ECEN 667
Power System Stability

Lecture 22: Power System Stabilizers

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Announcements

- Read Chapter 8
- Homework 7 is posted; due on Tuesday Nov 28
- Final is as per TAMU schedule. That is, Friday Dec 8 from 3 to 5pm
- We'll be doing power system stabilizers today, and will pickup with modal analysis next time

Overview of a Power System Stabilizer (PSS)

- A PSS adds a signal to the excitation system to improve the rotor damping
  - A common signal is proportional to speed deviation; other inputs, such as like power, voltage or acceleration, can be used
  - Signal is usually generated locally (e.g. from the shaft)
- Both local mode and inter-area mode can be damped. When oscillation is observed on a system or a planning study reveals poorly damped oscillations, use of participation factors helps in identifying the machine(s) where PSS has to be located
- Tuning of PSS regularly is important

Block Diagram of System with a PSS

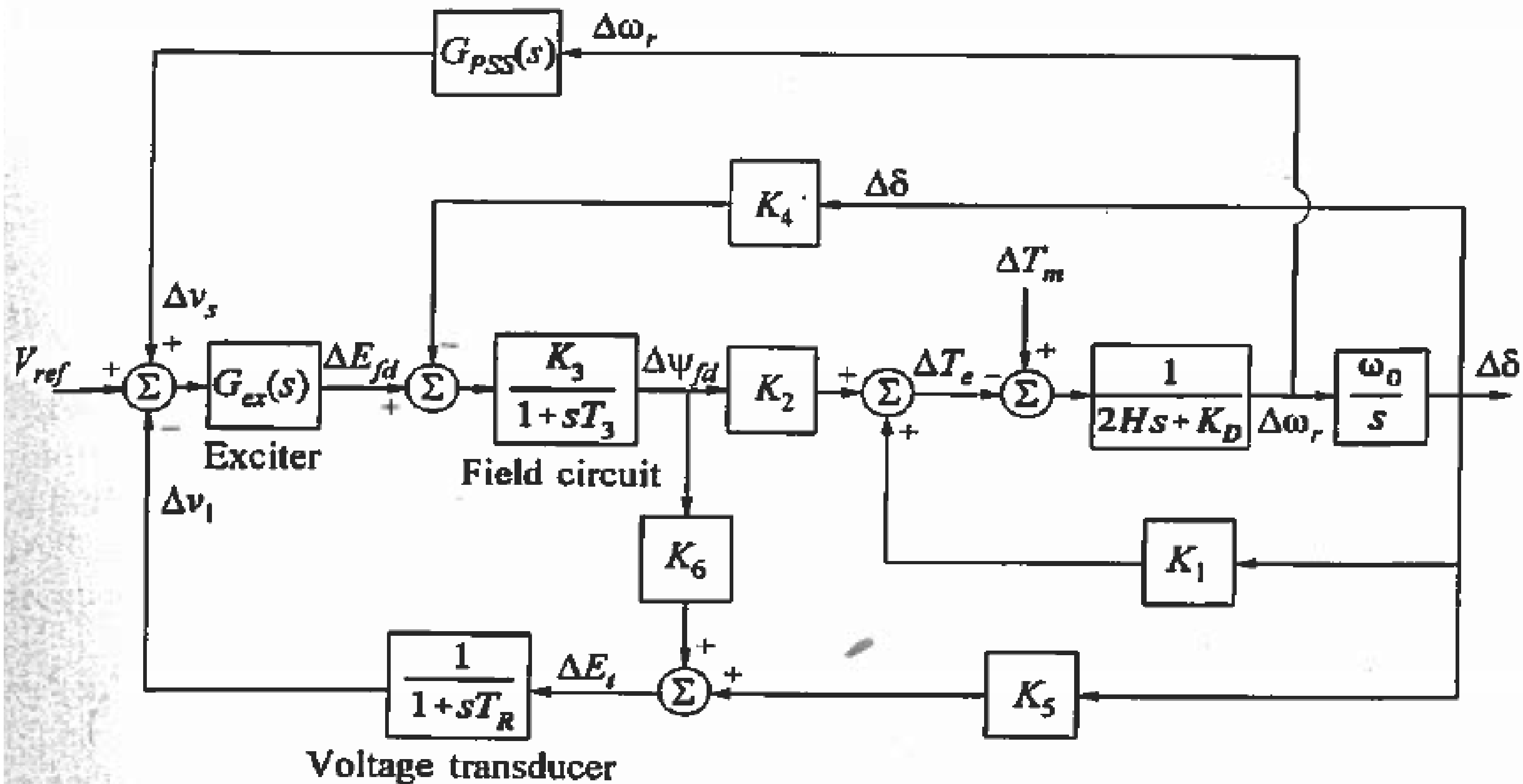
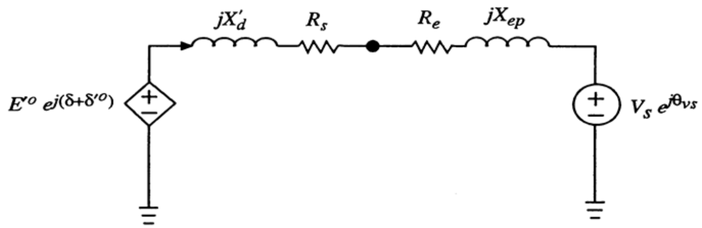


Figure 12.13 Block diagram representation with AVR and PSS

Image Source: Kundur, *Power System Stability and Control*

Power System Stabilizer Basics

- Stabilizers can be motivated by considering a classical model supplying an infinite bus



$$\frac{d\delta}{\mathrm{dt}} = \omega - \omega_s = \Delta\omega$$

$$\frac{2H}{\omega_0}\frac{d\Delta\omega}{dt} = T_M^0 - \frac{E'V_s}{X'_d + X_{ep}}\sin(\delta) - D\Delta\omega$$

- Assume internal voltage has an additional component

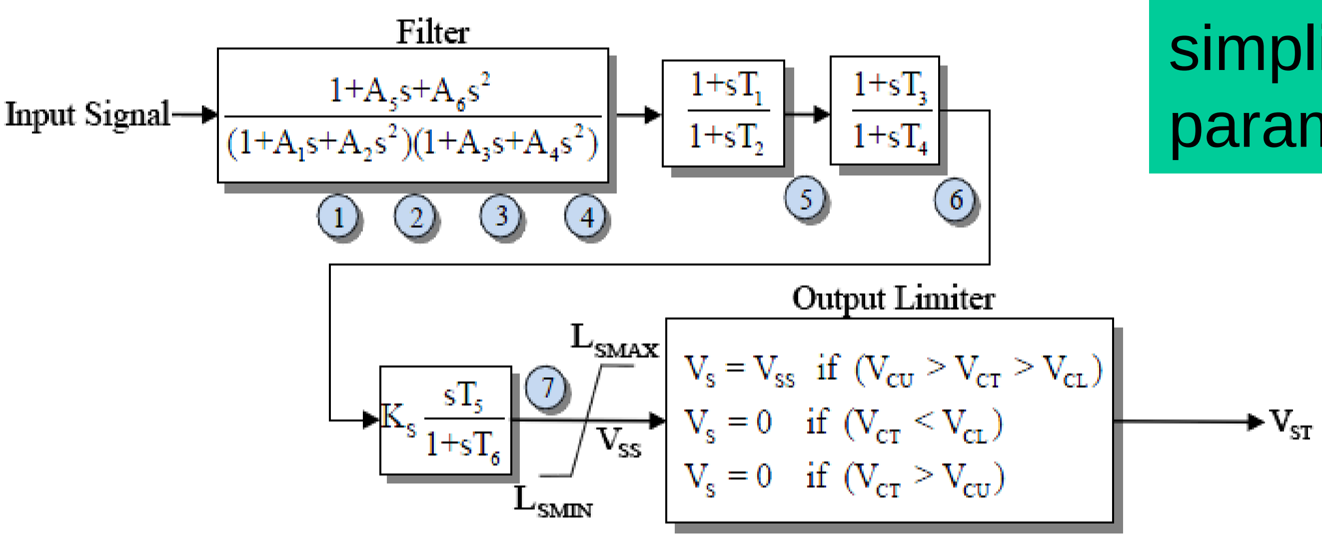
$$E' = E'_{org} + K\Delta\omega$$

- This will add additional damping if sin(δ) is positive
- In a real system there is delay, which requires compensation

Example PSS

- An example single input stabilizer is shown below (IEEEST)
  - The input is usually the generator shaft speed deviation, but it could also be the bus frequency deviation, generator electric power or voltage magnitude

