ECEN 667
Power System Stability

Lecture 7: Synchronous Machine Models

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Announcements

- Read Chapter 5 and Appendix A
- Homework 2 is how due on Tuesday (Sept 26)

Dq0 Reference Frame

- Stator is stationary, rotor is rotating at synchronous speed
- Rotor values need to be transformed to fixed reference frame for analysis
- Done using Park's transformation into what is known as the dq0 reference frame (direct, quadrature, zero)
  - Parks' 1929 paper voted 2nd most important power paper of 20th century (1st was Fortescue's sym. components paper)
- Convention used here is the q-axis leads the d-axis (which is the IEEE standard)
  - Others (such as Anderson and Fouad) use a q-axis lagging convention

Fundamental Laws

Kirchhoff's Voltage Law, Ohm's Law, Faraday's Law, Newton's Second Law

Stator

$$v_a = i_a r_s + \frac{d\lambda_a}{dt}$$

$$v_b = i_b r_s + \frac{d\lambda_b}{dt}$$

$$v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

Rotor

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{\text{shaft}}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

Dq0 transformations

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \triangleq T_{dqo} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \text{or} \quad i, \lambda$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = T_{dqo}^{-1} \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix}$$

Dq0 transformations

$$T_{dqo} \triangleq \frac{2}{3}\begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \cos\frac{P}{2}\theta_{shaft} & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

with the inverse,

$$T_{dqo}^{-1} = \begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \cos\frac{P}{2}\theta_{shaft} & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

Note that the transformation depends on the shaft angle.

Transformed System

Stator

$$v_d = r_s i_d - \omega\lambda_q + \frac{d\lambda_d}{dt}$$

$$v_q = r_s i_q + \omega\lambda_d + \frac{d\lambda_q}{dt}$$

$$v_o = r_s i_o + \frac{d\lambda_o}{dt}$$

Rotor

$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = r_{1d} i_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = r_{1q} i_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = r_{2q} i_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

Electrical & Mechanical Relationships

Electrical system:

$$v = iR + \frac{d\lambda}{dt} \quad \text{(voltage)}$$

$$vi = i^2R + i\frac{d\lambda}{dt} \quad \text{(power)}$$

Mechanical system:

$$J\left(\frac{2}{P}\right)\frac{d\omega}{dt} = T_m - T_e - T_{fw} \quad \text{(torque)}$$

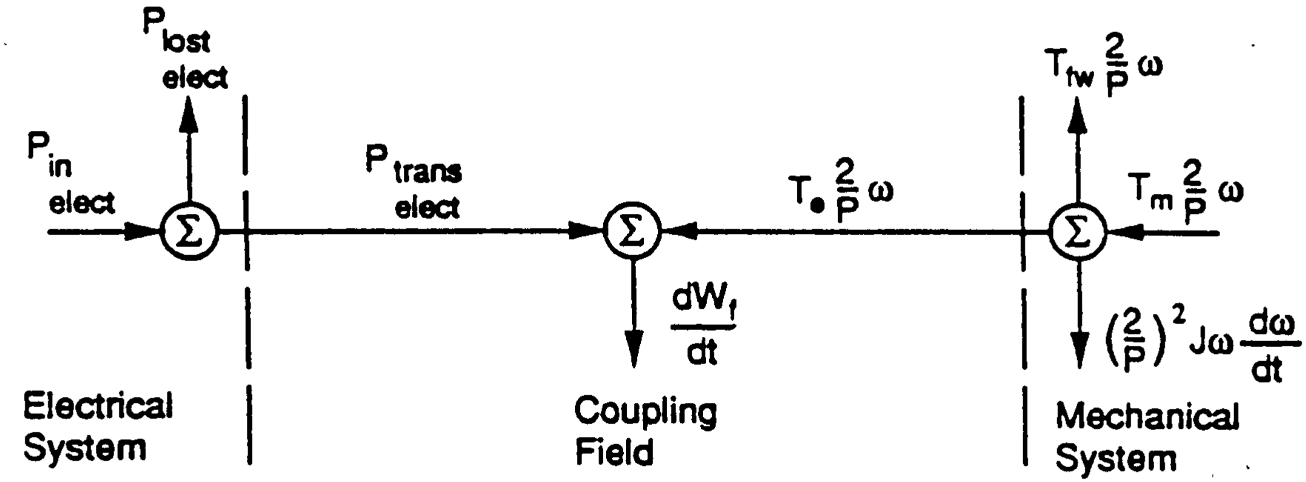
$$J\left(\frac{2}{P}\right)^2 \omega\frac{d\omega}{dt} = \frac{2}{P}\omega T_m - \frac{2}{P}\omega T_e - \frac{2}{P}\omega T_{fw} \quad \text{(power)}$$

