Geomagnetically Induced Current Sensitivity to Multiple Substation Grounding Resistances

Uyen Bui, Thomas J. Overbye, Komal Shetye
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Abstract--**The impact of geomagnetic disturbances (GMDs) on the high voltage electric grid can result in severe damage by producing geomagnetically induced currents (GICs). Protection against GMD therefore necessitates the accurate estimation of the GICs. GIC estimation can be challenging since they depend in part upon the substation grounding resistances, which are not always well known. To address this issue the paper proposes an algorithm to calculate the sensitivity of the GICs at any substation to these resistances. The method is demonstrated on several power grid models.**

I. Introduction

Power systems can suffer heavily from disturbances in the earth's magnetic field. Changes in the magnetic field can induce quasi-dc currents (with frequencies much less than 1 Hz) in high voltage, ac transmission lines [1]. As mentioned in [2], this causes half-cycle saturation in the transformers, causing harmonics and increases in the transformers' reactive power losses. The loss of reactive power support can lead to voltage collapse and/or transformer overheating, potentially resulting in equipment damage.

The effect of such catastrophic disturbances on power systems needs to be understood before preventative measures can be proposed. The electric field induced by geomagnetic disturbances (GMDs) is represented as dc voltage sources in series with the ac transmission lines [3]. The voltage sources are commonly represented as Norton equivalent current injections, denoted by $\boldsymbol{I}$. The geomagnetically induced currents (GICs) in the power system can then be calculated using $\boldsymbol{I}$, and the system's topology and parameters [4, 5, 6, 7]

$$\boldsymbol{V} = \boldsymbol{G}^{-1}\boldsymbol{I} \tag{1}$$

where vector $\boldsymbol{V}$ contains substation neutral voltages and bus voltages. Denote the number of substation neural voltages as s and m as the number of bus voltages. Thus $\boldsymbol{V}$ has size s+m, with the first s elements being the substation neural voltages. Then the GICs flowing into or out of the ground at a substation are calculated by dividing the substation's neutral voltage by its grounding resistance. As an example, the GIC flow into Substation A, which has the grounding resistance R_A, is

$$I_{\text{GIC,A}} = \frac{V_{\text{A}}}{R_{\text{A}}} \tag{2}$$

In Equation (1), matrix $\boldsymbol{G}$, which represents the system, is similar to the power flow bus admittance matrix except 1) it is a real matrix with just conductance values, 2) the conductance values are determined by the parallel combination of the three individual phase resistances, 3) $\boldsymbol{G}$ is augmented to include the substation neutral buses and substation grounding resistance values, 4) transmission lines with series capacitive compensation are omitted since series capacitors block dc flows, and 5) transformers are modeled with their winding resistance to the substation neutral when grounded.

The determination of $\boldsymbol{G}$ depends on several factors, some of which are readily available from standard power flow models or can be easily estimated. These include the network topology, transmission line resistances, transmission line series compensation, and the transformers' resistances and their configuration (wye or delta) [6]. However, the substation grounding resistance [2,8], which is an important factor of $\boldsymbol{G}$, might not be readily available. The grounding resistance can vary by more than an order of magnitude, from 0.05 to 1.5 ohms.

Previous work on this topic is conducted by Pirjola [9, 10] who is among the earliest researchers to investigate the effect of grounding resistance on GIC. Through a series of case-study computations, the author showed that for sufficiently small uncertainties in the grounding resistance, their impacts are insignificant in practical applications (where only the level of GIC, rather than its precise values, is needed). In addition, it was also shown that the interactions between different stations are modest. However, these conclusions are based on macro-adjustments to all grounding resistances simultaneously. In contrast, our research takes a different approach where only one (either local or neighboring) grounding resistance is adjusted at a time. Then the sensitivity of the GIC with respect to a grounding resistance can be explicitly quantified. With this approach, it is shown that the change in a GIC with respect to a grounding resistance might be nontrivial.

This paper is organized as follows. The method to calculate the GIC sensitivity to both local and neighboring grounding resistance is given in Section II. Then the algorithm is demonstrated on a 20-bus system and a 150-bus system, and insights are presented in Section III. Finally, a summary is given in the last section.

