ECEN 667
Power System Stability

Lecture 8: Synchronous Machine Models

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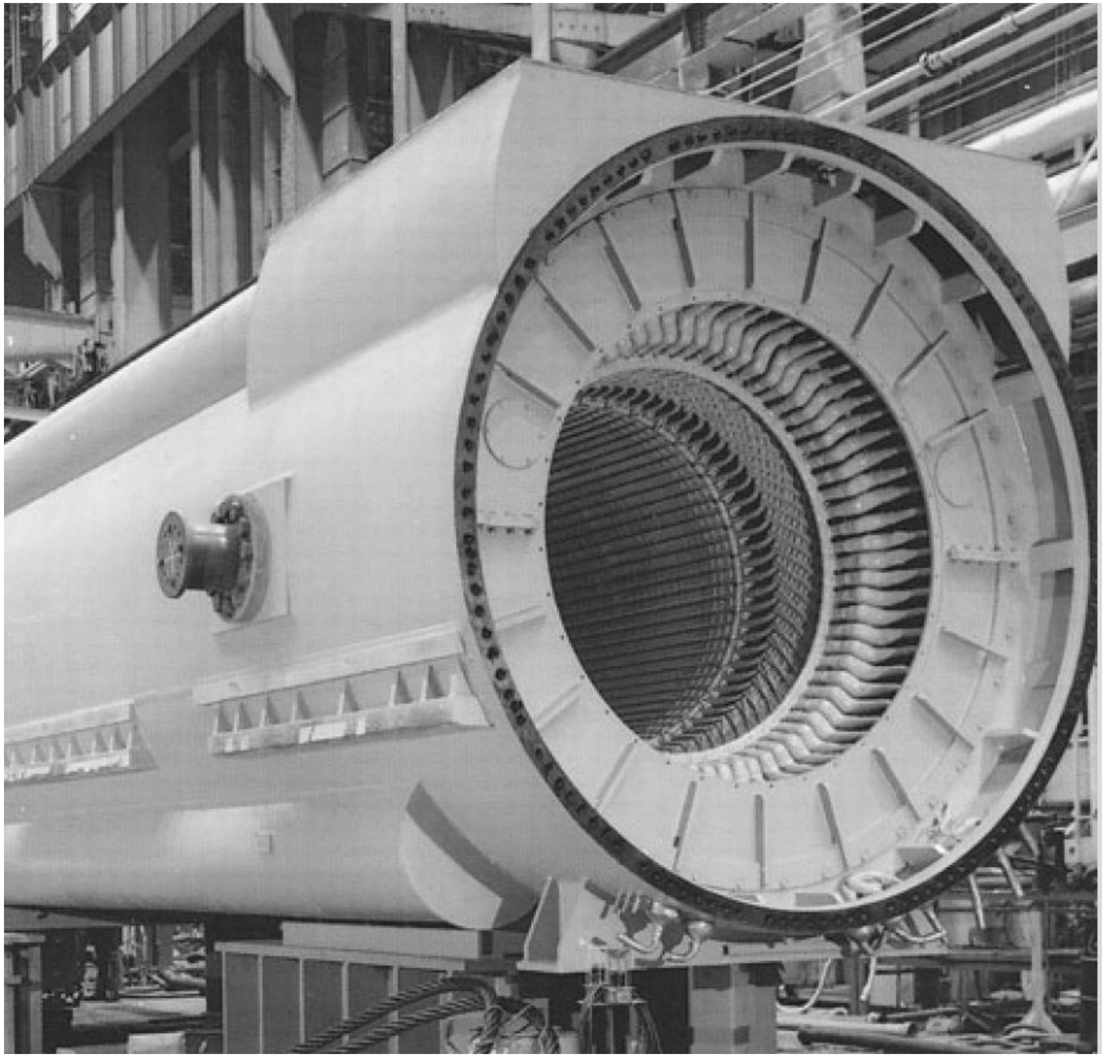
Announcements

- Read Chapter 5 and Appendix A
- Homework 2 is due today
- Homework 3 is posted, due on Thursday Oct 5

Synchronous Machine Stator

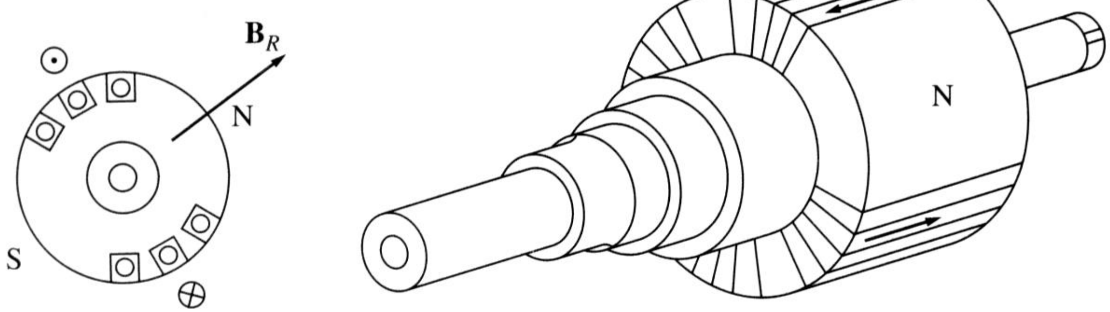


Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric.)

