Equivalent Line Limit Calculation Using Available Thermal Transfer Capability

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***Abstract*—Power system equivalents have been used for faster simulations, but not much interest was on calculating equivalent line limits and this limits the use of equivalents in simulations. Authors used total transfer capability in assigning equivalent line limits in previous papers and this paper utilizes available transfer capability in doing so. A detailed procedure of the proposed method is described with a small 4-bus case. An exact solution case and a non-exact solution case examples are provided. For non-exact solution cases, solutions are bounded with upper and lower estimates. Also, the proposed method applied to the IEEE 118-bus case and results are provided and compared with those with total transfer capability.**

Keywords—Equivalent line limits; available transfer capability; power system equivalent, power transfer distribution factors.

I. INTRODUCTION

Power system equivalents (PSEs) have been used for decades since the first equivalencing method named after the developer Ward was introduced in late 1940s [1]-[4]. This traditional Ward method and its variants are based on the Kron reduction which performs Gaussian elimination on a part of network (called external area) to equivalent the components in that area and retains the area of interest (called internal or study area) for study [5]. Boundary buses between the two areas are attached with newly created generators and loads to keep the line flow from the external area to the internal area. As the external buses are equivalenced with the Kron reduction, fictitious equivalent lines are created between their first neighbor buses and they do not have associated thermal limits. It is easy to make a small equivalent case where it is not possible to assign limits on equivalent lines that result in all the operating points same as those in the full system. In general, commercial tools assign zero to equivalent line limits which indicates any amount of power can flow on those lines. This aspect restricts the use of PSEs as line limits play a key role in many power system analyses. Other recent methods have been developed to create PSEs that capture different attributes of the full system such as power transfer distribution factors (PTDFs), locational marginal prices, and optimal power flows [6]-[8]. Even though other equivalencing methods have been developed for various applications not much of interest have been focused on limits of equivalent lines.

Recently, authors have produced a couple of methods that calculate equivalent line limits using total transfer capability [9]-[11]. Those methods use PTDFs to keep the total transfer capability (TTC) between the boundary buses. Those methods only use the network itself dropping all the bus injections hence are operating point independent. When there is no exact solution, they offer upper, lower and/or best estimates when equivalent line limits do not have exact solutions. Using unloaded systems is good in one aspect as they only focus on network and therefore may be more flexible with various flow patterns. However, a problem with using TTC for unloaded systems in calculating equivalent line limits occurs when there are movements of bus injections which frequently occur in equivalencing. After they assign limits on equivalent lines and reload the network and it can be challenging to solve power flow especially when many generators and loads in the external area are moved to boundary buses or into the study area in large cases. Of course, it is more difficult to solve optimal power flow without considering these movement of bus injections.

Therefore, this paper works with loaded systems considering all the bus injections and their movement in calculating equivalent line limits. The proposed method in the paper uses available transfer capability (ATC). According to the definition of ATC in the NERC document, ATC considers existing transmission commitments (ETC), capacity benefit margin (CBM) and transmission reliability margin (TRM) [12].

$$ATC = TTC - ETC - CBM - TRM \quad (1)$$

When calculating TTCs and ATCs, there are a few system constraints typically considered; voltage limits, angular stability limits, thermal limits, and pre- and post-contingency conditions. Since only the thermal limits are considered in this paper, available thermal transfer capability (ATTC) is defined as the ATC with only considering thermal limits. Also, the two margins are ignored for simplicity.

II. PROPOSED METHOD

A. Simulation Setup

The Max/Hungarian method presented in [9] is used in the paper in calculating equivalent line limits with ATTC. However, assumed simulation environment is the same as that in the top-down approach presented in [11] as it is better to be used for large-scale systems in future applications. This means that only the full case and its reduced equivalent case are needed and no

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extra information is required to know in assigning limits. Information such as the order of bus elimination or movement of bus injections from the external area to the study area are not needed in the simulation environment with the top-down approach. The limits of equivalent lines in the reduced case is the objective to calculate. All the data needed in assigning equivalent line limits are in the given two cases. This assumption will be beneficial for anybody who wants to use this proposed method as the only input is the full case and the corresponding equivalent case. Of course, the power flow of the full case needs to be solved without an error because this method uses ATTC which requires the amount of line flows.

