ECEN 615
Methods of Electric Power Systems Analysis

Lecture 17: Sensitivity Analysis, Least Squares

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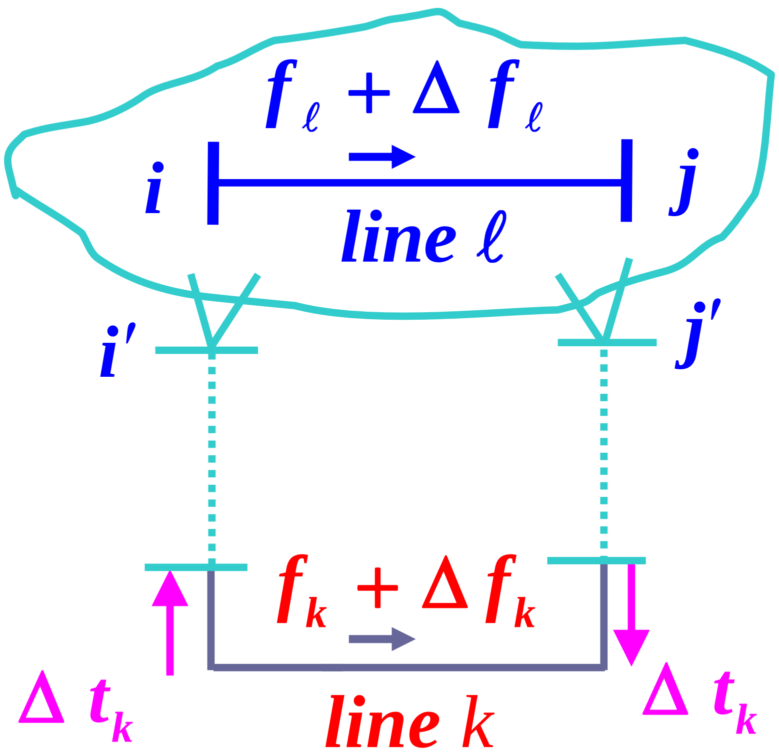
Announcements

- Read Chapter 9
  - We'll just briefly cover state estimation since it is covered by ECEN 614, but will use it as an example for least squares and QR factorization
- Homework 4 is due on Thursday Nov 1

LODFs Evaluation Revisited

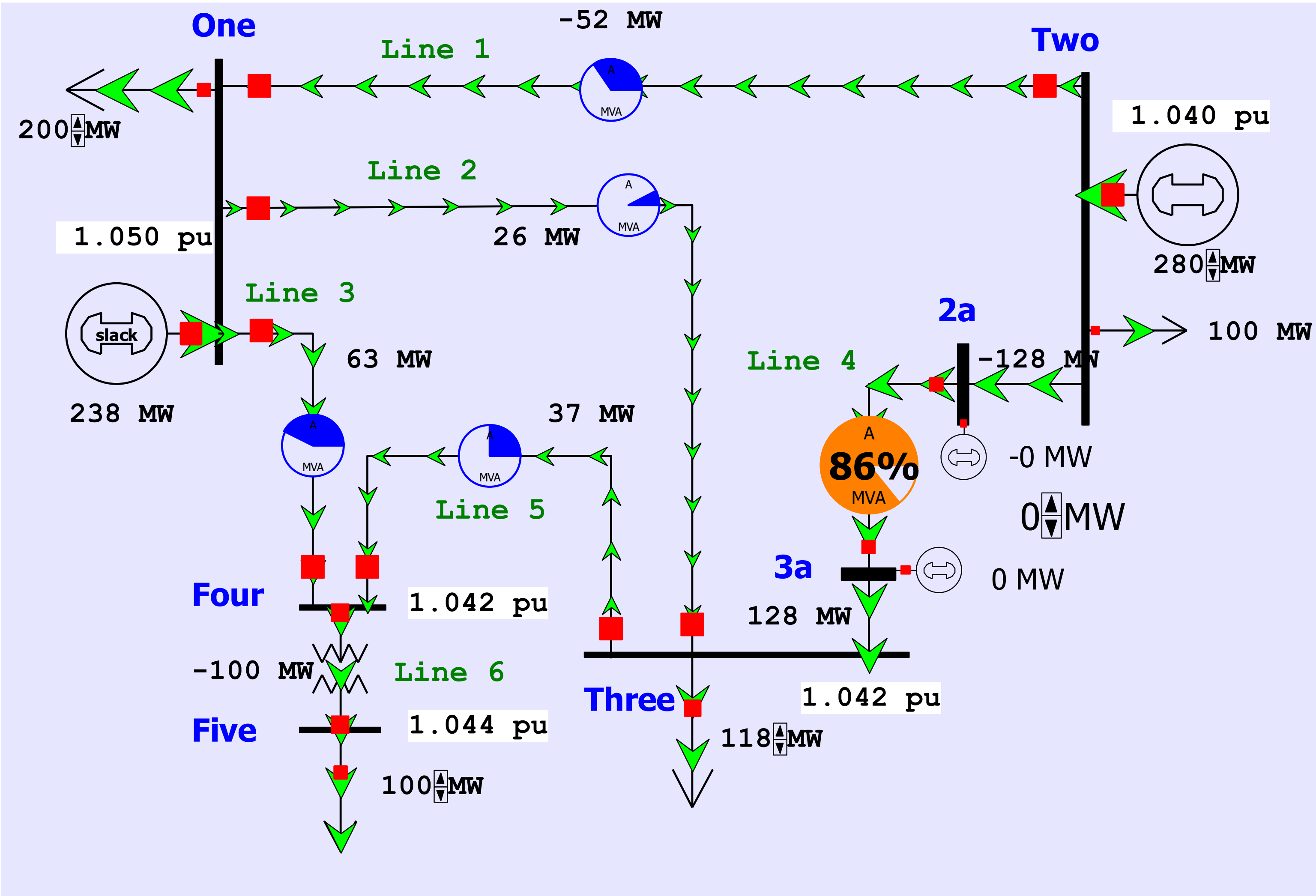
- We simulate the impact of the outage of line k by adding the basic transaction $w_k = \{i', j', \Delta t_k\}$