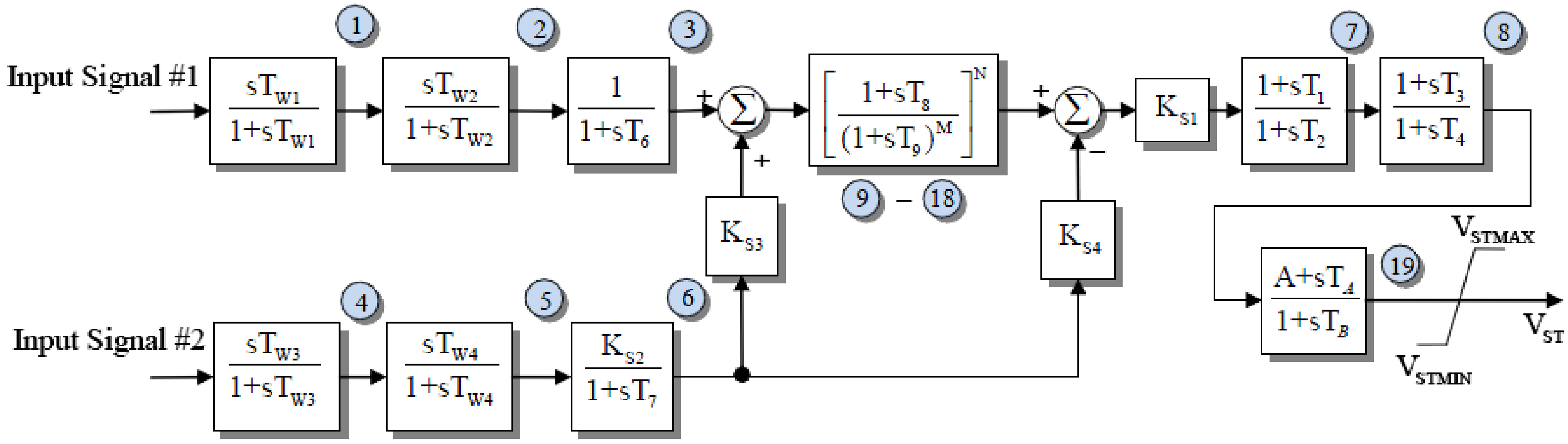
The model can be simplified by setting parameters to zero



V_{ST} is an input into the exciter

Example PSS

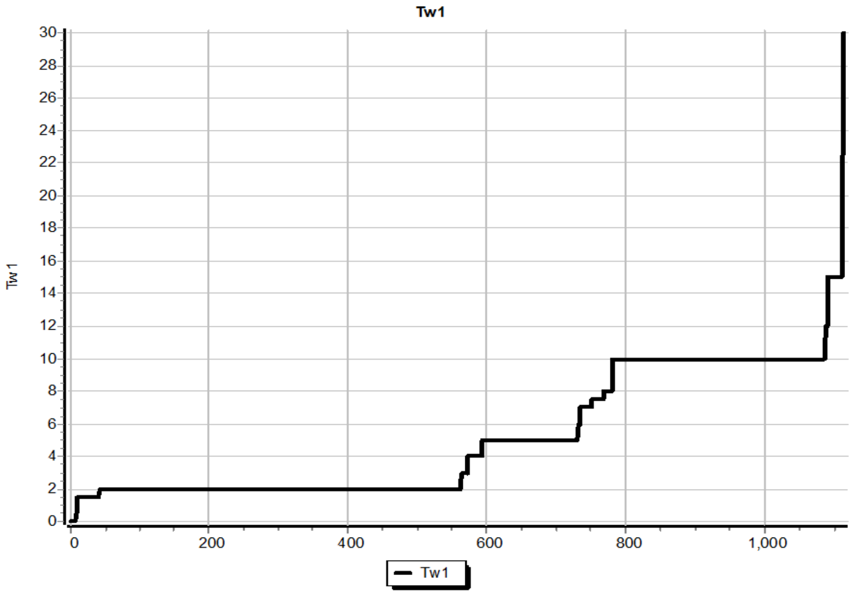
- Below is an example of a dual input PSS (PSS2A)
  - Combining shaft speed deviation with generator electric power is common
  - Both inputs have washout filters to remove low frequency components of the input signals

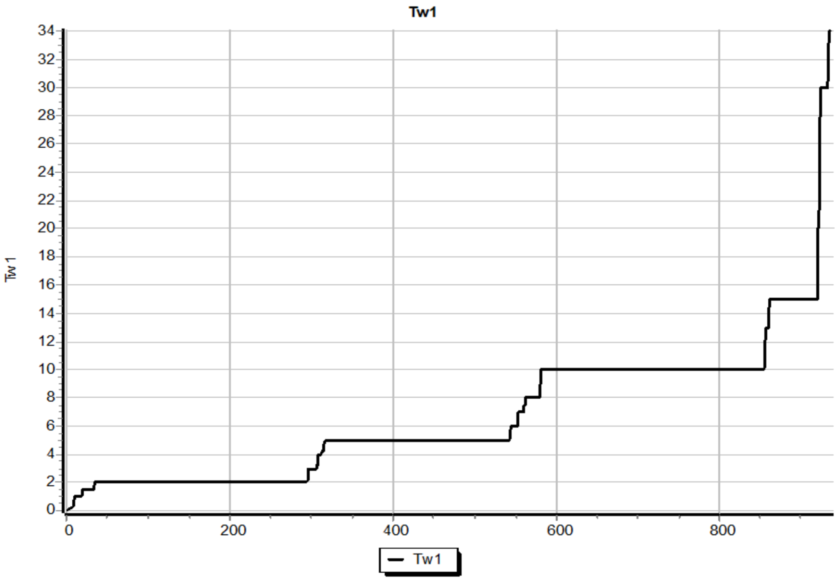


IEEE Std 421.5 describes the common stabilizers

Washout Parameter Variation

- The PSS2A is the most common stabilizer in both the 2015 EI and WECC cases. Plots show the variation in T_{W1} for EI (left) and WECC cases (right); for both the x-axis is the number of PSS2A stabilizers sorted by T_{W1}, and the y-axis is T_{W1} in seconds





PSS Tuning: Basic Approach

- The PSS parameters need to be selected to achieve the desired damping through a process known as tuning
- The next several slides present a basic method using a single machine, infinite bus (SMIB) representation
- Start with the linearized differential, algebraic model with controls **u** added to the states

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{y} + \mathbf{E}\Delta\mathbf{u}$$

$$\mathbf{0} = \mathbf{C}\Delta\mathbf{x} + \mathbf{D}\Delta\mathbf{y}$$

- If **D** is invertible then

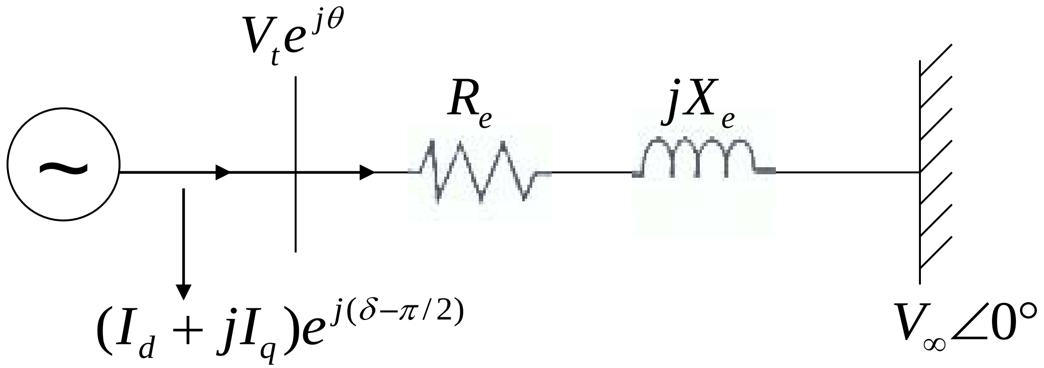
$$\Delta\dot{\mathbf{x}} = \left(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right)\Delta\mathbf{x} + \mathbf{E}\Delta\mathbf{u} = \mathbf{A}_{sys}\Delta\mathbf{x} + \mathbf{E}\Delta\mathbf{u}$$

PSS Tuning: Basic Approach

- Low frequency oscillations are considered the following approach
- A SMIB system is setup to analyze the local mode of oscillation (in the 1 to 3 Hz range)
  - A flux decay model is used with E_{fd} as the input
- Then, a fast-acting exciter is added between the input voltage and E_{fd}
- Certain constants, K_1 to K_6, are identified and used to tune a power system stabilizer

SMIB System (Flux Decay Model)