B. Criterion of Equivalent Line Limit Calculation

The criterion for the successful equivalent line limit calculation is bus-to-bus ATTC matching between the full and the equivalent cases. More specifically, the method tries to match the ATTCs between pairs of buses for equivalent lines in the equivalent case with those for the same bus pairs in the full case. Here, for equivalent line limit calculation, only the lines that are eliminated along with the external buses denoted by $\boldsymbol{\mathcal{L}} = \{l_i | i \in [1, L]\}$ are considered in calculating ATTCs in the full case. The retained lines do not need to participate in the process of equivalencing and calculating equivalent line limits as they are unchanged with their limit in the reduced case. Therefore, the ATTCs of only eliminated lines are used in calculating equivalent line limits. The ATTC for eliminated line l_i for transaction w, denoted as $T_{l_i}^w$ can be calculated with the limit of line l_i, denoted as F_{l_i}, PTDF of line l_i for the same transaction w, denoted as $\phi_{l_i}^w$, and the existing commitment on line l_i, denoted as M_{l_i}.

$$T_{l_i}^w = \begin{cases} \dfrac{F_{l_i} - M_{l_i}}{\phi_{l_i}^w}, & \phi_{l_i}^w > 0 \\ \infty, & \phi_{l_i}^w = 0 \\ \dfrac{-F_{l_i} - M_{l_i}}{\phi_{l_i}^w}, & \phi_{l_i}^w > 0 \end{cases} \tag{2}$$

When a PTDF value is close to zero, the transaction has little impact on the limiting line. It should be noted that since ATTC is dependent of the direction of transactions, the ATTC values of a transaction from bus *x* to bus *y* is different from those of a transaction from bus *y* to bus *x*.

The ATTC for transaction w is the minimum of all the ATTCs of the eliminated lines l_i.

$$T^w = \min_{l_i \in \boldsymbol{\mathcal{L}}} \left\{ T_{l_i}^w \right\} \tag{3}$$

This indicates how much more power can be transferred for transaction w through the network loaded with the already existing commitment before there is any violation on line limits in the system. PTDF values used in calculating ATTC are obtained with the lossless dc approximation which only considers real power [13].

C. Problem Formulation

When a bus or a group of adjacent buses, k, are eliminated in the full case and there are equivalent lines in the reduced case created between the first neighbor buses of k, as denoted by $\boldsymbol{S} = \{s_i | i \in [1, S]\}$. The set of equivalent lines are denoted by $\widetilde{\boldsymbol{\mathcal{L}}} = \{l_\tau | \tau \in [1, \tilde{L}]\}$ and there are $\tilde{L} = \binom{S}{2}$ number of limits to be determined. In the reduced case, the ATTC values of transactions w of bus pairs for equivalent lines only considering power flowing on equivalent lines l_τ can also be calculated as

$$\widetilde{T}_{l_\tau}^w = \begin{cases} \dfrac{\widetilde{F}_{l_\tau} - \widetilde{M}_{l_\tau}}{\widetilde{\phi}_{l_i}^w}, & \widetilde{\phi}_{l_\tau}^w > 0 \\ \infty, & \widetilde{\phi}_{l_\tau}^w = 0 \\ \dfrac{-\widetilde{F}_{l_\tau} - \widetilde{M}_{l_\tau}}{\widetilde{\phi}_{l_\tau}^w}, & \widetilde{\phi}_{l_\tau}^w > 0 \end{cases} \tag{4}$$

where tilde indicates values are from the reduced case and $\tilde{F}_{l_\tau}$ are the unknown equivalent line limits to be calculated.