II. Motivation and Method

Because the impact of GMDs on power systems is non-uniform due to heterogeneous grounding resistances and the cost of collecting accurate grounding resistance data might be high, identifying which substation are most sensitive, and thus requires more attention, could help make GMD protection more efficient. Our research addresses the above issue by

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finding a method to calculate the sensitivity of GIC to the substation grounding resistances in the system. In our previous paper [11], a simple and straightforward algorithm was developed to obtain the sensitivity of the GICs to the grounding resistance at a single substation. Although the algorithm performed well in the local resistance case, it could not calculate the sensitivity of GICs to neighboring substations' grounding resistances. Hence in this paper, we present a new algorithm utilizing the matrix inversion lemma and sparse vector methods to compute the sensitivity of GIC at any substation in the system to both its local and neighboring grounding resistances. The algorithm has two parts, one to find the sensitivity of GIC to the local resistance and the other for the neighboring resistances.

A. Sensitivity of GICs to local resistance

Consider a general system that has s substations and m buses. The variation of the approximated resistance R_i is interpreted as the variation of the corresponding admittance g_i, notated as $\tilde{g}_i$

$$g'_i = g_i + \tilde{g}_i \tag{3}$$

where g_i is the assumed grounding admittance of substation i and is the inverse of R_i. g'_i is the new g_i, and $\tilde{g}_i$ denotes the change or variation. This change leads to the change in admittance matrix $\boldsymbol{G}$ and is denoted as $\boldsymbol{G}'$.

$$\boldsymbol{G}' = \boldsymbol{G} + \tilde{g}_i \boldsymbol{e}_i \boldsymbol{e}_i^T \tag{4}$$

where $\boldsymbol{e}_i$ is an nx1 vector with element at row i being 1 and 0 otherwise; $\boldsymbol{e}_i^T$ is the transpose of e_i. Using the matrix inversion lemma, the inverse of admittance matrix is given as:

$$\boldsymbol{G}'^{-1} = \boldsymbol{G}^{-1} - \boldsymbol{G}^{-1}\boldsymbol{e}_i(\tilde{g}_i^{-1} + \boldsymbol{e}_i^T\boldsymbol{G}^{-1}\boldsymbol{e}_i)^{-1}\boldsymbol{e}_i^T\boldsymbol{G}^{-1} \tag{5}$$

Then, the GIC at substation i can be written in terms of the conductance g_i, matrix $\boldsymbol{G}$, and the current injection vector $\boldsymbol{I}$.

$$I_{\text{GIC},i} = g_i V_i = g_i e_i^T \boldsymbol{V} = g_i e_i^T \boldsymbol{G}^{-1}\boldsymbol{I} \tag{6}$$

Now, the percentage variation of GIC in terms of the percentage variation of grounding resistance is expressed as follows:

$$\frac{\frac{\Delta I_{\text{GIC},i}}{I_{\text{GIC},i}}}{\frac{\Delta R_i}{R_i}} = \frac{g_i^2\boldsymbol{R}_{ii} - g_i + g_i\tilde{g}_i\boldsymbol{R}_{ii} - \tilde{g}_i}{g_i + g_i\tilde{g}_i\boldsymbol{R}_{ii}} \tag{7}$$

In this equation, the driving point resistance $\boldsymbol{R}_{ii}$ is the element at i-th row, i-th column of resistance matrix $\boldsymbol{R}$, which is the inverse of admittance matrix $\boldsymbol{G}$. $\boldsymbol{G}$ is usually a sparse matrix so $\boldsymbol{R}_{ii}$ can be computed efficiently using the sparse vector method [12]. From (7) the change of the GIC due to the change in its assumed local substation resistance can be calculated. Taking the limit of (7) as the variation goes to 0 gives the desired sensitivity, which is the rate of change of GICs with respect to the variation of the assumed resistances.

$$S_{ii} := \lim_{\tilde{g}_i \to 0} \frac{\frac{\Delta I_{\text{GIC},i}}{I_{\text{GIC},i}}}{\frac{\Delta R_i}{R_i}} = g_i\boldsymbol{R}_{ii} - 1 = \frac{\boldsymbol{R}_{ii}}{R_i} - 1 \tag{8}$$

$\boldsymbol{R}_{ii}$ can be expressed as a function of the grounding resistance R_i and the Thevenin resistance $R_{\text{TH},i}$ looking into the network from substation i as follows:

$$\boldsymbol{R}_{ii} = \frac{1}{\frac{1}{R_i} + \frac{1}{R_{\text{TH},i}}} \tag{9}$$

Using (9), $\boldsymbol{R}_{ii}$ can be replaced by R_i and $R_{\text{TH},i}$ and (8) can be rewritten as:

$$S_{ii} = \frac{-1}{R_i + R_{\text{TH},i}} \tag{10}$$

This equation is the same as the GIC sensitivity to local substation grounding resistance derived in [11].