P is the number of poles (e.g., 2,4,6); T_{fw} is the friction and windage torque

Derive Torque

- Torque is derived by looking at the overall energy balance in the system
- Three systems: electrical, mechanical and the coupling magnetic field
  - Electrical system losses are in the form of resistance
  - Mechanical system losses are in the form of friction
- Coupling field is assumed to be lossless, hence we can track how energy moves between the electrical and mechanical systems

Energy Conversion



Look at the instantaneous power:

$$v_a i_a + v_b i_b + v_c i_c = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o$$

Change to Conservation of Power

$$P_{in\,elect} = v_a i_a + v_b i_b + v_c i_c + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q} + v_{2q} i_{2q}$$

$$P_{lost\,elect} = r_s\left(i_a^2 + i_b^2 + i_c^2\right) + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

$$P_{trans\,elect} = i_a \frac{d\lambda_a}{dt} + i_b \frac{d\lambda_b}{dt} + i_c \frac{d\lambda_c}{dt} + i_{fd} \frac{d\lambda_{fd}}{dt} + i_{1d} \frac{d\lambda_{1d}}{dt} + i_{1q} \frac{d\lambda_{1q}}{dt} + i_{2q} \frac{d\lambda_{2q}}{dt}$$

We are using $v = d\lambda/dt$ here

With the Transformed Variables

$$P_{in\ elect} = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q} + v_{2q} i_{2q}$$

$$P_{lost\ elect} = \frac{3}{2} r_s i_d^2 + \frac{3}{2} r_s i_q^2 + 3 r_s i_o^2 + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

With the Transformed Variables

$$P_{\substack{trans\\elect}} = -\frac{3}{2}\frac{P}{2}\frac{d\theta_{shaft}}{dt}\lambda_q i_d + \frac{3}{2}i_d\frac{d\lambda_d}{dt} + \frac{3}{2}\frac{P}{2}\frac{d\theta_{shaft}}{dt}\lambda_d i_q$$

$$+\frac{3}{2}i_q\frac{d\lambda_q}{dt} + 3i_o\frac{d\lambda_o}{dt} + i_{fd}\frac{d\lambda_{fd}}{dt} + i_{1d}\frac{d\lambda_{1d}}{dt}$$

$$+i_{1q}\frac{d\lambda_{1q}}{dt} + i_{2q}\frac{d\lambda_{2q}}{dt}$$

Change in Coupling Field Energy

$$\frac{dW_f}{dt} = \boxed{T_e \frac{2}{P}} \frac{d\theta}{dt} + \boxed{i_a} \frac{d\lambda_a}{dt} + \boxed{i_b} \frac{d\lambda_b}{dt}$$

$$+ \boxed{i_c} \frac{d\lambda_c}{dt} + \boxed{i_{fd}} \frac{d\lambda_{fd}}{dt} + \boxed{i_{1d}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{i_{1q}} \frac{d\lambda_{1q}}{dt} + \boxed{i_{2q}} \frac{d\lambda_{2q}}{dt}$$

First term on right is what is going on mechanically, other terms are what is going on electrically

This requires the lossless coupling field assumption

Change in Coupling Field Energy

For independent states $\theta, \lambda_a, \lambda_b, \lambda_c, \lambda_{fd}, \lambda_{1d}, \lambda_{1q}, \lambda_{2q}$

$$\frac{dW_f}{dt} = \boxed{\frac{\partial W_f}{\partial \theta}} \frac{d\theta}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_a}} \frac{d\lambda_a}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_b}} \frac{d\lambda_b}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_c}} \frac{d\lambda_c}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{fd}}} \frac{d\lambda_{fd}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{1d}}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_{1q}}} \frac{d\lambda_{1q}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{2q}}} \frac{d\lambda_{2q}}{dt}$$

Equate the Coefficients

$$T_e \frac{2}{P} = \frac{\partial W_f}{\partial \theta} \qquad i_a = \frac{\partial W_f}{\partial \lambda_a} \qquad \text{etc.}$$

There are eight such “reciprocity conditions for this model.

