *IEEE Transactions on Power Systems*, May 2003, pp. 819-829

# GGOV1: Selected Figures from 2003 Paper



May 18 2001 Test SW and NW Trips - Malin 500 kV Bus Frequency
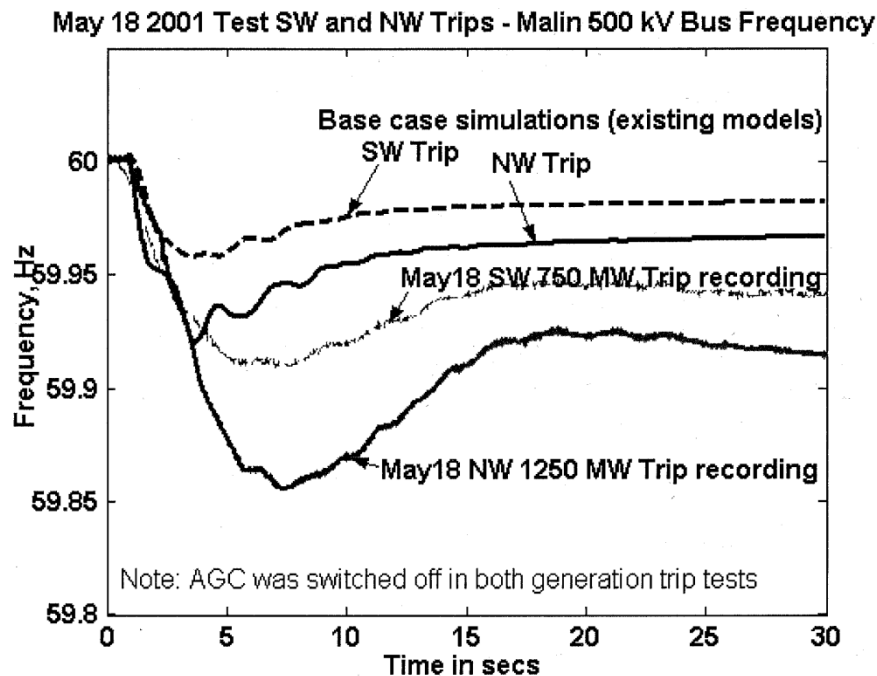
Fig. 1. Frequency recordings of the SW and NW trips on May 18, 2001. Also shown are simulations with existing modeling (base case).



June 3, 2002 - Diablo 950 MW Generation Trip : Malin 500 kV bus - frequency
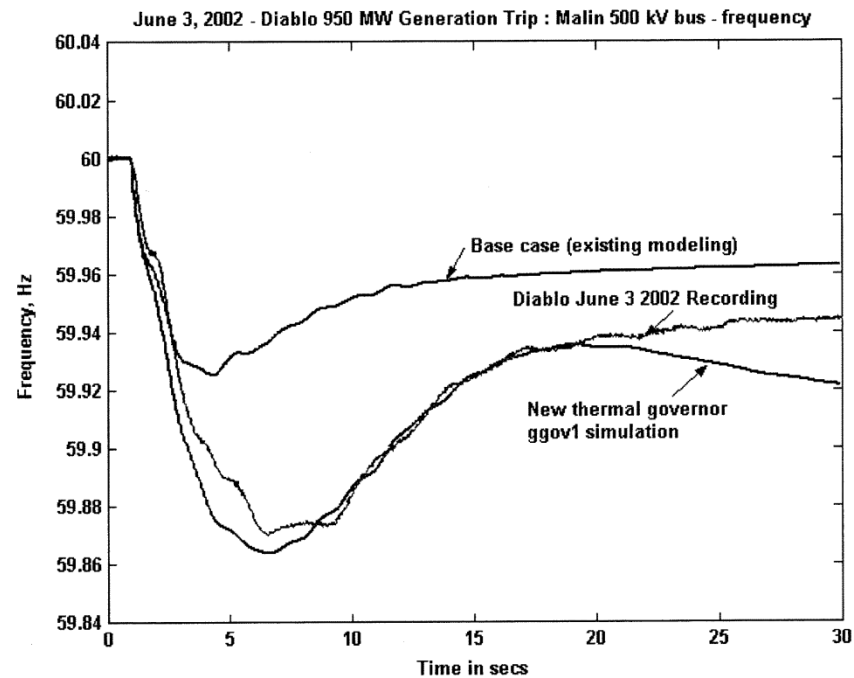
Governor model verification—950-MW Diablo generation trip on June 3, 2002.