B. Sensitivity of GICs to neighboring resistances

Next, the sensitivity of GIC at substation i to the variation of the substation j grounding resistance, with $j \neq i$, is considered. This variation has an effect on $\boldsymbol{G}$

$$\boldsymbol{G}' = \boldsymbol{G} + \tilde{g}_j \boldsymbol{e}_j \boldsymbol{e}_j^T \tag{11}$$

where $\tilde{g}_j$ is the variation of the grounding admittance at substation j. Reemploying the matrix inversion lemma for this case, the inverse of matrix $\boldsymbol{G}'$ is given as:

$$\boldsymbol{G}'^{-1} = \boldsymbol{G}^{-1} - \boldsymbol{G}^{-1}\boldsymbol{e}_j(\tilde{g}_j^{-1} + \boldsymbol{e}_j^T\boldsymbol{G}^{-1}\boldsymbol{e}_j)^{-1}\boldsymbol{e}_j^T\boldsymbol{G}^{-1} \tag{12}$$

Then, following the same procedure as in the derivation of local resistance in the previous section, the sensitivity of GICs at substation i to the grounding resistance of substation j is

$$\frac{\frac{\Delta I_{\text{GIC},i}}{I_{\text{GIC},i}}}{\frac{\Delta R_j}{R_j}} = \frac{kg_j + k\tilde{g}_j}{1 + \boldsymbol{R}_{jj}\tilde{g}_j} \tag{13}$$

In this equation, $\boldsymbol{R}_{jj}$ is the element at row j, column j of resistance matrix $\boldsymbol{R}$, and k is a scalar number given by

$$k = \boldsymbol{R}_{ij}\frac{\boldsymbol{e}_j^T\boldsymbol{G}^{-1}\boldsymbol{I}}{\boldsymbol{e}_i^T\boldsymbol{G}^{-1}\boldsymbol{I}} = \boldsymbol{R}_{ij}\frac{V_j}{V_i} \tag{14}$$

where $\boldsymbol{R}_{ij}$ is the element at row j, column i of $\boldsymbol{R}$, V_i and V_j are neutral dc voltages of substation i and j. Similar to (7), (13) is capable of providing the exact change of the GIC being studied given the change of the resistance. Taking the limit of

(13) as the variation $\tilde{g}_j$ goes to 0 provides the sensitivity S_{ij} of the GIC at substation i with respect to the variation of the assumed resistance at substation j. In contrast to the local sensitivity, which is always negative, this sensitivity can be positive or negative depending on the signs of V_i and V_j. If the sensitivity is positive, an increase in the assumed grounding resistance leads to an increase in GIC and vice versa.

$$S_{ij} := \lim_{\tilde{g}_j \to 0} \frac{\frac{\Delta I_{\mathrm{GIC},i}}{I_{\mathrm{GIC},i}}}{\frac{\Delta R_j}{R_j}} = g_j \boldsymbol{R}_{ij} \frac{V_j}{V_i} = \frac{\boldsymbol{R}_{ij}}{R_j} \frac{V_j}{V_i} \tag{15}$$