- SMIB with a flux decay machine model and a fast exciter



$$\dot{\delta} = \Delta\omega_{pu}\omega_s$$

$$\Delta\dot{\omega}_{pu} = \frac{1}{2H}[T_M - (E_q^{'}I_q + (X_d - X_d^{'})I_dI_q + D\Delta\omega_{pu})]$$

$$\dot{E}_q^{'} = \frac{1}{T_{do}^{'}}(-E_q^{'} - (X_d - X_d^{'})I_d + E_{fd})$$

Stator Equations

- Assume R_s=0, then the stator algebraic equations are:

$$X_q I_q - V_t \sin(\delta - \theta) = 0 \qquad (1)$$

$$E_q^{'} - V_t \cos(\delta - \theta) - X_d^{'} I_d = 0 \qquad (2)$$

$$(V_d + jV_q) e^{j(\delta - \pi/2)} = V_t e^{j\theta} \qquad (3)$$

$$\therefore \; V_d + jV_q = V_t e^{j\theta} e^{-j(\delta - \pi/2)} \qquad (4)$$

Expand the right hand side of (4)

$$V_d + jV_q = V_t \sin(\delta - \theta) + jV_t \cos(\delta - \theta) \qquad (5)$$

$$\therefore \; V_d = V_t \sin(\delta - \theta) \;\; and \;\; V_q = V_t \cos(\delta - \theta)$$

(1) *and* (2) *become* $\quad X_q I_q - V_d = 0 \qquad (6)$

$$E_q^{'} - V_q - X_d^{'} I_d = 0 \qquad (7)$$

Network Equations

- The network equation is (assuming zero angle at the infinite bus and no local load)

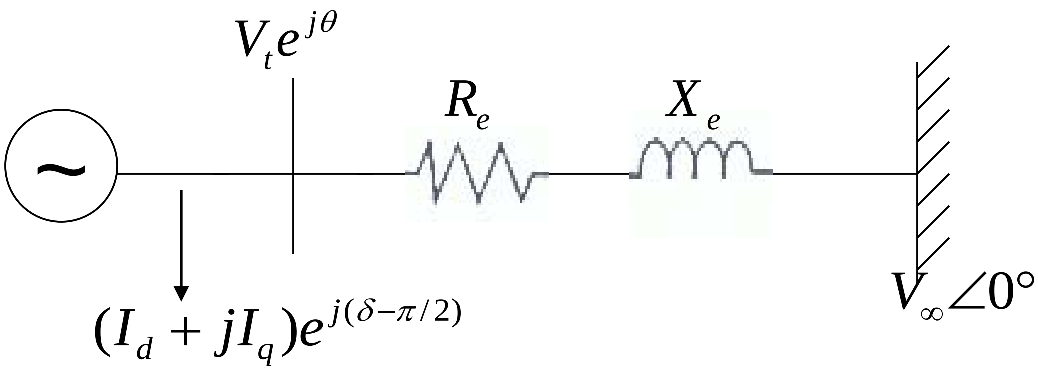
$$(I_d + jI_q)e^{j(\delta-\pi/2)} = \frac{(V_d + jV_q)e^{j(\delta-\pi/2)} - V_\infty \angle 0^\circ}{R_e + jX_e}$$

$$(I_d + jI_q) = \frac{(V_d + jV_q) - V_\infty e^{-j(\delta-\pi/2)}}{R_e + jX_e}$$

Simplifying,

$$R_e I_d - X_e I_q = V_d - V_\infty \sin\delta$$

$$X_e I_d + R_e I_q = V_q - V_\infty \cos\delta$$



Complete SMIB Model

$$\dot{E}_q' = \frac{1}{T_{do}'}\left(-E_q' - (X_d - X_d')I_d + E_{fd}\right)$$

$$\dot{\delta} = \Delta\omega_{pu}\omega_s \quad \text{note in the book } \nu = \omega_{pu} = \frac{\omega}{\omega_s}$$

Machine equations

$$\dot{\omega}_{pu} = \frac{1}{2H}\left[T_M - (E_q' I_q + (X_q - X_d')I_d I_q + D\Delta\omega_{pu})\right]$$

$$X_q I_q - V_d = 0$$

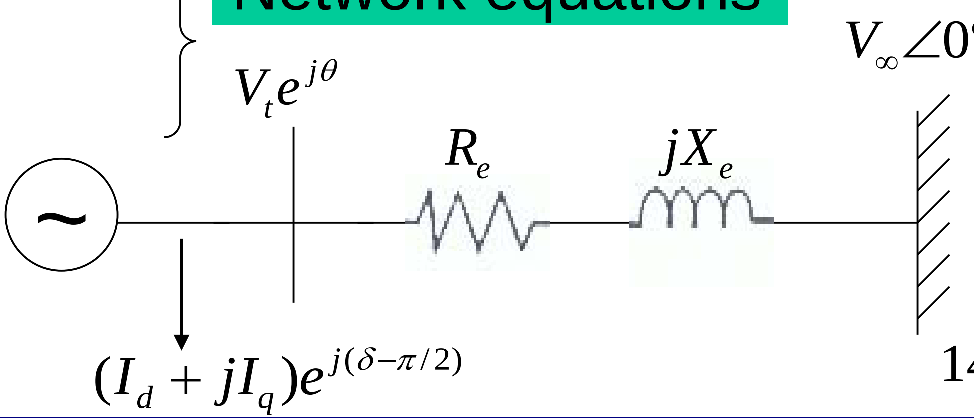
$$E_q' - V_q - X_d' I_d = 0$$

Stator equations

$$R_e I_d - X_e I_q = V_d - V_\infty \sin\delta$$

$$X_e I_d + R_e I_q = V_q - V_\infty \cos\delta$$

Network equations



Linearization of SMIB Model

Step 1: Linearize the stator equations

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \begin{bmatrix} 0 & X_q \\ -X_d^{'} & 0 \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta E_q^{'} \end{bmatrix}$$

Step 2: Linearize the network equations

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \begin{bmatrix} R_e & -X_e \\ X_e & R_e \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} + \begin{bmatrix} V_\infty \cos \delta^\circ \\ -V_\infty \sin \delta^\circ \end{bmatrix} \Delta\delta$$

Step 3: Equate the righthand sides of the above equations

$$\begin{bmatrix} R_e & -(X_e + X_q) \\ (X_e + X_d^{'}) & R_e \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta E_q^{'} \end{bmatrix} + \begin{bmatrix} -V_\infty \cos \delta^\circ \\ V_\infty \sin \delta^\circ \end{bmatrix} \Delta\delta$$

Notice this is equivalent to a generator at the infinite bus with modified resistance and reactance values

Linearization (contd)

Solve for $\Delta I_d, \Delta I_q$

$$\begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} X_e + X_q & -R_e V_\infty \cos\delta^\circ + V_\infty \sin\delta^\circ (X_q + X_e) \\ R_e & R_e V_\infty \sin\delta^\circ + V_\infty \cos\delta^\circ (X_d^{'} + X_e) \end{bmatrix} \begin{bmatrix} \Delta E_q^{'} \\ \Delta\delta \end{bmatrix} \quad (1)$$

The determinant is $\Delta = R_e^2 + (X_e + X_q)(X_e + X_d^{'})$

- <u>Final Steps involve</u>
  1. Linearizing Machine Equations

     $$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, I_d, I_q, E_{fd}, T_M) \qquad (2)$$

  2. Substitute (1) in the linearized equations of (2).

Linearization of Machine Equations

$$\begin{bmatrix} \Delta \dot{E}_q^{'} \\ \Delta \dot{\delta} \\ \Delta \omega_{pu} \end{bmatrix} = \begin{bmatrix} \frac{-1}{T_{do}^{'}} & 0 & 0 \\ 0 & 0 & \omega_s \\ \frac{-I_q^o}{2H} & 0 & -\frac{D\omega_s}{2H} \end{bmatrix} \begin{bmatrix} \Delta E_q^{'} \\ \Delta \delta \\ \Delta \omega_{pu} \end{bmatrix} \quad (1)$$

$$+ \begin{bmatrix} \frac{-1}{T_{do}^{'}}(X_d - X_d^{'}) & 0 \\ 0 & 0 \\ \frac{1}{2H} I_q^o (X_d^{'} - X_q) & \frac{1}{2H}(X_d^{'} - X_q) I_q^o - \frac{1}{2H} E_q^{'o} \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} + \begin{bmatrix} \frac{-1}{T_{do}^{'}} & 0 \\ 0 & 0 \\ 0 & \frac{1}{2H} \end{bmatrix} \begin{bmatrix} \Delta E_{fd} \\ \Delta T_M \end{bmatrix}$$

Symbolically we have

$$\Delta \dot{\mathbf{x}} = \mathbf{A}\Delta \mathbf{x} + \mathbf{B}\Delta \mathbf{u} + \mathbf{C}\Delta I_{d-q} \qquad (2)$$

$$\Delta \dot{\mathbf{I}}_{d-q} = \mathbf{D}\Delta \mathbf{x} \qquad (3)$$

- Substitute (3) in (2) to get the linearized model.

