# Incorporating the Geomagnetic Disturbance Models into the Existing Power System Test Cases 

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#### Abstract

Realistic public test cases can facilitate the studies on the GMDs impacts on the power system by providing a benchmark to validate the related analysis tools. Many standard test cases are available for different aspects of power system analysis. These cases are designed for ac analysis and do not contain the necessary inputs such as the substation grounding resistances and the geographic coordinates which are essential for GMD studies. In this paper, a framework is proposed to generate GMD-related parameters for the existing standard power system test cases. The substation geographic coordinates are the key parameters which are missing in the existing cases. The Kamada and Kawai algorithm and the Force-directed method are presented as two effective graph drawing algorithms to generate the geographic layout and determine the coordinates. The effectiveness of the proposed framework is evaluated through numerical results using the 20 -bus system and the IEEE 24 -bus system.


## I. Introduction

Solar coronal holes and coronal mass ejections can disturb the Earth's geomagnetic field. These geomagnetic disturbances (GMD) in turn induce electric fields which drive low frequency currents in the transmission lines. These geomagnetically induced currents (GICs) can cause increased harmonic currents and reactive power losses by causing transformers half-cycle saturation. This may cause voltage instability by a combination of two means. First, the increased transformer reactive power losses may lead directly to voltage instability. Second, the harmonic currents might cause relay misoperation and unintended disconnection of the reactive power providers such as static VAR compensators [1]-[4]. The 1989 Hydro-Québec blackout demonstrates the significance of G ICs a nd t heir potential impacts on power grid. In March13, 1989, Hydro-Québec network with $21,500 \mathrm{MW}$ generation and 2,000 kilometers of power lines went down for 9 hours and caused tens of millions of dollars damage to the utility and the costumers.

Considering the negative impacts of GMDs, proper mitigation programs are required to protect the system. One of the key challenges in GMD studies is the shortage of suitable test cases for evaluation purposes. Various power system test cases have been developed to validate the models associated with different aspects of power system such as power flow, dynamics, distributions, reliability [6]. These cases are designed for ac analysis and do not contain the necessary inputs such as substation grounding resistances and geographic coordinates which are essential for GMD studies. Hence, developing realistic test cases which include GIC-related parameters is
extremely useful for GMD studies. Reference [7] presents a 17-bus system designed for GIC calculation which models the Finish $400-\mathrm{kV}$ grid. A 20-bus test case is designed in [8] for GMDs studies which includes transformer models and two voltage levels. These cases do not contain the ac power flow parameters and, hence, cannot be used for steady-state voltage stability analysis under a GMD. Reference [14] proposes an algorithm to generate realistic synthetic power system test cases. These cases include both the ac power flow parameters and the GMD-related parameters.

The geographic coordinates are the key parameters which are missing in the standard cases and are essential for GIC analysis. Graph drawing techniques may be utilized to obtain the geographic layout and consequently determine the coordinates. A drawing of a graph is a pictorial representation of its vertices and edges. Very different layouts can be generated for the same graph with varying levels of understandability, usability and aesthetic. Various techniques have been developed for graph drawings each attempting to achieve different quality measures [9], [11], [12]. The common quality measures used for graph drawings are crossing number (number of edge pair that cross each other), the drawing area (the size of the smallest bounding box relative to the closest node distance) and symmetry display.

This paper proposes a framework to incorporate GMD modeling into the already-existing standard power system test cases. Kamada and Kawai (KK) Algorithm and Force-directed (FD) method are presented as two effective graph drawing algorithms. The geographic layout is developed using these techniques and the substation coordinates, the key parameters required for GIC analysis are obtained. The effectiveness of these techniques in retrieving the coordinates is evaluated through numerical results using the EPRI 20-bus test case. Moreover, the proposed procedure is applied to the IEEE24bus system and the necessary GMD-related parameters are defined for this case.

The paper is organized as follows: The GIC model is introduced in Section II. The proposed framework for determining the GIC-related parameters is presented in Section III. Section IV demonstrates the proposed technique through numerical results. Section V presents a conclusion and direction for future work.

## II. GIC Modeling

To calculate the voltage potential induced on the transmission line, the E-field is integrated over the length of the line. Assuming uniform E-field, the DC voltage on the line between bus $n$ and $m$ is expressed in:

$$
\begin{equation*}
V_{n m}=E^{N} L_{n m}^{N}+E^{E} L_{n m}^{E} \tag{1}
\end{equation*}
$$

where $L_{n m}^{N}$ and $L_{n m}^{E}$ denote the northward and eastward line distances; and $E^{N}$ and $E^{E}$ are the northward and eastward E-fields, respectively. The induced voltages are converted to the dc current injections through Norton Equivalent, and the total current injections are derived from Kirchhoff's current law (KCL) [4]. The vector of current injections is obtained by putting all the current injections together as given by $\mathbf{I}^{N o}=$ $\mathbf{C E}$ where $\mathbf{C}$ depends on the length, orientation and resistance of the lines.