# GGOV1 Block Diagram
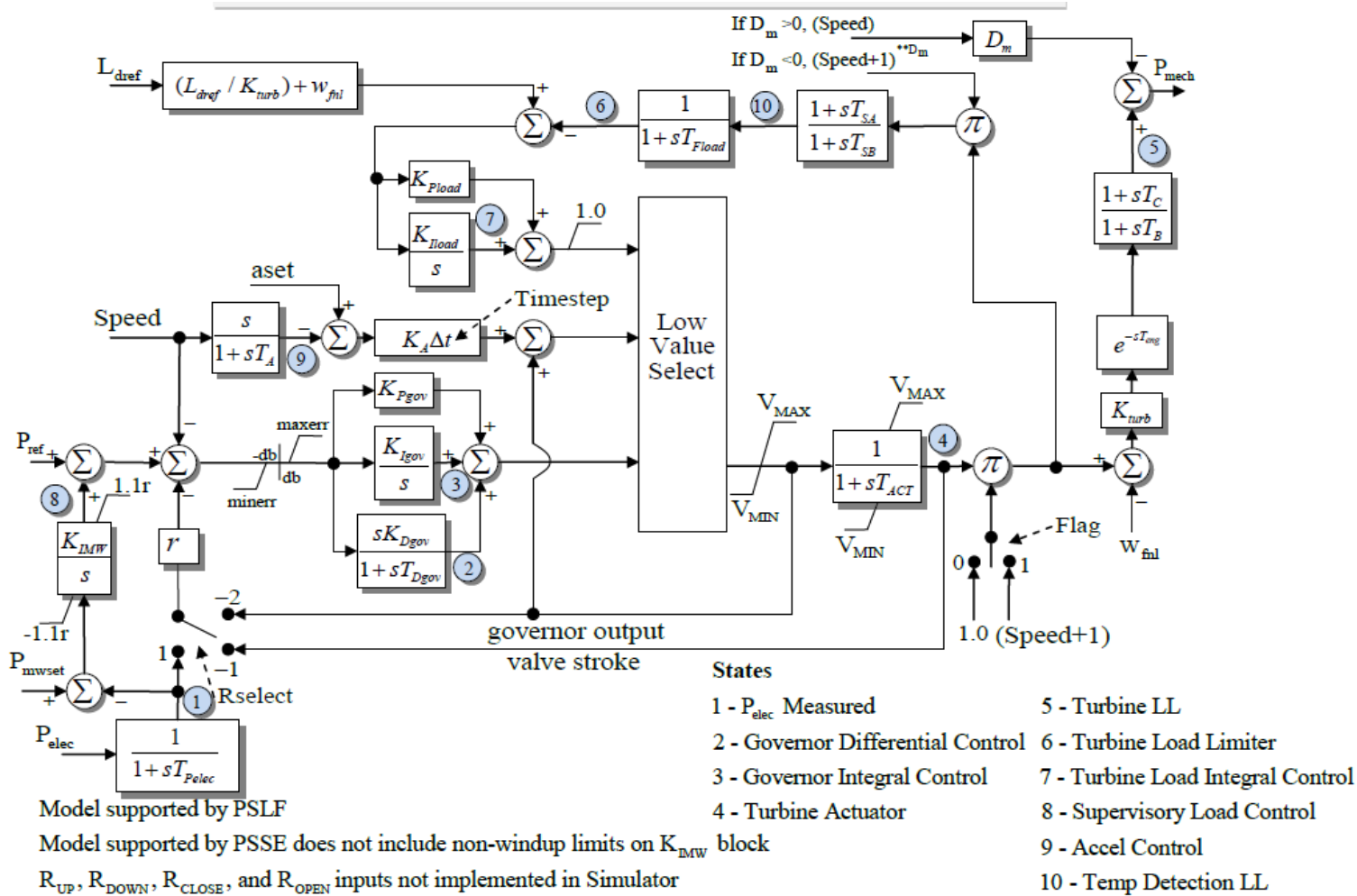
# Transient Stability Multimachine Simulations

- Next, we'll be putting the models we've covered so far together

- Later we'll add in new model types such as stabilizers, loads and wind turbines

- By way of history, prior to digital computers, network analyzers were used for system stability studies as far back as the 1930's (perhaps earlier)
  - For example see, J.D. Holm, "Stability Study of A-C Power Transmission Systems," AIEE Transactions, vol. 61, 1942, pp. 893-905

- Digital approaches started appearing in the 1960's

# Transient Stability Multimachine Simulations

- The general structure is as a set of differential-algebraic equations
  - Differential equations describe the behavior of the machines (and the loads and other dynamic devices)
  - Algebraic equations representing the network constraints
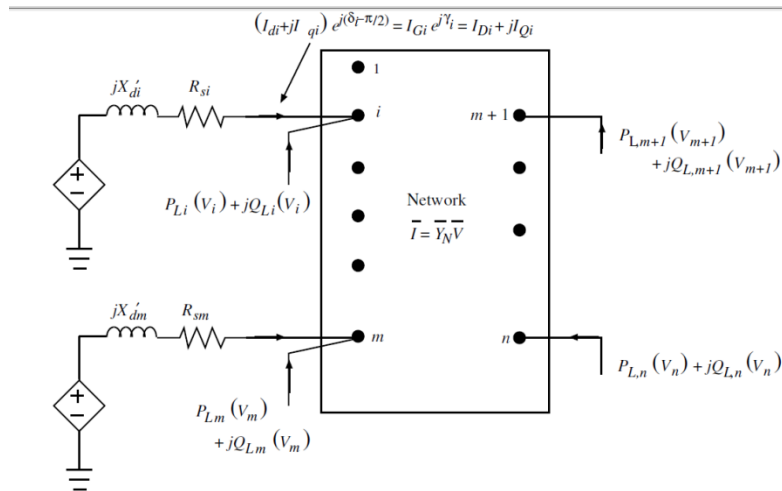