Image Source: Glover/Overbye/Sarma Book, Sixth Edition, Beginning of Chapter 8 Photo

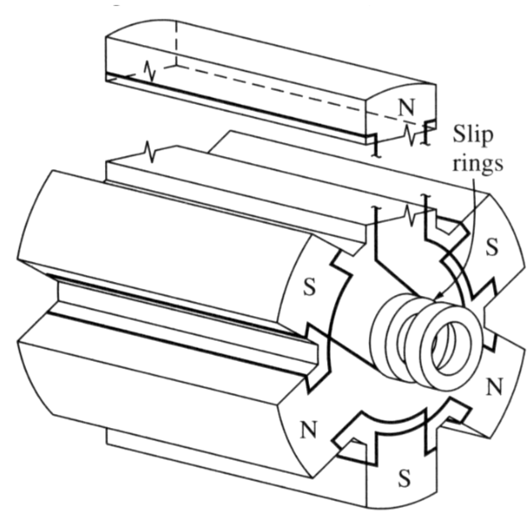
Synchronous Machine Rotors

- Rotors are essentially electromagnets



End view

Side view



Two pole (P) round rotor

Six pole salient rotor

Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Synchronous Machine Rotor

High pole salient rotor

Shaft

Part of exciter, which is used to control the field current

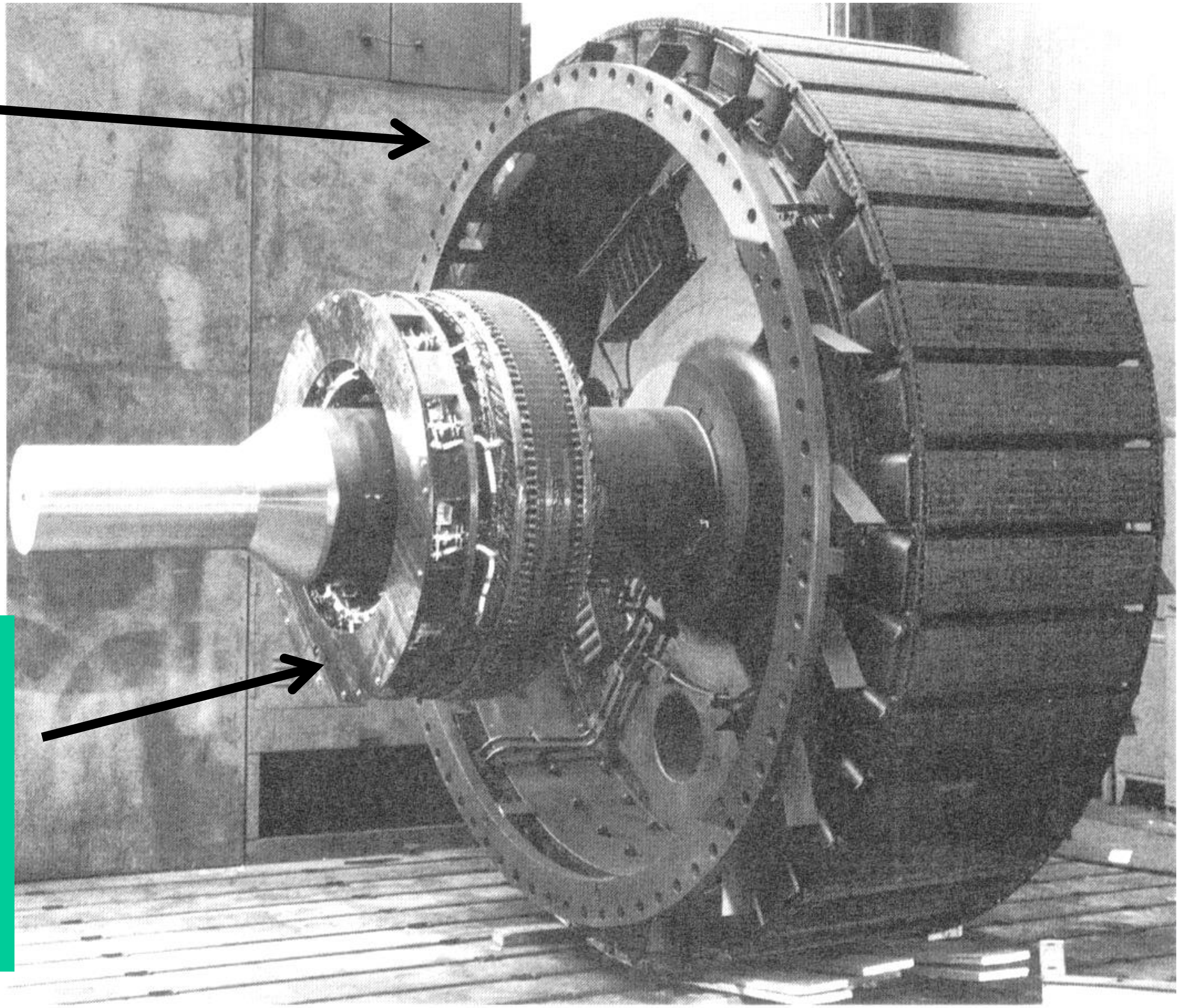
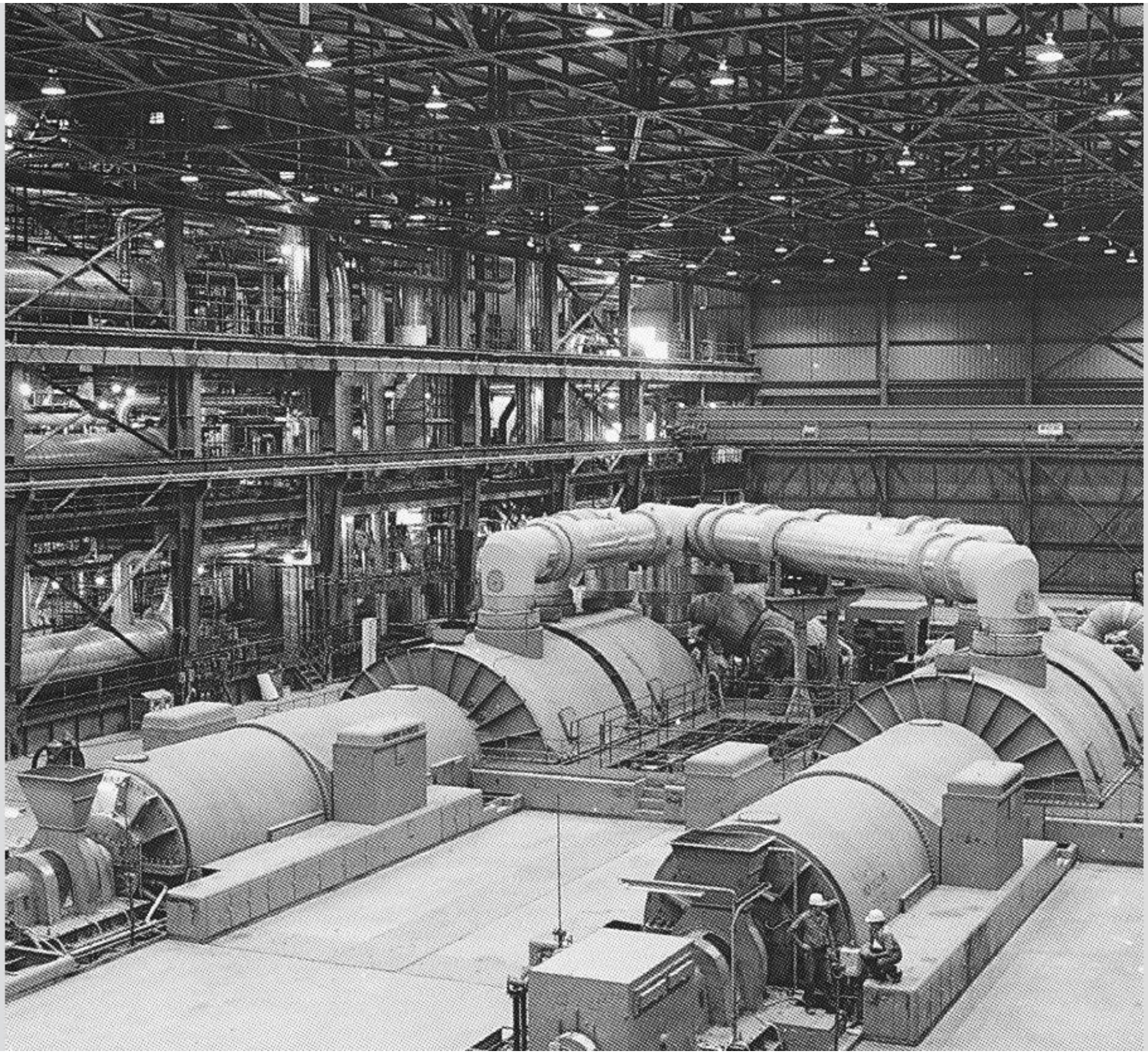


Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Synchronous Generators



1300-MW generating unit consisting of a cross-compound steam turbine and two 722-MVA synchronous generators (Courtesy of American Electric Power.)

Image Source: Glover/Overbye/Sarma Book, Sixth Edition, Beginning of Chapter 11 Photo

Fundamental Laws

Kirchhoff's Voltage Law, Ohm's Law, Faraday's Law, Newton's Second Law

Stator

$$v_a = i_a r_s + \frac{d\lambda_a}{dt}$$

$$v_b = i_b r_s + \frac{d\lambda_b}{dt}$$

$$v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

Rotor

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{\text{shaft}}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

Transformed System

Stator

$$v_d = r_s i_d - \omega\lambda_q + \frac{d\lambda_d}{dt}$$

$$v_q = r_s i_q + \omega\lambda_d + \frac{d\lambda_q}{dt}$$

$$v_o = r_s i_o + \frac{d\lambda_o}{dt}$$

Rotor

$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = r_{1d} i_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = r_{1q} i_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = r_{2q} i_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

Define Unscaled Variables

$$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$

ω_s is the rated synchronous speed
δ plays an important role!

$$\frac{d\lambda_d}{dt} = -r_s i_d + \omega\lambda_q + v_d$$

$$\frac{d\lambda_q}{dt} = -r_s i_q - \omega\lambda_d + v_q$$

$$\frac{d\lambda_o}{dt} = -r_s i_o + v_o$$

$$\frac{d\lambda_{fd}}{dt} = -r_{fd} i_{fd} + v_{fd}$$

$$\frac{d\lambda_{1d}}{dt} = -r_{1d} i_{1d} + v_{1d}$$

$$\frac{d\lambda_{1q}}{dt} = -r_{1q} i_{1q} + v_{1q}$$

$$\frac{d\lambda_{2q}}{dt} = -r_{2q} i_{2q} + v_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$J\frac{2}{p}\frac{d\omega}{dt} = T_m + \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_d i_q - \lambda_q i_d\right) - T_{f\omega}$$

Synchronous Machine Equations in Per Unit

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + V_o$$

$$\frac{1}{\omega_s}\frac{d\psi_{fd}}{dt} = -R_{fd} I_{fd} + V_{fd}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1d}}{dt} = -R_{1d} I_{1d} + V_{1d}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1q}}{dt} = -R_{1q} I_{1q} + V_{1q}$$

$$\frac{1}{\omega_s}\frac{d\psi_{2q}}{dt} = -R_{2q} I2 + V_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

Units of H are seconds

Sinusoidal Steady-State

$$V_a = \sqrt{2} V_s \cos\left(\omega_s t + \theta_{vs}\right)$$

$$V_b = \sqrt{2} V_s \cos\left(\omega_s t + \theta_{vs} - \frac{2\pi}{3}\right)$$

$$V_c = \sqrt{2} V_s \cos\left(\omega_s t + \theta_{vs} + \frac{2\pi}{3}\right)$$

$$I_a = \sqrt{2} I_s \cos\left(\omega_s t + \theta_{is}\right)$$

$$I_b = \sqrt{2} I_s \cos\left(\omega_s t + \theta_{is} - \frac{2\pi}{3}\right)$$

$$I_c = \sqrt{2} I_s \cos\left(\omega_s t + \theta_{is} + \frac{2\pi}{3}\right)$$

Here we consider the application
to balanced, sinusoidal conditions

Simplifying Using δ

- Recall that $\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$
- Hence
$$V_d = V_s \sin(\delta - \theta_{vs})$$
$$V_q = V_s \cos(\delta - \theta_{vs})$$
$$I_d = I_s \sin(\delta - \theta_{is})$$
$$I_q = I_s \cos(\delta - \theta_{is})$$

The conclusion is if we know δ, then we can easily relate the phase to the dq values!