The ATTC for transaction w in the reduced case is the minimum of all the ATTCs of the equivalent lines l_τ.

$$\widetilde{T}^w = \min_{l_\tau \in \widetilde{\boldsymbol{\mathcal{L}}}} \left\{ \widetilde{T}_{l_\tau}^w \right\} \tag{5}$$

Since the criterion of assigning equivalent line limits is to match ATTCs between bus pairs for equivalent lines with those for the same bus pairs in the full case, exact solution cases occur when

$$T^w = \widetilde{T}^w \tag{6}$$

As ATTC values are direction dependent, there should be a reference direction for transactions so that the calculation of PTDF values and ATTC values have a consistent sign convention. This can be done by solving the reduced case and set the direction of equivalent line flows as the reference for the transactions of interest. One may argue that the other direction also be considered. Combination of both directions of flow in each transaction can be computationally very heavy in large scale cases hence this paper focuses on a single direction based on the solved power flow.

(6) can be solved for the only unknown, equivalent line limit depending on the sign of PTDF value on the corresponding equivalent line in the reduced case as

$$\widetilde{F}_{l_\tau} = \begin{cases} \widetilde{T}_{l_\tau}^w \cdot \widetilde{\phi}_{l_i}^w + \widetilde{M}_{l_\tau}, & \widetilde{\phi}_{l_i}^w > 0 \\ \infty, & \widetilde{\phi}_{l_i}^w = 0 \\ -\widetilde{T}_{l_\tau}^w \cdot \widetilde{\phi}_{l_i}^w - \widetilde{M}_{l_\tau}, & \widetilde{\phi}_{l_i}^w < 0 \end{cases} \tag{7}$$

When the PTDF value of an equivalent line for a transaction is zero, which means there is on impact of the transaction on the line, the ATTC value for the transaction on the line becomes infinity as shown in (4). Therefore, the limit on the equivalent line should be infinity to accommodate the ATTC value.

III. EXAMPLE CASES

A 4-bus test case is used to illustrate the step-by-step procedure of the proposed method. Both an exact solution case and a non-exact solution case are demonstrated with the test case. When there is no exact solution, upper and lower estimates of equivalent line limits are calculated using the Max/Hungarian method presented in [9]. Also, the results of its application to the IEEE 118-bus case is provided.

A. 4-bus Test Case with Exact Solution

Fig. 1 depicts the full 4-bus case with line loadings on the left and PTDF values for a transaction from bus 2 to bus 3 on the right. Lines have a limit in MVA and a reactance in p.u. Bus 1 is the external area to be equivalenced making a reduced 3-bus case in Fig. 2.

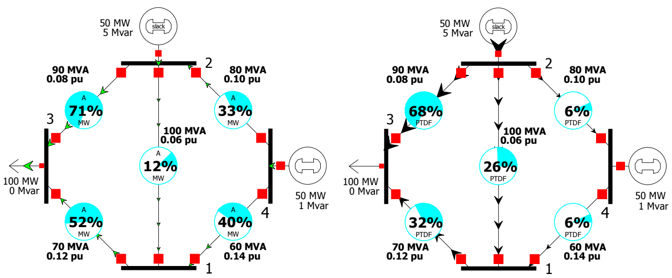

Fig. 1. Full 4-bus case with line loading (left) and PTDFs from bus 2 to bus 3 (right)

Fig. 2 also shows line loadings on the left and PTDF values for a transaction from bus 2 to bus 3 on the right.