Figure 7.2: *Interconnection of synchronous machine dynamic circuit and the rest of the network*

In EMTP applications the transmission line delays decouple the machines; in transient stability they are assumed to be coupled together by the algebraic network equations

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# General Form

- The general form of the problem is solving

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

where $\mathbf{x}$ is the vector of the state variables (such as the generator $\delta$'s), $\mathbf{y}$ is the vector of the algebraic variables (primarily the bus complex voltages), and $\mathbf{u}$ is the vector of controls (such as the exciter voltage setpoints)

# Transient Stability General Solution

- General solution approach is
  - Solve power flow to determine initial conditions
  - Back solve to get initial states, starting with machine models, then exciters, governors, stabilizers, loads, etc
  - Set t = $t_{start}$, time step = $\Delta t$, abort = false
  - While (t <= tend) and (not abort) Do Begin
    - Apply any contingency event
    - Solve differential and algebraic equations
    - If desired store time step results and check other conditions (that might cause the simulation to abort)
    - t = t + $\Delta t$
  - End while

# Algebraic Constraints

- The **g** vector of algebraic constraints is similar to the power flow equations, but usually rather than formulating the problem like in the power flow as real and reactive power balance equations, it is formulated in the current balance form

$$\mathbf{I}(\mathbf{x}, \mathbf{V}) = \mathbf{Y}\,\mathbf{V} \quad \text{or} \quad \mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x}, \mathbf{V}) = \mathbf{0}$$

where **Y** is the n × n bus admittance matrix $(\mathbf{Y} = \mathbf{G} + j\mathbf{B})$, **V** is the complex vector of the bus voltages, and **I** is the complex vector of the bus current injections

Simplest cases can have **I** independent of **x** and **V**, allowing a direct solution; otherwise we need to iterate

# Why Not Use the Power Flow Equations?

- The power flow equations were ultimately derived from
  $$\mathbf{I}(\mathbf{x}, \mathbf{V}) = \mathbf{Y}\,\mathbf{V}$$

- However, the power form was used in the power flow primarily because

  - For the generators the real power output is known and either the voltage setpoint (i.e., if a PV bus) or the reactive power output

  - In the quasi-steady state power flow time frame the loads can often be well approximated as constant power

  - The constant frequency assumption requires a slack bus

- These assumptions do not hold for transient stability

# Algebraic Equations for Classical Model

- To introduce the coupling between the machine models and the network constraints, consider a system modeled with just classical generators and impedance loads
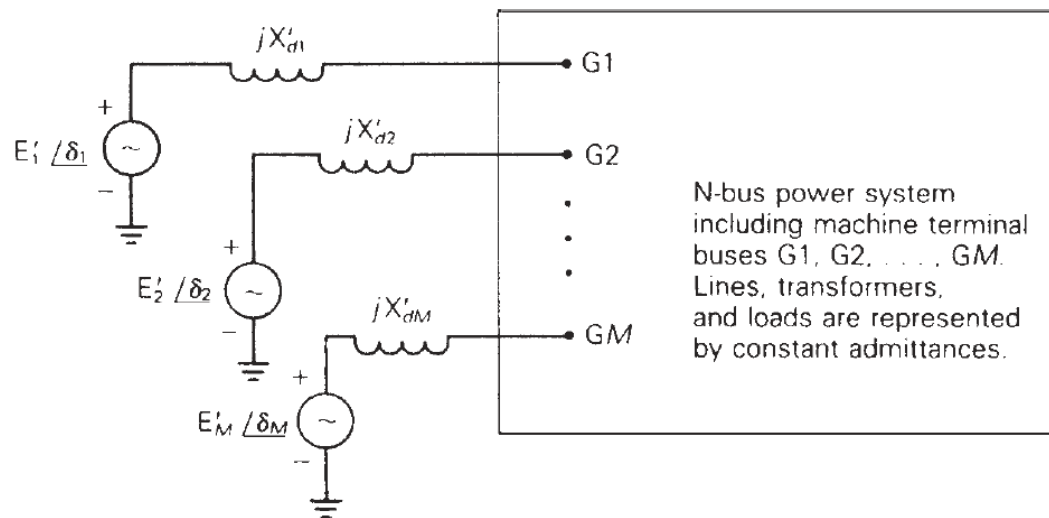
In this example because we are using the classical model all values are on the network reference frame

N-bus power system including machine terminal buses G1, G2, . . . , GM. Lines, transformers, and loads are represented by constant admittances.