- These algebraic equations can be written as complex equations,
$$\left(V_d + jV_q\right)e^{j(\delta - \pi/2)} = V_s e^{j\theta_{vs}}$$
$$\left(I_d + jI_q\right)e^{j(\delta - \pi/2)} = I_s e^{j\theta_{is}}$$

Summary So Far

- The model as developed so far has been derived using the following assumptions
  - The stator has three coils in a balanced configuration, spaced 120 electrical degrees apart
  - Rotor has four coils in a balanced configuration located 90 electrical degrees apart
  - Relationship between the flux linkages and currents must reflect a conservative coupling field
  - The relationships between the flux linkages and currents must be independent of θ_{shaft} when expressed in the dq0 coordinate system

Assuming a Linear Magnetic Circuit

- If the flux linkages are assumed to be a linear function of the currents then we can write

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \hline \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} L_{ss}\left(\theta_{shaft}\right) & L_{sr}\left(\theta_{shaft}\right) \\ \hline L_{rs}\left(\theta_{shaft}\right) & L_{rr}\left(\theta_{shaft}\right) \end{array}\right] \begin{bmatrix} i_a \\ i_b \\ i_c \\ \hline i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

The rotor self-inductance matrix L_{rr} is independent of θ_{shaft}

Conversion to dq0 for Angle Independence

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \\ \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} T_{dqo} L_{ss} T_{dqo}^{-1} & T_{dqo} L_{sr} \\ \hline L_{rs} T_{dqo}^{-1} & L_{rr} \end{array} \right] \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

Conversion to dq0 for Angle Independence

$$\lambda_d = \left(L_{\ell s} + L_{md}\right) i_d + L_{sfd} i_{fd} + L_{s1d} i_{1d}$$

$$\lambda_{fd} = \frac{3}{2} L_{sfd} i_d + L_{fdfd} i_{fd} + L_{fd1d} i_{1d}$$

$$\lambda_{1d} = \frac{3}{2} L_{s1d} i_d + L_{fd1d} i_{fd} + L_{1d1d} i_{1d}$$

$$\lambda_q = \left(L_{\ell s} + L_{mq}\right) i_q + L_{s1q} i_{1q} + L_{s2q} i_{2q}$$

$$\lambda_{1q} = \frac{3}{2} L_{s1q} i_q + L_{1q1q} i_{1q} + L_{1q2q} i_{2q}$$

$$\lambda_{2q} = \frac{3}{2} L_{s2q} i_q + L_{1q2q} i_{1q} + L_{2q2q} i_{2q}$$

$$\lambda_o = L_{\ell s} i_o$$

$$L_{md} \triangleq \frac{3}{2}\left(L_A + L_B\right),$$

$$L_{mq} \triangleq \frac{3}{2}\left(L_A - L_B\right)$$

For a round rotor machine L_B is small and hence L_{md} is close to L_{mq}. For a salient pole machine L_{md} is substantially larger

Convert to Normalized at f = ω_s

- Convert to per unit, and assume frequency of ω_s
- Then define new per unit reactance variables

$$X_{\ell s} \triangleq \frac{\omega_s L_{\ell s}}{Z_{BDQ}}, \quad X_{md} \triangleq \frac{\omega_s L_{md}}{Z_{BDQ}}, \quad X_{mq} \triangleq \frac{\omega_s L_{mq}}{Z_{BDQ}}$$

$$X_{fd} \triangleq \frac{\omega_s L_{fdfd}}{Z_{BFD}}, \quad X_{1d} \triangleq \frac{\omega_s L_{1d1d}}{Z_{B1D}}, \quad X_{fd1d} \triangleq \frac{\omega_s L_{fd1d} L_{sfd}}{Z_{BFD} L_{s1d}}$$

$$X_{1q} \triangleq \frac{\omega_s L_{1q1q}}{Z_{B1Q}}, \quad X_{2q} \triangleq \frac{\omega_s L_{2q2q}}{Z_{B2Q}}, \quad X_{1q2q} \triangleq \frac{\omega_s L_{1q2q} L_{s1q}}{Z_{B1Q} L_{s2q}}$$

$$X_{\ell fd} \triangleq X_{fd} - X_{md}, \quad X_{\ell 1d} \triangleq X_{1d} - X_{md}$$

$$X_{\ell 1q} \triangleq X_{1q} - X_{mq}, \quad X_{\ell 2q} \triangleq X_{2q} - X_{mq}$$

$$X_d \triangleq X_{\ell s} + X_{md}, \quad X_q \triangleq X_{\ell s} + X_{mq}$$

Key Simulation Parameters

- The key parameters that occur in most models can then be defined the following transient values

$$X'_d \triangleq X_{\ell s} + \frac{1}{\dfrac{1}{X_{md}} + \dfrac{1}{X_{\ell fd}}} = X_d - \frac{X_{md}^2}{X_{fd}}$$

$$X'_q \triangleq X_{\ell s} + \frac{1}{\dfrac{1}{X_{mq}} + \dfrac{1}{X_{\ell 1q}}} = X_q - \frac{X_{mq}^2}{X_{1q}}$$

$$T'_{do} \triangleq \frac{Xfd}{\omega_s R_{fd}}, \quad T'_{qo} \triangleq \frac{X_{1q}}{\omega_s R_{1q}}$$

