ECEN 667
Power System Stability

Lecture 16: Transient Stability Solutions

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Announcements

- Read Chapter 7
- Homework 5 is assigned today, due on Oct 31

Adding Network Equations

- Previous slides with the network equations embedded in the differential equations were a special case
- In general with the explicit approach we'll be alternating between solving the differential equations and solving the algebraic equations
- Voltages and currents in the network reference frame can be expressed using either polar or rectangular coordinates
- In rectangular with the book's notation we have

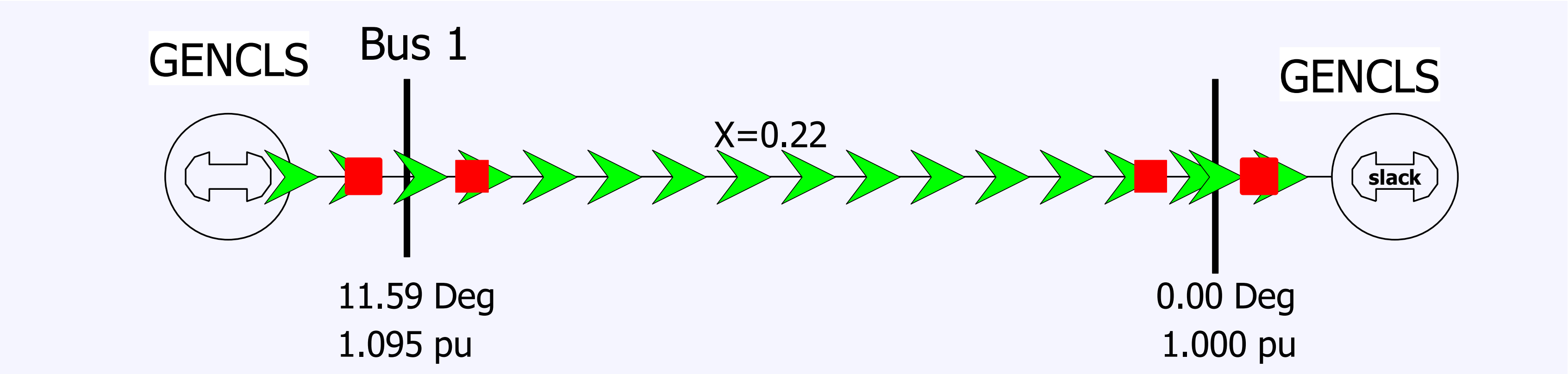
$$\bar{V}_i = V_{Di} + jV_{Qi}, \quad \bar{I}_i = I_{Di} + jI_{Qi}$$

Adding Network Equations

- Network equations will be written as **Y V- I(x,V) = 0**
  - Here **Y** is as from the power flow, except augmented to include the impact of the generator's internal impedance
  - Constant impedance loads are also embedded in **Y**; non-constant impedance loads are included in **I(x,V)**
- If **I** is independent of **V** then this can be solved directly: $\mathbf{V} = \mathbf{Y}^{-1}\mathbf{I}(\mathbf{x})$
- In general an iterative solution is required, which we'll cover shortly, but initially we'll go with just the direct solution

Two Bus Example, Except with No Infinite Bus

- To introduce the inclusion of the network equations, the previous example is extended by replacing the infinite bus at bus 2 with a classical model with $X_{d2}'=0.2$, $H_2=6.0$



PowerWorld Case B2_CLS_2Gen

Bus Admittance Matrix

- The network admittance matrix is

$$\mathbf{Y}_N = \begin{bmatrix} -j4.545 & j4.545 \\ j4.545 & -j4.545 \end{bmatrix}$$

- This is augmented to represent the Norton admittances associated with the generator models (X_{d1}'=0.3, X_{d2}'=0.2)

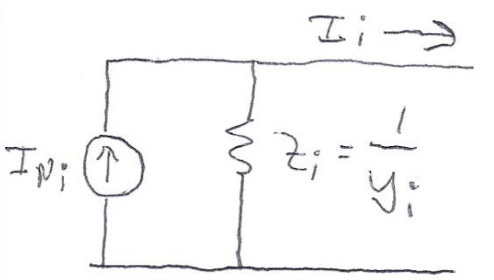
$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.3} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}$$

- In PowerWorld you can see this matrix by selecting Transient Stability, States/Manual Control, Transient Stability Ybus

Current Vector

- For the classical model the Norton currents are given by

$$\bar{I}_{Ni} = \frac{E_i'\angle\delta_i}{R_{s,i} + jX'_{d,i}}, \quad Y_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$



- The initial values of the currents come from the power flow solution
- As the states change (δ_i for the classical model), the Norton current injections also change

B2_CLS_Gen Initial Values

- The internal voltage for generator 1 is as before

$$\bar{I} = 1 - j0.3286$$

0.4179 radians

$$\bar{E}_1 = 1.0 + (j0.22 + j0.3)\bar{I} = 1.1709 + j0.52 = 1.281\angle 23.95^\circ$$

- We likewise solve for the generator 2 internal voltage

$$\bar{E}_2 = 1.0 - (j0.2)\bar{I} = 0.9343 - j0.2 = 0.9554\angle -12.08$$

- The Norton current injections are then

0.2108 radians

$$\bar{I}_{N1} = \frac{1.1709 + j0.52}{j0.3} = 1.733 - j3.903$$

$$\bar{I}_{N2} = \frac{0.9343 - j0.2}{j0.2} = -1 - j4.6714$$

Keep in mind the Norton current injections are not the current out of the generator

B2_CLS_Gen Initial Values

- To check the values, solve for the voltages, with the values matching the power flow values

$$\mathbf{V} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.733 - j3.903 \\ -1 - j4.671 \end{bmatrix}$$

$$= \begin{bmatrix} 1.072 + j0.22 \\ 1.0 \end{bmatrix}$$

Swing Equations

- With the network constraints modeled, the swing equations are modified to represent the electrical power in terms of the generator's state and current values

$$\mathrm{P}_{Ei} = E_{Di} I_{Di} + E_{Qi} I_{Qi}$$

$I_{Di}+jI_{Qi}$ is the current being injected into the network by the generator

- Then swing equation is then

$$\frac{d\delta_i}{dt} = \Delta\omega_{i.pu}\omega_s$$

$$\frac{d\Delta\omega_{i,pu}}{dt} = \frac{1}{2H_i}\left(P_{Mi} - \left(E_{Di} I_{Di} + E_{Qi} I_{Qi}\right) - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