We'll extend the figure slightly to include stator resistances, $R_{s,i}$

# Algebraic Equations for Classical Model

- Replace the internal voltages and their impedances by their Norton Equivalent

$$\overline{I}_i = \frac{E'_i \angle \delta_i}{R_{s,i} + jX'_{d,i}}, \quad Y_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$

- Current injections at the non-generator buses are zero since the constant impedance loads are included in **Y**
  - We'll modify this later when we talk about dynamic loads
- The algebraic constraints are then $\mathbf{I} - \mathbf{Y}\,\mathbf{V} = \mathbf{0}$

# Swing Equation

- Two first two differential equations for any machine correspond to the swing equation

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\frac{d\omega_i}{dt} = \frac{2H_i}{\omega_s}\frac{d\Delta\omega_i}{dt} = T_{Mi} - T_{Ei} - D_i\left(\Delta\omega_i\right)$$

with $T_{Ei} = \psi_{de,i}i_{qi} - \psi_{qe,i}i_{di}$

# Swing Equation Speed Effects

- There is often confusion about these equations because of whether speed effects are included
  - Recognizing that often $\omega \approx \omega_s$ (which is one per unit), some transient stability books have neglected speed effects

- For a rotating machine with a radial torque, power = torque times speed

- For a subtransient model

$$\bar{E}'' = \bar{V} + (R_s + jX'')\bar{I}, \quad E_d'' + jE_q'' = \left(-\psi_q'' + j\psi_d''\right)\omega$$

$$T_E = \psi_d'' I_q - \psi_q'' I \quad and$$

$$P_E = T_E\omega = \left(E_d'' + jE_q''\right)\left(I_d - jI_q\right) = E_d'' I_d + E_q'' I_q$$

# Classical Swing Equation

- Often in an introductory coverage of transient stability with the classical model the assumption is $\omega \approx \omega \approx \omega_s$ so the swing equation for the classical model is given as

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\frac{d\Delta\omega_i}{dt} = P_{Mi} - P_{Ei} - D_i\left(\Delta\omega_i\right)$$

with $\mathrm{P}_{Ei} = \left(E_i'\angle\delta_i\right)\left(E_i'\angle\delta_i - \bar{V}_i\right)Y_i$

- We'll use this simplification for our initial example

As an example of this initial approach see Anderson and Fouad, Power System Control and Stability, 2nd Edition, Chapter 2

# Numerical Solution

- There are two main approaches for solving

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

  - Partitioned-explicit: Solve the differential and algebraic equations separately (alternating between the two) using an explicit integration approach

  - Simultaneous-implicit: Solve the differential and algebraic equations together using an implicit integration approach

# Outline for Next Several Slides

- The next several slides will provide basic coverage of the solution process, partitioned explicit, then the simultaneous-implicit approach

- We'll start out with a classical model supplying an infinite bus, which can be solved by embedded the algebraic constraint into the differential equations

We'll start out just solving $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$

and then will extend to solving the full problem of

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

# Classical Swing Equation with Embedded Power Balance

- With a classical generator at bus i supplying an infinite bus with voltage magnitude $V_{inf}$, we can write the problem without algebraic constraints as

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i = \Delta\omega_{i.pu}\omega_s$$

$$\frac{d\Delta\omega_{i,pu}}{dt} = \frac{1}{2H_i}\left( P_{Mi} - \frac{E_i' V_{inf}}{X_{th}}\sin\delta_i - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

with $P_{Ei} = \dfrac{E_i' V_{inf}}{X_{th}}\sin\delta_i$

Note we are using the per unit speed approach

# Explicit Integration Methods

- As covered on the first day of class, there are a wide variety of explicit integration methods
  - We considered Forward Euler, Runge-Kutta, Adams-Bashforth
- Here we will just consider Euler's, which is easy to explain but not too useful, and a second order Runge-Kutta, which is commonly used