These values will be used in all the synchronous machine models

In a salient rotor machine X_{mq} is small so $X_q = X'_{q;}$ also X_{1q} is small so T'_{q0} is small

Key Simulation Parameters

- And the subtransient parameters

$$X''_d \triangleq X_{\ell s} + \frac{1}{\dfrac{1}{X_{md}} + \dfrac{1}{X_{\ell fd}} + \dfrac{1}{X_{\ell 1d}}}$$

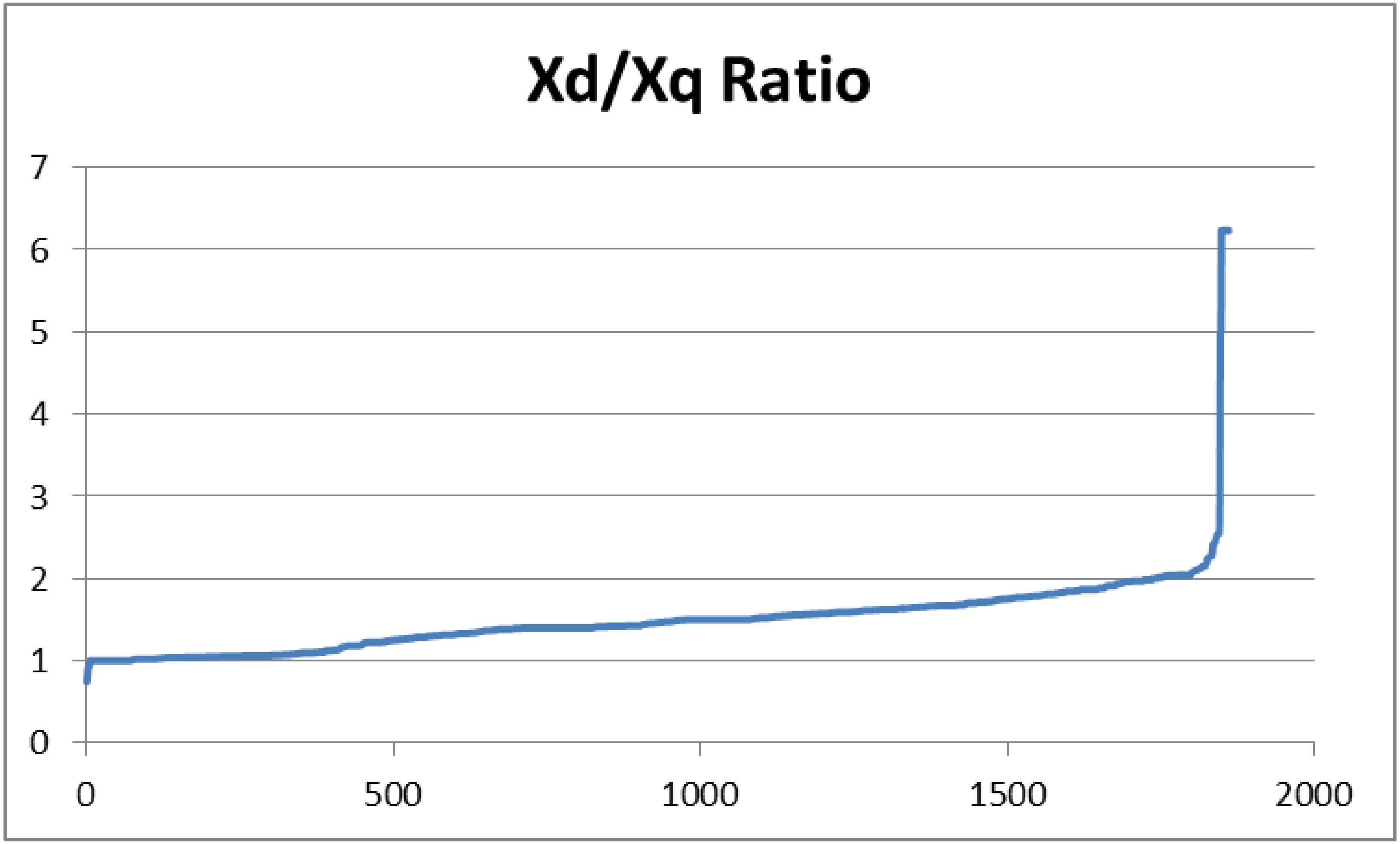
$$X''_q \triangleq X_{\ell s} + \frac{1}{\dfrac{1}{X_{mq}} + \dfrac{1}{X_{\ell 1q}} + \dfrac{1}{X_{\ell 2q}}}$$

$$T''_{do} = \frac{1}{\omega_s R_{1d}} \left(X \right.$$ [redacted]