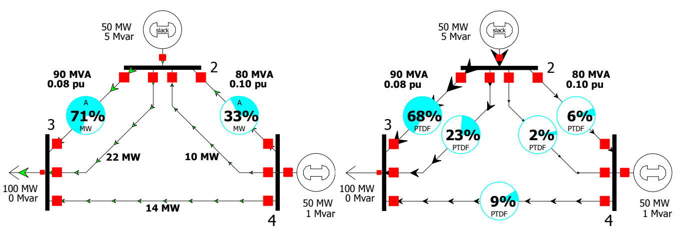

Fig. 2. Reduced 3-bus case with line loading (left) and PTDFs from bus 2 to bus 3 (right)

The ATTC value for a transaction from bus 2 to bus 3 considering only eliminated lines can be calculated using (2) and (3) as

$$
\begin{aligned}
T^{(2,3)} &= \min_{l_i \in \mathcal{L}} \left\{ T^{(2,3)}_{(1,2)},\ T^{(2,3)}_{(1,3)},\ T^{(2,3)}_{(1,4)} \right\} \\
&= \min_{l_i \in \mathcal{L}} \left\{ \frac{100-12.4}{0.258}, \frac{-70+36.3}{-0.323}, \frac{-60+23.9}{-0.065} \right\} \\
&= \min_{l_i \in \mathcal{L}} \left\{ 339.6,\ 104.5,\ 559.3 \right\} \\
&= 104.5
\end{aligned}
\tag{8}
$$

Likewise, the other two ATTC values can be calculated and all the ATTC values between the boundary buses are shown Table 1.

These ATTC values need to be matched with those in the reduced case for the same pair of transactions as follows.

$$
\min_{l_\tau \in \widetilde{\mathcal{L}}} \left\{ \widetilde{T}^{\,w}_{l_\tau} \right\} = T^{w} \tag{9}
$$

TABLE 1
ATTCS BETWEEN BOUNDARY BUSES IN 4-BUS CASE FOR EXACT SOLUTION

w	Binding line	T^{w} (MW)
(2, 3)	(1, 3)	104.5
(4, 2)	(1, 4)	103.3
(4, 3)	(1, 3)	83.6

The direction of transactions are determined by the direction of flows on the equivalent lines in the reduced case. (9) can be expanded for the 4-bus test case as

$$
\begin{aligned}
\min\left\{ \widetilde{T}^{(2,3)}_{(2,3)}, \widetilde{T}^{(2,3)}_{(2,4)}, \widetilde{T}^{(2,3)}_{(3,4)} \right\} &= T^{(2,3)} \\
\min\left\{ \widetilde{T}^{(4,2)}_{(2,3)}, \widetilde{T}^{(4,2)}_{(2,4)}, \widetilde{T}^{(4,2)}_{(3,4)} \right\} &= T^{(4,2)} \\
\min\left\{ \widetilde{T}^{(4,3)}_{(2,3)}, \widetilde{T}^{(4,3)}_{(2,4)}, \widetilde{T}^{(4,3)}_{(3,4)} \right\} &= T^{(4,3)}
\end{aligned}
\tag{10}
$$

Depending on the sign of PTDF values on the equivalent lines in the reduced case, one of the three equations in (4) can be used to formulate equality equations in (10). The first equation for transaction (2,3) in (10) can be rewritten using (7) and (8) as

$$
\begin{aligned}
&\min\left\{ \frac{\widetilde{F}_{(2,3)} - \widetilde{M}_{(2,3)}}{\widetilde{\phi}^{(2,3)}_{(2,3)}}, \frac{\widetilde{F}_{(2,4)} - \widetilde{M}_{(2,4)}}{\widetilde{\phi}^{(2,3)}_{(2,4)}}, \frac{-\widetilde{F}_{(3,4)} - \widetilde{M}_{(3,4)}}{\widetilde{\phi}^{(2,3)}_{(3,4)}} \right\} = 104.5 \\
&\min\left\{ \frac{\widetilde{F}_{(2,3)} - 22.0}{0.234}, \frac{\widetilde{F}_{(2,4)} + 9.7}{0.024}, \frac{-\widetilde{F}_{(3,4)} + 14.3}{-0.088} \right\} = 104.5
\end{aligned}
\tag{11}
$$

The denominators on the left hand side can be multiplied for each equivalent line and inequality constraints can be made as follows.