As defined above, $\boldsymbol{R}_{ii}$ and $\boldsymbol{R}_{ij}$ are elements of $\boldsymbol{G}^{-1}$. In a large system, $\boldsymbol{G}$ is usually sparse because each bus is only connected to a few other buses through the transmission lines. Hence the direct computation of $\boldsymbol{G}^{-1}$ will be redundant. As a result, the sparse vector method [12] is employed to obtain values of desired elements of $\boldsymbol{G}^{-1}$. The idea is that the elements of matrix $\boldsymbol{G}^{-1}$ needed to calculate all the sensitivities of the GIC at substation i are in the i$^{\text{th}}$ column of that matrix. Hence obtaining some particular sensitivity values of $I_{GIC,i}$ is equivalent to calculating some particular elements of the i$^{\text{th}}$ column of $\boldsymbol{G}^{-1}$. In (17), the i$^{\text{th}}$ column of $\boldsymbol{G}^{-1}$ is denoted as $\boldsymbol{x}$, and the i$^{\text{th}}$ column of unit matrix $\boldsymbol{I}$ is denoted as $\boldsymbol{b}$. Hence $\boldsymbol{b}$ is a vector in which all elements are 0 except the one at the i$^{\text{th}}$ row. Using factorization, (20) can be rewritten as the combination of (21) and (22). Then $\boldsymbol{x}$ can be determined using a forward and backward substitution. Fortunately, because $\mathbf{b}$ is a sparse vector and only few elements of $\boldsymbol{x}$ are needed, fast forward (FF) and fast backward (FB) substitution techniques can be used. To use these techniques, factorization paths are needed for both $\boldsymbol{L}$ and $\boldsymbol{U}$; each path is executed in the forward order for FF and the backward order for FB.

$$\boldsymbol{G}\boldsymbol{G}^{-1} = \boldsymbol{I} \tag{16}$$
$$\boldsymbol{G}(\boldsymbol{G}^{-1})_i = \boldsymbol{e}_i \tag{17}$$
$$\boldsymbol{G}\boldsymbol{x} = \boldsymbol{b} \tag{18}$$
$$\boldsymbol{G} = \boldsymbol{L}\boldsymbol{U} \tag{19}$$
$$\boldsymbol{L}\boldsymbol{U}\boldsymbol{x} = \boldsymbol{b} \tag{20}$$
$$\boldsymbol{L}\boldsymbol{y} = \boldsymbol{b} \tag{21}$$
$$\boldsymbol{U}\boldsymbol{x} = \boldsymbol{y} \tag{22}$$

III. Demonstration on Text Systems

A. 20-bus test system

The 20-bus system from [13] is used first to demonstrate the algorithm. The one-line diagram of the system is shown in Figure 2. The arrows represent the flow of the GICs for a one V/km eastward field, while the size of an arrow is proportional to the magnitude of the GIC on each of the lines. The locations of the eight substations in the case are labeled in the figure. The algorithms are applied to the two GMD scenarios considered in [13], namely, a uniform one V/km eastward field and a uniform one V/km northward field. Table 3 shows the assumed substation grounding resistance and the calculated GIC values for the two scenarios (reproduced from Tables I and VII of [13]). Substation 1 is modeled with a GIC blocking device and Substation 7 models a series capacitor location that has no connection to ground so they will be neglected in our calculation.

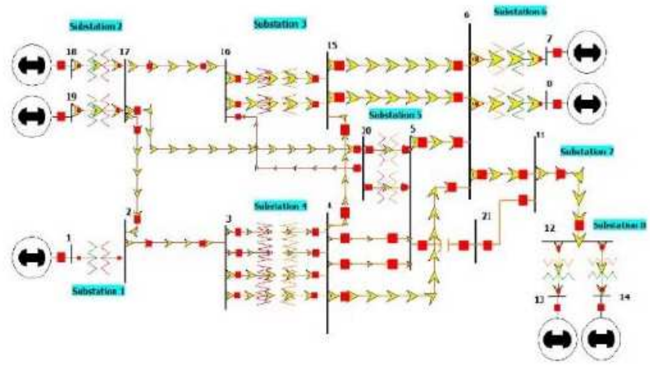

Figure 2: 20-bus GIC test system one-line showing 1 V/km eastward values

Table 3: 20-bus substation resistances and GIC flows

	Grounding resistance (Ω)	Eastward field GIC (A)	Northward field GIC (A)
Sub. 1	0.2 (but blocked)	0.00	0.00
Sub. 2	0.2	-189.29	115.63
Sub. 3	0.2	-109.49	139.85
Sub. 4	1.0	-124.58	19.98
Sub. 5	0.1	-65.46	-279.09
Sub. 6	0.1	354.52	-57.29
Sub. 7	No ground path	0.00	0.00
Sub. 8	0.1	134.30	60.9

