

# ECEN 667

## Power System Stability

### Lecture 20: Oscillations, Small Signal Stability Analysis

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# Announcements



- Read Chapter 7
- Homework 6 is due today
- Final is as per TAMU schedule. That is, Friday Dec 8 from 3 to 5pm

# Oscillations



- An oscillation is just a repetitive motion that can be either undamped, positively damped (decaying with time) or negatively damped (growing with time)
- If the oscillation can be written as a sinusoid then

$$e^{\alpha t} (a \cos(\omega t) + b \sin(\omega t)) = e^{\alpha t} C \cos(\omega t + \theta)$$

$$\text{where } C = \sqrt{A^2 + B^2} \text{ and } \theta = \tan\left(\frac{-b}{a}\right)$$

- And the damping ratio is defined as (see Kundur 12.46)

$$\xi = \frac{-\alpha}{\sqrt{\alpha^2 + \omega^2}}$$

The percent damping is just the damping ratio multiplied by 100; goal is sufficiently positive damping

# Power System Oscillations

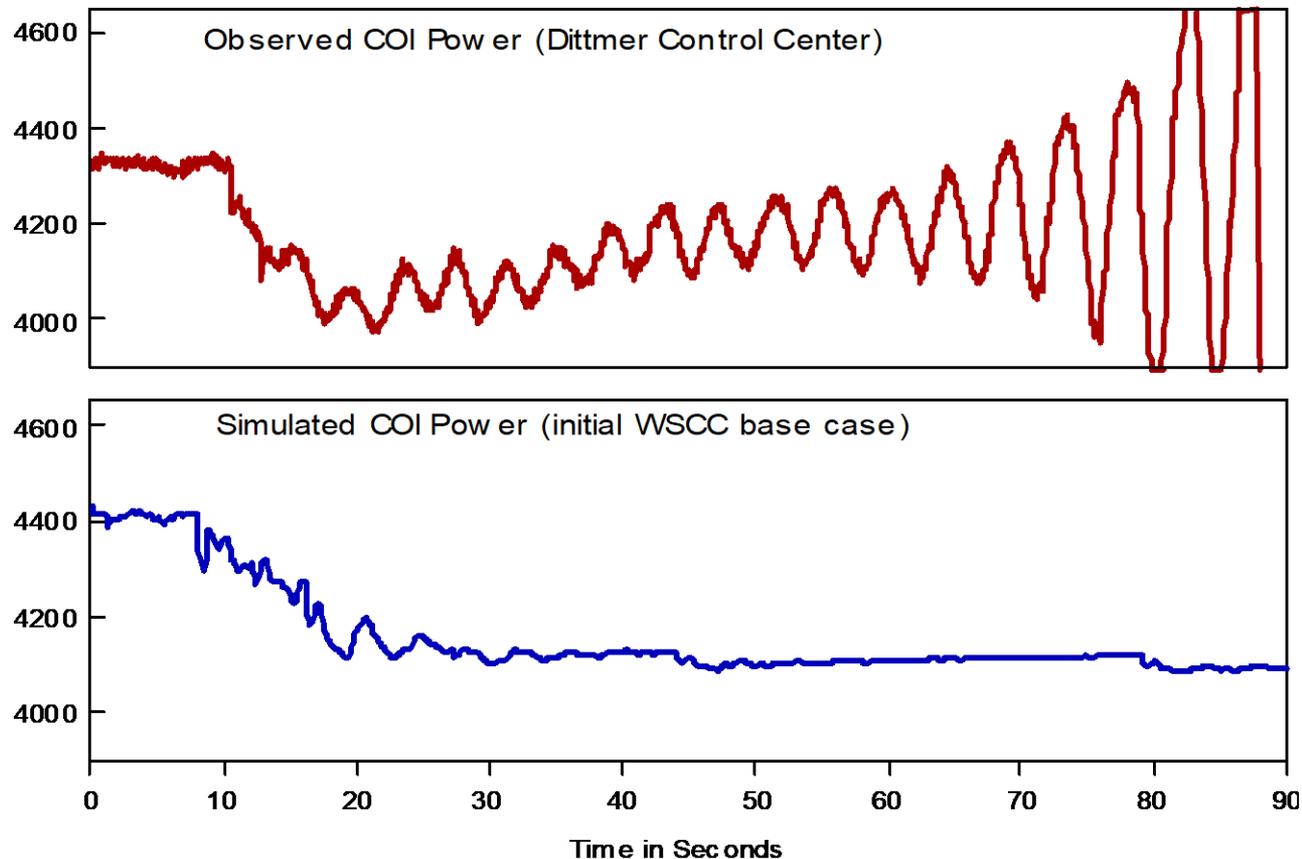


- Power systems can experience a wide range of oscillations, ranging from highly damped and high frequency switching transients to sustained low frequency ( $< 2$  Hz) inter-area oscillations affecting an entire interconnect
- Types of oscillations include
  - Transients: Usually high frequency and highly damped
  - Local plant: Usually from 1 to 5 Hz
  - Inter-area oscillations: From 0.15 to 1 Hz
  - Slower dynamics: Such as AGC, less than 0.15 Hz
  - Subsynchronous resonance: 10 to 50 Hz (less than synchronous)

# Example Oscillations



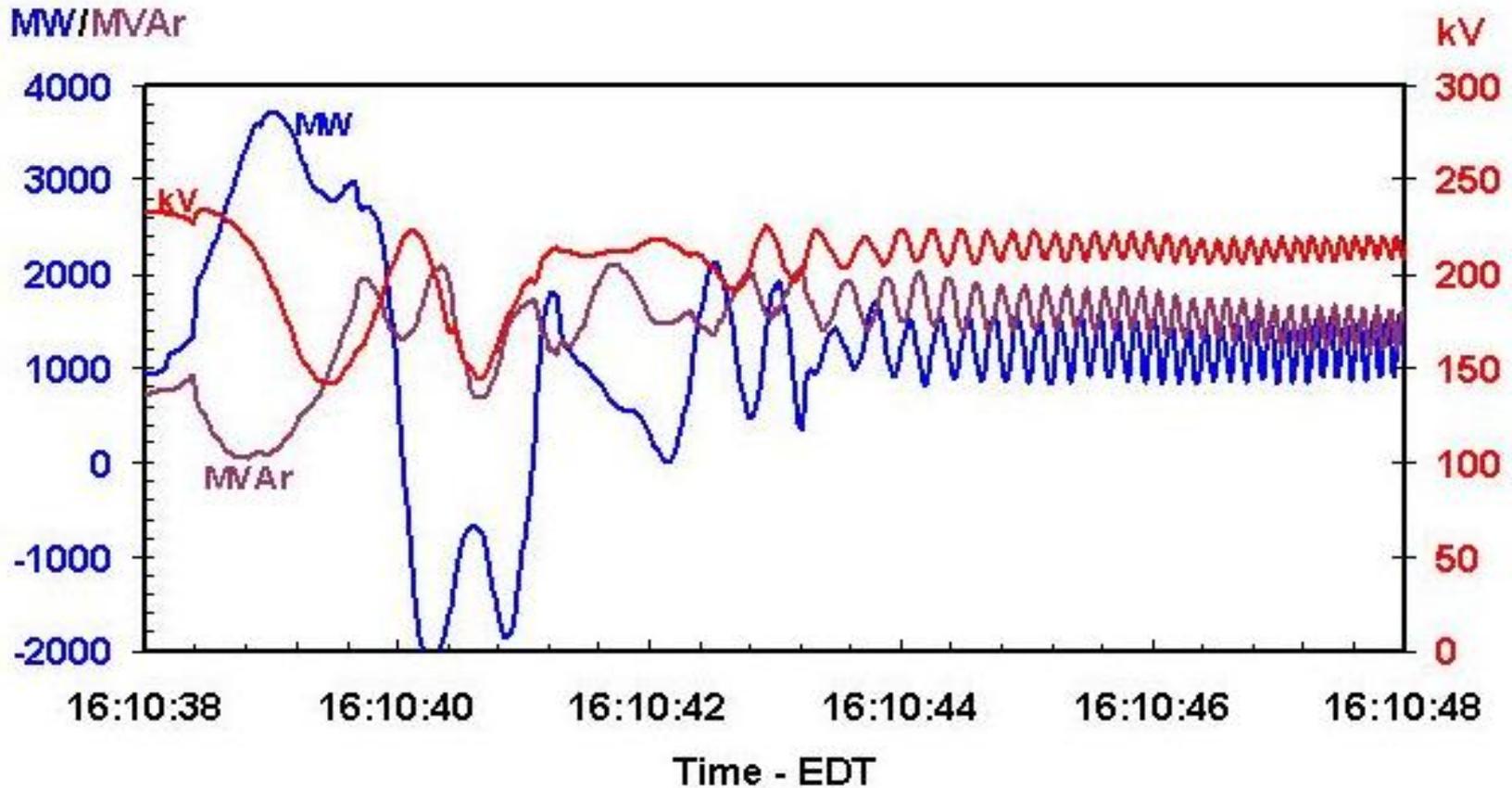
- The below graph shows an oscillation that was observed during a 1996 WECC Blackout