The nodal network equations are written using KCL and the bus voltages are obtained from the current injections as expressed in:

$$
\begin{equation*}
\mathbf{V}=\mathbf{G}^{-1} \mathbf{I}^{N o} \tag{2}
\end{equation*}
$$

where matrix $\mathbf{G}$ is similar to the bus admittance matrix except that it only captures the conductance values and is modified to include substation grounding resistances. The GICs are related to the bus voltages by Ohm's law:

$$
\begin{equation*}
\mathbf{I}=\mathbf{G}^{S} \mathbf{V}=\left(\mathbf{G}^{S} \mathbf{G}^{-1}\right) \mathbf{I}^{N o}=\left(\mathbf{G}^{S} \mathbf{G}^{-1} \mathbf{C}\right) \mathbf{E}=\mathbf{H E} \tag{3}
\end{equation*}
$$

where $\mathbf{G}^{S}$ is a diagonal matrix with the grounding resistances on its diagonal and $\mathbf{H}$ is the coefficient matrix defined as $\mathbf{H}:=\mathbf{G}^{S} \mathbf{G}^{-1} \mathbf{C}$. This linear model indicates that the GICs relate to the E-field through some constant coefficients. Matrix $\mathbf{H}$ depends only on the network resistances and topology.

The GIC model represents the whole network and the vector I includes the GICs at all transformers. Sometimes the whole network is not of interest, and it is desired to reduce the model to cover only specific transformers. To reduce the model, the entries of the $\mathbf{I}$ vector corresponding to the specified transformers are selected and the coefficient matrix is truncated accordingly.

## III. Determining the GiC-RElated Parameters

The key parameters required for GMD analysis, which are usually missing in standard test cases, are the substation grounding resistances and geographic coordinates. A rather simplistic model for determining the substation grounding resistances is used in [13]. In this model, the assumed resistance depends on the highest substation voltage level and its assumed size (based on the number of lines coming into the substation), with larger, higher voltage substations having lower values. Soil resistivity, which certainly can have an impact, is not included in this simplistic model. Ballpark values are usually substantially below $0.5 \Omega$ for 230 kV and above substations, and between 1 and $2 \Omega$ for the lower voltage substations.

The geographic coordinates may be obtained through developing a geographic layout of the system using the existing graph drawing techniques as described in the following.


Fig. 1. Example of using FD method for layout design.

## A. Force-directed Graph Drawings

Force-directed graph drawings is a method for drawing graphs in a way that looks pleasant to the eye [11]. The vertices of the graph are positioned in two-dimensional or three-dimensional space so that the edges are about the same length and the number of crossings is minimized. The algorithm assigns forces among the set of edges and the set of vertices and uses these forces to simulate the movement of the vertices or to minimize their energy. Attractive forces like springs are used to attract the vertices that are connected in the graph (based on Hooke's law). Repulsive forces like electrically charged particles are used to separate all pairs of vertices (based on Coulomb's law). The layout is obtained by solving for the equilibrium state of this system of forces. In equilibrium, the edges have similar lengths because of the spring attractions and the vertices are as far as possible from each other due to the electric repulsive forces.

The attractive and repulsive forces between a pair of vertices are defined as

$$
\left\{\begin{array}{l}
f_{a}(d)=\frac{d^{2}}{k}  \tag{4}\\
f_{r}(d)=-\frac{k^{2}}{d}
\end{array}\right.
$$

where $f_{a}$ and $f_{r}$ are respectively the attractive and repulsive forces and $d$ is the distance between the pair of vertices. $k$ is the optimal distance between the vertices as given by

$$
\begin{equation*}
k=C \sqrt{\frac{\text { area }}{\text { number of vertices }}} \tag{5}
\end{equation*}
$$

where $C$ is a constant. The algorithm reduces the number of edge-crossings significantly and the resulting layout is aesthetically pleasant. Fig. 1 illustrates an example of using FD method for layout design.

## B. Kamada and Kawai Algorithm

The FD method does not preserve the distances between the vertices and the edges, and the resulting layout has uniform lengths. Sometimes, it is desired to maintain the distances, especially in GMD applications where the line lengths have significant impact on the GIC flows. The Kamada and Kawai algorithm minimizes the difference between the ideal lengths and actual ones instead of minimizing the number of edge crossings [12]. Unlike the FD algorithm, no repulsive forces are considered between vertices. Spring forces are used between all pairs of vertices, with ideal spring lengths equal to
the vertices' graph-theoretic distance. The optimal layout is obtained by minimizing the total spring energy.