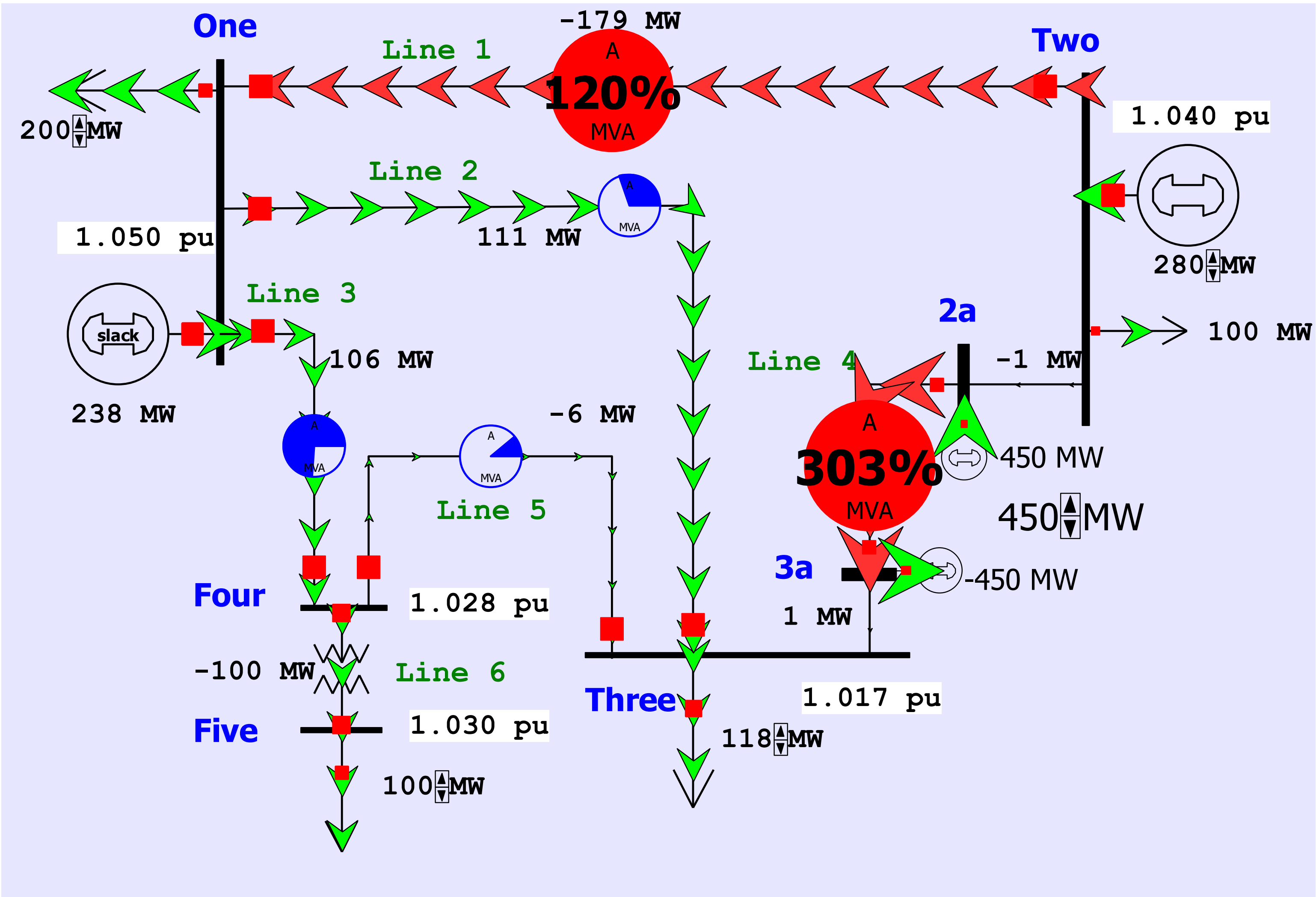
and selecting Δt_k in such a way that the flows on the dashed lines become exactly zero