# Example Oscillations



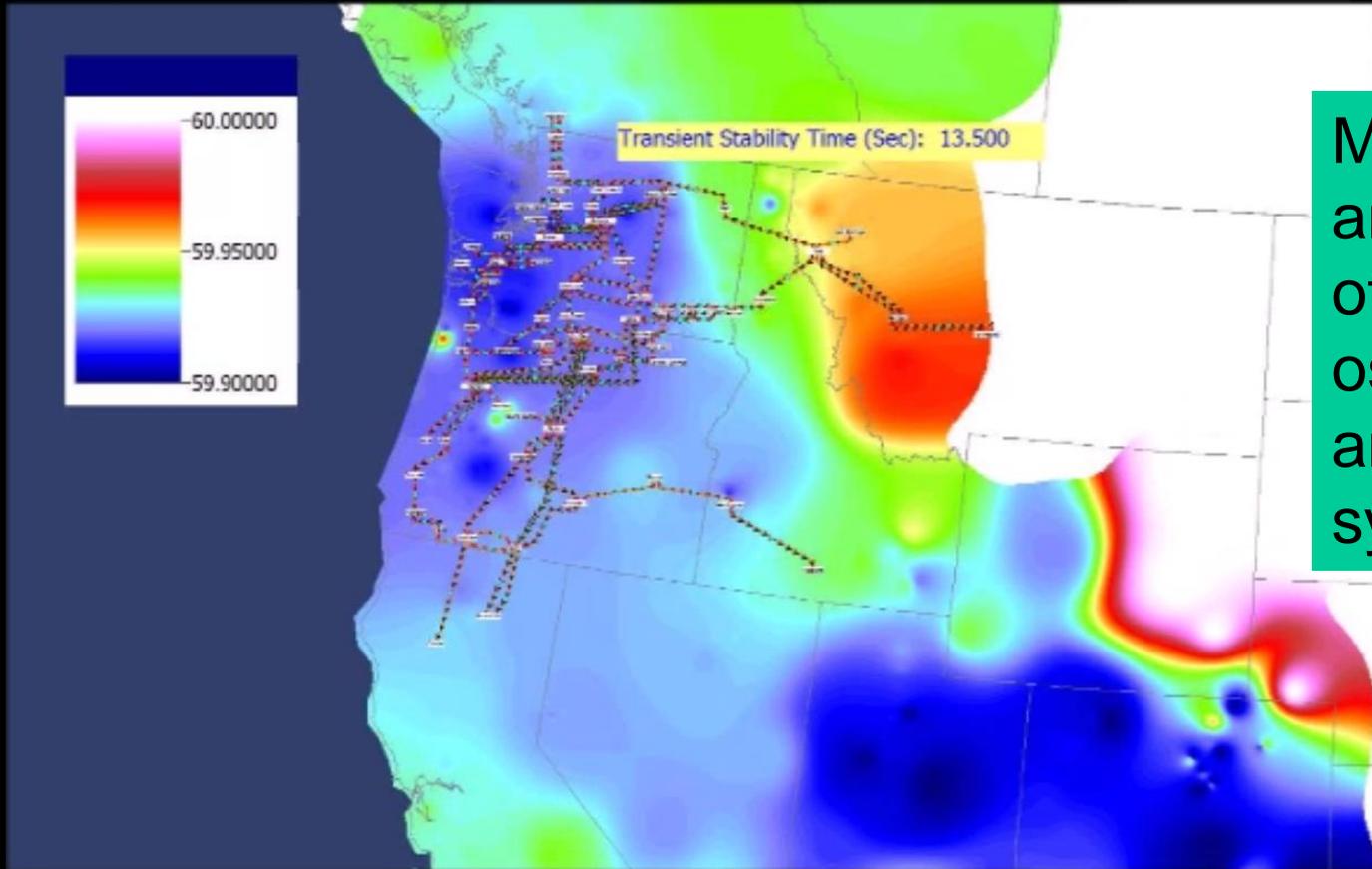
- The below graph shows oscillations on the Michigan/Ontario Interface on 8/14/03



# Fictitious System Oscillation



- Movies & TV



Movie shows an example of sustained oscillations in an equivalent system

# Forced Oscillations in WECC (from [1])



- Summer 2013 24 hour data: 0.37 Hz oscillations observed for several hours. Confirmed to be forced oscillations at a hydro plant from vortex effect.
- 2014 data: Another 0.5 Hz oscillation also observed. Source points to hydro unit as well. And 0.7 Hz. And 1.12 Hz. And 2 Hz.
- Resonance possible when system mode poorly damped and close. Resonance observed in model simulations.

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# Observing Modes and Damping



- With the advent of wide-scale PMU deployments, the modes and damping can be observed two ways
  - Event (ringdown) analysis – this requires an event
  - Ambient noise analysis – always available, but not as distinct