Two Bus, Two Generator Differential Equations

- The differential equations for the two generators are

$$\frac{d\delta_1}{dt} = \Delta\omega_{1.pu}\omega_s$$

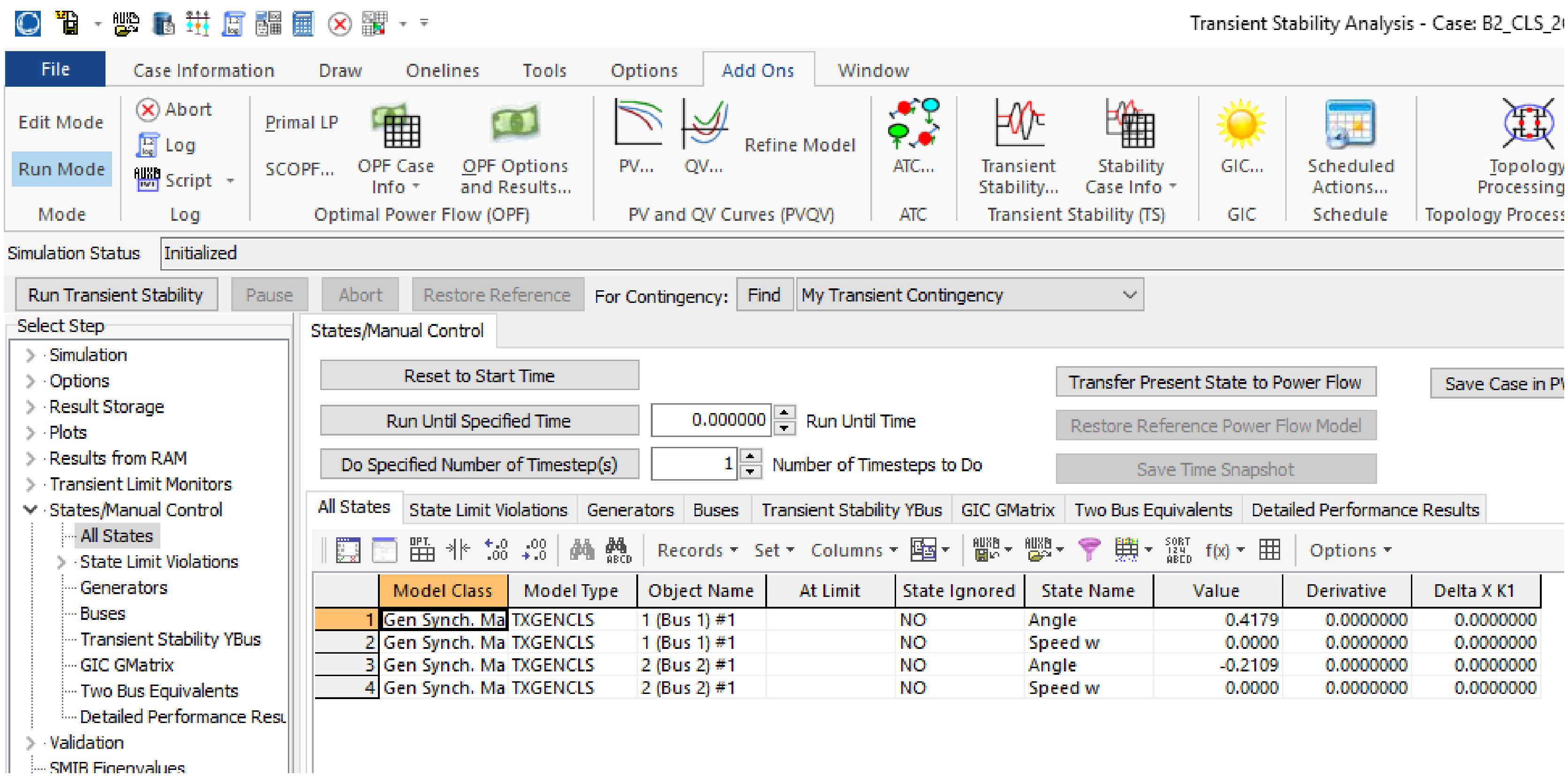
$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2H_1}\left(P_{M1} - \left(E_{D1}I_{D1} + E_{Q1}I_{Q1}\right)\right)$$

$$\frac{d\delta_2}{dt} = \Delta\omega_{2.pu}\omega_s$$

$$\frac{d\Delta\omega_{2,pu}}{dt} = \frac{1}{2H_2}\left(P_{M2} - \left(E_{D2}I_{D2} + E_{Q2}I_{Q2}\right)\right)$$

In this example
P_{M1} = 1 and
P_{M2} = -1

PowerWorld GENCLS Initial States



Solution at t=0.02

- Usually a time step begins by solving the differential equations. However, in the case of an event, such as the solid fault at the terminal of bus 1, the network equations need to be first solved
- Solid faults can be simulated by adding a large shunt at the fault location
  - Amount is somewhat arbitrary, it just needs to be large enough to drive the faulted bus voltage to zero
- With Euler's the solution after the first time step is found by first solving the differential equations, then resolving the network equations

Solution at t=0.02

- Using Y_{fault} = -j1000, the fault-on conditions become

$$\mathbf{V} = \begin{bmatrix} -j1007.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.733 - j3.903 \\ -1 - j4.671 \end{bmatrix}$$

$$= \begin{bmatrix} -0.006 - j0.001 \\ 0.486 - j0.1053 \end{bmatrix}$$

Solving for the currents into the network

$$I_1 = \frac{(1.1702 + j0.52) - V_1}{j0.3} = 1.733 - j3.900$$

$$I_2 = \frac{(0.9343 - j0.2) - (0.486 - j0.1053)}{j0.2} = -0.473 - j2.240$$

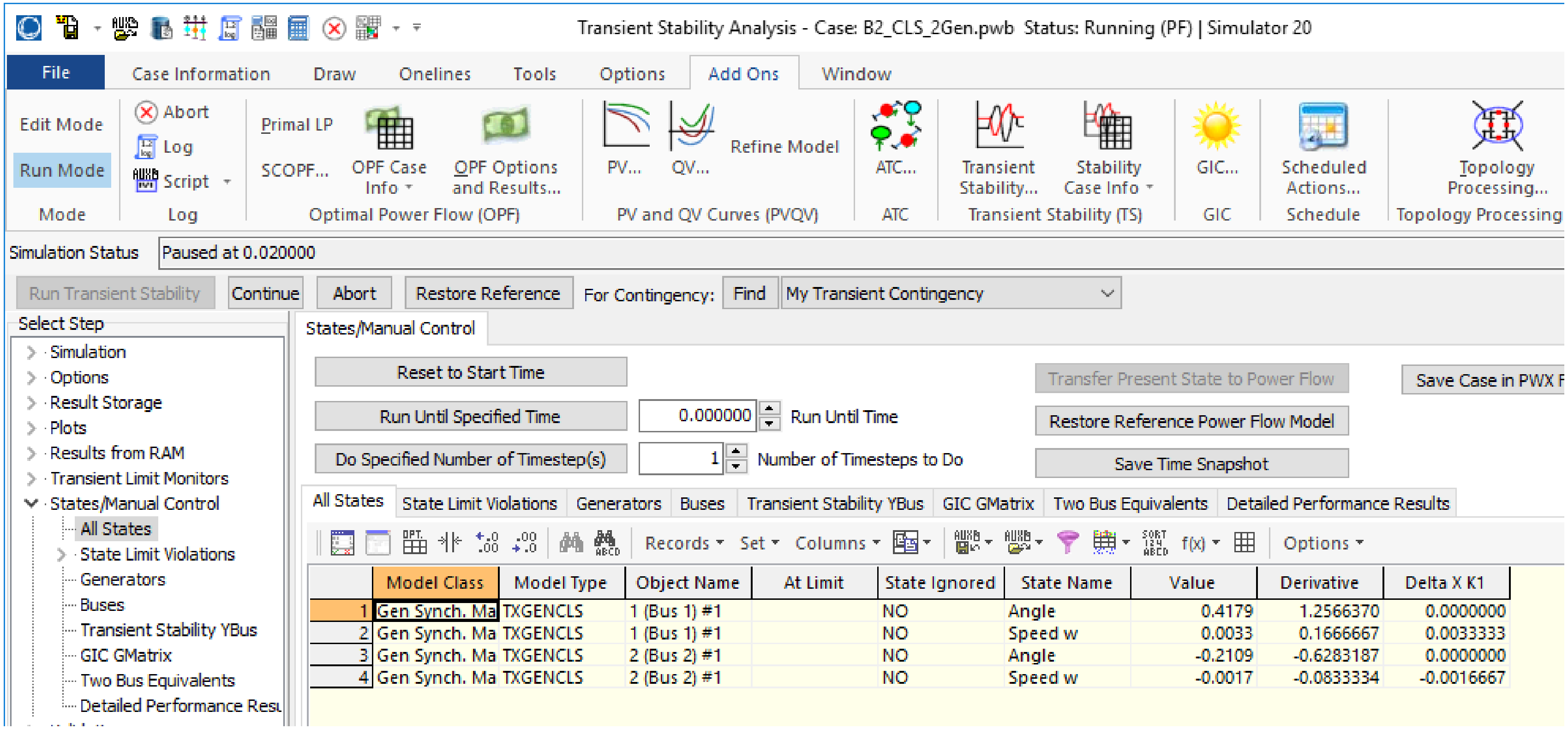
Solution at t=0.02

- Then the differential equations are evaluated, using the new voltages and currents
  - These impact the calculation of P_{Ei} with P_{E1}=0, P_{E2}=0