Linearized SMIB Model

$$\Delta \dot{E}_q^{'} = -\frac{1}{K_3 T_{do}^{'}} \Delta E_q^{'} - \frac{K_4}{T_{do}^{'}} \Delta \delta + \frac{1}{T_{do}^{'}} \Delta E_{fd}$$

$$\Delta \dot{\delta} = \omega_s \Delta \omega_{pu}$$

$$\Delta \dot{\omega}_{pu} = -\frac{K_2}{2H} \Delta E_q^{'} - \frac{K_1}{2H} \Delta \delta - \frac{D\omega_s}{2H} \Delta \omega_{pu} + \frac{1}{2H} \Delta T_M$$

- Excitation system is not yet included.
- $K_1 - K_4$ constants are defined on next slide

K1 – K4 Constants

$$\frac{1}{K_3} = 1 + \frac{(X_d - X_d^{'})(X_q + X_e)}{\Delta}$$

$$K_4 = \frac{V_\infty (X_d - X_d^{'})}{\Delta}\left[(X_q + X_e)\sin\delta^\circ - R_e\cos\delta^\circ\right]$$

$$K_2 = \frac{1}{\Delta}\left[I_q^o \Delta - I_q^o (X_d^{'} - X_q)(X_q + X_e) - R_e (X_d^{'} - X_q) I_d^o + R_e E_q^{'o}\right]$$

$$K_1 = -\frac{1}{\Delta}[I_q^o V_\infty (X_d^{'} - X_q)\{(X_q + X_e)\sin\delta^\circ - R_e\cos\delta^\circ\}$$
$$+ V_\infty\{(X_d^{'} - X_q) I_d^o - E_q^{'o}\}\{(X_d^{'} + X_e)\cos\delta^\circ - R_e\sin\delta^\circ\}]$$

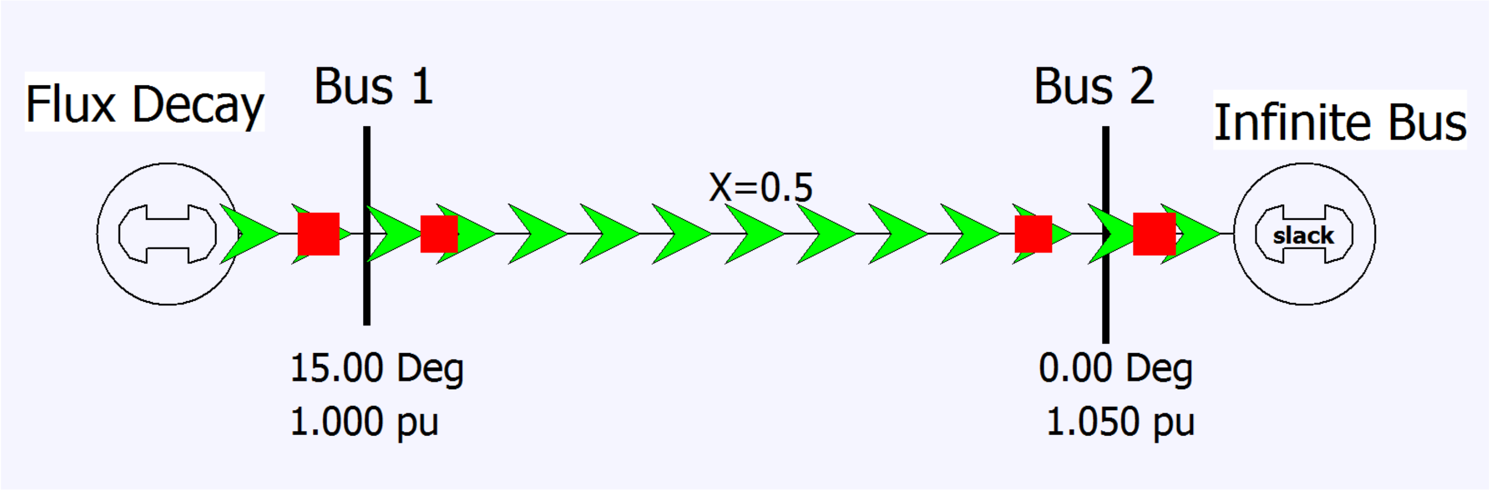
- $K_1 - K_4$ only involve machine and not the exciter.

Manual Control

- Without an exciter, the machine is on manual control.
- The previous matrix will tend to have two complex eigenvalues, corresponding to the electromechanical mode, and one real eigenvalue corresponding to the flux decay
- Changes in the operating point can push the real eigenvalue into the right-hand plane
  - This is demonstrated in the following example

Numerical Example

- Consider an SMIB system with Z_{eq} = j0.5, V_{inf} = 1.05, in which in the power flow the generator has S = 54.34 – j2.85 MVA with a V_t of 1∠15°
  - Machine is modeled with a flux decay model with (pu, 100 MVA) H=3.2, T'_{do}=9.6, X_d=2.5, X_q=2.1, X'_d=0.39, R_s=0, D=0



Saved as case B2_PSS_Flux

Initial Conditions

- The initial conditions are

$$I_G e^{j\gamma} = (I_d + jI_q)e^{j(\delta-\pi/2)} = \frac{1\angle 15^\circ - 1.05\angle 0^\circ}{j0.5} = 0.5443\angle 18^\circ$$

$$\delta(0) = angle\ of\ \overline{E}\ where\ \overline{E} = V_t e^{j\theta} + (R_s + jX_q)I_G e^{j\gamma}.$$

$$\overline{E} = 1\angle 15^\circ + (j2.1)(0.5443\angle 18^\circ)$$

$$= 1.4788\angle 65.5^\circ$$

$$I_d + jI_q = I_G e^{j\gamma} e^{-j(\delta-\pi/2)} = 0.5443\angle 42.48^\circ,$$

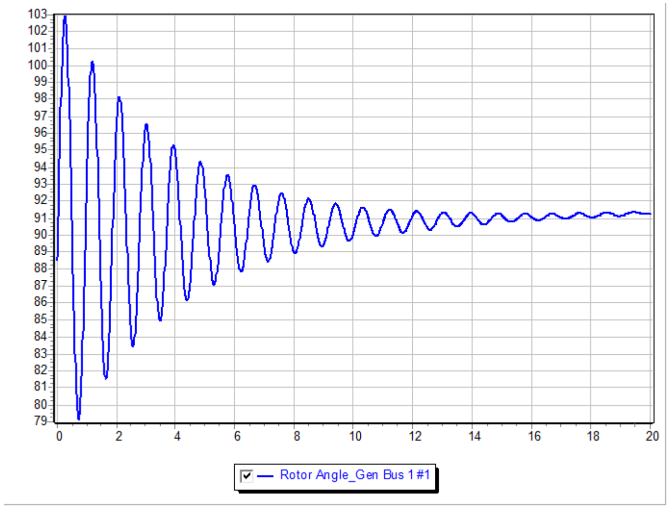
$$I_d = 0.4014,\ and\ I_q = 0.3676.$$

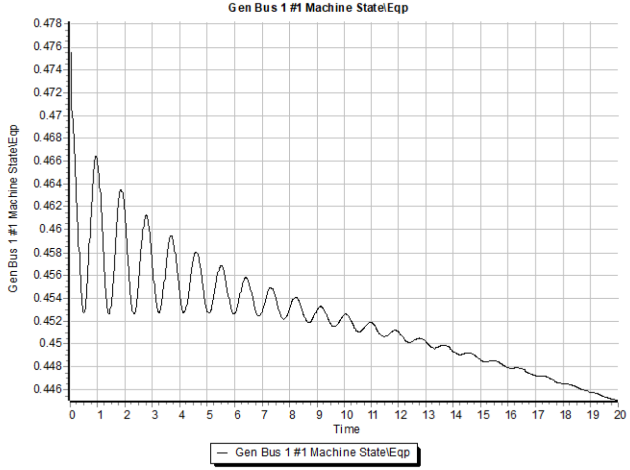
$$V_d + jV_q = Ve^{j\theta} e^{-j(\delta-\pi/2)} = 1\angle 39.48^\circ$$

Hence, $V_d = 0.7718,\ V_q = 0.6358$

SMIB Eigenvalues

- With the initial condition of Q = -2.85 Mvar the SMIB eigenvalues are $-0.223 \pm j7.374$, -0.0764
- Changing the Q to 20 Mvar gives eigenvalues of $-0.181 \pm j7.432$, -0.171
- Change the Q to –20 Mvar gives eigenvalues of $-0.256 \pm j6.981$, $+0.018$