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

Equate the Coefficients

$$\frac{\partial W_f}{\partial \theta_{shaft}} = \frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right) + T_e$$

$$\frac{\partial W_f}{\partial \lambda_d} = \frac{3}{2} i_d\,, \qquad \frac{\partial W_f}{\partial \lambda_q} = \frac{3}{2} i_q, \qquad \frac{\partial W_f}{\partial \lambda_o} = 3 i_o$$

$$\frac{\partial W_f}{\partial \lambda_{fd}} = i_{fd}\,, \qquad \frac{\partial W_f}{\partial \lambda_{1d}} = i_{1d}, \qquad \frac{\partial W_f}{\partial \lambda_{1q}} = i_{1q}, \qquad \frac{\partial W_f}{\partial \lambda_{2q}} = i_{2q}$$

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

Coupling Field Energy

- The coupling field energy is calculated using a path independent integration
  - For integral to be path independent, the partial derivatives of all integrands with respect to the other states must be equal

For example, $$\frac{3}{2}\frac{\partial i_d}{\partial \lambda_{fd}} = \frac{\partial i_{fd}}{\partial \lambda_d}$$

- Since integration is path independent, choose a convenient path
  - Start with a de-energized system so all variables are zero
  - Integrate shaft position while other variables are zero, hence no energy
  - Integrate sources in sequence with shaft at final θ_{shaft} value

Do the Integration

$$W_f = W_f^o + \int_{\theta_{shaft}^o}^{\theta_{shaft}} \left[\frac{3}{2} \frac{P}{2} \left(\hat{\lambda}_d i_q - \hat{\lambda}_q i_d \right) \right] d\hat{\theta}_{shaft}$$

$$+ \int_{\lambda_d^o}^{\lambda_d} \frac{3}{2} i_d d\hat{\lambda}_d + \int_{\lambda_q^o}^{\lambda_q} \frac{3}{2} i_q d\hat{\lambda}_q + \int_{\lambda_o^o}^{\lambda_o} 3 i_o d\hat{\lambda}_o$$

$$+ \int_{\lambda_{fd}^o}^{\lambda_{fd}} i_{fd} d\hat{\lambda}_{fd} + \int_{\lambda_{1d}^o}^{\lambda_{1d}} i_{1d} d\hat{\lambda}_{1d} + \int_{\lambda_{1q}^o}^{\lambda_{1q}} i_{1q} d\hat{\lambda}_{1q} + \int_{\lambda_{2q}^o}^{\lambda_{2q}} i_{2q} d\hat{\lambda}_{2q}$$

Torque

- Assume: i_q, i_d, i_o, i_{fd}, i_{1d}, i_{1q}, i_{2q} are independent of θ_{shaft} (current/flux linkage relationship is independent of θ_{shaft})
- Then W_f will be independent of θ_{shaft} as well
- Since we have

$$\frac{\partial W_f}{\partial \theta_{shaft}} = \frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right) + T_e = 0$$

$$T_e = -\frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right)$$

Define Unscaled Variables

$$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$

ω_s is the rated synchronous speed
δ plays an important role!

$$\frac{d\lambda_d}{dt} = -r_s i_d + \omega\lambda_q + v_d$$

$$\frac{d\lambda_q}{dt} = -r_s i_q - \omega\lambda_d + v_q$$

$$\frac{d\lambda_o}{dt} = -r_s i_o + v_o$$

$$\frac{d\lambda_{fd}}{dt} = -r_{fd} i_{fd} + v_{fd}$$

$$\frac{d\lambda_{1d}}{dt} = -r_{1d} i_{1d} + v_{1d}$$

$$\frac{d\lambda_{1q}}{dt} = -r_{1q} i_{1q} + v_{1q}$$

$$\frac{d\lambda_{2q}}{dt} = -r_{2q} i_{2q} + v_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$J\frac{2}{p}\frac{d\omega}{dt} = T_m + \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_d i_q - \lambda_q i_d\right) - T_{f\omega}$$

Convert to Per Unit

- As with power flow, values are usually expressed in per unit, here on the machine power rating

$$V_{Base} I_{Base} = P_{Base}$$

- Two common sign conventions for current: motor has positive currents into machine, generator has positive out of the machine
- Modify the flux linkage current relationship to account for the non power invariant "dqo" transformation