Table 4: 20-bus substation equivalent values

	Driving point resistance (Ω)	$R_{\mathrm{TH},i}$ (Ω)	S_{ii}	V_{TH} eastward (V)	V_{TH} northward (V)
Sub. 2	0.158	0.750	-0.210	-179.88	109.90
Sub. 3	0.115	0.272	-0.424	-51.61	65.95
Sub. 4	0.198	0.246	-0.802	-155.28	24.90
Sub. 5	0.076	0.321	-0.239	-27.53	-117.41
Sub. 6	0.075	0.302	-0.249	142.50	-23.02
Sub. 8	0.093	1.365	-0.068	196.69	89.20

The results from applying the algorithm for the local resistance case is presented in the S_{ii} column in Table 4 (since grounding resistance plays no role for Substations 1 and 7, they are omitted from the table). Noted that the sensitivity of each GIC to its local grounding resistance is independent of the GMD scenario, and depends only upon R_i and $\boldsymbol{R}_{ii}$ (see (8)); hence the values of S_{ii} in Table 4 are identical for the two scenarios. Most of S_{ii} values in the table are relatively low except those of Substations 3 and 4. This means the GICs at these substations are quite dependent on their local assumed grounding resistances while others are not. To illustrate, if Substation 4's grounding resistance value decreased by 50% (from 1.0 to 0.5Ω), the new GIC for an eastward field would change from -124.6 A to -208.1 A (the magnitude increases by 67.01%). On the other hand, if Substation 2's grounding resistance was also reduced by 50% (from 0.2 to 0.1 Ω), the Substation 2 GIC for the eastward field would only change from -189.3 to -211.6 A (the magnitude increases by 11.78%).

Using the algorithm for the neighboring resistance case, the sensitivities of GICs to neighboring substation grounding resistances for the one V/km eastward field scenario are given in Table 5, and those for the one V/km northward field scenario are given in Table 6. Substation 1 and Substation 7 are also omitted in those tables. As mentioned above, the sensitivity to local grounding resistance does not depend on GMD scenarios. In contrast, the sensitivity to neighboring grounding resistances changes with the magnitude and direction of the electric field. Thus for different GMD scenarios the sensitivities of a particular GIC to other substation resistances can be quite different and needs to be recomputed.

This can be illustrated by comparing the sensitivity values in Table 5 and Table 6. The entries in Table 6 show that the GIC at Substation 4 not only depends on the local grounding resistance ($S_{44} = -0.802$) but also on the assumed grounding resistance of Substation 5 ($S_{45} = -0.507$) in the northward field scenario. On the other hand, in Table 5, S_{45} is only 0.019 so the effect of R_5 is trivial to Substation 4's GIC in the eastward field scenario.

From both tables, it is apparent that the absolute value of the sensitivity of a GIC to the local resistance is usually greater than those of the sensitivities to nearby substation grounding resistances. However, there are exceptions. In Table 5, S_{55} is -0.238 but the sensitivities to the Substation 4 and Substation 6 resistances are 0.691 and -0.449 respectively. The fact that the absolute values of S_{54} and S_{56} are greater than that of S_{55} means if the GIC value at Substation 5 is important then accurate values of the resistances of Substation 4 and 6 are needed more than that of Substation 5. This situation also happens for the GICs at Substation 6 in Table 6 where the absolute value of S_{66} (-0.249) is smaller than those of S_{63} (-0.410) and S_{65} (0.404). In addition, in the case of the GICs at Substation 4, even though the absolute value of S_{45} (-0.507) is not greater than that of S_{44} (-0.802), it is still significant; hence the good accuracy for the grounding resistance of Substation 5 should still be considered in calculating $I_{\text{GIC},4}$. This example shows that the calculation of the sensitivity to neighboring substation grounding resistance is important.

Table 5: 20-bus system's sensitivity values for a uniform eastward field

	$I_{\text{GIC},2}$	$I_{\text{GIC},3}$	$I_{\text{GIC},4}$	$I_{\text{GIC},5}$	$I_{\text{GIC},6}$	$I_{\text{GIC},8}$
R_2	-0.210	0.121	0.028	0.250	-0.017	-0.004
R_3	0.040	-0.424	0.031	0.226	-0.052	-0.012
R_4	0.061	0.203	-0.802	0.691	-0.054	-0.013
R_5	0.015	0.040	0.019	-0.238	-0.015	-0.004
R_6	-0.030	-0.272	-0.044	-0.449	-0.249	0.133
R_8	-0.001	-0.009	-0.002	-0.015	0.019	-0.068