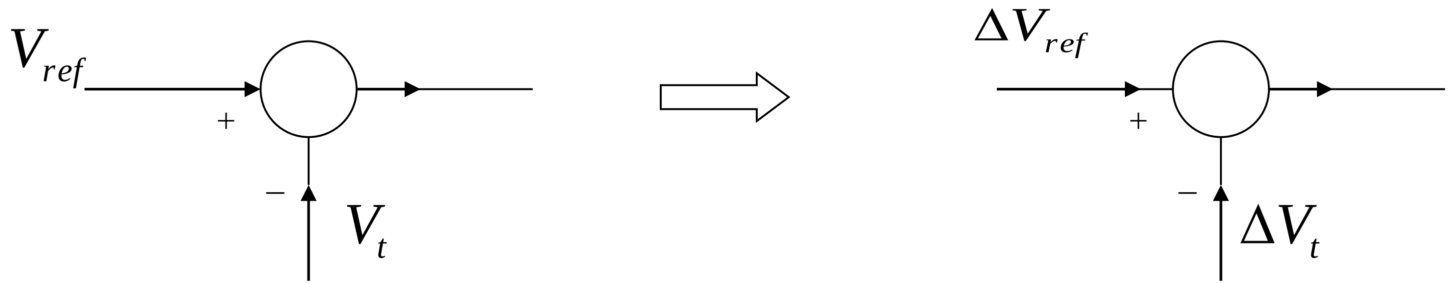




This condition is unstable in Eqp

Including Terminal Voltage

- The change in the terminal voltage magnitude also needs to be include since it is an input into the exciter



$$V_t = \sqrt{V_d^2 + V_q^2}$$

Differentiating

$$V_t^2 = V_d^2 + V_q^2$$

$$2V_t \Delta V_t = 2V_d^o \Delta V_d + 2V_q^o \Delta V_q$$

$$\Delta V_t = \frac{V_d^o}{V_t}\Delta V_d + \frac{V_q^o}{V_t}\Delta V_q$$

Computing

- While linearizing the stator algebraic equations, we had

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \begin{bmatrix} 0 & X_q \\ -X_d^{'} & 0 \end{bmatrix} \begin{bmatrix} \Delta E_q^{'} \\ \Delta \delta \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta E_q^{'} \end{bmatrix}$$

- Substitute this in expression for ΔV_t to get

$$\Delta V_t = K_5 \Delta \delta + K_6 \Delta E_q^{'}$$

$$K_5 = \frac{1}{\Delta} \left\{ \frac{V_d^o}{V_t} X_q [R_e V_\infty \sin \delta^\circ + V_\infty \cos \delta^\circ (X_d^{'} + X_e)] \right.$$

$$\left. + \frac{V_q^o}{V_t} [X_d^{'} (R_e V_\infty \cos \delta^\circ - V_\infty (X_q + X_e) \sin \delta^\circ)] \right\}$$

$$K_6 = \frac{1}{\Delta} \left\{ \frac{V_d^o}{V_t} X_q R_e - \frac{V_q^o}{V_t} X_d^{'} (X_q + X_e) \right\} + \frac{V_q^o}{V_t}$$

Heffron–Phillips Model (from 1952 and 1969)

- Add a fast exciter with a single differential equation

$$T_A \dot{E}_{fd} = -E_{fd} + K_A (V_{ref} - V_t)$$

- Linearize

$$T_A \Delta\dot{E}_{fd} = -\Delta E_{fd} + K_A (\Delta V_{ref} - \Delta V_t)$$

$$T_A \Delta\dot{E}_{fd} = -\Delta E_{fd} + K_A \Delta V_{ref} - K_A (K_5 \Delta\delta + K_6 \Delta E_q^{'})$$

- This is then combined with the previous three differential equations to give

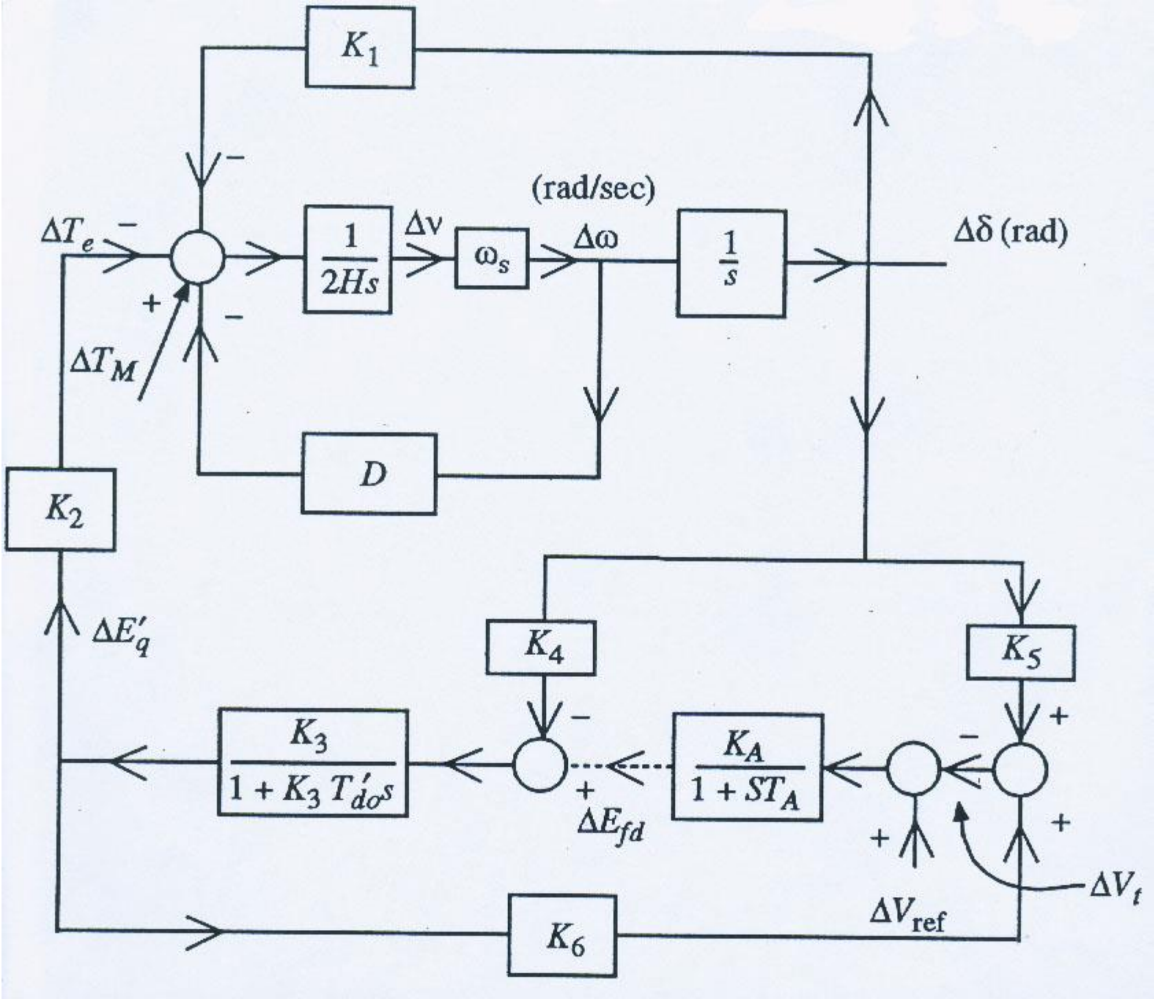
$$\dot{\mathbf{x}} = \mathbf{A}_{sys}\mathbf{x} + \mathbf{B}\Delta\mathbf{u}$$

$$\Delta\mathbf{u} = [\Delta T_M \quad \Delta V_{ref}]^T$$

SMIB Model

Block Diagram

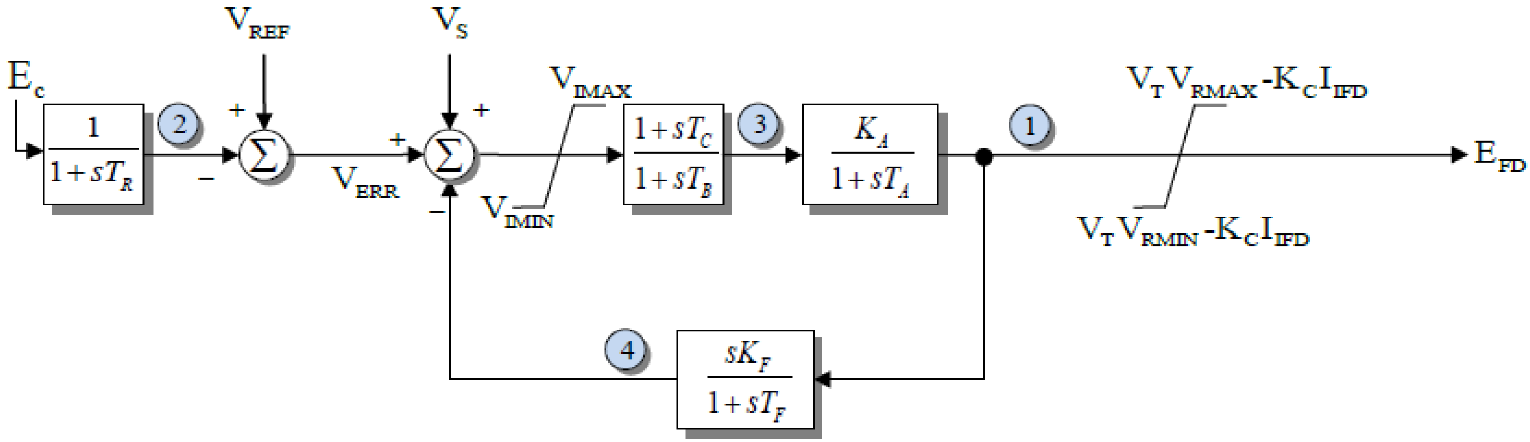
- K_1 to K_6 are affected by system loading