Convert to Per Unit

$$V_a \triangleq \frac{v_a}{V_{BABC}}, \quad V_b \triangleq \frac{v_b}{V_{BABC}}, \quad V_c \triangleq \frac{v_c}{V_{BABC}},$$

$$I_a \triangleq \frac{-i_a}{I_{BABC}}, \quad I_b \triangleq \frac{-i_b}{I_{BABC}}, \quad I_c \triangleq \frac{-i_c}{I_{BABC}}$$

$$\psi_a \triangleq \frac{\lambda_a}{\Lambda_{BABC}}, \quad \psi_b \triangleq \frac{\lambda_b}{\Lambda_{BABC}}, \quad \psi_c \triangleq \frac{\lambda_c}{\Lambda_{BABC}}$$

where V_{BABC} is rated RMS line-to-neutral stator voltage and

$$I_{BABC} \triangleq \frac{P_B}{3V_{BABC}}, \quad \Lambda_{BABC} \triangleq \frac{V_{BABC}}{\omega_B}$$

Convert to Per Unit

$$V_d \triangleq \frac{v_d}{V_{BDQ}}, \quad V_q \triangleq \frac{v_q}{V_{BDQ}}, \quad V_o \triangleq \frac{v_o}{V_{BDQ}},$$

$$I_d \triangleq \frac{-i_d}{I_{BDQ}}, \quad I_q \triangleq \frac{-i_q}{I_{BDQ}}, \quad I_o \triangleq \frac{-i_o}{I_{BDQ}}$$

$$\psi_d \triangleq \frac{\lambda_d}{\Lambda_{BDQ}}, \quad \psi_q \triangleq \frac{\lambda_q}{\Lambda_{BDQ}}, \quad \psi_o \triangleq \frac{\lambda_o}{\Lambda_{BDQ}}$$

where V_{BDQ} is rated peak line-to-neutral stator voltage and

$$I_{BDQ} \triangleq \frac{2P_B}{3V_{BDQ}}, \quad \Lambda_{BDQ} \triangleq \frac{V_{BDQ}}{\omega_B}$$

Convert to Per Unit

$$V_{fd} \triangleq \frac{v_{fd}}{V_{BFD}}, \quad V_{1d} \triangleq \frac{v_{1d}}{V_{B1D}}, \quad V_{1q} \triangleq \frac{v_{1q}}{V_{B1Q}}, \quad V_{2q} \triangleq \frac{v_{2q}}{V_{B2Q}}$$

$$I_{fd} \triangleq \frac{i_{fd}}{I_{BFD}}, \quad I_{1d} \triangleq \frac{i_{1d}}{I_{B1D}}, \quad I_{1q} \triangleq \frac{i_{1q}}{I_{B1Q}}, \quad I_{2q} \triangleq \frac{i_{2q}}{I_{B2Q}}$$

$$\psi_{fd} \triangleq \frac{\lambda_{fd}}{\Lambda_{BFD}}, \quad \psi_{1d} \triangleq \frac{\lambda_{1d}}{\Lambda_{B1D}}, \quad \psi_{1q} \triangleq \frac{\lambda_{1q}}{\Lambda_{B1Q}}, \quad \psi_{2q} \triangleq \frac{\lambda_{2q}}{\Lambda_{B2Q}}$$

Hence the ψ variables are just normalized flux linkages

Convert to Per Unit

Where the rotor circuit base voltages are

$$V_{BFD} \triangleq \frac{P_B}{I_{BFD}}, \qquad V_{B1D} \triangleq \frac{P_B}{I_{B1D}},$$

$$V_{B1Q} \triangleq \frac{P_B}{I_{B1Q}}, \qquad V_{B2Q} \triangleq \frac{P_B}{I_{B2Q}}$$

And the rotor circuit base flux linkages are

$$\Lambda_{BFD} \triangleq \frac{V_{BFD}}{\omega_B}, \qquad \Lambda_{B1D} \triangleq \frac{V_{B1D}}{\omega_B},$$

$$\Lambda_{B1Q} \triangleq \frac{V_{B1Q}}{\omega_B}, \qquad \Lambda_{B2Q} \triangleq \frac{V_{B2Q}}{\omega_B}$$