$$\begin{bmatrix} \delta_1(0.02) \\ \Delta\omega_1(0.02) \\ \delta_2(0.02) \\ \Delta\omega_1(0.02) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.0 \\ -0.211 \\ 0 \end{bmatrix} + 0.02 \begin{bmatrix} 0 \\ \frac{1}{6}(1-0) \\ 0 \\ \frac{1}{12}(-1-0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.00333 \\ -0.211 \\ -0.00167 \end{bmatrix}$$

- If solving with Euler's this is the final state value; using these state values the network equations are resolved, with the solution the same here since the δ's didn't vary

PowerWorld GENCLS at t=0.02



Solution Values Using Euler's

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

Time (Sec)	Gen 1 Rotor Angle	Gen 1 Speed (Hz)	Gen 2 Rotor Angle	Gen2 Speed (Hz)
0	23.9462	60	-12.0829	60
0.02	23.9462	60.2	-12.0829	59.9
0.04	25.3862	60.4	-12.8029	59.8
0.06	28.2662	60.6	-14.2429	59.7
0.08	32.5862	60.8	-16.4029	59.6
0.1	38.3462	61	-19.2829	59.5
0.1	38.3462	61	-19.2829	59.5
0.12	45.5462	60.9128	-22.8829	59.5436
0.14	52.1185	60.7966	-26.169	59.6017
0.16	57.8541	60.6637	-29.0368	59.6682
0.18	62.6325	60.5241	-31.426	59.7379
0.2	66.4064	60.385	-33.3129	59.8075
0.22	69.1782	60.2498	-34.6988	59.8751
0.24	70.9771	60.1197	-35.5982	59.9401
0.26	71.8392	59.9938	-36.0292	60.0031
0.28	71.7949	59.8702	-36.0071	60.0649

Solution at t=0.02 with RK2

- With RK2 the first part of the time step is the same as Euler's, that is solving the network equations with

$$\mathbf{x}(\mathrm{t}+\Delta\mathrm{t})^{(1)} = \mathbf{x}(\mathrm{t}) + \mathbf{k}_1 = \mathbf{x}(\mathrm{t}) + \Delta\mathrm{T}\mathbf{f}(\mathbf{x}(\mathrm{t}))$$

- Then calculate k2 and get a final value for $\mathbf{x}(\mathrm{t}+\Delta\mathrm{t})$

$$\mathbf{k}_2 = \Delta t \ \mathbf{f}\left(\mathbf{x}(t)+\mathbf{k}_1\right)$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{1}{2}\left(\mathbf{k}_1 + \mathbf{k}_2\right)$$

- Finally solve the network equations using the final value for $\mathbf{x}(\mathrm{t}+\Delta\mathrm{t})$

Solution at t=0.02 with RK2

- From the first half of the time step

$$x(0.02)^{(1)} = \begin{bmatrix} 0.418 \\ 0.00333 \\ -0.211 \\ -0.00167 \end{bmatrix}$$

- Then

$$\mathbf{k}_2 = \Delta t\ \mathbf{f}\left(\mathbf{x}(t)+\mathbf{k}_1\right) = 0.02\begin{bmatrix} 1.256 \\ \frac{1}{6}(1-0) \\ -0.628 \\ \frac{1}{12}(-1-0) \end{bmatrix} = \begin{bmatrix} 0.0251 \\ 0.00333 \\ -0.0126 \\ -0.00167 \end{bmatrix}$$

Solution at t=0.02 with RK2

- The new values for the Norton currents are

$$\overline{I}_{N1} = \frac{1.281\angle 24.69^\circ}{j0.3} = 1.851 - j3.880$$

$$\overline{I}_{N2} = \frac{0.9554\angle -12.43^\circ}{j0.2} = -1.028 - j4.665$$

$$\mathbf{V}(0.02) = \begin{bmatrix} -j1007.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.851 - j3.880 \\ -1.028 - j4.665 \end{bmatrix}$$

$$= \begin{bmatrix} -0.006 - j0.001 \\ 0.486 - j0.108 \end{bmatrix}$$

Solution Values Using RK2

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

Time (Sec)	Gen 1 Rotor Angle	Gen 1 Speed (Hz)	Gen 2 Rotor Angle	Gen2 Speed (Hz)
0	23.9462	60	-12.0829	60
0.02	24.6662	60.2	-12.4429	59.9
0.04	26.8262	60.4	-13.5229	59.8
0.06	30.4262	60.6	-15.3175	59.7008
0.08	35.4662	60.8	-17.8321	59.6008
0.1	41.9462	61	-21.0667	59.5008
0.1	41.9462	61	-21.0667	59.5008
0.12	48.7754	60.8852	-24.4759	59.5581
0.14	54.697	60.7538	-27.4312	59.6239
0.16	59.6315	60.6153	-29.8931	59.6931
0.18	63.558	60.4763	-31.8509	59.7626
0.2	66.4888	60.3399	-33.3109	59.8308
0.22	68.4501	60.2071	-34.286	59.8972
0.24	69.4669	60.077	-34.789	59.9623
0.26	69.5548	59.9481	-34.8275	60.0267
0.28	68.7151	59.8183	-34.4022	60.0916

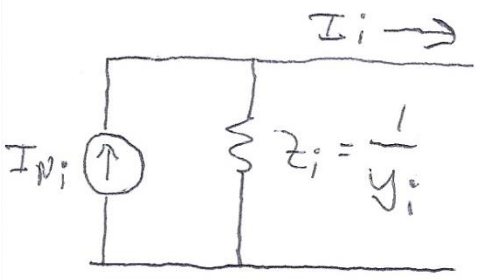
Angle Reference

- The initial angles are given by the angles from the power flow, which are based on the slack bus's angle
- As presented the transient stability angles are with respect to a synchronous reference frame
  - Sometimes this is fine, such as for either shorter studies, or ones in which there is little speed variation
  - Oftentimes this is not best since the when the frequencies are not nominal, the angles shift from the reference frame
- Other reference frames can be used, such as with respect to a particular generator's value, which mimics the power flow approach; the selected reference has no impact on the solution

Subtransient Models

- The Norton current injection approach is what is commonly used with subtransient models in industry
- If subtransient saliency is neglected (as is the case with GENROU and GENSAL in which $X''_d = X''_q$) then the current injection is

$$I_{Nd} + jI_{Nq} = \frac{E''_d + jE''_q}{R_s + jX''} = \frac{\left(-\psi''_q + j\psi''_d\right)\omega}{R_s + jX''}$$



  - Subtransient saliency can be handled with this approach, but it is more involved (see Arrillaga, *Computer Analysis of Power Systems*, section 6.6.3)

Subtransient Models

- Note, the values here are on the dq reference frame
- We can now extend the approach introduced for the classical machine model to subtransient models
- Initialization is as before, which gives the δ's and other state values
- Each time step is as before, except we use the δ's for each generator to transfer values between the network reference frame and each machine's dq reference frame
  - The currents provide the coupling