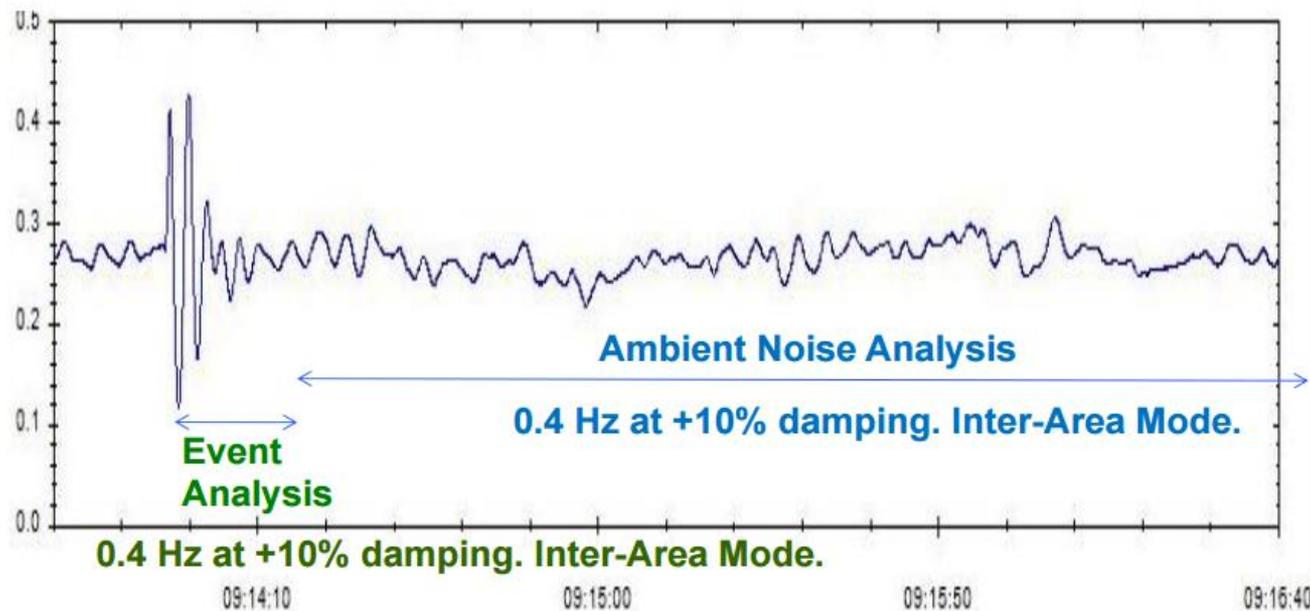


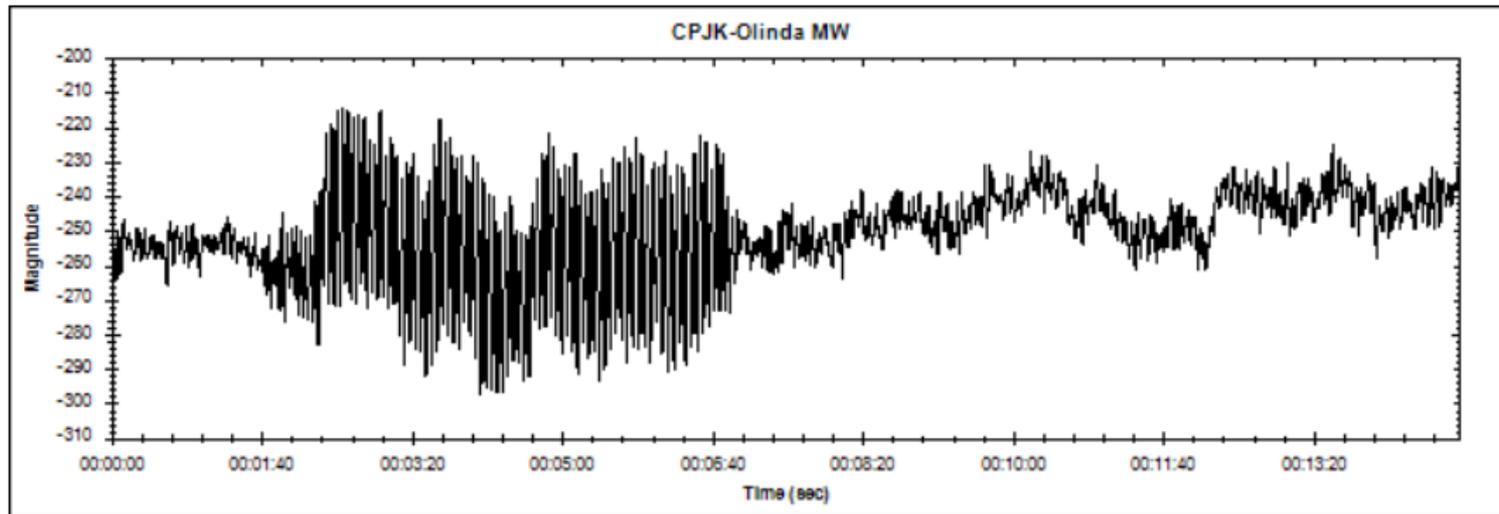
Image Source: M. Venkatasubramanian, "Oscillation Monitoring System", June 2015  
<http://www.energy.gov/sites/prod/files/2015/07/f24/3.%20Mani%20Oscillation%20Monitoring.pdf>

# Resonance with Interarea Mode [1]



- Resonance effect high when:
  - Forced oscillation frequency near system mode frequency
  - System mode poorly damped
  - Forced oscillation location near the two distant ends of mode
- Resonance effect medium when
  - Some conditions hold
- Resonance effect small when
  - None of the conditions holds

# Medium Resonance on 11/29/2005



- 20 MW 0.26 Hz Forced Oscillation in Alberta Canada
- 200 MW Oscillations on California-Oregon Inter-tie
- System mode 0.27 Hz at 8% damping
- Two out of the three conditions were true.

1. M. Venkatasubramanian, "Oscillation Monitoring System", June 2015

<http://www.energy.gov/sites/prod/files/2015/07/f24/3.%20Mani%20Oscillation%20Monitoring.pdf>