$$
\widetilde{F}_{(2,3)} > 46.50,\quad \widetilde{F}_{(2,4)} > -7.16,\quad \widetilde{F}_{(3,4)} > 23.50 \tag{12}
$$

This can be done for the other two equations and all the inequality constraints can be put in a matrix as

$$
\Psi = \begin{bmatrix} 46.50 & -7.16 & 23.50 \\ -19.15 & 34.54 & 25.46 \\ 39.28 & 27.81 & 30.72 \end{bmatrix} \tag{13}
$$

Each row corresponds to each transactions and each column corresponds to each equivalent line limit. Also, all the entries in each column has to be smaller than or equal to the corresponding equivalent line limit. One entry for each row must have an equality constraint so that the ATTC for the transaction can be determined. Moreover, one entry for each column must have an equality constraint so that the corresponding equivalent line limit can be determined.

Criterion for exact solution: if the maximum entry in each column belongs to a different row, those maximum values are the exact solution for the equivalent line limits. This is because they satisfy all the equality and inequality constraints.

Often, just choosing the maximum value in each column provides an exact solution. Hence, this 4-bus test case has an exact solution as $\tilde{F}_{(2,3)} = 46.50\ MW$, $\tilde{F}_{(2,4)} = 34.54\ MW$, and $\tilde{F}_{(3,4)} = 31.72\ MW$.

From experiment of other cases, entries in the Psi matrix can be a negative value which is not possible when using TTCs for unloaded systems. Negative entries may be converted to zero as a line limit should be a positive value from a practical point of view.

The ATTC values with the exact solution limits can be calculated in the reduced case using (4) and (5) to see if they really yield the exact ATTC values from the full case. The ATTC value for transaction from bus 2 to bus 3 is calculated for validation as follows.

$$\begin{aligned}\tilde{T}^{(2,3)} &= \min\left\{\tilde{T}_{(1,2)}^{(2,3)},\ \tilde{T}_{(1,3)}^{(2,3)},\ \tilde{T}_{(1,4)}^{(2,3)}\right\} \\ &= \min\left\{\frac{46.5-22.0}{0.2234}, \frac{34.5+9.7}{0.024}, \frac{-30.7+14.3}{-0.88}\right\} \quad (14) \\ &= \min\{104.5,\ 1849.4,\ 186.3\} \\ &= 104.5\end{aligned}$$

B. 4-bus Test Case with Non-Exact Solution

Not every elimination of a bus or a group of buses results in an exact solution case. This is because equivalent line limits are not calculated the way equivalent line admittances are calculated by Kron reduction during the equivalencing process. Fig. 3 shows a full 4-bus test case with line loading on the left and PTDF values for a transaction from bus 2 to bus 3 on the right. Bus 1 is the external bus to be eliminated from the full case to create a reduced 3-bus case in Fig. 4.

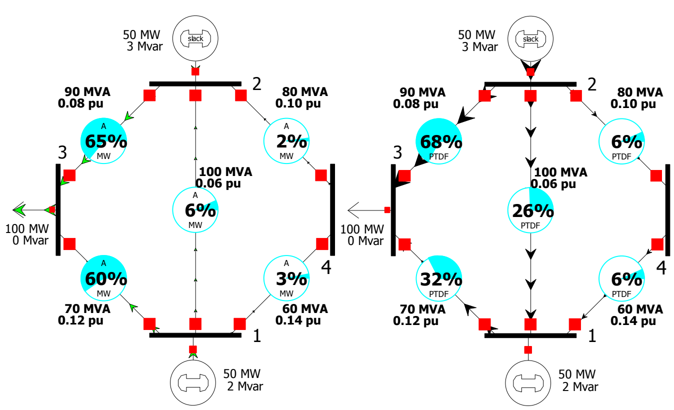

Fig. 3. Full 4-bus case with line loading (left) and PTDFs from bus 2 to bus 3 (right)

Fig. 4 also shows line loading on the left and PTDF values for the same transaction as the full case on the right. The difference from the exact solution case above is that bus 1 now has a generator attached to it injecting power into the system and bus 4 does not have a generator any more.