Table 6: 20-bus system's sensitivity values for a uniform northward field

	$I_{\text{GIC},2}$	$I_{\text{GIC},3}$	$I_{\text{GIC},4}$	$I_{\text{GIC},5}$	$I_{\text{GIC},6}$	$I_{\text{GIC},8}$
R_2	-0.210	0.058	0.107	-0.036	-0.066	0.006
R_3	0.085	-0.424	0.249	-0.068	-0.410	0.035
R_4	0.016	0.025	-0.802	-0.026	-0.054	0.005
R_5	-0.104	-0.135	-0.507	-0.238	0.404	-0.035
R_6	-0.008	-0.034	-0.044	0.017	-0.249	-0.047
R_8	0.001	0.003	0.004	-0.002	-0.053	-0.068

Even though the 20-bus test case only has few substations, there are still two remarks prompted by Tables 5 and 6. First, the absolute value of the sensitivity of a GIC to the local resistance is frequently greater than most of the sensitivities to other substation grounding resistances. This result implies that, regularly, the accurate value of the local resistance has the largest share in the calculation of GIC. Secondly, most of the external sensitivity values in both tables are trivial (smaller than 0.1) and the non-trivial ones are only a handful. In addition, all of them are sensitivity of GIC to grounding resistances of substations that are directly connected or two hops away from the substation being studied. This observation reveals that only grounding resistances from nearby substations contribute significantly to the sensitivity of GIC; this aspect will be examined further in the following section.

B. 150-bus synthetic system

For GMD studies there are only few realistic test cases that are not restricted by data confidentiality. To address this issue [14] provides a method to generate a completely synthetic transmission system for GMD studies. The approach utilizes public energy and census data to form the basis for generation, load, and geographic substation placement. The approach is to build synthetic power system models that match statistical characteristics of the actual grids.

The next system considered here is the 150 bus system from [14]. This model contains 98 substations with a geographic footprint covering approximately 35ºN to 36.5ºN latitude and 90ºW to 82ºW longitude; the wide geographic area makes it better for GIC analysis. Nominal voltages of 500 kV and 230 kV were used in the model. Since the most of the substation grounding resistances in this system are not known, their values are assigned based on a heuristic that considered the substation's highest nominal voltage and its number of buses nominal voltage level and the number of buses in them. The larger, higher voltage substations have lower resistance values.

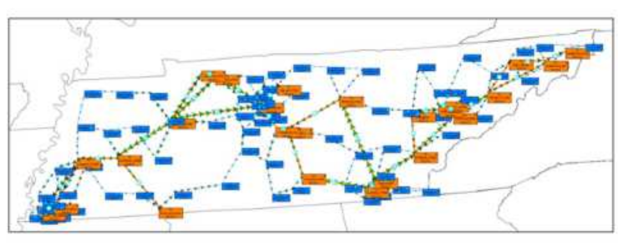

Figure 3: One-line diagram of 150-bus GMD test case

Table 7: 150-bus system's 15 substations with the largest GICs and their local sensitivity values for a one V/km eastward field

Substation number	Assume R (Ω)	GIC (A)	Sensitivity
44	0.18	147.53	-0.356
93	0.11	-147.17	-0.429
78	0.18	-134.92	-0.461
91	0.12	112.58	-0.366
42	0.18	97.65	-0.427
95	0.11	-87.84	-0.41
70	0.18	-69.05	-0.44
94	0.12	-66.41	-0.45
96	0.15	50.95	-0.227
85	0.18	-42.29	-0.402
97	0.11	41.41	-0.463
26	0.18	-39.91	-0.383
89	0.18	38.94	-0.361
21	0.18	37.92	-0.359
92	0.12	30.62	-0.369

For this example, assume a uniform one V/km eastward electric field is applied to this synthetic system. Note here only the 500 kV substations are examined for GIC sensitivity analysis. Figure 3 shows the system substations, with the orange ones represented the 500 kV and the blue ones have the 230 kV. Among the 98 substations in the network there are twenty-seven 500 kV substations; their GIC magnitudes range from 1.27 to 147.53 A; Table 7 shows the fifteen substations with the highest GICs.