The Δt_k is zeroing out the flow on the dashed lines; if we simulated in power flow the flow on the line itself would be quite high

LODFs Evaluation Revisited Five Bus Example



LODFs Evaluation Revisited Five Bus Example



Multiple Line LODFs

- LODFs can also be used to represent multiple device contingencies, but it is usually more involved than just adding the effects of the single device LODFs
- Assume a simultaneous outage of lines k_1 and k_2
- Now setup two transactions, w_{k1} (with value Δt_{k1})and w_{k2} (with value Δt_{k2}) so

$$f_{k1} + \Delta f_{k1} + \Delta f_{k2} - \Delta t_{k1} = 0$$
$$f_{k2} + \Delta f_{k1} + \Delta f_{k2} - \Delta t_{k2} = 0$$

$$f_{k1} + \varphi_{k1}^{(w_{k1})} \Delta t_{k1} + \varphi_{k1}^{(w_{k2})} \Delta t_{k2} - \Delta t_{k1} = 0$$

$$f_{k2} + \varphi_{k2}^{(w_{k1})} \Delta t_{k1} + \varphi_{k2}^{(w_{k2})} \Delta t_{k1} - \Delta t_{k2} = 0$$

Multiple Line LODFs

- Hence we can calculate the simultaneous impact of multiple outages; details for the derivation are given in C.Davis, T.J. Overbye, "Linear Analysis of Multiple Outage Interaction," *Proc. 42nd HICSS*, 2009
- Equation for the change in flow on line ℓ for the outage of lines k_1 and k_2 is

$$\Delta f_\ell = \begin{bmatrix} d_\ell^{k1} & d_\ell^{k2} \end{bmatrix} \begin{bmatrix} 1 & -d_{k1}^{k2} \\ -d_{k2}^{k1} & 1 \end{bmatrix}^{-1} \begin{bmatrix} f_{k1} \\ f_{k2} \end{bmatrix}$$