Convert to Per Unit

$$R_s \triangleq \frac{r_s}{Z_{BDQ}}, \qquad R_{fd} \triangleq \frac{r_{fd}}{Z_{BFD}}, \qquad R_{1d} \triangleq \frac{r_{1d}}{Z_{B1D}},$$

$$R_{1q} \triangleq \frac{r1q}{Z_{B1Q}}, \qquad R_{2q} \triangleq \frac{r_{2q}}{Z_{B2Q}},$$

$$Z_{BDQ} \triangleq \frac{V_{BDQ}}{I_{BDQ}}, \qquad Z_{BFD} \triangleq \frac{V_{BFD}}{I_{BFD}}, \qquad Z_{B1D} \triangleq \frac{V_{B1D}}{I_{B1D}},$$

$$Z_{B1Q} \triangleq \frac{V_{B1Q}}{I_{B1Q}}, \qquad Z_{B2Q} \triangleq \frac{V_{B2Q}}{I_{B2Q}}$$

Convert to Per Unit

- Almost done with the per unit conversions! Finally define inertia constants and torque

$$H \triangleq \frac{\frac{1}{2} J \left(\omega_B \frac{2}{P}\right)^2}{S_B}, M = \frac{2H}{\omega_s}$$

$$T_M \triangleq \frac{T_m}{T_B}, \quad T_{ELEC} \triangleq \frac{T_e}{T_B}, \quad T_{FW} \triangleq \frac{T_{fw}}{T_B}, \quad T_B \triangleq \frac{S_B}{\omega_B \frac{2}{P}}$$

Synchronous Machine Equations

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + V_o$$

$$\frac{1}{\omega_s}\frac{d\psi_{fd}}{dt} = -R_{fd} I_{fd} + V_{fd}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1d}}{dt} = -R_{1d} I_{1d} + V_{1d}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1q}}{dt} = -R_{1q} I_{1q} + V_{1q}$$

$$\frac{1}{\omega_s}\frac{d\psi_{2q}}{dt} = -R_{2q} I 2 + V_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

Sinusoidal Steady-State

$$V_a = \sqrt{2} V_s \cos(\omega_s t + \theta_{vs})$$

$$V_b = \sqrt{2} V_s \cos\left(\omega_s t + \theta_{vs} - \frac{2\pi}{3}\right)$$

$$V_c = \sqrt{2} V_s \cos\left(\omega_s t + \theta_{vs} + \frac{2\pi}{3}\right)$$

$$I_a = \sqrt{2} I_s \cos(\omega_s t + \theta_{is})$$

$$I_b = \sqrt{2} I_s \cos\left(\omega_s t + \theta_{is} - \frac{2\pi}{3}\right)$$

$$I_c = \sqrt{2} I_s \cos\left(\omega_s t + \theta_{is} + \frac{2\pi}{3}\right)$$

Here we consider the application
to balanced, sinusoidal conditions

Transforming to dq0

$$V_d = \left(\frac{\sqrt{2}V_s V_{BABC}}{V_{BDQ}}\right)\sin\left(\frac{P}{2}\theta_{shaft} - \omega_s t - \theta_{vs}\right)$$

$$V_q = \left(\frac{\sqrt{2}V_s V_{BABC}}{V_{BDQ}}\right)\cos\left(\frac{P}{2}\theta_{shaft} - \omega_s t - \theta_{vs}\right)$$

$$V_o = 0$$

$$I_d = \left(\frac{\sqrt{2}I_s I_{BABC}}{I_{BDQ}}\right)\sin\left(\frac{P}{2}\theta_{shaft} - \omega_s t - \theta_{is}\right)$$

$$I_q = \left(\frac{\sqrt{2}I_s I_{BABC}}{I_{BDQ}}\right)\cos\left(\frac{P}{2}\theta_{shaft} - \omega_s t - \theta_{is}\right)$$

$$I_o = 0$$

Simplifying Using δ

- Recall that $\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$
- Hence

$$V_d = V_s \sin(\delta - \theta_{vs})$$
$$V_q = V_s \cos(\delta - \theta_{vs})$$
$$I_d = I_s \sin(\delta - \theta_{is})$$
$$I_q = I_s \cos(\delta - \theta_{is})$$

The conclusion is if we know δ, then we can easily relate the phase to the dq values!

