Name:	Answers
-------	---------

ECEN 460

Exam #2

Tuesday, November 14, 2017 75 Minutes

Closed book, closed notes
Two 8.5 by 11 inch note sheets allowed (front and back for both)
Calculators allowed

1	25
---	----

Total _____/ 100

1. (25 points total)

A generator bus with a 1.0 per unit voltage supplies a constant power load through a lossless transmission line with per unit (100 MVA base) impedance of j0.05 and no line charging. Assume the per unit constant power load is $P_L = 2.0$ and $Q_L = 1.0$.

- (18 pts) a) Starting with an initial voltage guess of 1.0∠0°, determine the first iteration value of the load bus voltage (magnitude and angle) using the Newton-Raphson power flow method.
- (7 pts) b) Now assume that the generator's per unit voltage is increased to 1.05 per unit and assume that the constant power load is replaced by a voltage dependent load of

$$P_{L} = 2.0 + 1.0 | V_{L} |$$

$$Q_{L} = 1.0 + 1.0 | V_{L} |$$

where $|V_L|$ is the load bus voltage magnitude. Repeat part a) using this voltage dependent load with an initial voltage guess of 1.05 \angle 0°.

Answer

The power balance equations at the load bus (assumed to be bus 2) are

$$P_2(\mathbf{x}) = |V_1||V_2|(20\sin\theta_2) + 2 = 0$$

$$Q_2(\mathbf{x}) = |V_1||V_2|(-20\cos\theta_2) + |V_2|^2(20) + 1 = 0$$

where
$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix}$$

Then
$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} |V_1| |V_2| (20\cos\theta_2) & |V_1| (20\sin\theta_2) \\ |V_1| |V_2| (20\sin\theta_2) & |V_1| (-20\cos\theta_2) + 2|V_2| (20) \end{bmatrix}$$

$$\rightarrow \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1.0 \end{bmatrix} \rightarrow \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1.0 \end{bmatrix} - \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.1 \\ 0.95 \end{bmatrix}$$

For part b) add the voltage dependence

$$P_2(\mathbf{x}) = |V_1||V_2|(20\sin\theta_2) + (2+|V_2|) = 0$$

$$Q_2(\mathbf{x}) = |V_1||V_2|(-20\cos\theta_2) + |V_2|^2(20) + (1+|V_2|) = 0$$

Then
$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} |V_1| |V_2| (20\cos\theta_2) & |V_1| (20\sin\theta_2) + 1 \\ |V_1| |V_2| (20\sin\theta_2) & |V_1| (-20\cos\theta_2) + 2|V_2| (20) + 1 \end{bmatrix}$$

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1.05 \end{bmatrix} \rightarrow \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1.05 \end{bmatrix} - \begin{bmatrix} 22.05 & 1 \\ 0 & 22 \end{bmatrix}^{-1} \begin{bmatrix} 3.05 \\ 2.05 \end{bmatrix} = \begin{bmatrix} -0.134 \\ 0.957 \end{bmatrix}$$

2. (25 points total)

The fuel-cost curves for a three generator system with a load of 1000 MW are

$$C_1(P_{G1}) = 600 + 25 * P_{G1} + 0.01 * (P_{G1})^2$$

$$C_2(P_{G2}) = 500 + 30 * P_{G2} + 0.025 * (P_{G2})^2$$

$$C_3(P_{G3}) = 1500 + 40 * P_{G3} + 0.01 * (P_{G3})^2$$

Generator limits are: $0 \le P_{G1} \le 200$ $200 \le P_{G2} \le 800$ $200 \le P_{G3} \le 600$

For this problem you can either 1) directly solve it to get the economic dispatch, giving the values of all the variables, or 2) use the lambda iteration method to determine the values of λ^{M} , $P_{G1}(\lambda^{\text{M}})$, $P_{G2}(\lambda^{\text{M}})$, and $P_{G3}(\lambda^{\text{M}})$, after two iterations. If you decide to use the lambda iteration method show the values of all variables at each iteration and use starting values of $\lambda^{\text{L}} = 40$ and $\lambda^{\text{H}} = 60$. Regardless of your method be sure to consider the generator limits; you may ignore any penalty factors.

Answer: The incremental costs are $IC_1=25+0.02P_{G1}$, $IC_2=30+0.05P_{G2}$, $IC_3=40+0.02P_{G3}$.

For solution option 1 the problem can be solved quickly if one recognizes that P_{G1} will be at its max limit (since at 200 MW its incremental cost is only 29). Then with the constraint that $P_{G3}=800$ - P_{G2} , solving for equal incremental costs gives $30+0.05P_{G2}=40+0.02*(800-P_{G2})$, which gives $P_{G2}=26/0.07=371.4$ MW, $P_{G3}=428.6$ MW, and $\lambda=\$48.6$ /MWh.