Let $n$ be the number of vertices and $p_{1}, p_{2}, \cdots, p_{n}$ be the particles in a plane representing the vertices $v_{1}, v_{2}, \cdots, v_{n}$ respectively. The energy of the system is given by

$$
\begin{equation*}
E=\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{1}{2} k_{i j}\left(\left|p_{i}-p_{j}\right|-l_{i j}\right) \tag{6}
\end{equation*}
$$

where $l_{i j}$ is the desired length of the spring between $p_{i}$ and $p_{j}$ and is calculated by

$$
\begin{equation*}
l_{i j}=\frac{L_{0}}{\max _{i \leq j} d_{i j}} d_{i j} \tag{7}
\end{equation*}
$$

where $d_{i j}$ is the distance between $v_{i}$ and $v_{j}$ and $L_{0}$ is the length of the display area. $k_{i j}$ is the strength of the spring between $p_{i}$ and $p_{j}$ as expressed in:

$$
\begin{equation*}
k_{i j}=\frac{K^{s p r}}{d_{i j}^{2}} \tag{8}
\end{equation*}
$$

where $K^{s p r}$ is a constant. For a two-dimensional space, the particle $p_{i}$ is represented by the rectangular coordinates $\left(x_{i}, y_{i}\right)$ and the system energy in (6) is given by

$$
\begin{align*}
E= & \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{1}{2} k_{i j}\left\{\left(x_{i}-x_{j}\right)^{2}+\left(y_{i}-y_{j}\right)^{2}\right. \\
& \left.+l_{i j}^{2}-2 l_{i j} \sqrt{\left(x_{i}-x_{j}\right)^{2}+\left(y_{i}-y_{j}\right)^{2}}\right\} \tag{9}
\end{align*}
$$

The necessary condition of the local minimum is as follow:

$$
\begin{equation*}
\frac{\partial E}{\partial x_{m}}=\frac{\partial E}{\partial y_{m}}=0, \quad 1 \leq m \leq n \tag{10}
\end{equation*}
$$

The partial derivative of the energy with respect to $x$ and $y$ is expressed in:

$$
\begin{align*}
\frac{\partial E}{\partial x_{m}} & =\sum_{i \nsupseteq} k_{m i}\left\{\left(x_{m}-x_{i}\right)-\frac{l_{m i}\left(x_{m}-x_{i}\right)}{\sqrt{\left(x_{m}-x_{i}\right)^{2}+\left(y_{m}-y_{i}\right)^{2}}}\right\} \\
\frac{\partial E}{\partial y_{m}} & =\sum_{i \nsupseteq} k_{m i}\left\{\left(y_{m}-y_{i}\right)-\frac{l_{m i}\left(x_{m}-x_{i}\right)}{\sqrt{\left(x_{m}-x_{i}\right)^{2}+\left(y_{m}-y_{i}\right)^{2}}}\right\} \tag{11}
\end{align*}
$$

This gives rise to a system of $2 n$ nonlinear equations and Newton-Raphson may be used for solving it.

Note that KK requires the line lengths as input. However, the lengths are not usually available for the synthetic cases. To address this, the line conductance may be used as a criterion to determine the length. The line conductance depends on its conductor type, the conductor bundling structure and the length. The conductor type and the bundling structure depend on the voltage level. Heuristics may be developed to get the resistance per meter for different voltage levels through the statistical analysis of the real power systems. Using the resistance (available in standard test cases) and the estimated resistance per line, the line length can be estimated.


Fig. 2. One-line diagram of the 20 -bus system.


Fig. 3. The geographic layout of the 20-bus system using the available coordinates.

## IV. Numerical Results

In this section, the geographic layouts of two power systems are developed using the KK and BF methods. The first system to study is the 20-bus system in [8] with the one-line diagram shown in Fig. 2. This case is designed specifically for GMD applications and contains substation geographic coordinates. Fig. 3 illustrates the geographic layout of the system using the available coordinates.

The Kamada and Kawai algorithm is utilized to develop a geographic layout as shown in Fig. 4. The coordinates are not provided to the algorithm and are to be estimated. Instead, the line lengths are calculated from the coordinates and are given to the algorithm:

$$
\begin{align*}
& a=\sin ^{2}\left(\frac{\phi_{2}-\phi_{1}}{2}\right)+\cos \left(\phi_{1}\right) \cos \left(\phi_{2}\right) \sin ^{2}\left(\frac{\lambda_{2}-\lambda_{1}}{2}\right) \\
& c=2 \operatorname{atan} 2(\sqrt{a}, \sqrt{1-a}) \\
& d=R c \tag{12}
\end{align*}
$$



Fig. 4. Geographic layout of the 20 -bus system obtained from the KK algorithm.