- These algebraic equations can be written as complex equations,

$$\left(V_d + jV_q\right)e^{j(\delta - \pi/2)} = V_s e^{j\theta_{vs}}$$
$$\left(I_d + jI_q\right)e^{j(\delta - \pi/2)} = I_s e^{j\theta_{is}}$$

Summary So Far

- The model as developed so far has been derived using the following assumptions
  - The stator has three coils in a balanced configuration, spaced 120 electrical degrees apart
  - Rotor has four coils in a balanced configuration located 90 electrical degrees apart
  - Relationship between the flux linkages and currents must reflect a conservative coupling field
  - The relationships between the flux linkages and currents must be independent of θ_{shaft} when expressed in the dq0 coordinate system

Two Main Types of Synchronous Machines

- Round Rotor
  - Air-gap is constant, used with higher speed machines
- Salient Rotor (often called Salient Pole)
  - Air-gap varies circumferentially
  - Used with many pole, slower machines such as hydro
  - Narrowest part of gap in the d-axis and the widest along the q-axis

Assuming a Linear Magnetic Circuit

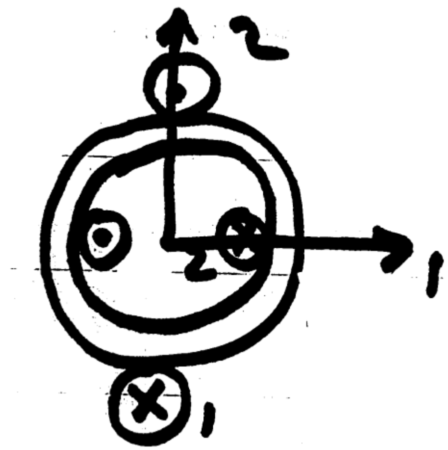
- If the flux linkages are assumed to be a linear function of the currents then we can write

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \hline \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} L_{ss}\left(\theta_{shaft}\right) & L_{sr}\left(\theta_{shaft}\right) \\ \hline L_{rs}\left(\theta_{shaft}\right) & L_{rr}\left(\theta_{shaft}\right) \end{array} \right] \begin{bmatrix} i_a \\ i_b \\ i_c \\ \hline i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

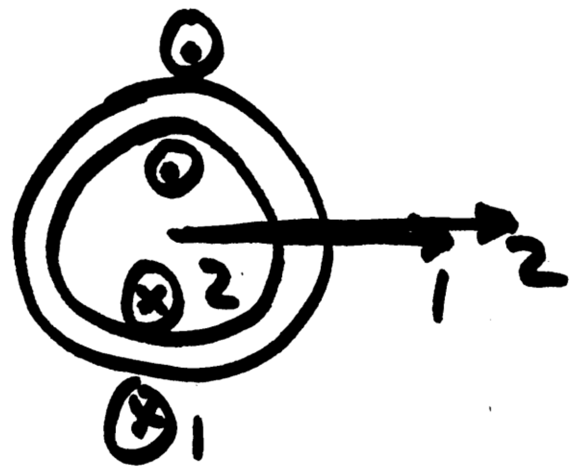
The rotor self-inductance matrix L_{rr} is independent of θ_{shaft}

Inductive Dependence on Shaft Angle

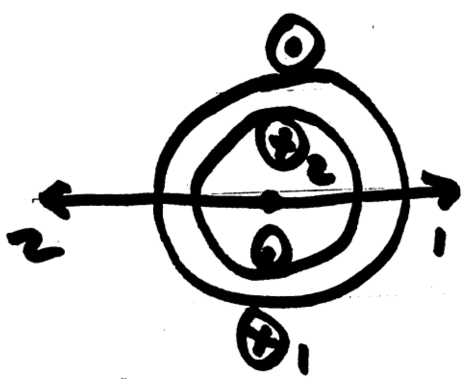
$L_{12} = 0$



$L_{12} = +\ \text{maximum}$



$L_{12} = -\ \text{maximum}$



Stator Inductances

- The self inductance for each stator winding has a portion that is due to the leakage flux which does not cross the air gap, L_{ls}
- The other portion of the self inductance is due to flux crossing the air gap and can be modeled for phase a as

$$L_A + L_B \cos(P\theta_{shaft})$$

- Mutual inductance between the stator windings is modeled as

$$\frac{1}{2} L_A - L_B \cos(P\theta_{shaft} + offset)$$

The offset angle is either $2\pi/3$ or $-2\pi/3$

Conversion to dq0 for Angle Independence

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \\ \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} T_{dqo} L_{ss} T_{dqo}^{-1} & T_{dqo} L_{sr} \\ \hline L_{rs} T_{dqo}^{-1} & L_{rr} \end{array} \right] \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