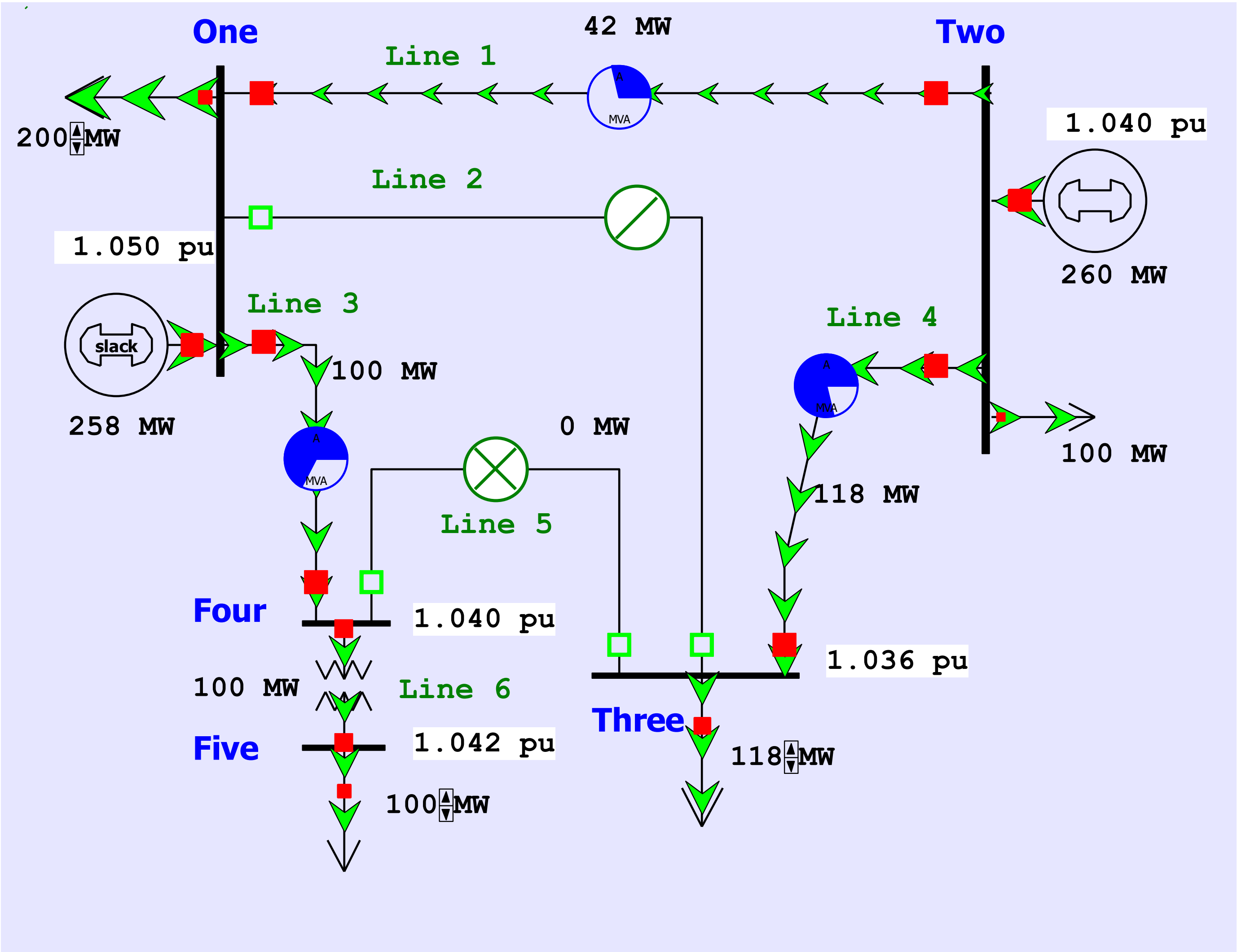
Multiple Line LODFs

- Example: Five bus case, outage of lines 2 and 5 to flow on line 4.

$$\Delta f_\ell = \begin{bmatrix} d_\ell^{k1} & d_\ell^{k2} \end{bmatrix} \begin{bmatrix} 1 & -d_{k1}^{k2} \\ -d_{k2}^{k1} & 1 \end{bmatrix}^{-1} \begin{bmatrix} f_{k1} \\ f_{k2} \end{bmatrix}$$

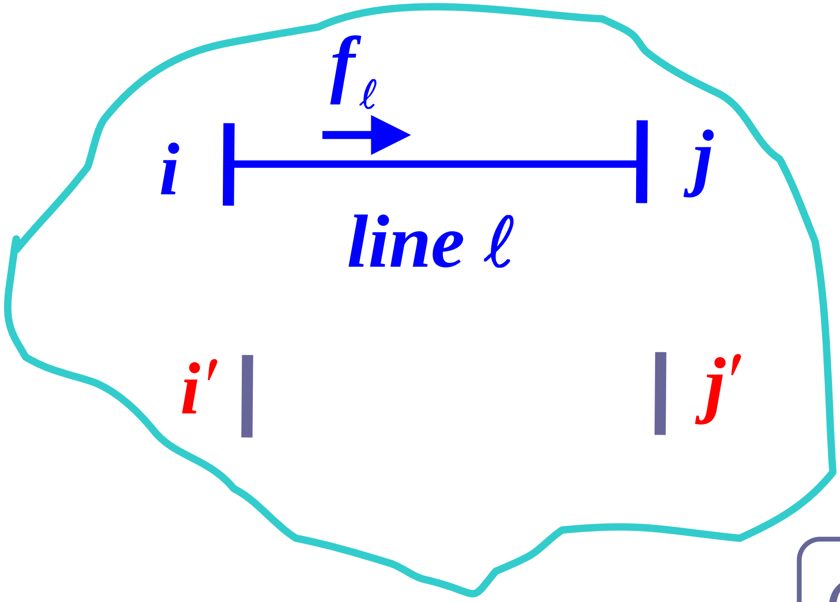
$$\Delta f_\ell = \begin{bmatrix} 0.4 & 0.25 \end{bmatrix} \begin{bmatrix} 1 & -0.75 \\ -0.6 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.336 \\ -0.331 \end{bmatrix} = 0.005$$