# An On-line Oscillation Detection Tool

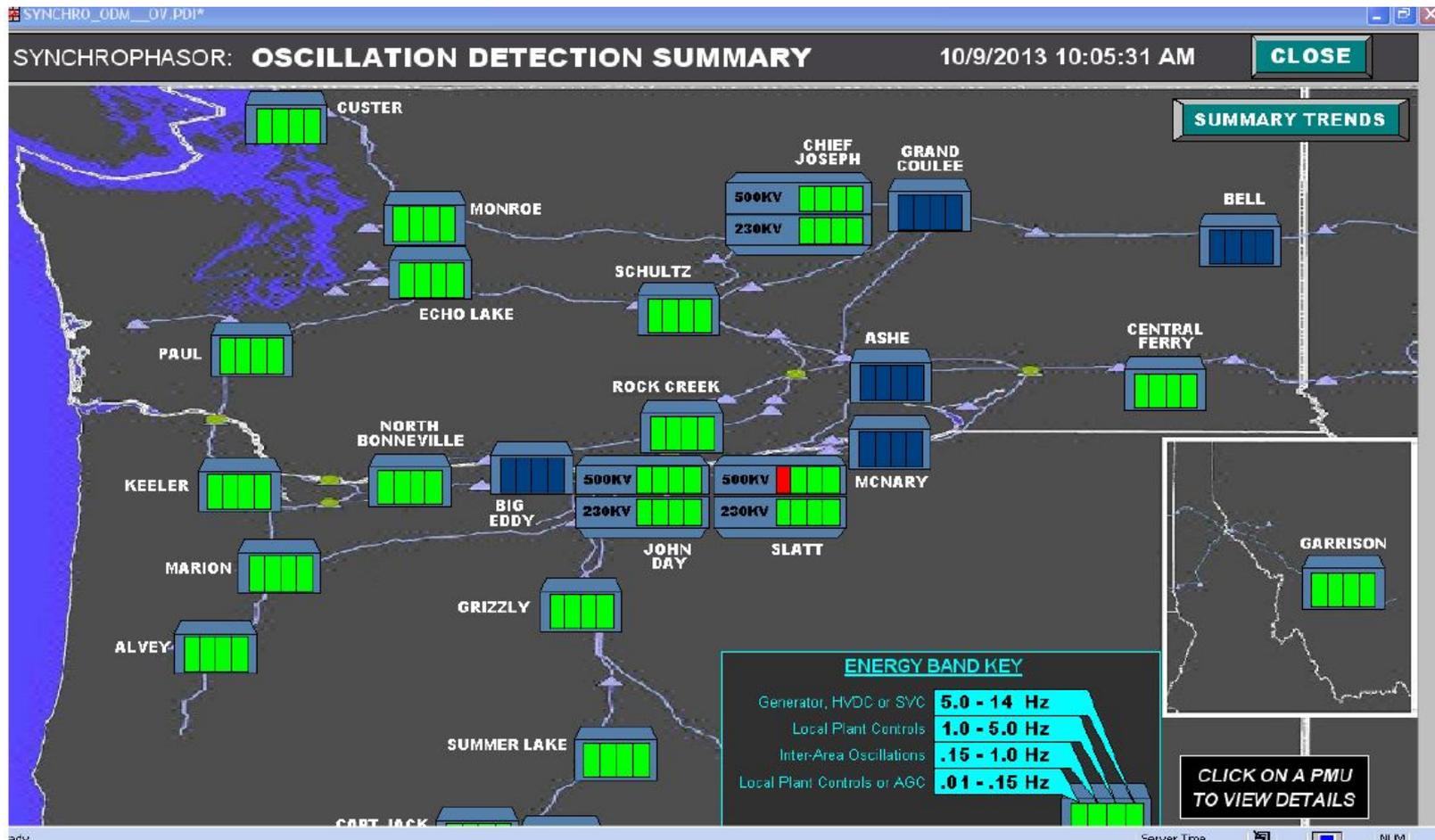


Image source: WECC Joint Synchronized Information Subcommittee Report, October 2013

# Small Signal Stability Analysis



- Small signal stability is the ability of the power system to maintain synchronism following a small disturbance
  - System is continually subject to small disturbances, such as changes in the load
- The operating equilibrium point (EP) obviously must be stable
- Small system stability analysis (SSA) is studied to get a feel for how close the system is to losing stability and to get additional insight into the system response
  - There must be positive damping

# Model Based SSA



- Assume the power system is modeled in our standard form as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

- The system can be linearized about an equilibrium point

$$\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x} + \mathbf{B} \Delta \mathbf{y}$$

$$\mathbf{0} = \mathbf{C} \Delta \mathbf{x} + \mathbf{D} \Delta \mathbf{y}$$

- Eliminating  $\Delta \mathbf{y}$  gives

$$\Delta \dot{\mathbf{x}} = \left( \mathbf{A} - \mathbf{B} \mathbf{D}^{-1} \mathbf{C} \right) \Delta \mathbf{x} = \mathbf{A}_{\text{sys}} \Delta \mathbf{x}$$

If there are just classical generator models then  $\mathbf{D}$  is the power flow Jacobian; otherwise it also includes the stator algebraic equations

# Model Based SSA

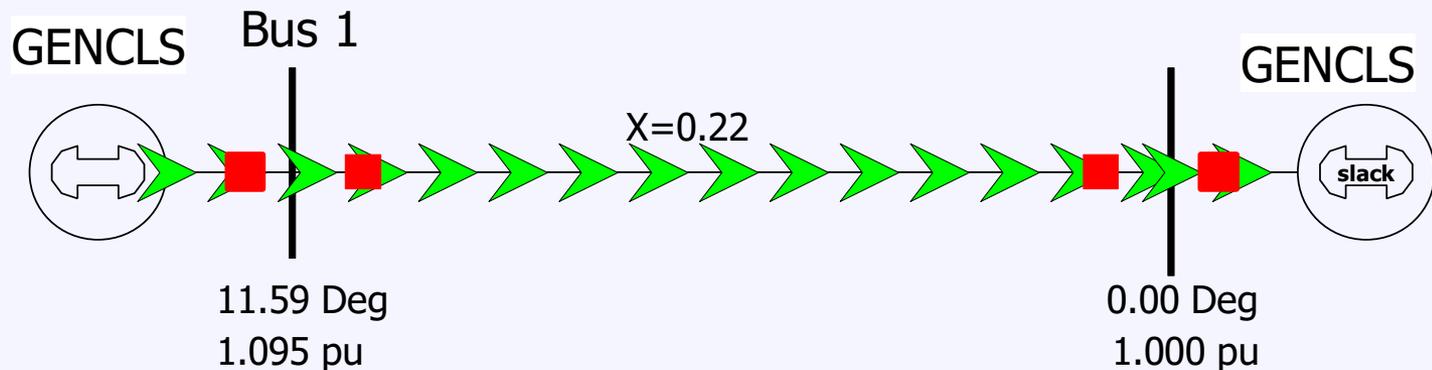


- The matrix  $\mathbf{A}_{\text{sys}}$  can be calculated doing a partial factorization, just like what was done with Kron reduction
- SSA is done by looking at the eigenvalues (and other properties) of  $\mathbf{A}_{\text{sys}}$

# SSA Two Generator Example



- Consider the two bus, two classical generator system from lectures 18 and 20 with  $X_{d1}'=0.3$ ,  $H_1=3.0$ ,  $X_{d2}'=0.2$ ,  $H_2=6.0$



- Essentially everything needed to calculate the **A**, **B**, **C** and **D** matrices was covered in lecture 19

# SSA Two Generator Example



- The  $\mathbf{A}$  matrix is calculated differentiating  $\mathbf{f}(\mathbf{x}, \mathbf{y})$  with respect to  $\mathbf{x}$  (where  $\mathbf{x}$  is  $\delta_1, \Delta\omega_1, \delta_2, \Delta\omega_2$ )

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu} \omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2H_1} (P_{M1} - P_{E1} - D_1 \Delta\omega_{1,pu})$$

$$\frac{d\delta_2}{dt} = \Delta\omega_{2,pu} \omega_s$$

$$\frac{d\Delta\omega_{2,pu}}{dt} = \frac{1}{2H_2} (P_{M2} - P_{E2} - D_2 \Delta\omega_{2,pu})$$

$$P_{Ei} = (E_{Di}^2 - E_{Di} V_{Di}) G_i + (E_{Qi}^2 - E_{Qi} V_{Qi}) G_i + (E_{Di} V_{Qi} - E_{Qi} V_{Di}) B_i$$

$$E_{Di} + jE_{Qi} = E_i' (\cos \delta_i + j \sin \delta_i)$$

# SSA Two Generator Example



- Giving

$$\mathbf{A} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.761 & 0 & 0 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0 & 0 & -0.389 & 0 \end{bmatrix}$$

- **B**, **C** and **D** are as calculated previously for the implicit integration, except the elements in **B** are not multiplied by  $\Delta t/2$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.2889 & 0.6505 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0.3893 \end{bmatrix}$$

# SSA Two Generator Example



- The **C** and **D** matrices are

$$\mathbf{C} = \begin{bmatrix} -3.903 & 0 & 0 & 0 \\ -1.733 & 0 & 0 & 0 \\ 0 & 0 & -4.671 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 7.88 & 0 & -4.54 \\ -7.88 & 0 & 4.54 & 0 \\ 0 & -4.54 & 0 & 9.54 \\ 4.54 & 0 & -9.54 & 0 \end{bmatrix}$$

- Giving

$$\mathbf{A}_{sys} = \mathbf{A} - \mathbf{BD}^{-1}\mathbf{C} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.229 & 0 & 0.229 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0.114 & 0 & -0.114 & 0 \end{bmatrix}$$