Two Bus Example with Two GENROU Machine Models

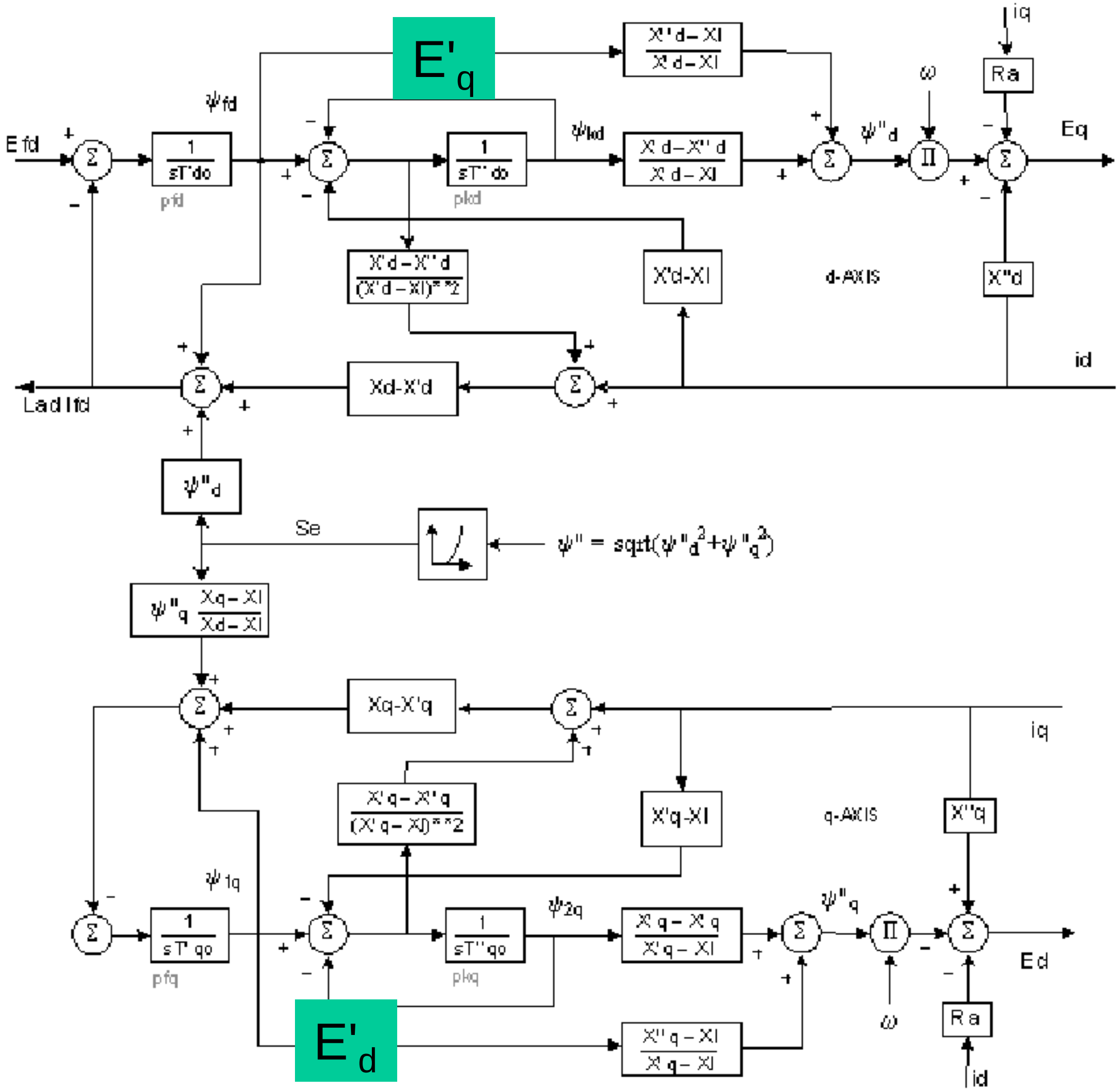
- Use the same system as before, except with we'll model both generators using GENROUs
  - For simplicity we'll make both generators identical except set H_1=3, H_2=6; other values are X_d=2.1, X_q=0.5, X'_d=0.2, X'_q=0.5, X''_q=X''_d=0.18, X_l=0.15, T'_{do} = 7.0, T'_{qo}=0.75, T''_{do}=0.035, T''_{qo}=0.05; no saturation
  - With no saturation the value of the δ's are determined (as per Lecture 11) by solving

$$|E|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$

  - Hence for generator 1

$$|E_1|\angle\delta_1 = 1.0946\angle 11.59° + \left(j0.5\right)\left(1.052\angle -18.2°\right) = 1.431\angle 30.2°$$

GENROU Block Diagram



Two Bus Example with Two GENROU Machine Models

- Using the approach from Lecture 11 the initial state vector is

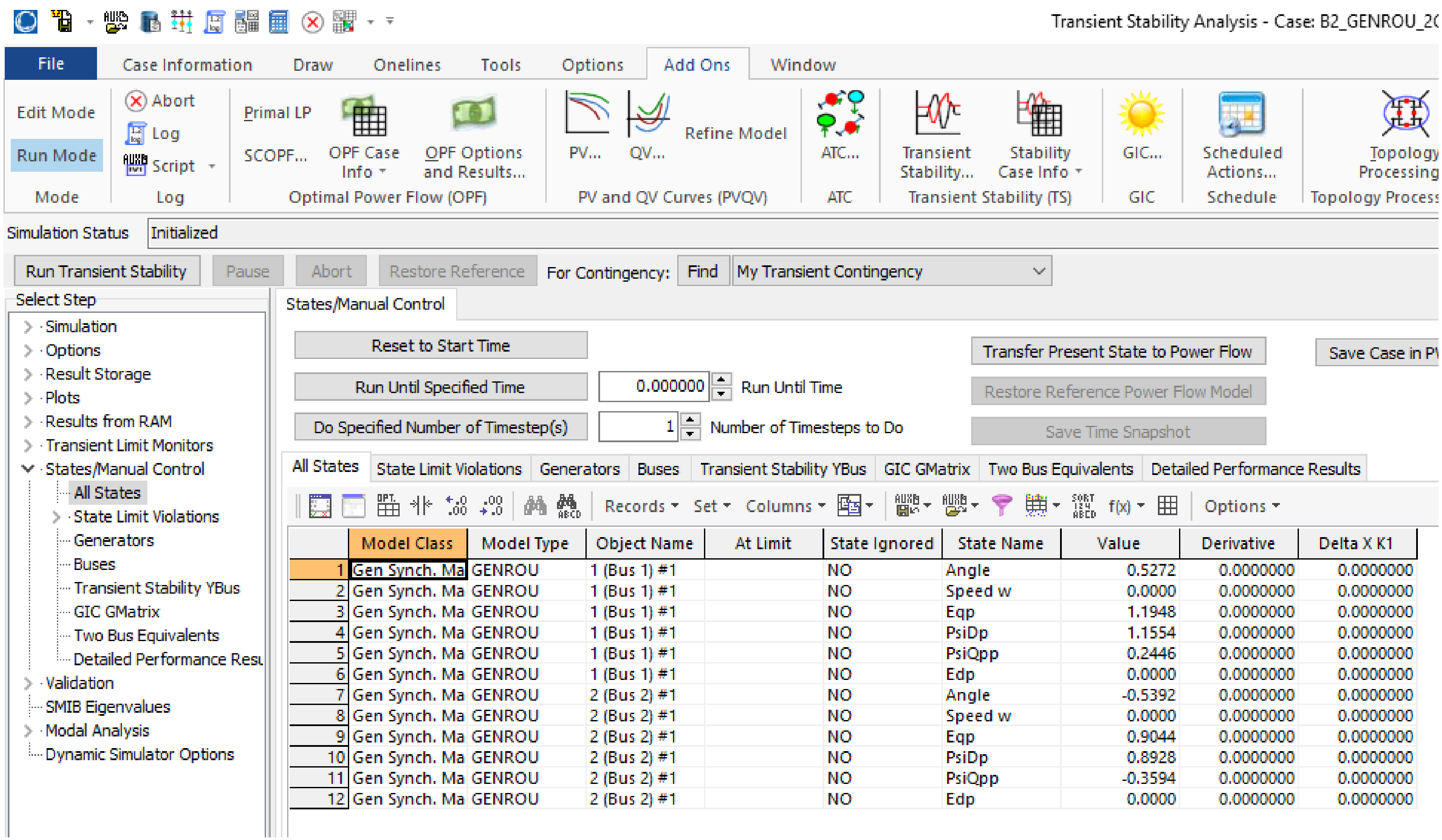
$$\mathbf{x}(0) = \begin{bmatrix} \delta_1 \\ \Delta\omega_1 \\ E'_{q1} \\ \psi_{1d1} \\ \psi_{2q1} \\ E'_{d1} \\ \delta_2 \\ \Delta\omega_2 \\ E'_{q2} \\ \psi_{1d2} \\ \psi_{2q2} \\ E'_{d2} \end{bmatrix} = \begin{bmatrix} 0.5273 \\ 0.0 \\ 1.1948 \\ 1.1554 \\ 0.2446 \\ 0 \\ -0.5392 \\ 0 \\ 0.9044 \\ 0.8928 \\ -0.3594 \\ 0 \end{bmatrix}$$