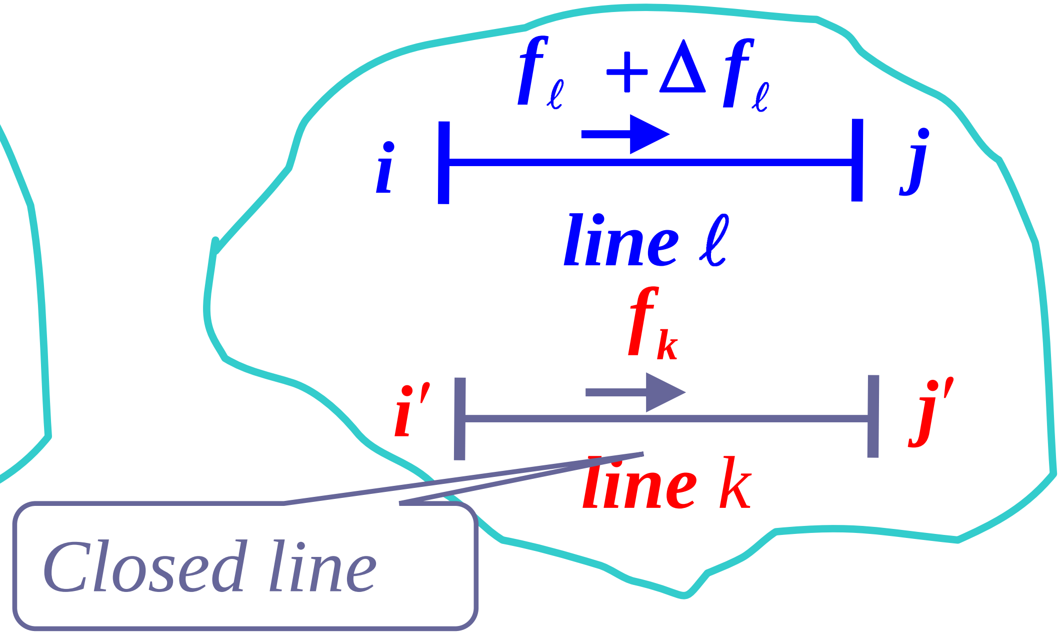
Multiple Line LODFs



Flow goes from 117.5 to 118.0

Line Closure Distribution Factors (LCDFs)





base case

line k addition case

$$LCDF_{\ell}^{k} = \frac{\Delta f_\ell}{f_k} = LCDF_{\ell,k}$$

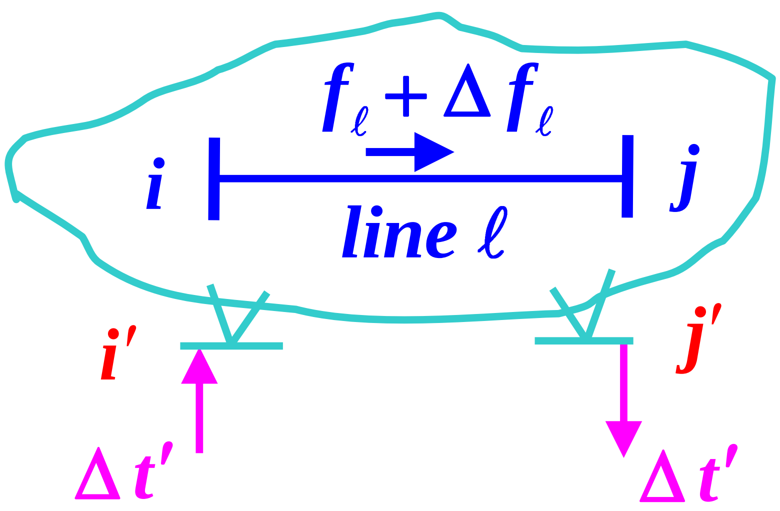
LCDF Definition

- The line closure distribution factor (LCDF), $LCDF_{\ell,k}$, for the closure of line k (or its addition if it does not already exist) is the portion of the line active power flow on line k that is distributed to line ℓ due to the closure of line k
- Since line k is currently open, the obvious question is, "what flow on line k?"
- Answer (in a dc power flow sense) is the flow that will occur when the line is closed (which we do not know)