On diagram Δv is used to indicate what we've been calling $\Delta\omega_{pu}$

Add an EXST1 Exciter Model

- Set the parameters to K_A = 400, T_A=0.2, all others zero with no limits and no compensation



- Hence this simplified exciter is represented by a single differential equation

$$T_A \dot{E}_{fd} = -E_{fd} + K_A (V_{REF} - V_t + V_s)$$

V_s is the input from the stabilizer, with an initial value of zero

Initial Conditions (contd)

- From the stator algebraic equation,

$$E_q^{'} = V_q + X_d^{'} I_d$$

$$= 0.63581 + (0.39)(0.4014) = 0.7924$$

$$E_{fd} = E_q^{'} + (X_d - X_d^{'}) I_d$$

$$= 0.7924 + (2.5 - 0.39)0.4014 = 1.6394$$

$$V_{REF} = V_t + \frac{E_{fd}}{K_A} = 1 + \frac{1.6394}{400} = 1.0041$$

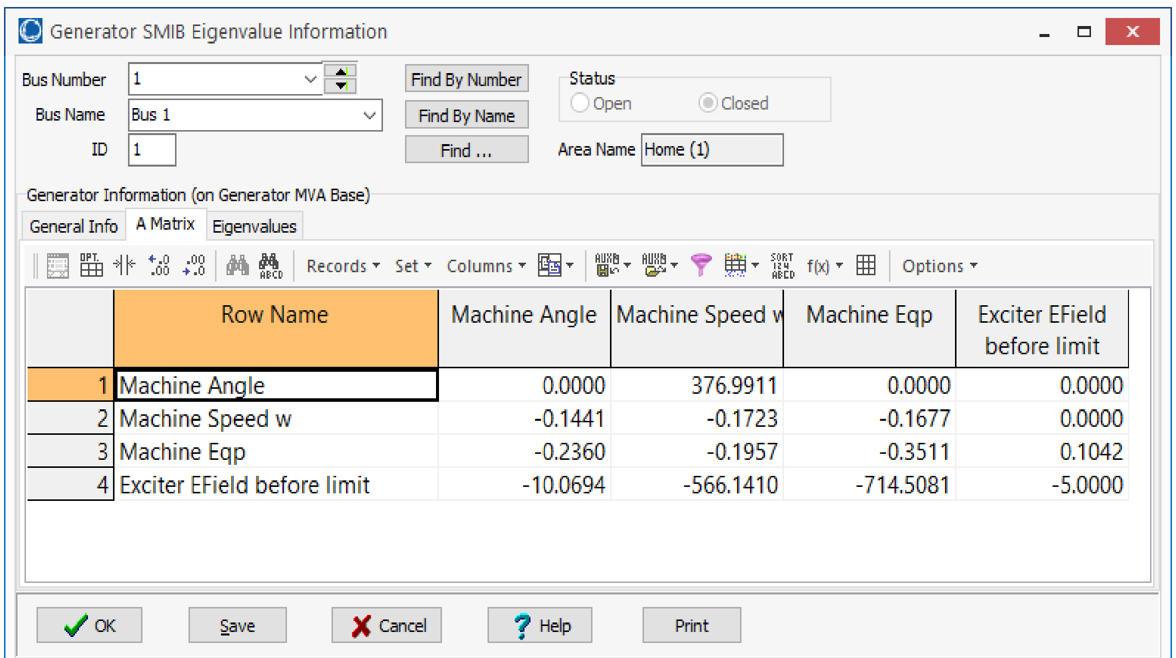
$$\omega_s = 377, \; T_M = E_q^{'} I_q + (X_q - X_d^{'}) I_d I_q$$

$$= (0.7924)(0.3676) + (2.1 - 0.39)(0.4014)(0.3676)$$

$$= 0.5436 \quad \left(checks\ with\ (V_t V_\infty \sin\theta) / X_e \right)$$

SMIB Results

- Doing the SMIB gives a matrix that closely matches the book's matrix from Example 8.7
  - The variable order is different; the entries in the w column are different because of the speed dependence in the swing equation and the Norton equivalent current injection



	Row Name	Machine Angle	Machine Speed w	Machine Eqp	Exciter EField before limit
1	Machine Angle	0.0000	376.9911	0.0000	0.0000
2	Machine Speed w	-0.1441	-0.1723	-0.1677	0.0000
3	Machine Eqp	-0.2360	-0.1957	-0.3511	0.1042
4	Exciter EField before limit	-10.0694	-566.1410	-714.5081	-5.0000

The ω values are different from the book because of speed dependence included in the stator voltage and swing equations

Computation of K1 – K6 Constants

- Calculating the values with the formulas gives

$$\Delta = R_e^2 + (X_e + X_q)(X_e + X_d^{'}) = 2.314$$

$$\frac{1}{K_3} = 1 + \frac{(X_d + X_d^{'})(X_q + X_e)}{\Delta} = 3.3707$$

$$K_3 = 0.296667$$

$$K_4 = \frac{V_\infty (X_d - X_d^{'})}{\Delta}[(X_q + X_e)\sin\delta^\circ - R_e\cos\delta^\circ] = 2.26555$$

$$K_2 = \frac{1}{\Delta}[I_q^o\Delta - I_q^o(X_d^{'} - X_q)(X_q + X_e) - R_e(X_d^{'} - X_q)I_d^o + R_e E_q^{'o}] = 1.0739$$

Similarly K_1, K_5, and K_6 are calculated as

$$K_1 = 0.9224\ ,\ K_5 = 0.005\ ,\ K_6 = 0.3572$$

Damping of Electromechanical Modes

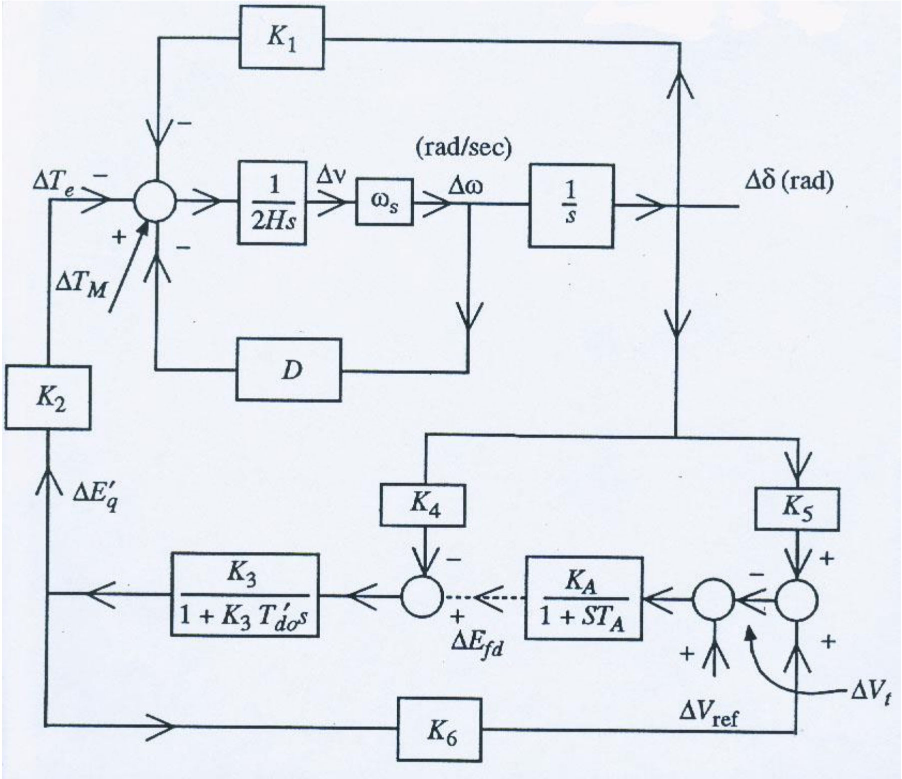
- Damping can be considered using either state-space analysis or frequency analysis
- With state-space analysis the equations can be written as

$$\begin{bmatrix} \Delta \dot{E}_q' \\ \Delta \dot{\delta} \\ \Delta \dot{\omega}_{pu} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{K_3 T_{do}'} & -\dfrac{K_4}{T_{do}'} & 0 \\ 0 & 0 & \omega_s \\ -\dfrac{K_2}{2H} & -\dfrac{K_1}{2H} & -\dfrac{D\omega_s}{2H} \end{bmatrix} \begin{bmatrix} \Delta E_q' \\ \Delta \delta \\ \Delta \omega_{pu} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{T_{do}'} \\ 0 \\ 0 \end{bmatrix} \Delta E_{fd}$$