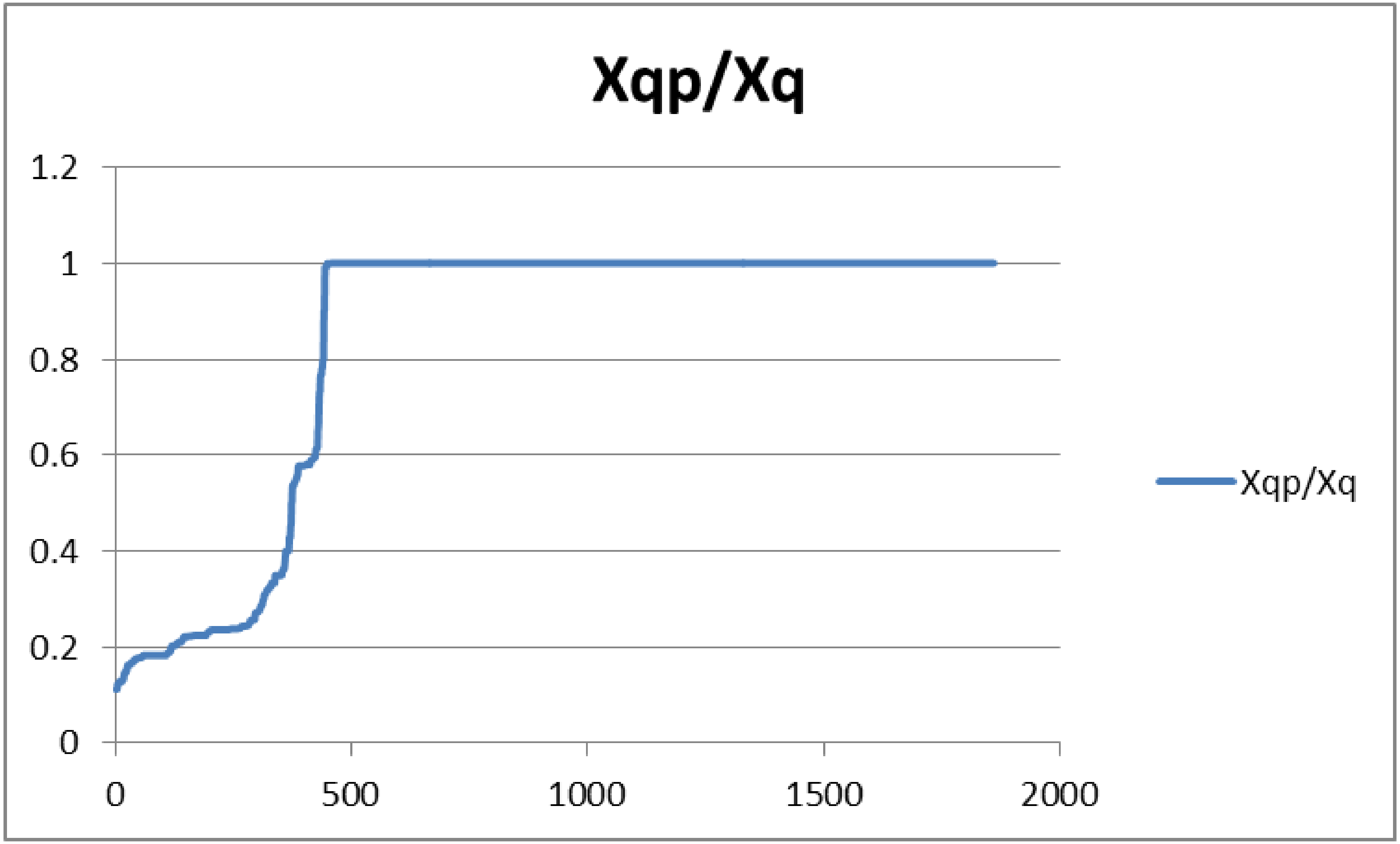
$$\frac{\quad}{X_{md}} + \frac{\quad}{X}$$ [redacted]

These values will be used in the subtransient machine models. It is common to assume $X''_d = X''_q$

Example Xd/Xq Ratios for a WECC Case



Example X'q/Xq Ratios for a WECC Case



About 75% are Clearly Salient Pole Machines!

Internal Variables

- Define the following variables, which are quite important in subsequent models

$$E'_q \triangleq \frac{X_{md}}{X_{fd}} \psi_{fd}$$

$$E'_d \triangleq \frac{X_{mq}}{X_{1q}} \psi_{1q}$$

$$E_{fd} \triangleq \frac{X_{md}}{R_{fd}} V_{fd}$$

Hence E'_q and E'_d are scaled flux linkages and E_{fd} is the scaled field voltage

Dynamic Model Development

- In developing the dynamic model not all of the currents and fluxes are independent
  - In the book formulation only seven out of fourteen are independent
- Approach is to eliminate the rotor currents, retaining the terminal currents (I_d, I_q, I_0) for matching the network boundary conditions

Rotor Currents

- Use new variables to solve for the rotor currents

$$\psi_d = -X_d'' I_d + \frac{\left(X_d'' - X_{\ell s}\right)}{\left(X_d' - X_{\ell s}\right)} E_q' + \frac{\left(X_d' - X_d''\right)}{\left(X_d' - X_{\ell s}\right)} \psi_{1d}$$

$$I_{fd} = \frac{1}{X_{md}} \left[E_q' + \left(X_d - X_d'\right)\left(I_d - I_{1d}\right) \right]$$

$$I_{1d} = \frac{X_d' - X_d''}{\left(X_d' - X_{\ell s}\right)^2} \left[\psi_{1d} + \left(X_d' - X_{\ell s}\right) I_d - E_q' \right]$$

Rotor Currents

$$\psi_q = -X_q'' I_q - \frac{\left(X_q'' - X_{\ell s}\right)}{\left(X_q' - X_{\ell s}\right)} E_d' + \frac{\left(X_q' - X_q''\right)}{\left(X_q' - X_{\ell s}\right)} \psi_{2q}$$

$$I_{1q} = \frac{1}{X_{mq}} \left[-E_d' + \left(X_q - X_q'\right)\left(I_q - I_{2q}\right) \right]$$

$$I_{2q} = \frac{X_q' - X_q''}{\left(X_q' - X_{\ell s}\right)^2} \left[\psi_{2q} + \left(X_q' - X_{\ell s}\right) I_q + E_d' \right]$$

$$\psi_o = X_{\ell s}\left(-I_o\right)$$

Final Complete Model

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + Vo$$

These first three equations define what are known as the stator transients; we will shortly approximate them as algebraic constraints

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)\left[I_d - \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left(\psi_{1d} + \left(X'_d - X_{\ell s}\right)I_d - E'_q\right)\right] + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\left[I_q - \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