LCDF Evaluation

- We simulate the impact of the closure of line k by imposing the additional basic transaction

$$w_k = \{ i', j', \Delta t_k \}$$



on the base case network and we select Δt_k so that

$$\Delta t_k = - f_k$$

LCDF Evaluation

- For the other parts of the network, the impacts of the addition of line k are the same as the impacts of adding the basic transaction w_k

$$\Delta f_\ell = \varphi_\ell^{(w_k)} \Delta t_k = -\varphi_\ell^{(w_k)} f_k$$

- Therefore, the definition is

$$LCDF_{\ell,k} = \frac{\Delta f_\ell}{f_k} = -\varphi_\ell^{(w_k)}$$

- The post-closure flow f_k is determined (in a dc power flow sense) as the flow that would occur from the angle difference divided by (1 + $\varphi_k^{(w_k)}$)

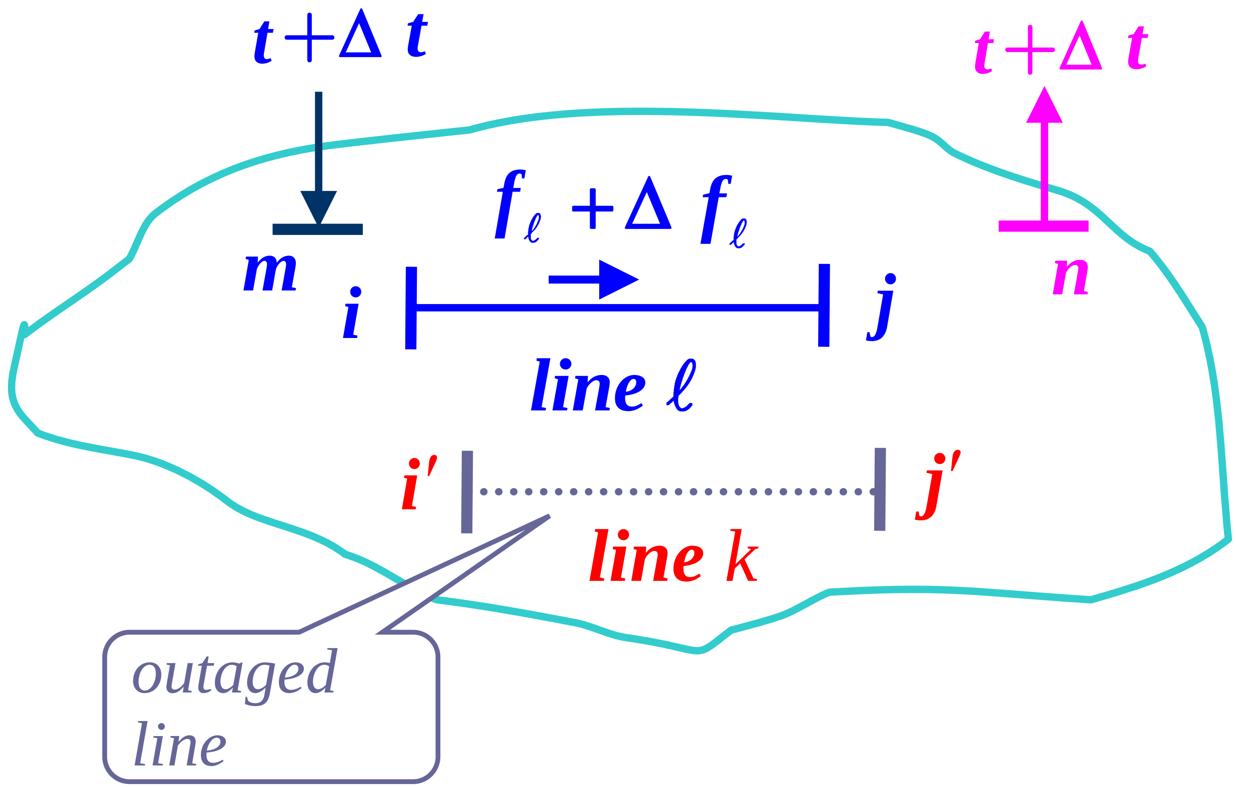
Outage Transfer Distribution Factor

- The outage transfer distribution factor (OTDF) is defined as the PTDF with the line k outaged
- The OTDF applies only to the post-contingency configuration of the system since its evaluation explicitly considers the line k outage