The change in E_{fd} comes from the exciter

State-Space Analysis

- With no exciter there are three loops
  - Top complex pair of eigenvalues loop
  - Bottom loop through ΔE_q' contributes positive damping



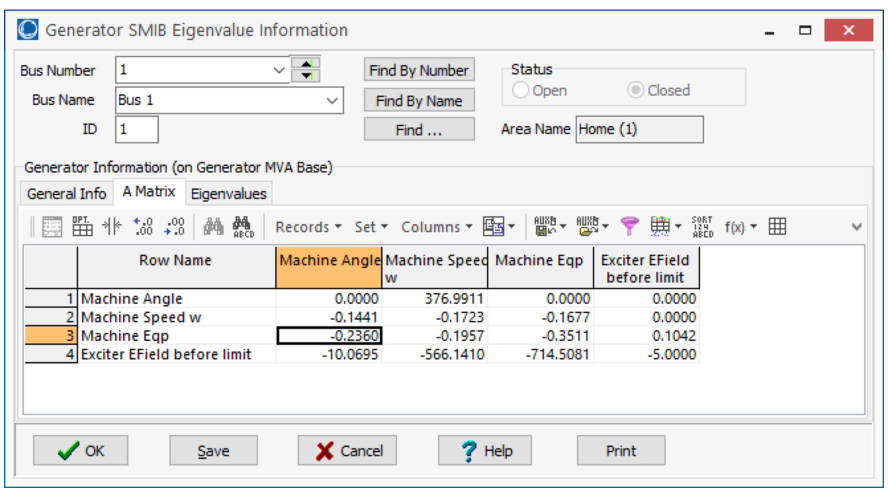
- Adding the exciter gives

$$\Delta \dot{E}_{fd} = \frac{1}{T_A}\left(\Delta E_{fd} - K_A K_5 \Delta\delta - K_A K_6 \Delta E'_q + K_A \Delta V_{ref}\right)$$

B2_PSS_Flux Example

- The SMIB can be used to plot how the eigenvalues change as the parameters (like K_A) are varied

$$\begin{bmatrix} \Delta \dot{E}_q' \\ \Delta \dot{\delta} \\ \Delta \dot{\omega}_{pu} \\ \Delta \dot{E}_{fd} \end{bmatrix} = \begin{bmatrix} -\frac{1}{K_3 T_{do}'} & -\frac{K_4}{T_{do}'} & 0 & \frac{1}{T_{do}'} \\ 0 & 0 & \omega_s & 0 \\ -\frac{K_2}{2H} & -\frac{K_1}{2H} & -\frac{D\omega_s}{2H} & 0 \\ -\frac{K_A K_6}{T_A} & -\frac{K_A K_5}{T_A} & 0 & \frac{-1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta E_q' \\ \Delta \delta \\ \Delta \omega_{pu} \\ \Delta E_{fd} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_A}{T_A} \end{bmatrix} \Delta V_{ref}$$



	Row Name	Machine Angle	Machine Speed w	Machine Eqp	Exciter EField before limit
1	Machine Angle	0.0000	376.9911	0.0000	0.0000
2	Machine Speed w	-0.1441	-0.1723	-0.1677	0.0000
3	Machine Eqp	-0.2360	-0.1957	-0.3511	0.1042
4	Exciter EField before limit	-10.0695	-566.1410	-714.5081	-5.0000

Values for K_1 to K_6 can be verified for previous case using the PowerWorld SMIB matrix

Verified K Values

- The below equations verify the results provided on the previous slide SMIB matrix with the earlier values

$$-\frac{K_1}{2H} = -0.1441 \rightarrow K_1 = 0.2882 \times 3.2 = 0.922$$

$$-\frac{K_2}{2H} = -0.1677 \rightarrow K_2 = 0.3354 \times 3.2 = 1.073$$

$$-\frac{1}{K_3 T'_{do}} = -0.3511 \rightarrow K_3 = \frac{1}{0.3511 \times 9.6} = 0.297$$

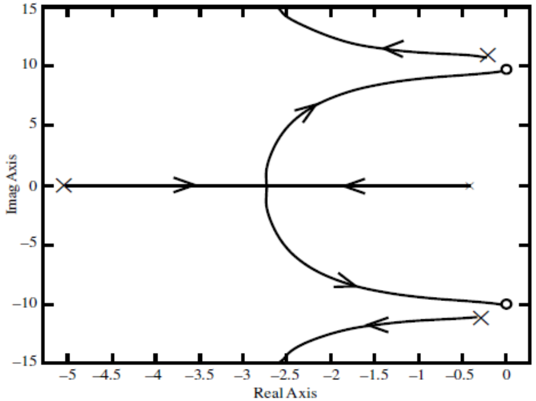
$$-\frac{K_4}{T'_{do}} = -0.236 \rightarrow K_4 = 0.236 \times 9.6 = 2.266$$

$$-\frac{K_A K_5}{T_A} = -10.069 \rightarrow K_5 = \frac{10.069 \times 0.2}{400} = 0.005$$

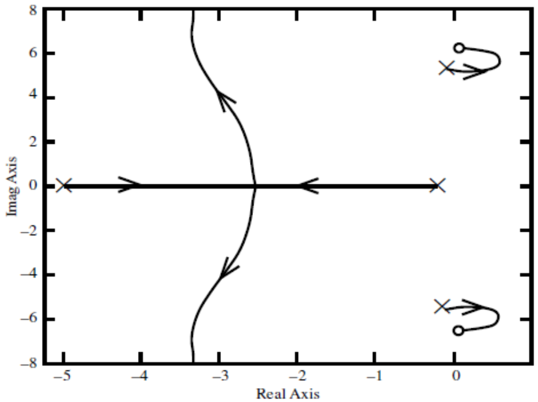
$$-\frac{K_A K_6}{T_A} = -714.5 \rightarrow K_5 = \frac{714.5 \times 0.2}{400} = 0.357$$

Root Locus for Varying K_A

- This can be considered a feedback system, with the general root locus as shown



(a)



(b)

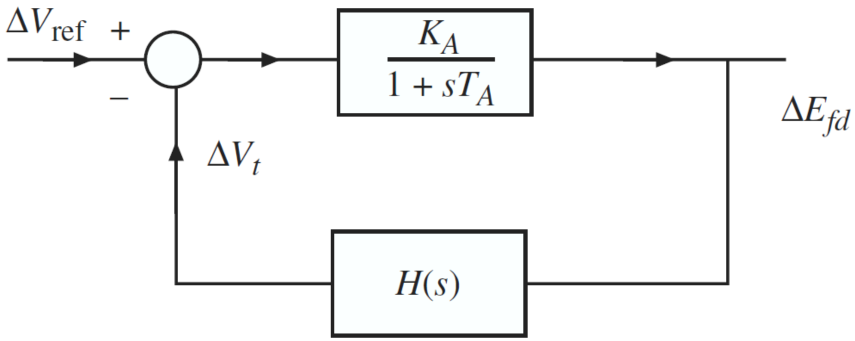
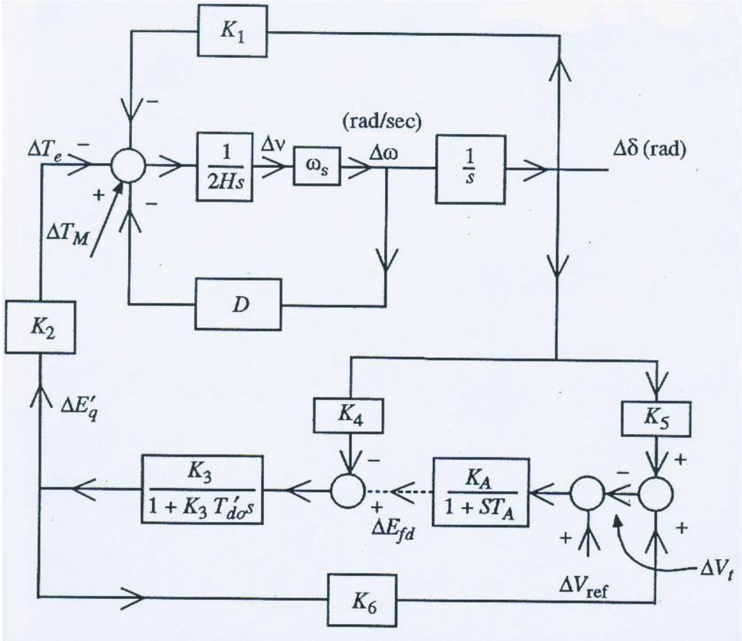