Final Complete Model

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right)I_d$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right)I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

T_{FW} is the friction and windage component that we'll consider later

Final Complete Model

$$\psi_d = -X''_d I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$\psi_q = -X''_q I_q - \frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$\psi_o = -X_{\ell s} I_o$$

Single-Machine Steady-State

$$0 = R_s I_d + \psi_q + V_d \qquad (\omega = \omega_s)$$

$$0 = R_s I_q - \psi_d + V_q$$

$$0 = R_s I_o + V_o$$

$$0 = -E'_q - \left(X_d - X'_d\right) I_d + E_{fd}$$

$$0 = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right) I_d$$

$$0 = -E'_d + \left(X_q - X'_q\right) I_q$$

$$0 = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right) I_q$$

$$0 = \omega - \omega_s$$

$$0 = T_m - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

$$\psi_d = E'_q - X''_d I_d$$

$$\psi_q = -X''_q I_q - E'_d$$

$$\psi_o = -X_{\ell s} I_o$$

The key variable we need to determine the initial conditions is actually δ, which doesn't appear explicitly in these equations!

Field Current

- The field current, I_{fd}, is defined in steady-state as

$$I_{fd} = E_{fd} / X_{md}$$

- However, what is usually used in transient stability simulations for the field current is

$$I_{fd} X_{md}$$

- So the value of X_{md} is not needed

Single-Machine Steady-State

- Previous derivation was done assuming a linear magnetic circuit
- We'll consider the nonlinear magnetic circuit (section 3.5) but will first do the steady-state condition (3.6)
- In steady-state the speed is constant (equal to ω_s), δ is constant, and all the derivatives are zero
- Initial values are determined from the terminal conditions: voltage magnitude, voltage angle, real and reactive power injection

Single-Machine Steady-State

$$0 = R_s I_d + \psi_q + V_d \qquad (\omega = \omega_s)$$

$$0 = R_s I_q - \psi_d + V_q$$

$$0 = R_s I_o + V_o$$

$$0 = -E'_q - (X_d - X'_d) I_d + E_{fd}$$

$$0 = -\psi_{1d} + E'_q - (X'_d - X_{\ell s}) I_d$$

$$0 = -E'_d + (X_q - X'_q) I_q$$

$$0 = -\psi_{2q} - E'_d - (X'_q - X_{\ell s}) I_q$$

$$0 = \omega - \omega_s$$

$$0 = T_m - (\psi_d I_q - \psi_q I_d) - T_{FW}$$

$$\psi_d = E'_q - X''_d I_d$$

$$\psi_q = -X''_q I_q - E'_d$$

$$\psi_o = -X_{\ell s} I_o$$

The key variable we need to determine the initial conditions is actually δ, which doesn't appear explicitly in these equations!

Determining δ without Saturation

- Recalling the relation between δ and the stator values

$$\left(V_d + jV_q\right)e^{j(\delta-\pi/2)} = V_s e^{j\theta s}$$

$$\left(I_d + jI_q\right)e^{j(\delta-\pi/2)} = I_s e^{j\phi s}$$

- We then combine the equations for V_d and V_q and get

$$\left(V_d + jV_q\right)e^{j(\delta-\pi/2)} = -\left(R_s + jX_q\right)\left(I_d + jI_q\right)e^{j(\delta-\pi/2)} + \tilde{E}$$

where

$$\tilde{E} = j\left[\left(X_q - X'_d\right)I_d + E'_q\right]e^{j(\delta-\pi/2)}$$

Determining δ without Saturation

- In order to get the initial values for the variables we need to determine δ
- We'll eventually consider two approaches: the simple one when there is no saturation, and then later a general approach for models with saturation
- To derive the simple approach we have

$$V_d = R_s I_d + E'_d + X'_q I_q$$

$$V_q = -R_s I_q + E'_q - X'_d I_d$$

Determining δ without Saturation

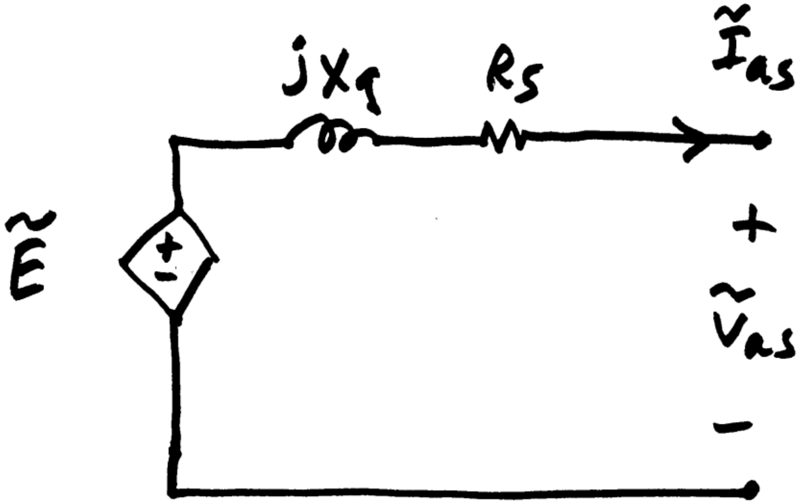
$$\text{Since } j = e^{j(\pi/2)}$$

$$\tilde{E} = \left[\left(X_q - X_d'\right)I_d + E_q'\right]e^{j\delta}$$

- Then in terms of the terminal values

$$\tilde{E} = \tilde{V}_{as} + (R_s + jX_q)\tilde{I}_{as}$$

$$\textit{The angle on } \tilde{E} = \delta$$

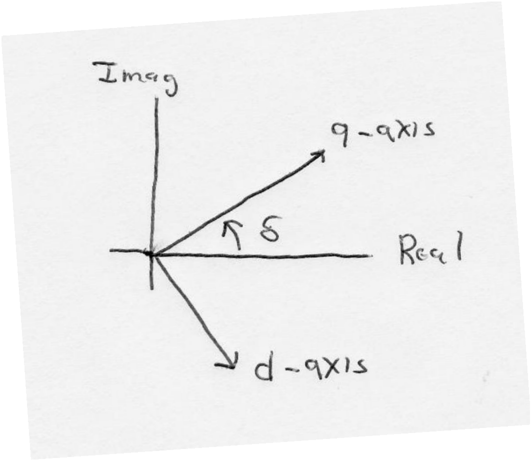


D-q Reference Frame

- Machine voltage and current are "transformed" into the d-q reference frame using the rotor angle, δ
  - Terminal voltage in network (power flow) reference frame are $V_S = V_t = V_r + jV_i$