$$\left(\varphi_{\ell}^{(w)} \right)^{k}$$

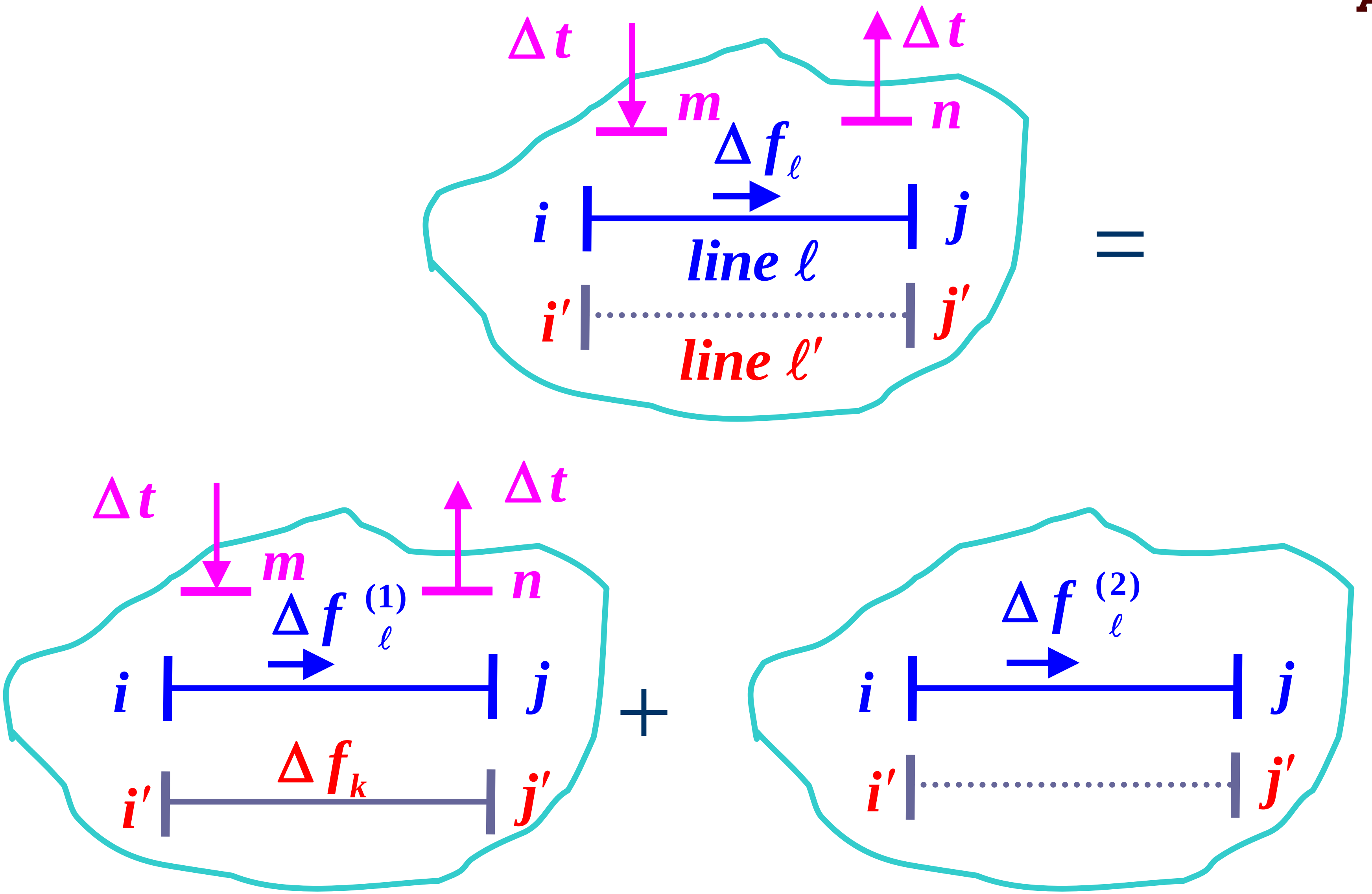
- This is a quite important value since power system operation is usually contingency constrained