Conversion to dq0 for Angle Independence

$$\lambda_d = \left(L_{\ell s} + L_{md}\right) i_d + L_{sfd} i_{fd} + L_{s1d} i_{1d}$$

$$\lambda_{fd} = \frac{3}{2} L_{sfd} i_d + L_{fdfd} i_{fd} + L_{fd1d} i_{1d}$$

$$\lambda_{1d} = \frac{3}{2} L_{s1d} i_d + L_{fd1d} i_{fd} + L_{1d1d} i_{1d}$$

$$\lambda_q = \left(L_{\ell s} + L_{mq}\right) i_q + L_{s1q} i_{1q} + L_{s2q} i_{2q}$$

$$\lambda_{1q} = \frac{3}{2} L_{s1q} i_q + L_{1q1q} i_{1q} + L_{1q2q} i_{2q}$$

$$\lambda_{2q} = \frac{3}{2} L_{s2q} i_q + L_{1q2q} i_{1q} + L_{2q2q} i_{2q}$$

$$\lambda_o = L_{\ell s} i_o$$

$$L_{md} \triangleq \frac{3}{2}\left(L_A + L_B\right),$$

$$L_{mq} \triangleq \frac{3}{2}\left(L_A - L_B\right)$$

For a round rotor machine L_B is small and hence L_{md} is close to L_{mq}. For a salient pole machine L_{md} is substantially larger

Convert to Normalized at f = ω_s

- Convert to per unit, and assume frequency of ω_s
- Then define new per unit reactance variables

$$X_{\ell s} \triangleq \frac{\omega_s L_{\ell s}}{Z_{BDQ}}, \quad X_{md} \triangleq \frac{\omega_s L_{md}}{Z_{BDQ}}, \quad X_{mq} \triangleq \frac{\omega_s L_{mq}}{Z_{BDQ}}$$

$$X_{fd} \triangleq \frac{\omega_s L_{fdfd}}{Z_{BFD}}, \quad X_{1d} \triangleq \frac{\omega_s L_{1d1d}}{Z_{B1D}}, \quad X_{fd1d} \triangleq \frac{\omega_s L_{fd1d} L_{sfd}}{Z_{BFD} L_{s1d}}$$

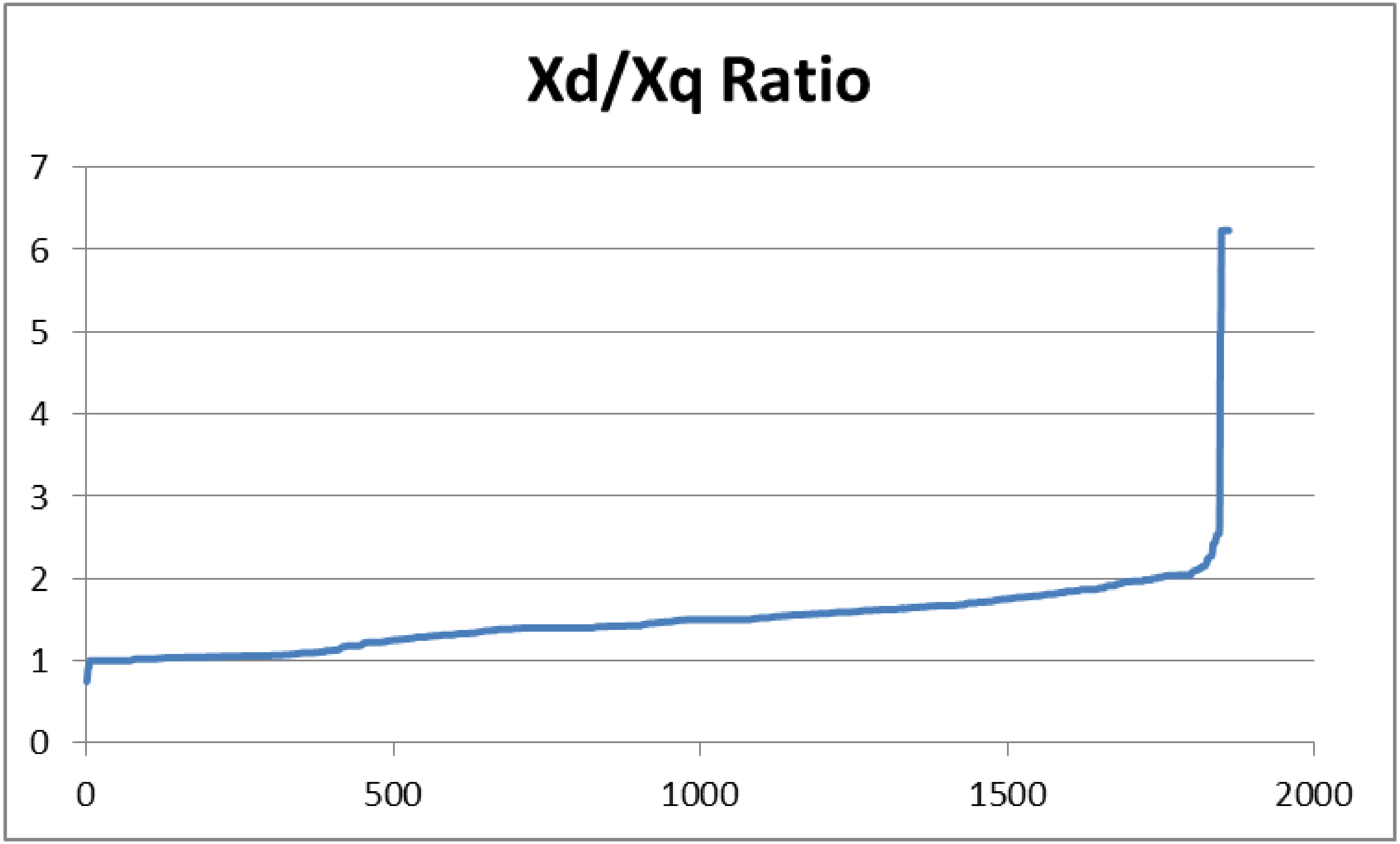
$$X_{1q} \triangleq \frac{\omega_s L_{1q1q}}{Z_{B1Q}}, \quad X_{2q} \triangleq \frac{\omega_s L_{2q2q}}{Z_{B2Q}}, \quad X_{1q2q} \triangleq \frac{\omega_s L_{1q2q} L_{s1q}}{Z_{B1Q} L_{s2q}}$$