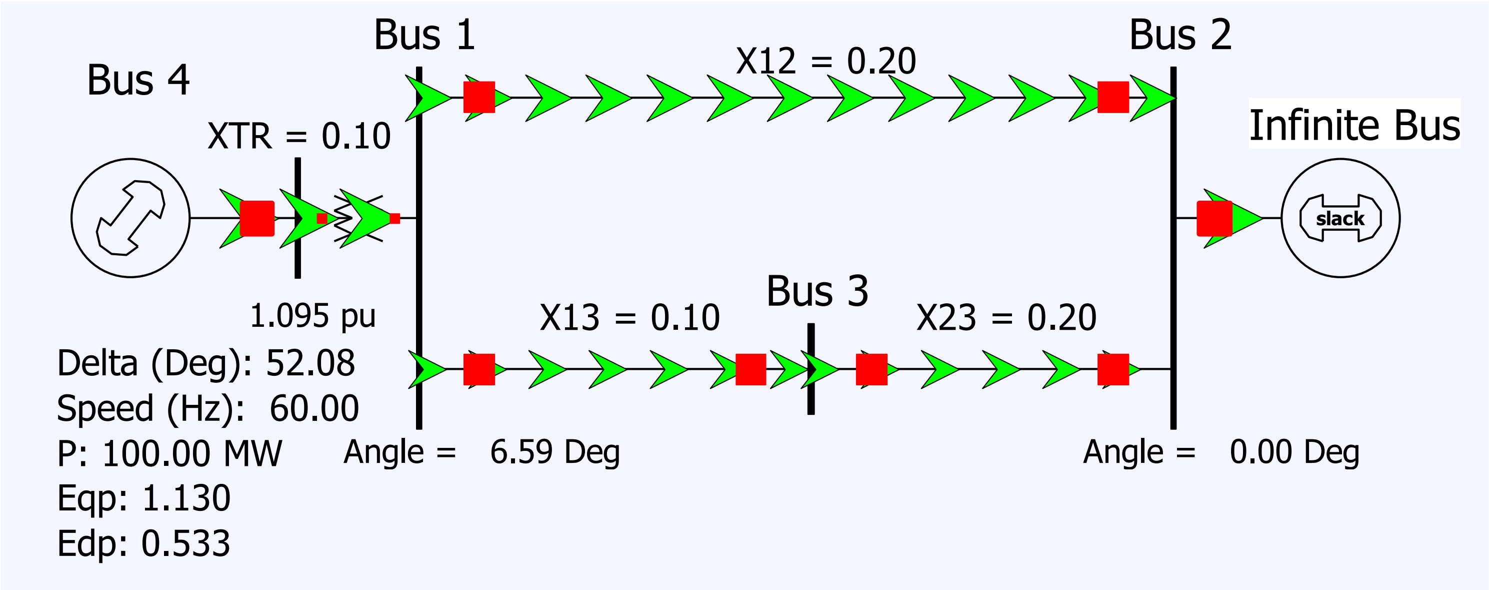
$$\begin{bmatrix} V_r \\ V_i \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix}$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$



A Steady-State Example (11.10)

- Assume a generator is supplying 1.0 pu real power at 0.95 pf lagging into an infinite bus at 1.0 pu voltage through the below network. Generator pu values are R_s=0, X_d=2.1, X_q=2.0, X'_d=0.3, X'_q=0.5



A Steady-State Example, cont.

- First determine the current out of the generator from the initial conditions, then the terminal voltage

$$\tilde{I} = 1.0526\angle -18.20^\circ = 1 - j0.3288$$

$$\tilde{V}_s = 1.0\angle 0^\circ + \left(j0.22 \right)\left(1.0526\angle -18.20^\circ\right)$$
$$= 1.0946\angle 11.59^\circ = 1.0723 + j0.220$$

A Steady-State Example, cont.

- We can then get the initial angle and initial dq values

$$\tilde{E} = 1.0946\angle 11.59^\circ + \left(j2.0\right)\left(1.052\angle -18.2^\circ\right) = 2.814\angle 52.1^\circ$$

$$\rightarrow \delta = 52.1^\circ$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix}\begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix}\begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

- Or $V_d + jV_q = V_s e^{j\theta} e^{j(\pi/2-\delta)} = 1.0945\angle(11.6 + 90 - 52.1)$

$$= 1.0945\angle 49.5^\circ = 0.710 + j0.832$$

A Steady-State Example, cont.

- The initial state variable are determined by solving with the differential equations equal to zero.

$$E_q^{'} = V_q + R_s I_q + X_d' I_d = 0.8326 + \left(0.3\right)\left(0.9909\right) = 1.1299$$

$$E_d^{'} = V_d - R_s I_d - X_q' I_q = 0.7107 - (0.5)(0.3553) = 0.5330$$

$$E_{fd} = E_q^{'} + (X_d - X_d')I_d = 1.1299 + (2.1 - 0.3)(0.9909) = 2.9135$$