Note that this is a salient pole machine with $X'_q=X_q$; hence E'_d will always be zero

The initial currents in the dq reference frame are $I_{d1}=0.7872$, $I_{q1}=0.6988$, $I_{d2}=0.2314$, $I_{q2}=-1.0269$

Initial values of $\psi''_{q1}= -0.2236$, and $\psi''_{d1} = 1.179$

PowerWorld GENROU Initial States



	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value	Derivative	Delta X K1
1	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Angle	0.5272	0.0000000	0.0000000
2	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Speed w	0.0000	0.0000000	0.0000000
3	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Eqp	1.1948	0.0000000	0.0000000
4	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	PsiDp	1.1554	0.0000000	0.0000000
5	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.2446	0.0000000	0.0000000
6	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Edp	0.0000	0.0000000	0.0000000
7	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Angle	-0.5392	0.0000000	0.0000000
8	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Speed w	0.0000	0.0000000	0.0000000
9	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Eqp	0.9044	0.0000000	0.0000000
10	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8928	0.0000000	0.0000000
11	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.3594	0.0000000	0.0000000
12	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Edp	0.0000	0.0000000	0.0000000

Solving with Euler's

- We'll again solve with Euler's, except with Δt set now to 0.01 seconds (because now we have a subtransient model with faster dynamics)
  - We'll also clear the fault at t=0.05 seconds
- For the more accurate subtransient models the swing equation is written in terms of the torques

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\frac{d\omega_i}{dt} = \frac{2H_i}{\omega_s}\frac{d\Delta\omega_i}{dt} = T_{Mi} - T_{Ei} - D_i\left(\Delta\omega_i\right)$$

with $T_{Ei} = \psi''_{d,i} i_{qi} - \psi''_{q,i} i_{di}$

Other equations are solved based upon the block diagram

Norton Equivalent Current Injections

- The initial Norton equivalent current injections on the dq base for each machine are

$$I_{Nd1} + jI_{Nq1} = \frac{\left(-\psi''_{q1} + j\psi''_{d1}\right)\omega_1}{jX''_1} = \frac{\left(-0.2236 + j1.179\right)(1.0)}{j0.18}$$

$$= 6.55 + j1.242$$

$$I_{ND1} + jI_{NQ1} = 2.222 - j6.286$$

$$I_{Nd2} + jI_{Nq2} = 4.999 + j1.826$$

$$I_{ND2} + jI_{NQ2} = -1 - j5.227$$

Recall the dq values are on the machine's reference frame and the DQ values are on the system reference frame

Moving between DQ and dq

- Recall

$$\begin{bmatrix} I_{di} \\ I_{qi} \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_{Di} \\ I_{Qi} \end{bmatrix}$$

- And

$$\begin{bmatrix} I_{Di} \\ I_{Qi} \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_{di} \\ I_{qi} \end{bmatrix}$$

The currents provide the key coupling between the two reference frames

Bus Admittance Matrix

- The bus admittance matrix is as from before for the classical models, except the diagonal elements are augmented using

$$Y_i = \frac{1}{R_{s,i} + jX''_{d,i}}$$

$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.18} & 0 \\ 0 & \frac{1}{j0.18} \end{bmatrix} = \begin{bmatrix} -j10.101 & j4.545 \\ j4.545 & -j10.101 \end{bmatrix}$$

Algebraic Solution Verification

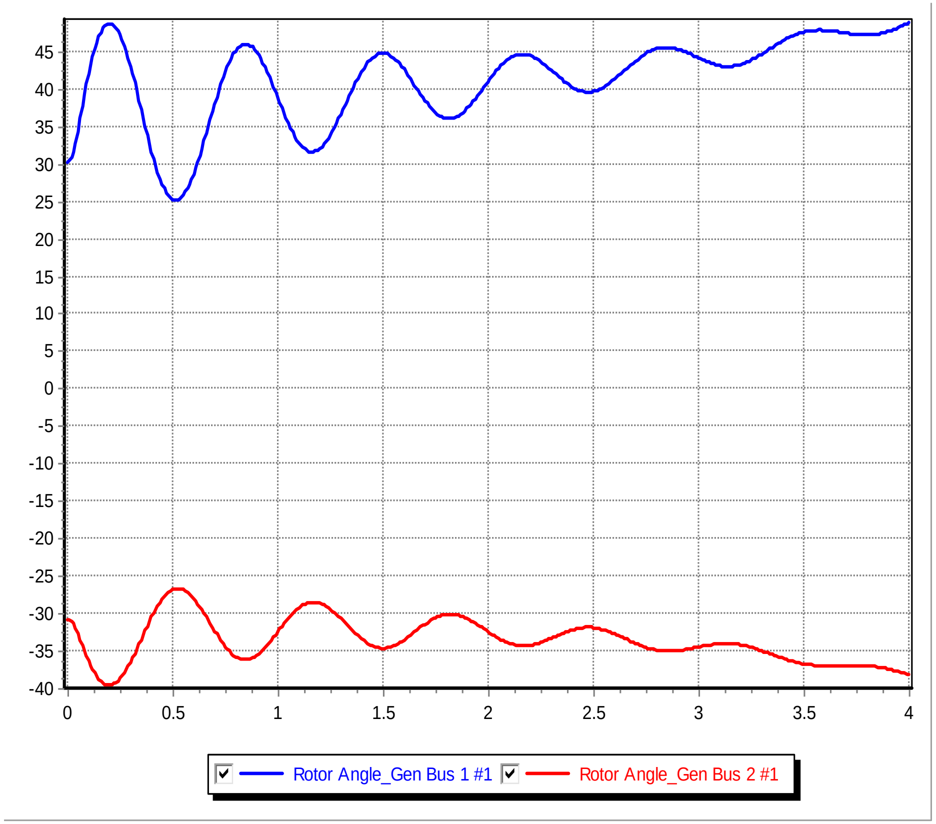
- To check the values solve (in the network reference frame)

$$\mathbf{V} = \begin{bmatrix} -j10.101 & j4.545 \\ j4.545 & -j10.101 \end{bmatrix}^{-1} \begin{bmatrix} 2.222 - j6.286 \\ -1 - j5.227 \end{bmatrix}$$