$$X_{\ell fd} \triangleq X_{fd} - X_{md}, \quad X_{\ell 1d} \triangleq X_{1d} - X_{md}$$

$$X_{\ell 1q} \triangleq X_{1q} - X_{mq}, \quad X_{\ell 2q} \triangleq X_{2q} - X_{mq}$$

$$X_d \triangleq X_{\ell s} + X_{md}, \quad X_q \triangleq X_{\ell s} + X_{mq}$$

Example Xd/Xq Ratios for a WECC Case



Normalized Equations

$$\psi_d = X_d(-I_d) + X_{md}I_{fd} + X_{md}I_{1d}$$

$$\psi_{fd} = X_{md}(-I_d) + X_{fd}I_{fd} + c_d X_{md}I_{1d}$$

$$\psi_{1d} = X_{md}(-I_d) + c_d X_{md}I_{fd} + X_{1d}I_{1d}$$

$$c_d \approx 1$$

$$\psi_q = X_q(-I_q) + X_{mq}I_{1q} + X_{mq}I_{2q}$$

$$\psi_{1q} = X_{mq}(-I_q) + X_{1q}I_{1q} + c_q X_{mq}I_{2q}$$

$$\psi_{2q} = X_{mq}(-I_q) + c_q X_{mq}I_{1q} + X_{2q}I_{2q}$$

$$c_q \approx 1$$

$$\psi_o = X_{\ell s}(-I_o)$$

$$c_d \triangleq \frac{X_{fd1d}}{X_{md}}, \quad c_q \triangleq \frac{X_{1q2q}}{X_{mq}}$$

Key Simulation Parameters

- The key parameters that occur in most models can then be defined the following transient values

$$X'_d \triangleq X_{\ell s} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}} = X_d - \frac{X_{md}^2}{X_{fd}}$$

$$X'_q \triangleq X_{\ell s} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}} = X_q - \frac{X_{mq}^2}{X_{1q}}$$

$$T'_{do} \triangleq \frac{Xfd}{\omega_s R_{fd}}, \quad T'_{qo} \triangleq \frac{X_{1q}}{\omega_s R_{1q}}$$

These values will be used in all the synchronous machine models

In a salient rotor machine X_{mq} is small so $X_q = X'_{q;}$ also X_{1q} is small so T'_{q0} is small