Fig. 5. Comparison of the line lengths obtained from the KK algorithm with the actual ones for the 20 -bus system.
where $\phi$ and $\lambda$ are latitude and longitude respectively, $d$ is the distance between points 1 and 2 (in mile), and $R$ is the Earth radius, i.e. $6,371 \mathrm{~km}$. Figure 4 illustrates the resulting layout obtained from the KK algorithm. Comparing this layout with the actual one in Fig. 3, it is observed that the layout developed by KK algorithm preserves the lengths, but does not capture the original layout. This is because developing the layout from only the line lengths does not provide a unique solution and additional information is required to retrieve the original layout.

The line lengths are calculated for the layout developed by the KK algorithm and are compared with the actual ones derived from the coordinates. This comparison is illustrated in Fig. 5. The obtained lengths agree well with the actual ones, except for some occasional mismatches.

Next, the geographic layout is calculated through the FD algorithm as illustrated in Fig. 6. The resulting layout has only one crossing and the line lengths are almost uniform. The algorithm uses only the incident matrix as input and the


Fig. 6. Geographic layout of the 20 -bus system obtained from the FD method.
actual line lengths are ignored.
The second system to study is the IEEE 24-bus system with the one-line diagram shown in Fig. 7. This system is designed for ac analysis and does not contain the substation geographic coordinates. To make it suitable for GMD studies, the KK algorithm is utilized to develop a geographic layout of the system and consequently obtain the substation coordinates. The resulting layout is shown in Fig. 8. The KK algorithm takes the incident matrix and the line lengths as input. The required line lengths are collected from the available data in [15]. Note that the line lengths are not usually available for the synthetic standard cases and the line conductances may be used to estimate them as described in Section III. Alternatively, the FD method may be used instead of KK algorithm to get the layout since it does not require the line lengths.

The line lengths obtained from the KK layout are compared with the actual lengths in Fig. 9. There is relatively good agreement between the obtained and actual lengths.

Next, the layout is obtained using the FD method as shown in Fig. 10. Unlike the KK layout, that had many crossings and looked very crowded; this layout has only one crossing and appears aesthetically pleasant. The line lengths are almost uniform and bear no correlation with the actual lengths.

Pearson correlation coefficient is used to measure the correlation between the actual lengths and those obtained from the investigated layout designs. The definition of Pearson coefficient is as follow:

$$
\begin{equation*}
\rho_{X, Y}=\frac{\operatorname{cov}(X, Y)}{\sigma_{X} \sigma_{Y}} \tag{13}
\end{equation*}
$$

where $\rho_{X, Y}$ and $\operatorname{cov}(X, Y)$ are the Pearson correlation and covariance between signals $X$ and $Y$, respectively, and $\sigma$ is the standard deviation. Table I presents the correlation coefficients for both drawing methods and the two investigated systems. It is observed that the KK algorithm provides much better correlation than the FD method. Moreover, the correlations are higher for the EPRI 20-bus case than the IEEE 24-buse system. This could relate to the fact that the 20 -bus case


Fig. 7. One-line diagram of the IEEE 24-bus system.


Fig. 8. Geographic layout of the IEEE 24-bus system obtained from the KK algorithm.
was originally designed for GMD studies and has an actual geographic layout. On the other hand, the 24-bus system may not have been designed based on a real geographic layout, and therefore, there might be no feasible layout that can correlate well with the available line lengths. Further exploration into the feasibility of the geographic layout given a set of line lengths will be an interesting future direction.

## V. Conclusions

In this paper, a framework is proposed to incorporate the GMD modeling into the already existing standard power system cases. The geographic coordinates are the key parameters that are missing in the standard cases and are essential for


Fig. 9. Comparison of the line lengths obtained from the KK algorithm with the actual ones for the 24-bus system.


Fig. 10. Geographic layout of the IEEE 24-bus system obtained from FD method.

GMD studies. KK and BF are presented as two effective graph drawing techniques to generate the geographic layout and consequently get the coordinates. The proposed framework is applied to the 20-bus and the IEEE 24-bus systems and their coordinates are determined. Numerical results indicate that the layout obtained from KK preserves the line lengths, while BF provides a layout which is aesthetically pleasing, but its resulting lengths have little correlation with the actual lengths.

The study suggests several directions for future research: First, the algorithm can be applied to other standard test cases such as the IEEE 118-bus or the 300-bus systems. Second, the algorithm for estimating the line lengths from their resistances were described briefly. This algorithm can be further refined and statistical analysis of the actual systems may be utilized to verify its effectiveness. Last, the effectiveness of the test cases generated from the proposed framework may be validated by performing GMD studies on the generated case and evaluating the results.

TABLE I
Grounding Resistances of the 20-bus Test Case.

| Test Case | Kamada-Kawai | Force-directed |
| :--- | :---: | :---: |
| IEEE 24-bus | 0.694 | 0.1711 |
| EPRI 20-bus | 0.8554 | 0.3441 |

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