# Forward Euler

- Recall the Forward Euler approach is approximate

$$\dot{\mathbf{x}} \;=\; \mathbf{f}(\mathbf{x}(t)) \;=\; \frac{d\mathbf{x}}{dt} \text{as} \; \frac{\Delta\mathbf{x}}{\Delta t}$$
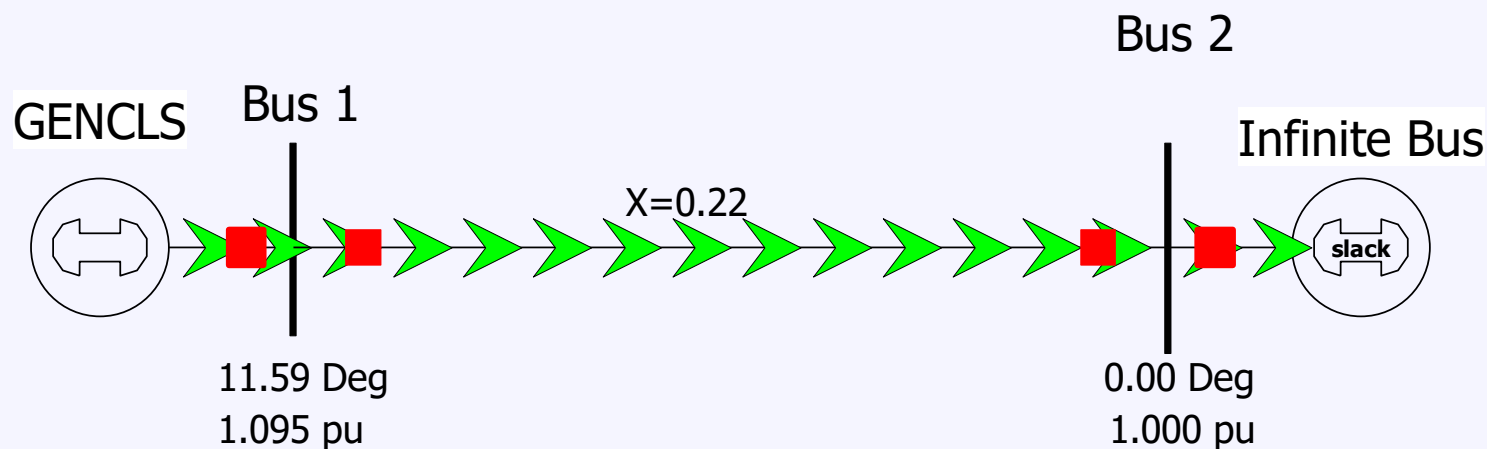
Then

$$\mathbf{x}(t + \Delta t) \;\approx\; \mathbf{x}(t) + \Delta t\,\mathbf{f}(\mathbf{x}(t))$$

- Error with Euler's varies with the square of the time step

# Infinite Bus GENCLS Example using the Forward Euler's Method

- Use the four bus system from before, except now gen 4 is modeled with a classical model with $X_d'$=0.3, H=3 and D=0; also we'll reduce to two buses with equivalent line reactance, moving the gen from bus 4 to 1



In this example Xth = (0.22 + 0.3), with the internal voltage $\bar{E}'_1 = 1.281\angle 23.95°$ giving E'$_1$=1.281 and $\delta_1$= 23.95°

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# Infinite Bus GENCLS Example

- The associated differential equations for the bus 1 generator are

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times 3}\left( 1 - \frac{1.281}{0.52}\sin\delta_1 \right)$$

- The value of $P_{M1} = 1$ is determined from the initial conditions, and would stay constant in this simple example without a governor

- The value $\delta_1 = 23.95°$ is readily verified as an equilibrium point (which is 0.418 radians)

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# Infinite Bus GENCLS Example

- Assume a solid three phase fault is applied at the generator terminal, reducing $P_{E1}$ to zero during the fault, and then the fault is self-cleared at time $T^{clear}$, resulting in the post-fault system being identical to the pre-fault system

  - During the fault-on time the equations reduce to

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times3}(1-0)$$

That is, with a solid fault on the terminal of the generator, during the fault $P_{E1} = 0$

# Euler's Solution

- The initial value of **x** is

$$\mathbf{x}(0) = \begin{bmatrix} \delta(0) \\ \omega_{pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

- Assuming a time step $\Delta t = 0.02$ seconds, and a $T^{clear}$ of 0.1 seconds, then using Euler's

$$\mathbf{x}(0.02) = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + 0.02 \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.00333 \end{bmatrix}$$

- Iteration continues until $t = T^{clear}$

Note Euler's assumes $\delta$ stays constant during the first time step

# Euler's Solution

- At t = T$^{\text{clear}}$ the fault is self-cleared, with the equations changing to

$$\frac{d\delta}{dt} = \Delta\omega_{pu}\omega_s$$

$$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{6}\left(1 - \frac{1.281}{0.52}\sin\delta\right)$$

- The integration continues using the new equations

# Euler's Solution Results (Δt=0.02)

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

| Time | Gen 1 Rotor Angle, Degrees | Gen 1 Speed (Hz) |
|---|---|---|
| 0 | 23.9462 | 60 |
| 0.02 | 23.9462 | 60.2 |
| 0.04 | 25.3862 | 60.4 |
| 0.06 | 28.2662 | 60.6 |
| 0.08 | 32.5862 | 60.8 |
| 0.1 | 38.3462 | 61 |
| 0.1 | 38.3462 | 61 |
| 0.12 | 45.5462 | 60.8943 |
| 0.14 | 51.9851 | 60.7425 |
| 0.16 | 57.3314 | 60.5543 |
| 0.18 | 61.3226 | 60.3395 |
| 0.2 | 63.7672 | 60.1072 |
| 0.22 | 64.5391 | 59.8652 |
| 0.24 | 63.5686 | 59.6203 |
| 0.26 | 60.8348 | 59.3791 |
| 0.28 | 56.3641 | 59.1488 |

This is saved as PowerWorld case B2_CLS_Infinite. The integration method is set to Euler's on the Transient Stability, Options, Power System Model page