$$= \begin{bmatrix} 1.072 + j0.22 \\ 1.0 \end{bmatrix}$$

Results

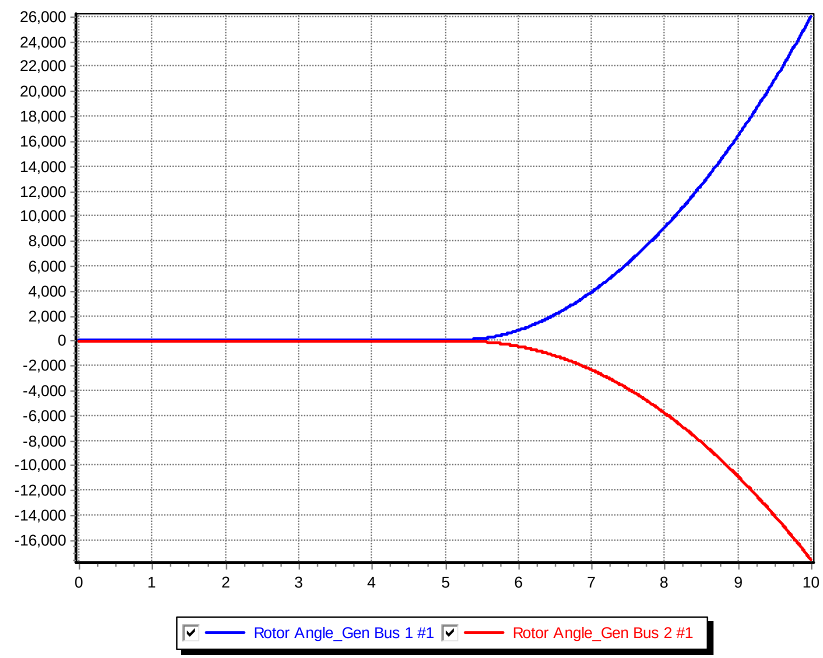
- The below graph shows the results for four seconds of simulation, using Euler's with Δt=0.01 seconds



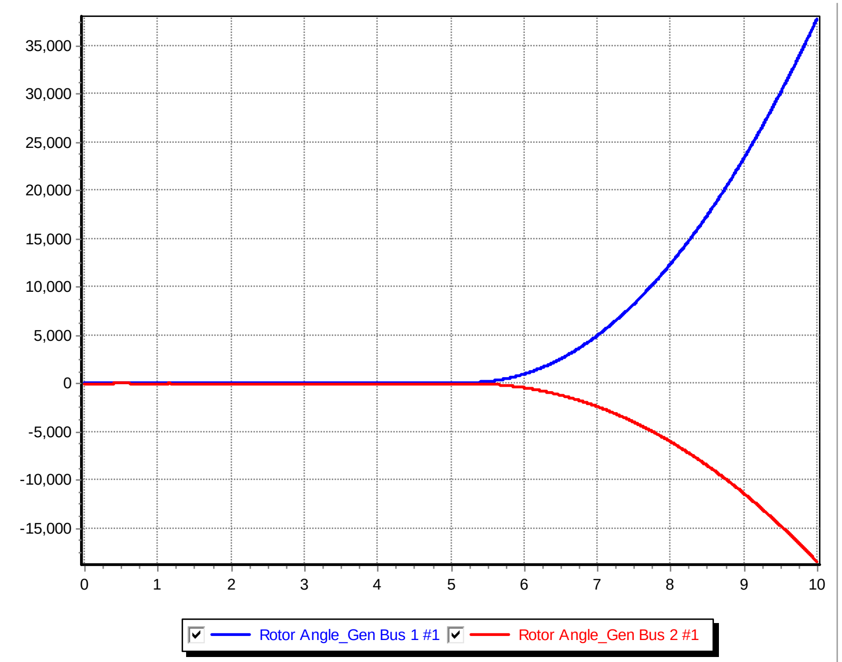
PowerWorld case is B2_GENROU_2GEN_EULER

Results for Longer Time

- Simulating out 10 seconds indicates an unstable solution, both using Euler's and RK2 with Δt=0.005, so it is really unstable!



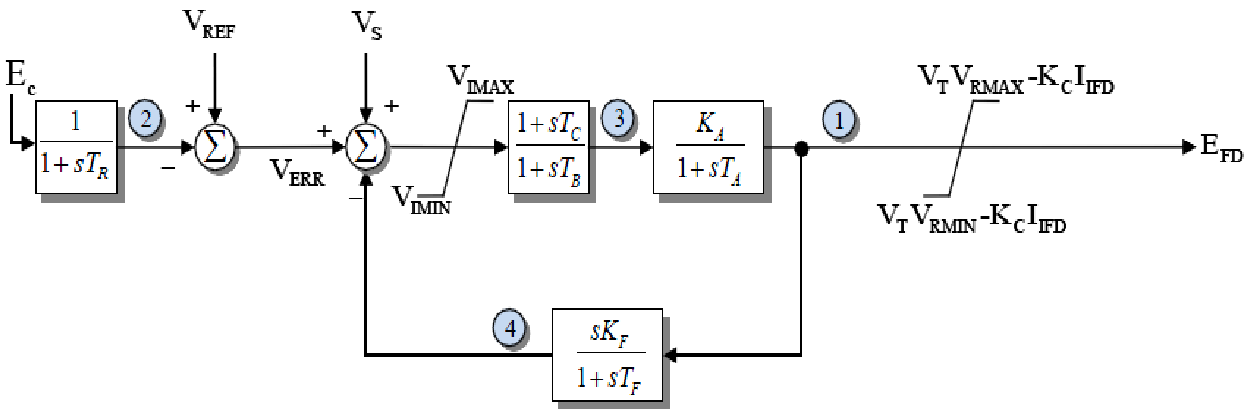
Euler's with Δt=0.01



RK2 with Δt=0.005

Adding More Models

- In this situation the case is unstable because we have not modeled exciters
- To each generator add an EXST1 with T_R=0, T_C=T_B=0, K_f=0, K_A=100, T_A=0.1



  - This just adds one differential equation per generator

$$\frac{dE_{FD}}{dt} = \frac{1}{T_A}\left(K_A\left(V_{REF} - |V_t|\right) - E_{FD}\right)$$

Two Bus, Two Gen With Exciters

- Below are the initial values for this case from PowerWorld

All States | State Limit Violations | Generators | Buses | Transient Stability YBus | GIC GMatrix | Two Bus Equivalents

Records ▾ Set ▾ Columns ▾ f(x) ▾

	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value
1	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Angle	0.5273
2	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Speed w	0.0000
3	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Eqp	1.1948
4	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiDp	1.1554
5	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.2446
6	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Edp	0.0000
7	Gen Exciter	EXST1	1 (Bus 1) #1		NO	EField before lim	2.6904
8	Gen Exciter	EXST1	1 (Bus 1) #1		YES	Sensed Vt	1.0946
9	Gen Exciter	EXST1	1 (Bus 1) #1		YES	VLL	0.0269
10	Gen Exciter	EXST1	1 (Bus 1) #1		NO	VF	0.0000
11	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Angle	-0.5392
12	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Speed w	0.0000
13	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Eqp	0.9044
14	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8928
15	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.3594
16	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Edp	0.0000
17	Gen Exciter	EXST1	2 (Bus 2) #1		NO	EField before lim	1.3441
18	Gen Exciter	EXST1	2 (Bus 2) #1		YES	Sensed Vt	1.0000
19	Gen Exciter	EXST1	2 (Bus 2) #1		YES	VLL	0.0134
20	Gen Exciter	EXST1	2 (Bus 2) #1		NO	VF	0.0000