# SSA Two Generator

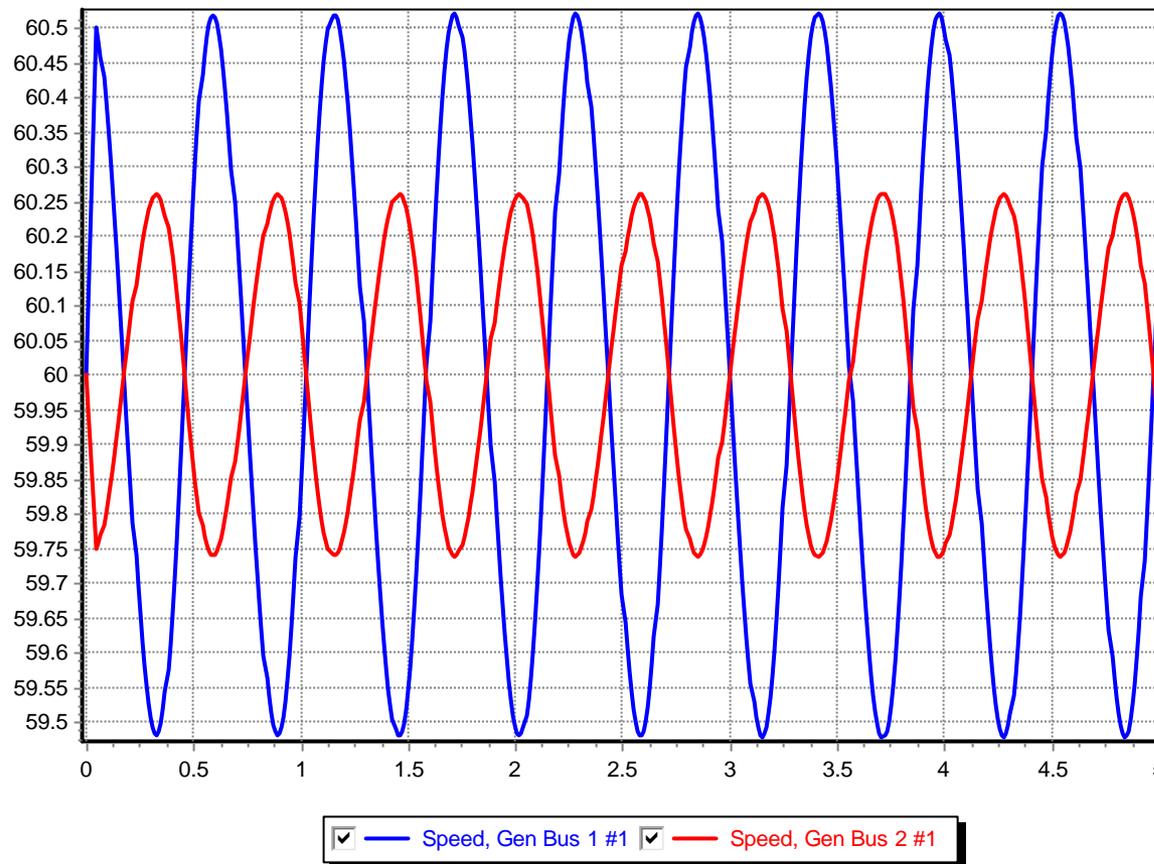


- Calculating the eigenvalues gives a complex pair and two zero eigenvalues
- The complex pair, with values of  $\pm j11.39$  corresponds to the generators oscillating against each other at 1.81 Hz
- One of the zero eigenvalues corresponds to the lack of an angle reference
  - Could be rectified by redefining angles to be with respect to a reference angle (see book 226) or we just live with the zero
- Other zero is associated with lack of speed dependence in the generator torques