When a bus with bus injection is being eliminated during an equivalencing process, the Ward equivalent and its variant split the bus injection and attach a portion of it to its first neighbor buses to maintain flow on retained lines unaffected. Another way is to just remove it from the external area when the bus injection is far from the study area so that it does not have much impact on fidelity of simulation results. Also, some application moves the bus injection as a whole to a bus in the study area and attach loads in other buses to keep the same line flows [14]-[15]. No matter how bus injections from the external area are redistributed into the study area during equivalencing, the proposed method does not need to know. It just needs a full case and a reduced case to assign equivalent line limits to keep the ATTC values as close as those in the full case.

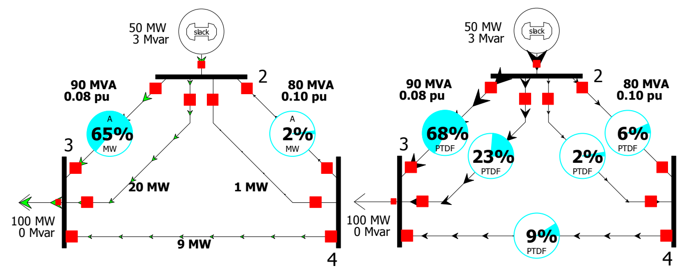

Fig. 4. Reduced 3-bus case with line loading (left) and PTDFs from bus 2 to bus 3 (right)

When a bus with bus injection is being eliminated during an equivalencing process, the Ward equivalent and its variant split the bus injection and attach a portion of it to its first neighbor buses to maintain flow on retained lines unaffected. Another way is to just remove it from the external area when the bus injection is far from the study area so that it does not have much impact on fidelity of simulation results. Also, some application moves the bus injection as a whole to a bus in the study area and attach loads in other buses to keep the same line flows [14]-[15]. No matter how bus injections from the external area are redistributed into the study area during equivalencing, the proposed method does not need to know. It just needs a full case and a reduced case to assign equivalent line limits to keep the ATTC values as close as those in the full case.

The ATTC values between the boundary buses of bus 1 can be calculated using (2) and (2) as shown in Table 2. As this is different line flows from the exact solution case, the ATTC values are also different.

TABLE 2
ATTCS BETWEEN BOUNDARY BUSES IN 4-BUS CASE FOR NON-EXACT SOLUTION

w	Binding line	T^w (MW)
(2, 3)	(1, 3)	87.01
(4, 2)	(1, 4)	176.31
(4, 3)	(1, 3)	69.61

The Psi matrix for inequality constraints can be obtained with (10) as follows.

$$\Psi = \begin{bmatrix} 40.47 & 1.48 & 16.60 \\ -15.78 & 43.09 & 28.04 \\ 34.46 & 15.71 & 22.62 \end{bmatrix} \quad (15)$$

Now, choosing the maximum value in each column does not meet the *Criterion for exact solution* as both the 2nd and 3rd column has their maximum values at the 2nd row. This does not satisfy equality constraints for transaction (3,4), which is the 3rd row. The power flow for the transaction is overestimated as no entry in the row is enforced. Therefore, this set of entries is the upper estimate of equivalent line limits; $\tilde{F}_{(2,3)} = 40.47\ MW$, $\tilde{F}_{(2,4)} = 43.09\ MW$, and $\tilde{F}_{(3,4)} = 28.04\ MW$.