# Generator 1 Delta: Euler's

- The below graph shows the generator angle for varying values of Δt; numerical instability is clearly seen
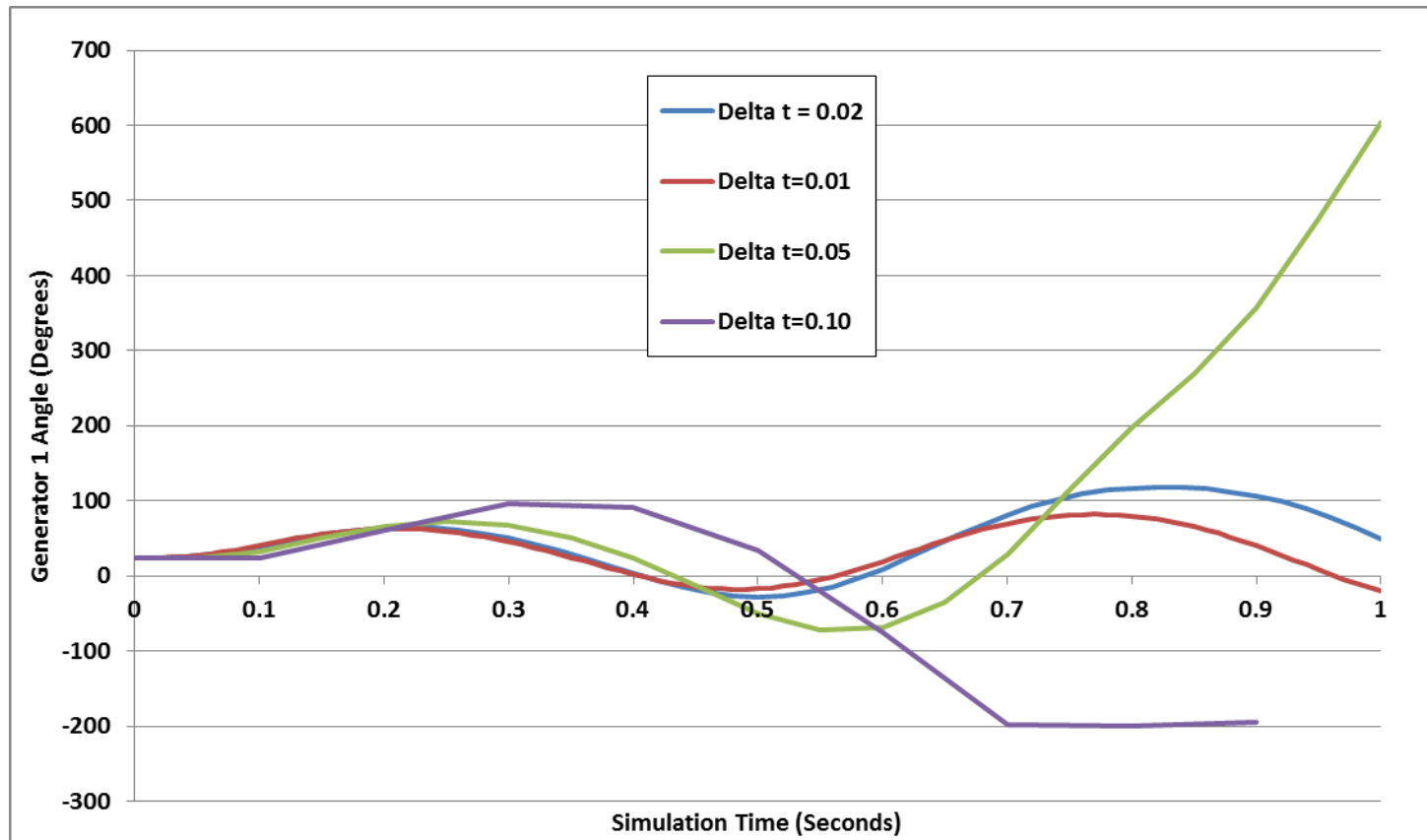
# Second Order Runge-Kutta

- Runge-Kutta methods improve on Euler's method by evaluating $\mathbf{f}(\mathbf{x})$ at selected points over the time step

- One approach is a second order method (RK2) in which

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)$$

where

$$\mathbf{k}_1 = \Delta t \, \mathbf{f}(\mathbf{x}(t))$$

$$\mathbf{k}_2 = \Delta t \, \mathbf{f}(\mathbf{x}(t) + \mathbf{k}_1)$$

This is also known as Heun's method or as the Improved Euler's or Modified Euler's Method

- That is, $\mathbf{k}_1$ is what we get from Euler's; $\mathbf{k}_2$ improves on this by reevaluating at the estimated end of the time step