# SSA Two Generator Speeds



- The two generator system response is shown below for a small disturbance

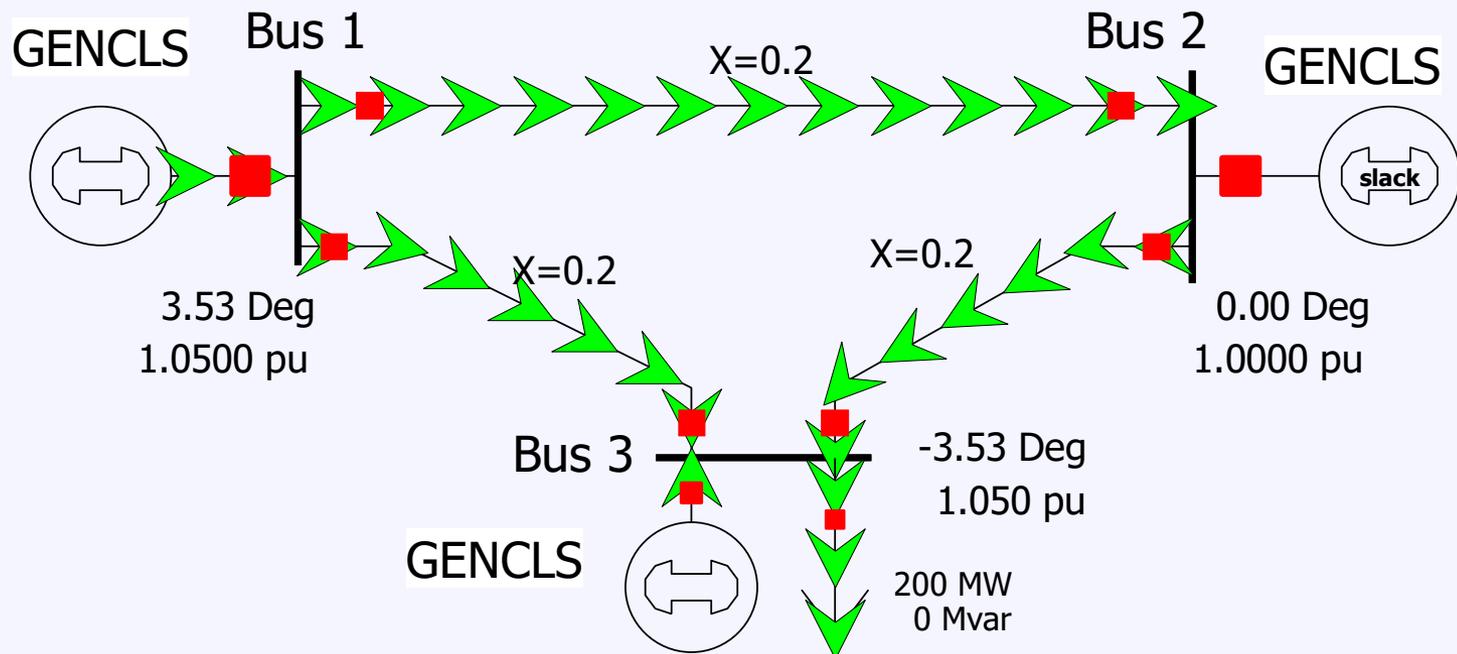


Notice the actual response closely matches the calculated frequency

# SSA Three Generator Example



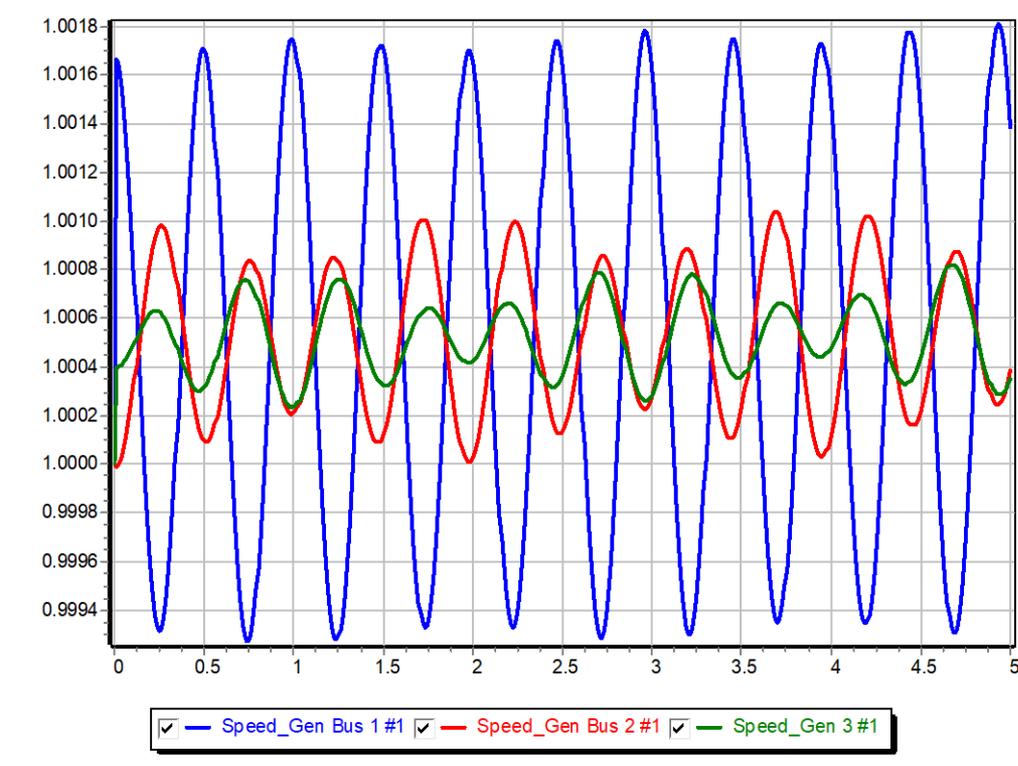
- The two generator system is extended to three generators with the third generator having  $H_3$  of 8 and  $X_{d3}'=0.3$



# SSA Three Generator Example



- Using SSA, two frequencies are identified: one at 2.02 Hz and one at 1.51 Hz



The oscillation is started with a short, self-clearing fault

Shortly we'll discuss modal analysis to determine the contribution of each mode to each signal

# Visualizing the Oscillations with PowerWorld



- Visualization of results can be key to understanding and explaining power system dynamics
- The PowerWorld transient stability contour toolbar allows for the rapid creation of a time-sequenced oneline contours of transient stability results
- These displays can then be made into a movie by either
  - Capturing the screen as the contours are creating using screen recording software such as Camtasia
  - Or having Simulator automatically store the contour images as jpegs and then creating a movie using software such as Microsoft Movie Maker

# Contour Toolbar



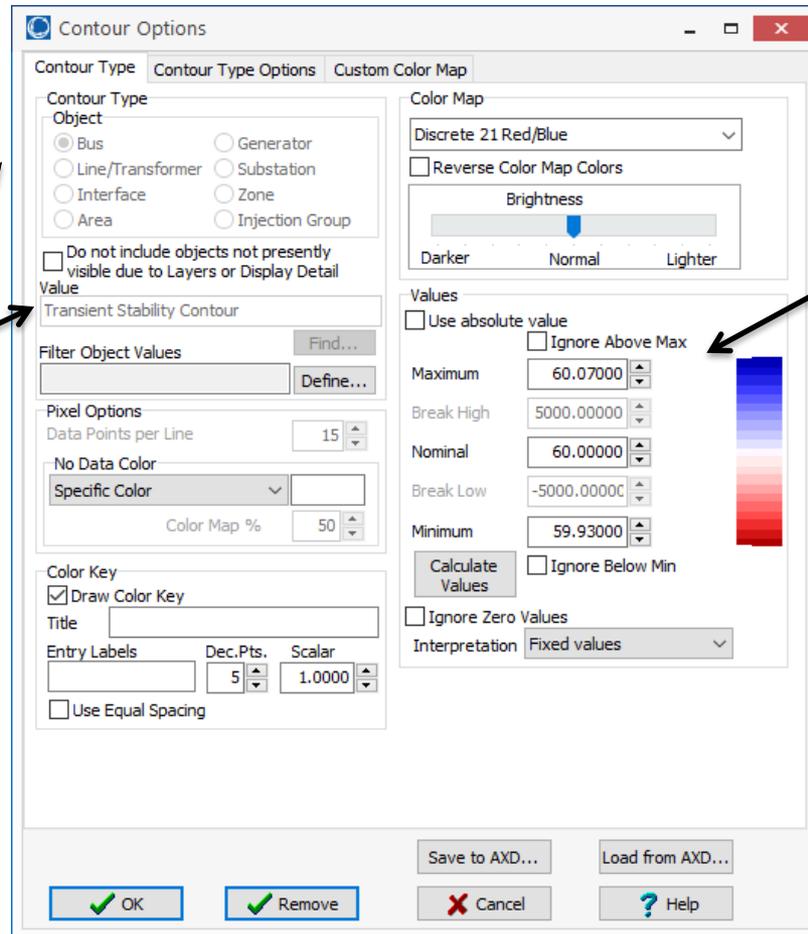
- The contour toolbar uses stored transient stability results, and hence it is used only after the transient stability solution has finished
  - It can be used with either RAM or hard drive results
- It requires having a oneline with objects associated with the desired results (buses, generators, substations, etc)
- The contour toolbar is shown by either
  - Selecting **Add-ons, Stability Case Info, Show Transient Contour Toolbar**
  - On the Transient Stability Analysis Form select the **Show Transient Contour Toolbar** button at the bottom of the form

# Contour Toolbar Buttons and Fields



- Several buttons and fields control the creation of the contour images
- The **Options** menu is used to specify three options
  - **Contour Options** is used to display the Contour Options dialog; it must be used to specify the contouring options, including the maximum/nominal/minimum values
    - When the Contour Options dialog is shown the Contour Type and Value fields are disabled since these values are specified on the toolbar
    - Other options can be set to customize the contour

# Contour Toolbar Contour Options



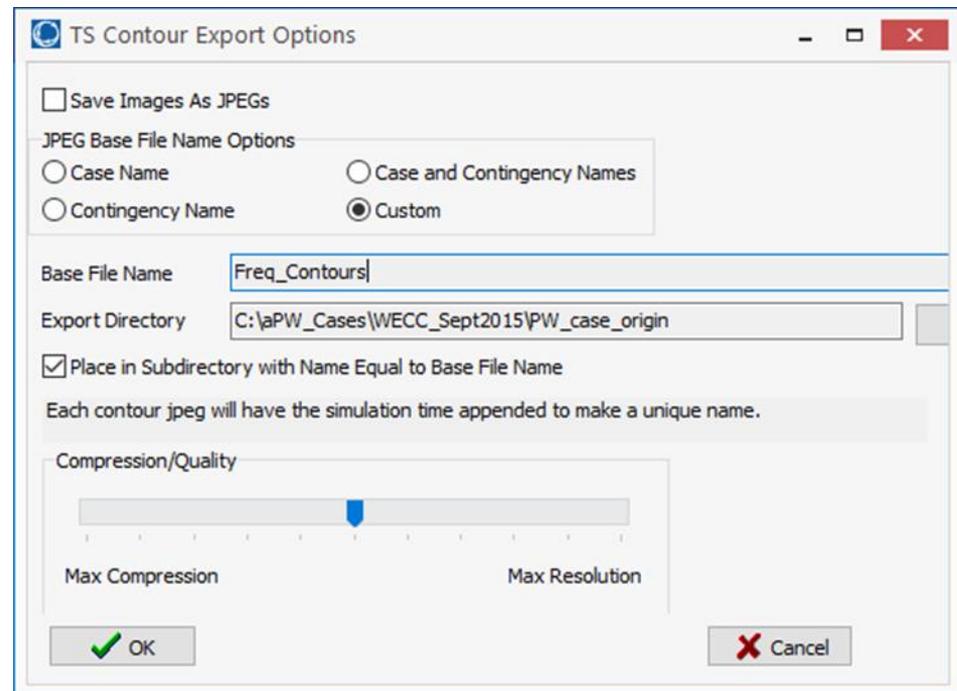
These fields are disabled

The Maximum, Nominal and Minimum values need to be set. These should be set taking into account how the values vary throughout the transient stability run.