Lower estimates can be obtained by enforcing the equality constraint for all transactions. This causes some of the inequality constraints to be violated as this solution underestimate the power flow in some transactions. Given the Psi matrix for the non-exact solution case, the Hungarian method is utilized for underestimate [16]. The Hungarian method also known as Kuhn-Munkres algorithm is a combinatorial optimization algorithm to solve assignment problems. The first step is to create a matrix with limit violation cost for each entry. The limit violation cost is defined by the sum of violation for all entries in each column. Let the Psi be an m by n matrix and then the procedure to obtain the matrix with limit violation cost is as follows.

a. Create a matrix with the same size as the Psi matrix.

b. With the 1st entry in each column in Psi, compare with all the other entries in its column.

c. Sum the differences between itself and larger entries in its column and put the value in the new matrix.

d. Repeat b and c for all the other entries in each column.

The matrix with limit violation cost for the 4-bus non-exact solution case is obtained with the above procedure as

$$\Lambda = \begin{bmatrix} 0 & 55.83 & 17.46 \\ 105.29 & 0 & 0 \\ 6.02 & 27.38 & 5.43 \end{bmatrix} \tag{16}$$

Now, this becomes a resource assignment problem where one entry in each row and each column has to be chosen to that the sum of all the chosen entries is minimized. Applying the Hungarian method, the selected entries are zero in the 1st and the 2nd column and 5.43 for the 3rd column. This combination minimizes the sum of limit violation cost. Therefore, the lower estimates for equivalent line limits are $\tilde{F}_{(2,3)} = 40.47\ MW$, $\tilde{F}_{(2,4)} = 43.09\ MW$, and $\tilde{F}_{(3,4)} = 22.62\ MW$. The limit for equivalent line (3,4) is the only difference from the overestimates.

The ATTC values with the upper estimates and lower estimates are calculated in the reduced case using (4) and (5) to see how much ATTCs are overestimated and underestimated, respectively, comparing with those in the full case as shown in Table 3.

TABLE 3
COMPARISON OF ATTCS BETWEEN THE FULL AND THE REDUCED CASE FOR NON-EXACT SOLUTION

w	Full 4-bus case	Reduced 3-bus case	
	T^w (MW)	T^w /w overestimate	T^w /w underestimate
(2,3)	87.01	87.02	87.02
(4,2)	176.31	176.31	126.37
(4,3)	69.61	97.15	69.60

When the maximum entry in each column in the Psi matrix is chosen for upper estimates, the ATTC for transaction from bus 4 to bus 3 is overestimated about 39.6% while with the lower estimates determined with the Hungarian method, the ATTC for transaction from bus 4 to bus 2 is underestimated about 28.3%.

C. IEEE 118-bus Case

The proposed method is applied to the IEEE 118-bus case. The buses to be eliminated are selected as the same buses used in [10] to compare the results. While [10] uses the unloaded IEEE 118-bus case only considering the network itself, this paper uses the whole network. The Ward equivalencing is used to make a reduced 62-bus case and bus injections in the external area are redistributed into the boundary buses in the reduced case. Also, the same grouping of maximal adjacent external buses is performed as [10] resulting in 31 mutually independent subgroups of buses to be eliminated. Fig. 5 shows the result of the eliminating 31 subgroups from the IEEE 118-bus case using ATTC. The numbers of x-axis indicate the bus numbers in each subgroup. Number of boundary buses of each group is shown with circles and normalized rms ATC values for each group in the reduced cases are shown with squares.