According to Table 7, the sensitivities to local grounding resistance of all fifteen substations all have values larger than 0.1, indicating that the assumed resistance dominates in determining the GIC. Moreover, more than half of them (8 out of 15) have sensitivities at or above 0.4. For example Substation 78 has an assumed grounding resistance of 0.18 Ω and sensitivity of -0.461; its GIC magnitude is 134.92 A. If the assumed grounding resistance is increased from 0.18 to 0.5 Ω, then the magnitude would drop to 73.98 A.

Besides the local grounding resistance, GICs in these substations are also dependent on the assumed resistances of the neighboring substations. Similar to the 20-bus system case, most of the external sensitivities in this case are very small and can be ignored. However, there are a handful of values that are non-trivial and are presented in Table 9. The result reaffirms that the influence of external grounding resistances on the calculation of GIC at a particular substation cannot be overlooked. In addition, the last column in Table 9 shows the relative distance between the substations. If the value is 1 then the two substations are directly connected. If it is 2 then they are connected through one other substation. The values in this column again indicates that GICs are only affected by the grounding resistances of nearby substations.

Table 9: 150-bus system external sensitivity values of 15 substations

Substation of GIC	Substation of resistance	Sensitivity	Number of hops
42	44	0.209	1
85	93	0.173	1
85	95	0.147	1
95	70	0.134	1
95	78	0.286	1
70	78	0.359	1
70	95	0.133	1
21	91	0.358	1
21	93	-0.183	1
92	42	0.489	1
92	44	0.546	2

C. Insights about sensitivity

A natural question from the above analysis is whether it is possible to recover the grounding resistance from GIC measurements. Assuming the above sensitivity analysis is carried out, and the sensitivity of GIC to the local grounding resistance dominates those due to neighboring grounding resistances, then the effect of the latter can be ignored and (7) can be used to solve for the local grounding resistance from its GIC measurement. For instance, consider Substation 44 of the 150-bus case example in the previous section. The local sensitivity of GIC at Substation 44 is calculated to be -0.356, while its sensitivities to grounding resistances of nearby Substations 92, 53, 48 are 0.017, 0.002, and 0.002, respectively. Apparently, the absolute value of the local sensitivity is significantly larger than those of the external sensitivities so we can assume that these neighboring grounding resistances have trivial impact on the GIC. Hence, the inaccuracy of the GIC at Substation 44 is solely due to the difference between the assumed value and the actual value of the local grounding resistance. Consequently, giving the GIC's measured value, the actual grounding resistance of Substation 44 can be obtained from (7). To test this statement, the assigned actual values and the assumed values of grounding resistances at Substations 44, 92, 53, and 48 are given in Table 10. The GIC values at Substation 44 are obtained from simulations. The first simulation uses assumed grounding resistances and produces a GIC value of 147.53 A while the second one uses actual resistance values and gives a GIC value of 176.47 A. Solving (7) for $\tilde{g}_{44}$, the result is 4.742. From there one can calculate the desired value of Substation 44's grounding resistance, which is equal to 0.097 Ω. This result is very close to the actual R_{44} provided in Table 10 (0.1 Ω). It also shows that even though the error of the neighboring resistance values are nontrivial, they do not have significant effect on the calculation of the local grounding resistance's actual value.

Table 10: Resistance values of Substation 44, 92, 53, and 48

	Sensitivity of GIC to the assumed R	Assumed R (Ω)	Actual R (Ω)	ΔR (Ω)
R_{44}	-0.356	0.18	0.1	0.08
R_{92}	0.017	0.12	0.2	0.08
R_{53}	0.002	0.18	0.5	0.32
R_{48}	0.002	0.18	0.5	0.32

It can be observed from the above example that, given a GIC measurement at a substation, if the local sensitivity considerably dominates the sensitivities due to external resistances, then the actual value of the local grounding resistance can be estimated with high accuracy. This observation is useful because measuring GIC is generally easier than measuring the grounding resistance. It would help in the estimation of unknown parameters in a power system and improve the accuracy of GIC calculation.

IV. CONCLUSION

This paper extends the previous work of [11] by providing an algorithm for calculating GIC sensitivity at an arbitrary substation to grounding resistances of any substations in the system. Demonstration of the algorithm on two systems, one with 20 buses and one with 150 buses, reveals valuable insights on how utility company can protect their power grid from GMD effectively.

V. ACKNOWLEDGMENTS

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