- Error varies with the cubic of the time step

# Second Order Runge-Kutta (RK2)

- Again assuming a time step $\Delta t = 0.02$ seconds, and a $T^{clear}$ of 0.1 seconds, then using Heun's approach

$$\mathbf{x}(0) = \begin{bmatrix} \delta(0) \\ \Delta\omega_{pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

$$\mathbf{k}_1 = 0.02 \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.00333 \end{bmatrix}, \quad \mathbf{x}(0) + \mathbf{k}_1 = \begin{bmatrix} 0.418 \\ 0.00333 \end{bmatrix}$$

$$\mathbf{k}_2 = 0.02 \begin{bmatrix} 1.257 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0.0251 \\ 0.00333 \end{bmatrix}$$

$$\mathbf{x}(0.020) = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2) = \begin{bmatrix} 0.431 \\ 0.00333 \end{bmatrix}$$

# RK2 Solution Results ($\Delta$t=0.02)

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

| Time | Gen 1 Rotor Angle, Degrees | Gen 1 Speed (Hz) |
|---|---|---|
| 0 | 23.9462 | 60 |
| 0.02 | 24.6662 | 60.2 |
| 0.04 | 26.8262 | 60.4 |
| 0.06 | 30.4262 | 60.6 |
| 0.08 | 35.4662 | 60.8 |
| 0.1 | 41.9462 | 61 |
| 0.1 | 41.9462 | 61 |
| 0.12 | 48.6805 | 60.849 |
| 0.14 | 54.1807 | 60.6626 |
| 0.16 | 58.233 | 60.4517 |
| 0.18 | 60.6974 | 60.2258 |
| 0.2 | 61.4961 | 59.9927 |
| 0.22 | 60.605 | 59.7598 |
| 0.24 | 58.0502 | 59.5343 |
| 0.26 | 53.9116 | 59.3241 |
| 0.28 | 48.3318 | 59.139 |

This is saved as PowerWorld case B2_CLS_Infinite. The integration method should be changed to Second Order Runge-Kutta on the Transient Stability, Options, Power System Model page

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- The below graph shows the generator angle for varying values of Δt; much better than Euler's but still the beginning of numerical instability with larger values of Δt
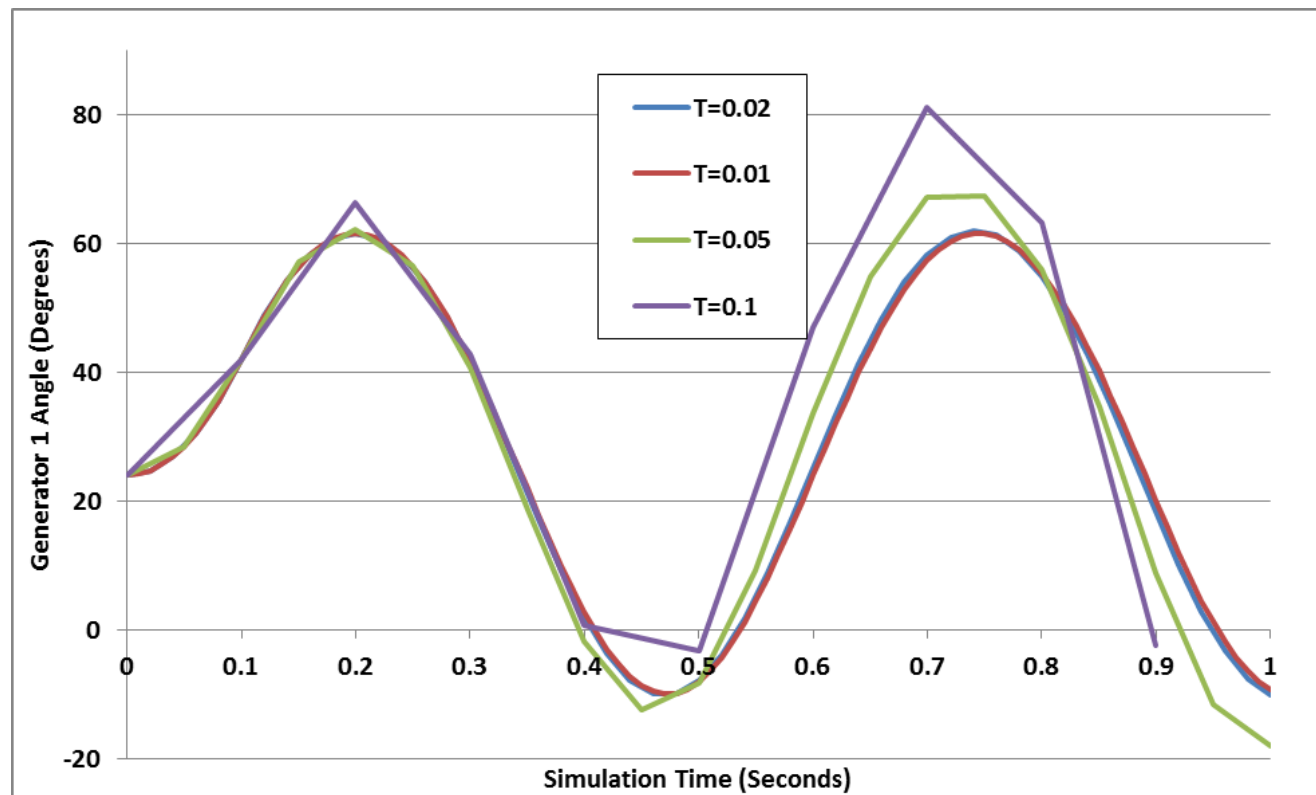
# Adding Network Equations