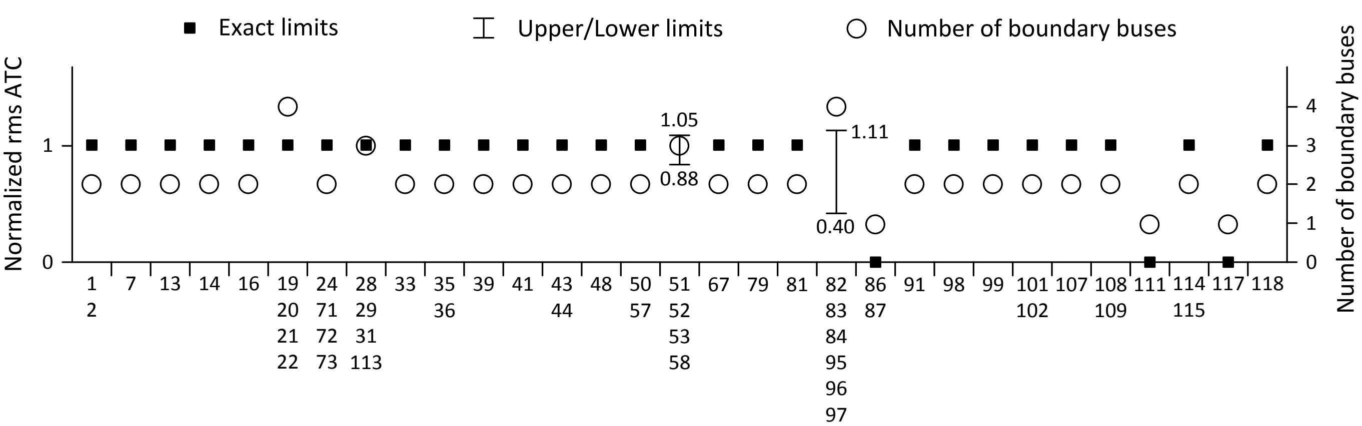

Fig. 5 Result of eliminating 31 subgroups of 56 buses in IEEE 118-bus case using ATTC

An interesting point in this result is that the groups with non-exact solution are different from [10]. When using TTC in [10], a subgroup with buses 19, 20, 21 and 22, and the other with buses 28, 29, 31 and 113 are the non-exact solution cases while using ATTC in this paper, a subgroup with buses 51,52, 53 and 58, and the other with buses 82,83,84,95,96 and 97 result in upper and lower limits. Since subgroup eliminations is used here, the number of fill-ins does not need to be considered when equivalencing as it is in [10].

There are three subgroups with just one boundary bus. These subgroups do not have any associated equivalent lines, thus their normalized rms ATC is zero.

Table 4 compares line loading between the full case and the reduced case when solved for power flow. The reduced case with lower limits have one additional line overloaded compared to the full case. This line is one of the equivalent lines when the subgroup of buses 82, 83, 84, 95, 96 and 97 is eliminated and its boundary buses are 77, 80, 85 and 94. However, this line is not overloaded when the reduced case with assign with upper line limits.

TABLE 4
COMPARISON BETWEEN FULL AND REDUCED CASE FOR LINE LOADING

	From	To	CID	Limit (MVA)	Loading (%)
Full case	8	30	1	100	117.1
Eq. case w/ lower limits	8	30	1	100	122.1
	80	85	99	4.7	166.5
Eq. case w/ upper limits	8	30	1	100	122.1
	80	85	99	7.9	33.1

IV. COMPUTATIONAL ASPECT

Let S be the number of boundary buses of a group of adjacent buses that are being eliminated. There are $\binom{S}{2}$ number of equivalent lines created between the boundary buses. The number of transaction to consider is also the same. The number of PTDF calculation is $\binom{S}{2}\left[\binom{S}{2}+S\right]$ for both the full case and the reduced case. The number of ATTC calculations in the full case is $\binom{S}{2}^2$. For non-exact solution cases, the Hungarian method requires the computation complexity in the order of $\binom{S}{2}^3$ which is not small. For larger cases, this could be a problematic as the number of boundary buses increases. Choosing the group of external buses with fewer number of boundary buses would be wise for faster simulations.

V. CONCLUSIONS

This paper introduced a way to assign limits on equivalent lines by trying to keep available thermal transfer capability in the reduced case as close as possible to those in the full case. Given the full case and the reduced case, the proposed method can calculate exact equivalent line limits if possible. When exact solution does not exist, the method provides upper and lower estimates. Testing with larger cases should be the next step since computation would be very expensive with the current method as the number of boundary buses increase. Also, larger cases are harder to solve power flow as more amount of bus injections are moved to boundary buses.

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