# Contour Toolbar Export Option



- **Export** is used to display the TS Contour Export Options dialog, which provides the option of saving the contours as jpeg format files
  - On the dialog select **Save Images as JPEGs** to save the files
  - Each jpeg file will have a base file name with the associated time in seconds appended



# Contour Toolbar Value Meaning Option



- The **Value Meaning** option is used to indicate which values will actually be contoured
  - **Actual Value**: Good for things like frequency
  - **Percent of Initial**: Contours the percent of the initial value; useful for values with widely different initial values, like generator MW outputs
  - **Deviation from Initial**: Contours the deviation from the initial; good for voltages
  - **Percent Deviation**: Percentage deviation
- Contour limits should be set appropriately

# Creating a Contour Sequence

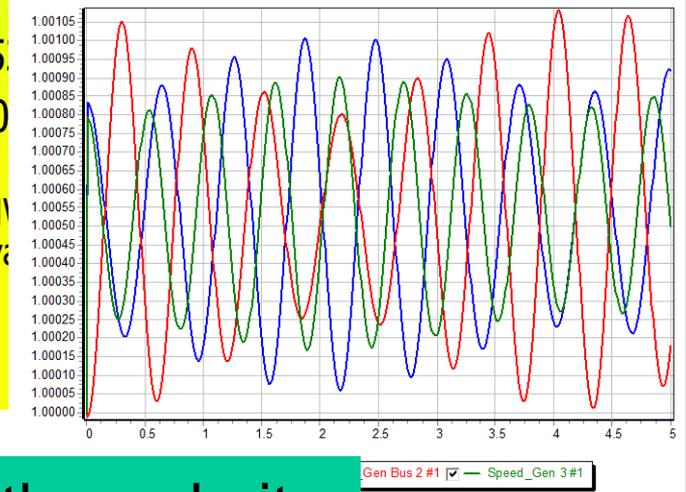
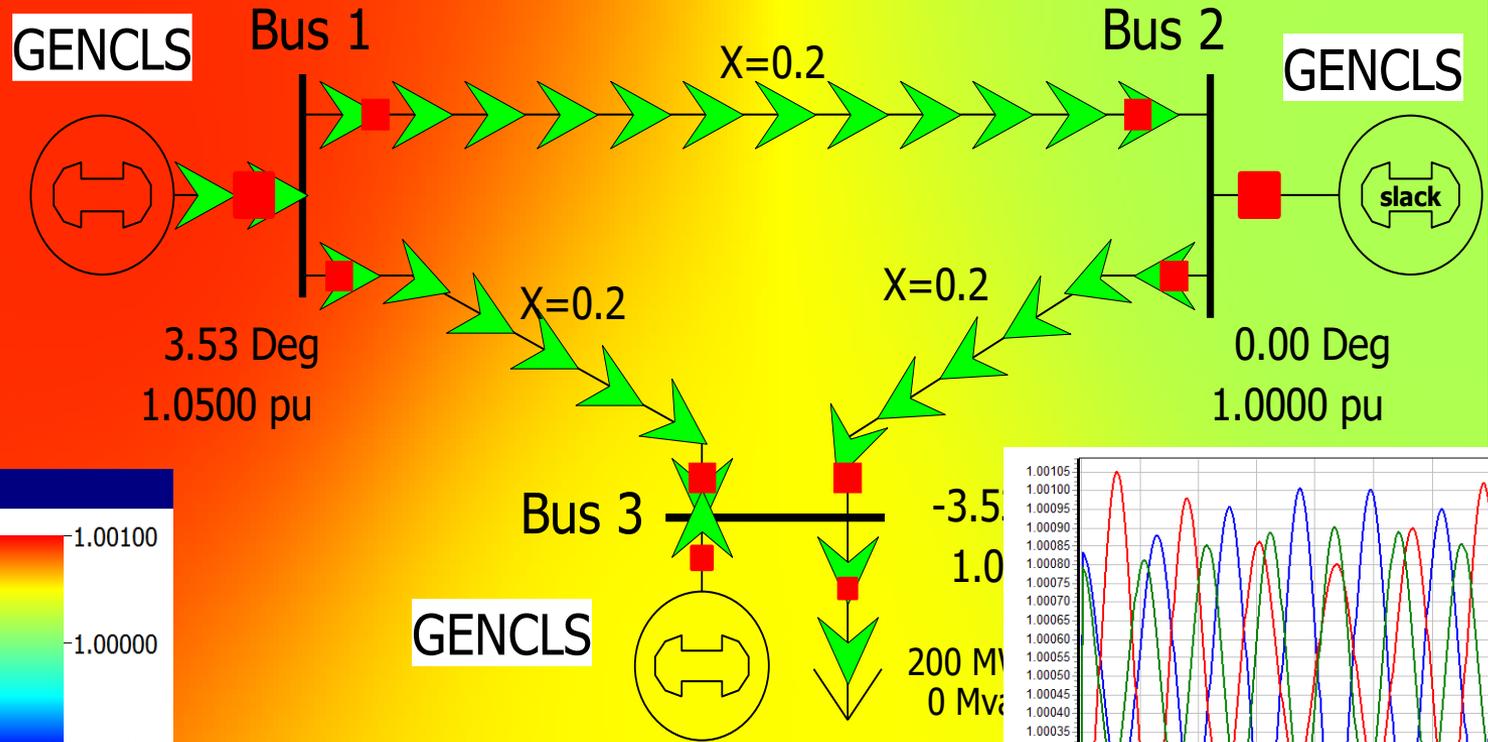


- A typical way to use the contour tool bar is to
  - Solved the desired transient stability contingency, making sure to save the associated contour fields
  - Display the toolbar
  - Set the options
  - Select the play button which will automatically create the sequence of contours
    - Contours can be saved with either screen recording software or as a series of jpegs
- Images can be combined as a movie using a tool such as Microsoft Movie Maker

# Three Bus Example (With $H_1$ set to 6.0 and $H_3$ to 4.0)



Transient Stability Time (Sec): 5.000



A low resolution copy of the movie is on the website

# Comtrade Format (IEEE Std. C37.111)



- Comtrade is a standard for exchanging power system time-varying data
  - Originally developed for power system transient results such as from digital fault recorders (DFRs), but it can be used for any data
- Comtrade is now being used for the exchange of PMU data and transient stability results
- Three variations on the standard (1991, 1999 and 2013 format)
- PowerWorld now allows transient stability results to be quickly saved in all three Comtrade Formats

# Three Bus Example Comtrade Results



PowerWorld,Transient Stability,1991

```
3,      3A,      0D
1,Gen Bus 1 #1_Speed,,, ,1.0054131589E-9,      1,0,      0,999998
2,Gen Bus 2 #1_Speed,,, ,1.09404544294E-9,      0.999988,0,      0,999998
3,Gen 3 #1_Speed,,, ,9.01581660036E-10,      1,0,      0,999998
```

```
60
0
0,503
04/11/16,00:00:00.000000
04/11/16,00:00:00.000000|
ASCII
```

```
1,      0,      0, 11169,      0,
2,      0,      0, 11169,      0,
3,      10000,828905,      0,879676,
4,      10000,828905,      0,879676,
5,      20000,825941,      5448,875444,
6,      30000,820131,      16181,867114,
7,      40000,811594,      32198,854686,|
8,      50000,800330,      53337,838422,
9,      60000,786339,      79270,818589,
10,     70000,769859,109779,795318,
11,     80000,751125,144320,769138,
12,     90000,730138,182784,740181,
13,    100000,707255,224625,709109,
14,    110000,682474,269299,676185,
```