Figure 8.9: Small-signal model viewed as a feedback system

Frequency Domain Analysis

- Alternatively we could use frequency domain analysis
- Ignoring T_A gives $$\frac{\Delta E_q'(s)}{\Delta\delta(s)} = \frac{-K_3\left(K_4 + K_A K_5\right)}{1 + K_A K_3 K_6 + sK_3 T_{do}'}$$
- If K_5 is positive, then the response is similar to the case without an exciter
- If K_5 is negative, then with a sufficiently large K_A the electromechanical modes can become unstable



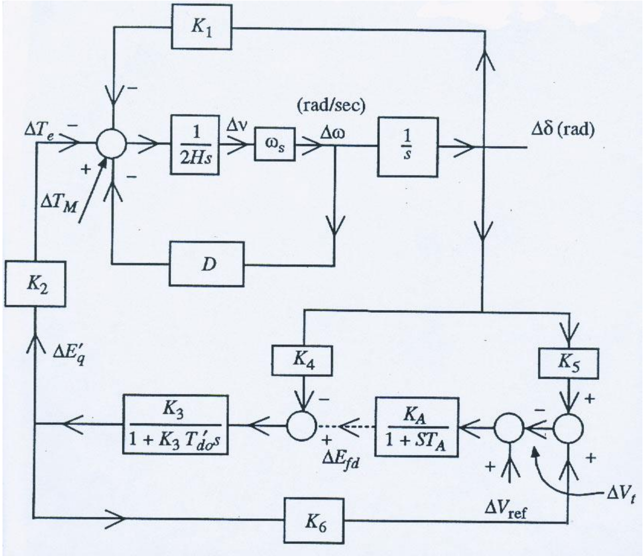
Frequency Domain Analysis

- Thus a fast acting exciter can be bad for damping, but has over benefits, such as minimizing voltage deviations
- Including T_A

$$\frac{\Delta E_q'(s)}{\Delta\delta(s)} = \frac{-K_3\left(K_4\left(1+sT_A\right)+K_A K_5\right)}{K_A K_3 K_6 + \left(1+sK_3 T_{do}^{'}\right)\left(1+sT_A\right)}$$

- Including the torque angle gives

$$\frac{\Delta T_e(s)}{\Delta\delta(s)} = K_2 \frac{\Delta E_q'(s)}{\Delta\delta(s)} = H(s)$$



Torque-Angle Loop

- The torque-angle loop is as given in Figure 8.11; with damping neglected it has two imaginary eigenvalues

$$\lambda_{1,2} = \pm j\sqrt{\frac{K_1\omega_s}{2H}}$$

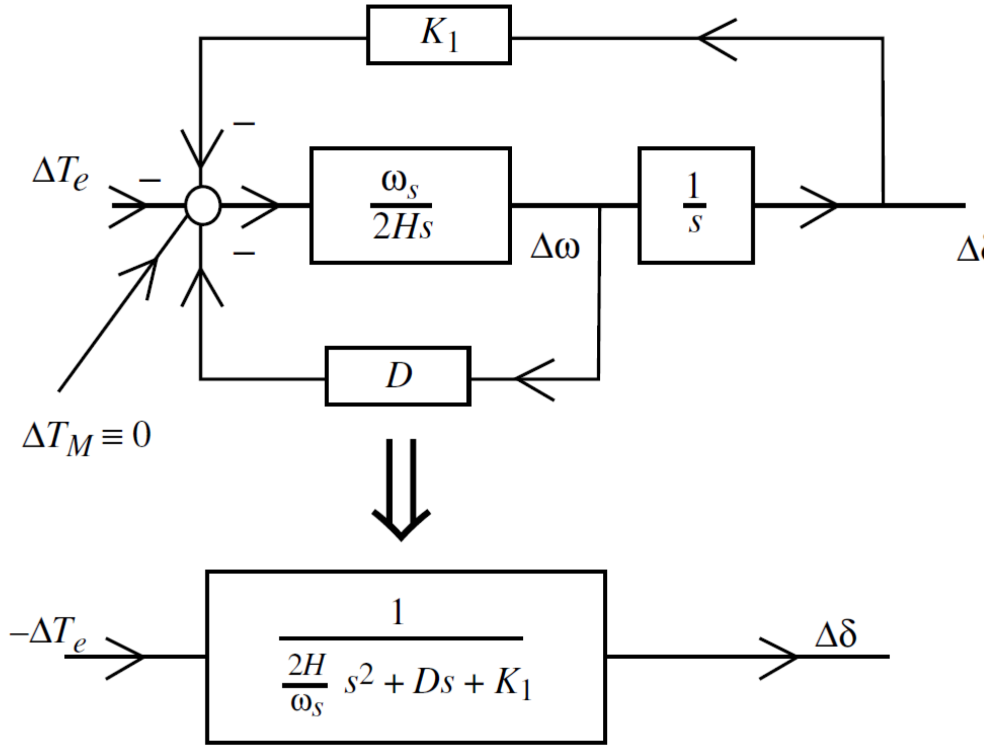


Figure 8.11: Torque-angle loop

Torque-Angle Loop with Other Dynamics

- The other dynamics can be included as in Figure 8.12

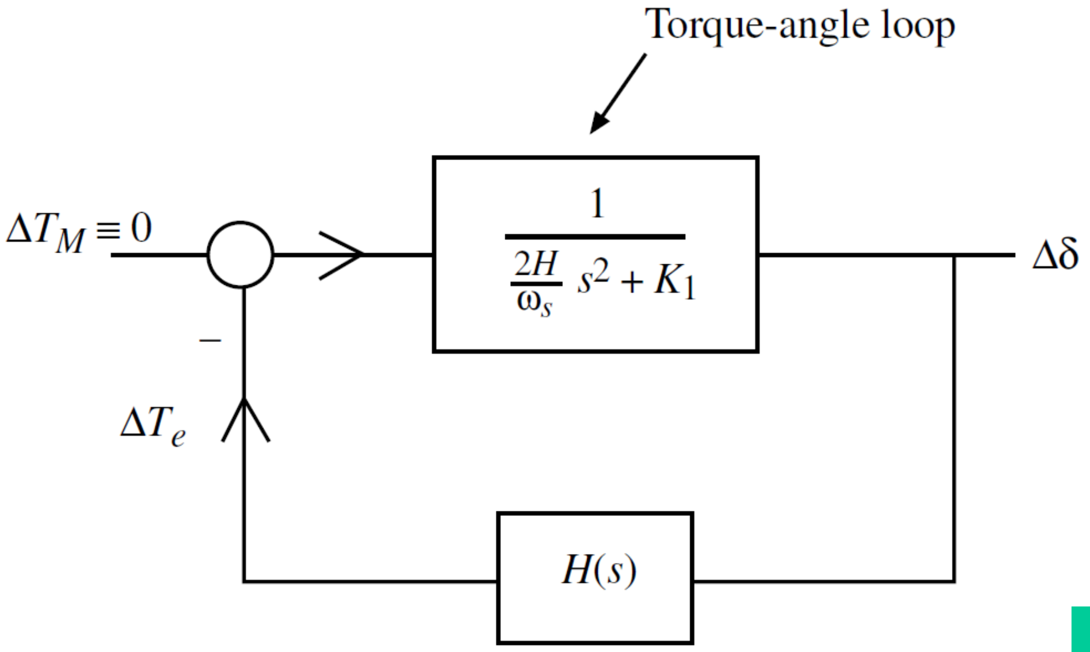


Figure 8.12: Torque-angle loop with other dynamics added

$$\frac{2H}{\omega_s} s^2 \Delta\delta + K_1 \Delta\delta + H(s)\Delta\delta = 0$$

H(s) contributes both synchronizing torque and damping torque

Torque-Angle Loop with Other Dynamics

- Previously we had

$$\frac{\Delta T_e(s)}{\Delta\delta(s)} = H(s) = \frac{-K_2K_3\left(K_4\left(1+sT_A\right)+K_AK_5\right)}{K_AK_3K_6+\left(1+sK_3T'_{do}\right)\left(1+sT_A\right)}$$

- If we neglect K_4 (which has little effect at 1 to 3 Hz) and divide by K_3 we get

$$\frac{\Delta T_e(s)}{\Delta\delta(s)} = H(s) = \frac{-K_2K_AK_5}{K_AK_6+\frac{1}{K_3}+s\left(\frac{T_A}{K_3}+T'_{do}\right)+s^2T'_{do}T_A}$$

Synchronizing Torque

- The synchronizing torque is determined by looking at the response at low frequencies (ω≈0)

$$H(j\omega = 0) = \frac{-K_2 K_A K_5}{K_A K_6 + \frac{1}{K_3}}$$

- With high K_A, this is approximately

$$H(j\omega = 0) = \frac{-K_2 K_5}{K_6}$$

$$\frac{2H}{\omega_s} s^2 \Delta\delta + K_1 \Delta\delta + H(s)\Delta\delta = 0$$

The synchronizing torque is dominated by K_1; it is enhanced if K_5 is negative