For the second solution option, writing the power in terms of λ gives

$$P(\lambda) = \frac{\lambda - 25}{0.02} + \frac{\lambda - 30}{0.05} + \frac{\lambda - 40}{0.02} - 1000$$

For the given λ range of 40 to 60, P_{G1} will be at its max limit so this becomes

$$P(\lambda) = \frac{\lambda - 30}{0.05} + \frac{\lambda - 40}{0.02} - 800$$

Then with
$$\lambda_0^L = 40, \lambda_0^H = 60, \lambda_0^M = 50$$

First iteration,
$$P(\lambda_0^M) = 100 \rightarrow \lambda_1^H = 50 \rightarrow \lambda_1^M = 45$$

Second iteration,
$$P(\lambda_1^M) = -250 \rightarrow \lambda_2^L = 45 \rightarrow \lambda_2^M = 47.5$$

3. (Short Answer: 20 points total – five points each)

A. Name at least two of the approximations that are used to develop the dc power flow?

No line resistance so G terms are zero, small angle differences so $cos(\theta) = 1$, $sin(\theta) = \theta$, unity voltge magnitude, reactive power ignored

B. In two or three sentences explain how a three-phase synchronous motor works.

AC currents in stator windings setup a rotating magnetic field. On rotor magnetic field is either created by the dc field current or a permanent magnetic. Rotor operates at synchronous speed, with the magnetic poles on the rotor following the opposite magnetic poles due to the stator rotating magnetic field.

C. Assume the per unit losses on a small three generator system (with G_1 , G_2 and G_3) can be approximated as $0.03(P_{G1})^2 + 0.08(P_{G2})^2 - 0.01$ ($P_{G1} \times P_{G2}$). If the per unit generator outputs are $P_{G1} = 2$, $P_{G2} = 1$, $P_{G3} = 2.5$, what are the penalty factor for each generator?

$$L_{i} = \frac{1}{1 - \frac{\partial P_{Losses}}{\partial P_{Gi}}} \rightarrow L_{1} = \frac{1}{1 - (0.06P_{G1} - 0.01P_{G2})} = 1.124$$

$$L_2 = \frac{1}{1 - (0.16P_{G2} - 0.01P_{G2})} = 1.163, \quad L_3 = \frac{1}{1 - 0} = 1.0$$

D. Explain how you could use power flow analysis to approximate the penalty factor for a generator

Solve the power flow to determine the initial losses, change the real power output of generator i by a small about, ΔP_{Gi} , to get the new losses, and set ΔP_{Losses} as the difference. Then

$$L_{i} \approx \frac{1}{1 - \frac{\Delta P_{Losses}}{\Delta P_{Gi}}}$$

4. (30 points total)

True/False – Two points each. Circle T if statement is true, F if statement is False.

- **T** F 1. Nonlinear equations (e.g., power flow) can have multiple solutions.
- T $\underline{\mathbf{F}}$ 2. Because the transmission line reactance is almost always positive, the power flow Jacobian matrix is guaranteed to never be singular.
- **T** F 3. In the power flow the slack bus voltage angle does not change.
- T $\underline{\mathbf{F}}$ 4. The dc power flow solves for the per unit voltage magnitudes, assuming that the voltage angles are constant at zero degrees.
- T **F** 5. While perhaps interesting from a theoretical perspective, machine magnetic saturation is seldom encountered in practice.
- **T** F 6. In the ERCOT LMP markets generators can be paid more than they offer into the market.
- T $\underline{\mathbf{F}}$ 7. As discussed in class, the high electricity prices around the country are due to the historically high prices of natural gas.
- T $\underline{\mathbf{F}}$ 8. In economic dispatch the slack bus penalty factor is always zero.
- T **F** 9. While ERCOT provides electricity to the vast majority of Texas, its maximum electric load has never exceeded 5 GW.
- T $\underline{\mathbf{F}}$ 10. As proven in class, any system with less than four players is guaranteed to have a single Nash equilibrium.
- **T** F 11. The LMPs in the ERCOT real-time market can exceed \$500/MWh.
- T **E** 12. Assuming it is coded correctly, the final operating cost from a security constrained optimal power flow (SCOPF) solution will always be less than the final operating cost from an Optimal Power Flow (OPF) for the same system.
- **T** F 13. In the power flow voltage dependent loads can modify the power flow Jacobian.
- **T** F 14. Texas has more installed wind power than any other state.
- T $\underline{\mathbf{F}}$ 15. The ballpark figure from class is one pound of coal can provide about one MWh of electricity.