The 1991 format is just ascii using four files; the 1999 format extends to allow data to be stored in binary format; the 2013 format extends to allow a single file format

# Large System Studies



- The challenge with large systems, which could have more than 100,000 states, is the sheer size
  - Most eigenvalues are associated with the local plants
  - Computing all the eigenvalues is computationally challenging, order  $n^3$
- Specialized approaches can be used to calculate particular eigenvalues of large matrices
  - See Kundur, Section 12.8 and associated references

# Single Machine Infinite Bus

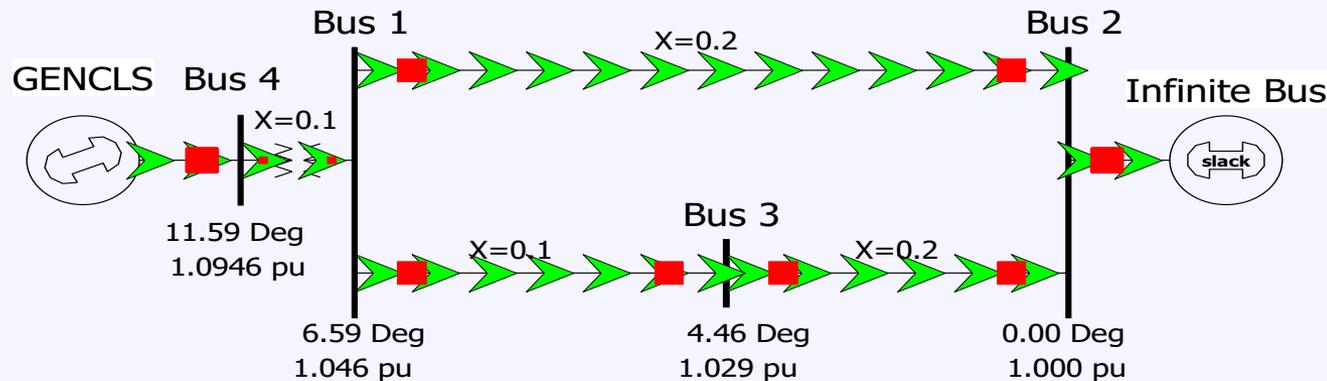


- A quite useful analysis technique is to consider the small signal stability associated with a single generator connected to the rest of the system through an equivalent transmission line
- Driving point impedance looking into the system is used to calculate the equivalent line's impedance
  - The  $Z_{ii}$  value can be calculated quite quickly using sparse vector methods
- Rest of the system is assumed to be an infinite bus with its voltage set to match the generator's real and reactive power injection and voltage

# Small SMIB Example



- As a small example, consider the 4 bus system shown below, in which bus 2 really is an infinite bus



- To get the SMIB for bus 4, first calculate  $Z_{44}$

$$Y_{bus} = j \begin{bmatrix} -25 & 0 & 10 & 10 \\ 0 & 1 & 0 & 0 \\ 10 & 0 & -15 & 0 \\ 10 & 0 & 0 & -13.33 \end{bmatrix} \rightarrow Z_{44} = j0.1269$$

$Z_{44}$  is  $Z_{th}$  in parallel with  $jX'_{d,4}$  (which is  $j0.3$ ) so  $Z_{th}$  is  $j0.22$

# Small SMIB Example



- The infinite bus voltage is then calculated so as to match the bus  $i$  terminal voltage and current

$$\bar{V}_{\text{inf}} = \bar{V}_i - Z_i \bar{I}_i$$

$$\text{where } \left( \frac{P_i + jQ_i}{\bar{V}_i} \right)^* = \bar{I}_i$$

While this was demonstrated on an extremely small system for clarity, the approach works the same for any size system

- In the example we have

$$\left( \frac{P_4 + jQ_4}{\bar{V}_4} \right)^* = \left( \frac{1 + j0.572}{1.072 + j0.220} \right)^* = 1 - j0.328$$

$$\bar{V}_{\text{inf}} = (1.072 + j0.220) - (j0.22)(1 - j0.328)$$

$$\bar{V}_{\text{inf}} = 1.0$$

# Calculating the A Matrix



- The SMIB model **A** matrix can then be calculated either analytically or numerically
  - The equivalent line's impedance can be embedded in the generator model so the infinite bus looks like the "terminal"
- This matrix is calculated in PowerWorld by selecting Transient Stability, SMIB Eigenvalues
  - Select Run SMIB to perform an SMIB analysis for all the generators in a case
  - Right click on a generator on the SMIB form and select Show SMIB to see the Generator SMIB Eigenvalue Dialog
  - These two bus equivalent networks can also be saved, which can be quite useful for understanding the behavior of individual generators

# Example: Bus 4 SMIB Dialog



- On the SMIB dialog, the General Information tab shows information about the two bus equivalent

Generator SMIB Eigenvalue Information

Bus Number: 4  
Bus Name: Bus 4  
ID: 1  
Area Name: Home (1)

Find By Number  
Find By Name  
Find ...

Status:  Open  Closed

Generator Information (on Generator MVA Base)

General Info | A Matrix | Eigenvalues

Generator MVA Base: 100.000

Infinite Bus Voltage Magnitude (pu): 1.0000  
Infinite Bus Angle (deg): -0.0000

Terminal Current Magnitude (pu): 1.0526  
Terminal Current Angle (deg): -18.193

Terminal Voltage Magnitude (pu): 1.0946  
Terminal Voltage Angle (deg): 11.5942

Network Impedance on Generator MVA Base  
Network R (Gen Base): 0.00000  
Network X (Gen Base): 0.22000

Network Impedance on System MVA Base  
Network R (System Base): 0.00000  
Network X (System Base): 0.22000

OK Save Cancel Help Print

PowerWorld case B4\_SMIB

# Example: Bus 4 SMIB Dialog



- On the SMIB dialog, the **A Matrix** tab shows the  $A_{\text{sys}}$  matrix for the SMIB generator

Generator SMIB Eigenvalue Information

Bus Number: 4  
 Bus Name: Bus 4  
 ID: 1

Status:  Open  Closed  
 Area Name: Home (1)

Generator Information (on Generator MVA Base)

General Info | **A Matrix** | Eigenvalues

Row Name	Machine Angle	Machine Speed w
1 Machine Angle	0.0000	376.9911
2 Machine Speed w	-0.3753	0.0000

- In this example  $A_{21}$  is showing

$$\frac{\partial \Delta \omega_{4,pu}}{\partial \delta_4} = \frac{1}{2H_4} \left( \frac{-\partial P_{E,4}}{\partial \delta_4} \right) = - \left( \frac{1}{6} \right) \left( \left( \frac{-1}{0.3 + 0.22} \right) (-1.2812 \cos(23.94^\circ)) \right)$$

$$= -0.3753$$

# Example: Bus 4 SMIB Dialog



- On the SMIB dialog, the Eigenvalues tab shows the  $A_{sys}$  matrix eigenvalues and participation factors (which we'll cover shortly)

Generator SMIB Eigenvalue Information

Bus Number: 4  
Bus Name: Bus 4  
ID: 1  
Area Name: Home (1)

Generator Information (on Generator MVA Base)

General Info | A Matrix | **Eigenvalues**

	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Speed w
1	0.0000	11.8945	11.8945	0.0000	1.8931	0.5282	1.8931	0.7071	0.7071
2	0.0000	-11.8945	11.8945	0.0000	-1.8931	-0.5282	1.8931	0.7071	0.7071

- Saving the two bus SMIB equivalent, and putting a short, self-cleared fault at the terminal shows the 1.89 Hz, undamped response