Because of the zero values the other differential equations for the exciters are included but treated as ignored

Case is B2_GENROU_2GEN_EXCITER

Viewing the States

- PowerWorld allows one to single-step through a solution, showing the **f**(**x**) and the $\mathbf{K}_1$ values
  - This is mostly used for education or model debugging

All States | State Limit Violations | Generators | Buses | Transient Stability YBus | GIC GMatrix | Two Bus Equivalents

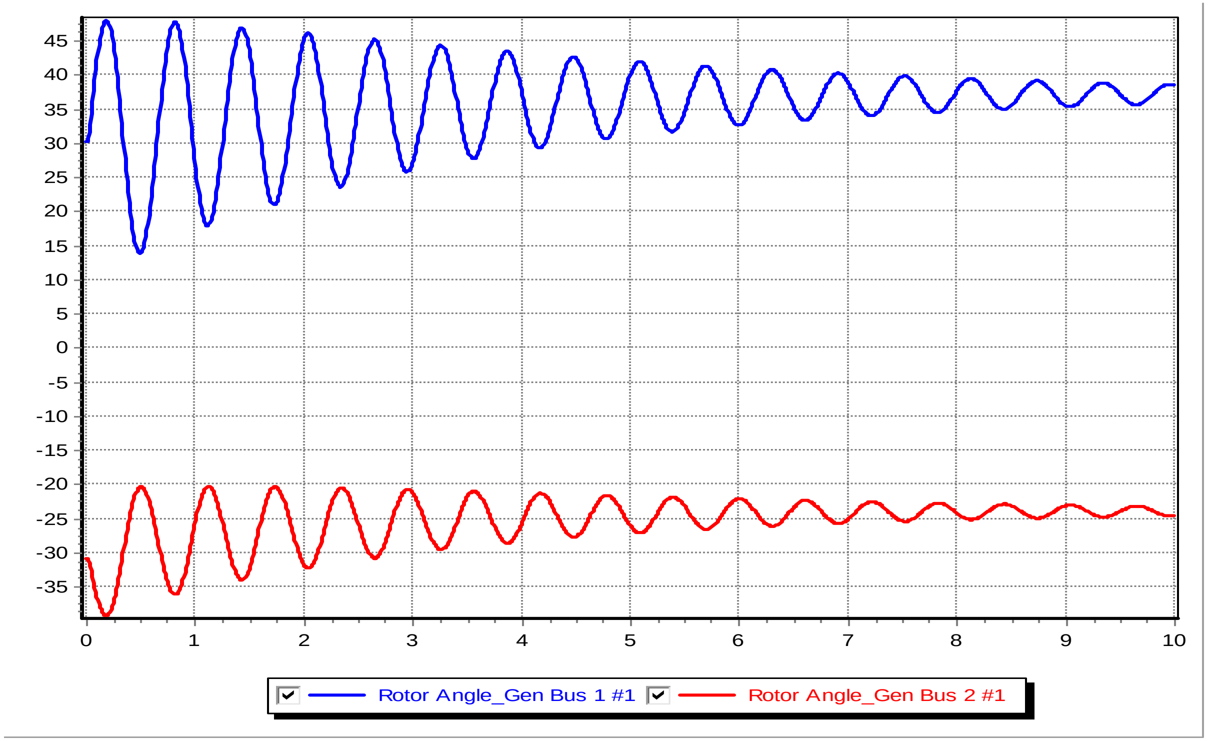
Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value	Derivative	Delta X K1
1	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Angle	0.5288	0.6283185	0.0015708
2	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Speed w	0.0017	0.1666667	0.0016667
3	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Eqp	1.1813	-1.4246850	-0.0135115
4	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiDp	1.0788	-6.1374236	-0.0766226
5	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.1276	-7.0939033	-0.1170377
6	Gen Synch. Mac	GENROU	1 (Bus 1) #1		NO	Edp	0.0000	0.0000000	0.0000000
7	Gen Exciter	EXST1	1 (Bus 1) #1		NO	EField before lim	3.4214	65.7861970	0.7309577
8	Gen Exciter	EXST1	1 (Bus 1) #1		YES	Sensed Vt	0.0000	0.0000000	0.0000000
9	Gen Exciter	EXST1	1 (Bus 1) #1		YES	VLL	0.1000	0.0000000	0.0000000
10	Gen Exciter	EXST1	1 (Bus 1) #1		NO	VF	0.0000	0.0000000	0.0000000
11	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Angle	-0.5400	-0.2896794	-0.0007854
12	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Speed w	-0.0008	-0.0833331	-0.0007684
13	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Eqp	0.9010	-0.2497156	-0.0033918
14	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8661	-2.1684713	-0.0267221
15	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.2480	8.9252864	0.1113928
16	Gen Synch. Mac	GENROU	2 (Bus 2) #1		NO	Edp	0.0000	0.0000000	0.0000000
17	Gen Exciter	EXST1	2 (Bus 2) #1		NO	EField before lim	2.2097	77.9031593	0.8655907
18	Gen Exciter	EXST1	2 (Bus 2) #1		YES	Sensed Vt	0.5032	0.0000000	0.0000000
19	Gen Exciter	EXST1	2 (Bus 2) #1		YES	VLL	0.1000	0.0000000	0.0000000
20	Gen Exciter	EXST1	2 (Bus 2) #1		NO	VF	0.0000	0.0000000	0.0000000

Derivatives shown are evaluated at the end of the time step

Two Bus Results with Exciters

- Below graph shows the angles with Δt=0.01 and a fault clearing at t=0.05 using Euler's
  - With the addition of the exciters case is now stable



Constant Impedance Loads

- The simplest approach for modeling the loads is to treat them as constant impedances, embedding them in the bus admittance matrix
  - Only impact the Ybus diagonals
- The admittances are set based upon their power flow values, scaled by the inverse of the square of the power flow bus voltage

$$\bar{S}_{load,i} = \bar{V}_i \bar{I}^*_{load,i} = \left|\bar{V}_i\right|^2 \left(G_{load,i} - jB_{load,i}\right)$$

$$G_{load,i} - jB_{load,i} = \frac{\bar{S}_{load,i}}{\left|\bar{V}_i\right|^2}$$

Note the positive sign comes from the sign convention on $\bar{\mathrm{I}}_{\mathrm{load,i}}$

In PowerWorld the default load model is specified on Transient Stability, Options, Power System Model

Example 7.4 Case (WSCC 9 Bus)

- PowerWorld Case Example_7_4 duplicates the example 7.4 case from the book, with the exception of using different generator models

Violations | Generators | Buses | Transient Stability YBus | GIC GMatrix | Two Bus Equivalents

Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

	Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6	Bus 7	Bus 8	Bus 9
1	Bus1	0.000 - j42.361			-0.000 + j17.361					
2	Bus 2		0.000 - j27.111					-0.000 + j16.000		
3	Bus 3			0.000 - j23.732						-0.000 + j17.065
4	Bus 4	-0.000 + j17.361			3.307 - j39.309	-1.365 + j11.604	-1.942 + j10.511			
5	Bus 5				-1.365 + j11.604	3.814 - j17.843		-1.188 + j5.975		
6	Bus 6				-1.942 + j10.511		4.102 - j16.133			-1.282 + j5.588
7	Bus 7		-0.000 + j16.000			-1.188 + j5.975		2.805 - j35.446	-1.617 + j13.698	
8	Bus 8							-1.617 + j13.698	3.741 - j23.642	-1.155 + j9.784
9	Bus 9			-0.000 + j17.065			-1.282 + j5.588		-1.155 + j9.784	2.437 - j32.154