- Previous slides with the network equations embedded in the differential equations were a special case

- In general with the explicit approach we'll be alternating between solving the differential equations and solving the algebraic equations

- Voltages and currents in the network reference frame can be expressed using either polar or rectangular coordinates

- In rectangular with the book's notation we have

$$\bar{V}_i = V_{Di} + jV_{Qi}, \quad \bar{I}_i = I_{Di} + jI_{Qi}$$

# Adding Network Equations

- Network equations will be written as $\mathbf{Y}\,\mathbf{V} - \mathbf{I(x,V)} = \mathbf{0}$
  - Here $\mathbf{Y}$ is as from the power flow, except augmented to include the impact of the generator's internal impedance
  - Constant impedance loads are also embedded in $\mathbf{Y}$; non-constant impedance loads are included in $\mathbf{I(x,V)}$

- If $\mathbf{I}$ is independent of $\mathbf{V}$ then this can be solved directly: $\mathbf{V} = \mathbf{Y}^{-1}\mathbf{I(x)}$

- In general an iterative solution is required, which we'll cover shortly, but initially we'll go with just the direct solution

# Two Bus Example, Except with No Infinite Bus

- To introduce the inclusion of the network equations, the previous example is extended by replacing the infinite bus at bus 2 with a classical model with $X_{d2}'=0.2$, $H_2=6.0$
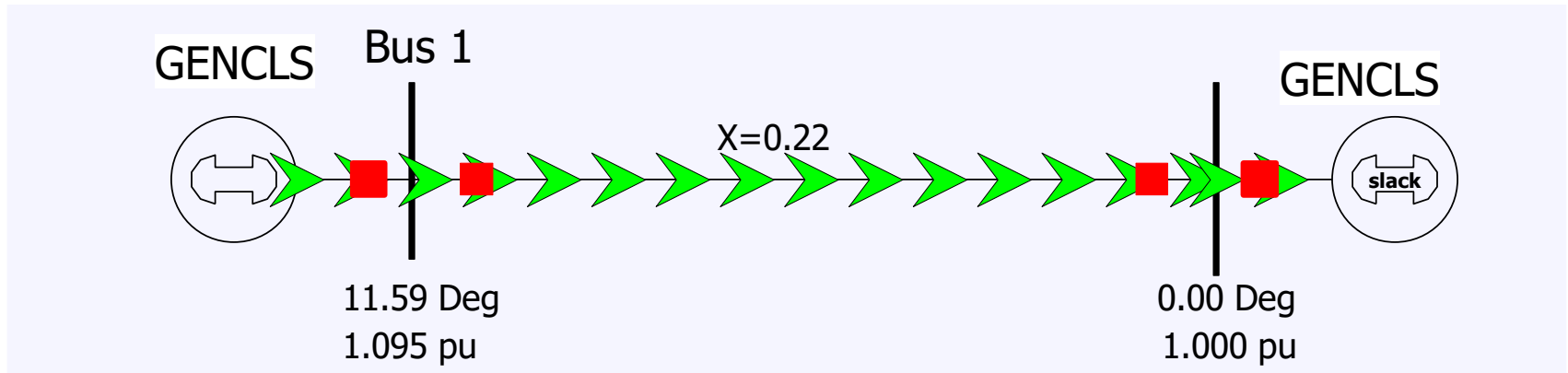


PowerWorld Case B2_CLS_2Gen

# Bus Admittance Matrix

- The network admittance matrix is

$$\mathbf{Y}_N = \begin{bmatrix} -j4.545 & j4.545 \\ j4.545 & -j4.545 \end{bmatrix}$$

- This is augmented to represent the Norton admittances associated with the generator models ($X_{d1}'$=0.3, $X_{d2}'$=0.2)

$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \dfrac{1}{j0.3} & 0 \\ 0 & \dfrac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}$$
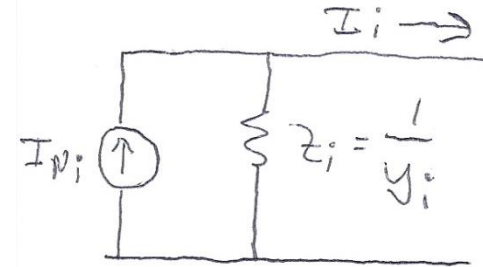
- In PowerWorld you can see this matrix by selecting Transient Stability, States/Manual Control, Transient Stability Ybus

# Current Vector

- For the classical model the Norton currents are given by

$$\bar{I}_{Ni} = \frac{E'_i \angle \delta_i}{R_{s,i} + jX'_{d,i}}, \quad Y_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$



- The initial values of the currents come from the power flow solution

- As the states change ($\delta_i$ for the classical model), the Norton current injections also change