Bus 5 Example: Without the load $Y_{55} = 2.553 - j17.339$

$$\bar{S}_{load,5} = 1.25 + j0.5 \text{ and } \left|\bar{V}_5\right| = 0.996$$

$$\mathbf{Y}_{55} = 2.553 - j17.579 + \frac{(1.25 - j0.5)}{|0.996|^2} = 3.813 - j17.843$$

Nonlinear Network Equations

- With constant impedance loads the network equations can usually be written with **I** independent of **V**, then they can be solved directly (as we've been doing)

$$\mathbf{V} = \mathbf{Y}^{-1}\,\mathbf{I}(\mathbf{x})$$

- In general this is not the case, with constant power loads one common example
- Hence a nonlinear solution with Newton's method is used
- We'll generalize the dependence on the algebraic variables, replacing **V** by **y** since they may include other values beyond just the bus voltages

Nonlinear Network Equations

- Just like in the power flow, the complex equations are rewritten, here as a real current and a reactive current
  YV – I(x,y) = 0
- The values for bus i are

$$g_{Di}(\mathbf{x},\mathbf{y}) = \sum_{k=1}^{n} \left(G_{ik} V_{Dk} - B_{ik} V_{QK} \right) - I_{NDi} = 0$$

$$g_{Qi}(\mathbf{x},\mathbf{y}) = \sum_{k=1}^{n} \left(G_{ik} V_{Qk} + B_{ik} V_{DK} \right) - I_{NQi} = 0$$

- For each bus we add two new variables and two new equations
- If an infinite bus is modeled then its variables and equations are omitted since its voltage is fixed

This is a rectangular formulation; we also could have written the equations in polar form

Nonlinear Network Equations

- The network variables and equations are then

$$\mathbf{y} = \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ \vdots \\ V_{Dn} \\ V_{Qn} \end{bmatrix} \qquad \mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{bmatrix} \sum_{k=1}^{n} \left(G_{1k} V_{Dk} - B_{1k} V_{QK} \right) - I_{ND1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{ik} V_{Qk} + B_{ik} V_{DK} \right) - I_{NQ1}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{2k} V_{Dk} - B_{2k} V_{QK} \right) - I_{ND2}(\mathbf{x},\mathbf{y}) = 0 \\ \vdots \\ \sum_{k=1}^{n} \left(G_{nk} V_{Dk} - B_{nk} V_{QK} \right) - I_{NDn}(\mathbf{x},\mathbf{y}) = 0 \\ \sum_{k=1}^{n} \left(G_{nk} V_{Qk} + B_{nk} V_{DK} \right) - I_{NQn}(\mathbf{x},\mathbf{y}) = 0 \end{bmatrix}$$

Nonlinear Network Equation Newton Solution

The network equations are solved using a similar procedure to that of the Netwon-Raphson power flow

Set $v = 0$; make an initial guess of $\mathbf{y}$, $\mathbf{y}^{(v)}$

While $\left\|\mathbf{g}(\mathbf{y}^{(v)})\right\| > \varepsilon$ Do

$$\begin{aligned}\mathbf{y}^{(v+1)} &= \mathbf{y}^{(v)} - \mathbf{J}(\mathbf{y}^{(v)})^{-1}\mathbf{g}(\mathbf{y}^{(v)}) \\ v &= v+1\end{aligned}$$

End While

Network Equation Jacobian Matrix

- The most computationally intensive part of the algorithm is determining and factoring the Jacobian matrix, $\mathbf{J}(\mathbf{y})$

$$\mathbf{J}(\mathbf{y}) = \begin{bmatrix} \dfrac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \dfrac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \dfrac{\partial g_{D1}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \\ \dfrac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \dfrac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \dfrac{\partial g_{Q1}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \\ \vdots & \ddots & \ddots & \vdots \\ \dfrac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{D1}} & \dfrac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{Q1}} & \cdots & \dfrac{\partial g_{Qn}(\mathbf{x},\mathbf{y})}{\partial V_{Qn}} \end{bmatrix}$$

Network Jacobian Matrix

- The Jacobian matrix can be stored and computed using a 2 by 2 block matrix structure
- The portion of the 2 by 2 entries just from the $\mathbf{Y}_{bus}$ are

$$\begin{bmatrix} \dfrac{\partial \hat{g}_{Di}(\mathbf{x},\mathbf{y})}{\partial V_{Dj}} & \dfrac{\partial \hat{g}_{Di}(\mathbf{x},\mathbf{y})}{\partial V_{Qj}} \\ \dfrac{\partial \hat{g}_{Qi}(\mathbf{x},\mathbf{y})}{\partial V_{Dj}} & \dfrac{\partial \hat{g}_{Qi}(\mathbf{x},\mathbf{y})}{\partial V_{Qj}} \end{bmatrix} = \begin{bmatrix} G_{ij} & -B_{ij} \\ B_{ij} & G_{ij} \end{bmatrix}$$

The "hat" was added to the g functions to indicate it is just the portion from the $\mathbf{Y}_{bus}$

- The major source of the current vector voltage sensitivity comes from non-constant impedance loads; also dc transmission lines

Example: Constant Current and Constant Power Load

- As an example, assume the load at bus k is represented with a ZIP model

$$P_{Load,k} = P_{BaseLoad,k}\left(P_{z,k}\left|\bar{V}_k^{2}\right| + P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(Q_{z,k}\left|\bar{V}_k^{2}\right| + Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right)$$

The base load values are set from the power flow

- The constant impedance portion is embedded in the $\mathbf{Y}_{\text{bus}}$

$$\hat{P}_{Load,k} = P_{BaseLoad,k}\left(P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right) = \left(P_{BL,i,k}\left|\bar{V}_k\right| + P_{BL,p,k}\right)$$

$$\hat{Q}_{Load,k} = Q_{BaseLoad,k}\left(Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right) = \left(Q_{BL,i,k}\left|\bar{V}_k\right| + Q_{BL,p,k}\right)$$

- Usually solved in per unit on network MVA base

Example: Constant Current and Constant Power Load

- The current is then

$$\bar{I}_{Load,k} = I_{D,Load,k} + jI_{Q,Load,k} = \left(\frac{\hat{P}_{Load,k} + j\hat{Q}_{Load,k}}{\bar{V}_k} \right)^*$$

$$= \left(\frac{\left(P_{BL,i,k}\sqrt{V_{DK}^2 + V_{QK}^2} + P_{BL,p,k} \right) - j\left(Q_{BL,i,k}\sqrt{V_{DK}^2 + V_{QK}^2} + Q_{BL,p,k} \right)}{V_{Dk} - jV_{Qk}} \right)$$

- Multiply the numerator and denominator by $V_{DK}+jV_{QK}$ to write as the real current and the reactive current

Example: Constant Current and Constant Power Load

$$I_{D,Load,k} = \frac{V_{Dk}P_{BL,p,k} + V_{QK}Q_{BL,p,k}}{V_{DK}^2 + V_{QK}^2} + \frac{V_{Dk}P_{BL,i,k} + V_{QK}Q_{BL,i,k}}{\sqrt{V_{DK}^2 + V_{QK}^2}}$$

$$I_{Q,Load,k} = \frac{V_{Qk}P_{BL,p,k} - V_{DK}Q_{BL,p,k}}{V_{DK}^2 + V_{QK}^2} + \frac{V_{Qk}P_{BL,i,k} - V_{DK}Q_{BL,i,k}}{\sqrt{V_{DK}^2 + V_{QK}^2}}$$

- The Jacobian entries are then found by differentiating with respect to V_{DK} and V_{QK}
  - Only affect the 2 by 2 block diagonal values
- Usually constant current and constant power models are replaced by a constant impedance model if the voltage goes too low, like during a fault