Outage Transfer Distribution Factor (OTDF)



$$\left(\varphi_{\ell}^{(w)}\right)^{k} \triangleq \left.\frac{\Delta f_{\ell}}{\Delta t}\right|_{k\ outaged}$$

OTDF Evaluation



OTDF Evaluation

- Since $\Delta f_{\ell}^{(1)} = \varphi_{\ell}^{(w)} \Delta t$

  and $\Delta f_{k} = \varphi_{k}^{(w)} \Delta t$

  then $\Delta f_{\ell}^{(2)} = d_{\ell}^{k} \Delta f_{k} = d_{\ell}^{k} \varphi_{k}^{(w)} \Delta t$

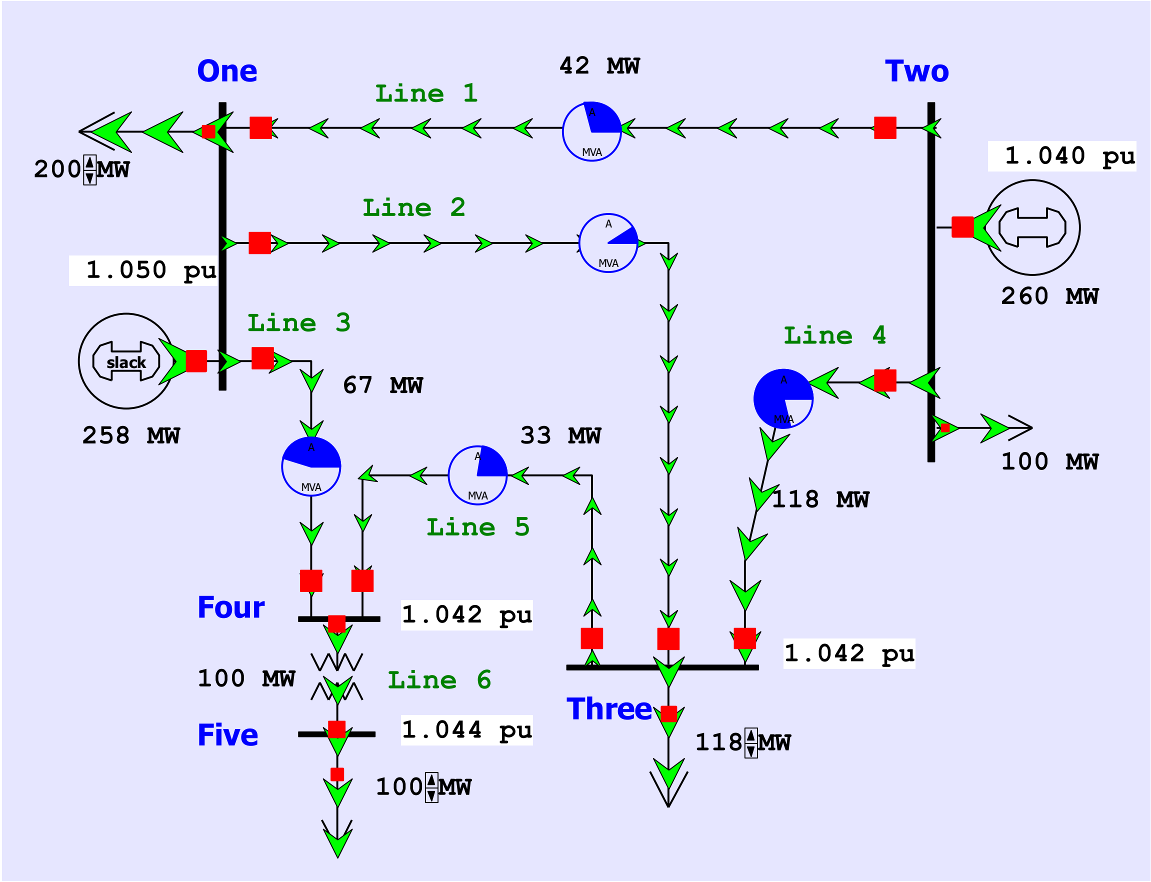
  so that

$$\Delta f_{\ell} = \Delta f_{\ell}^{(1)} + \Delta f_{\ell}^{(2)} = \left[\varphi_{\ell}^{(w)} + d_{\ell}^{k} \varphi_{k}^{(w)} \right] \Delta t$$

$$\left(\varphi_{\ell}^{(w)} \right)^{k} = \varphi_{\ell}^{(w)} + d_{\ell}^{k} \varphi_{k}^{(w)}$$

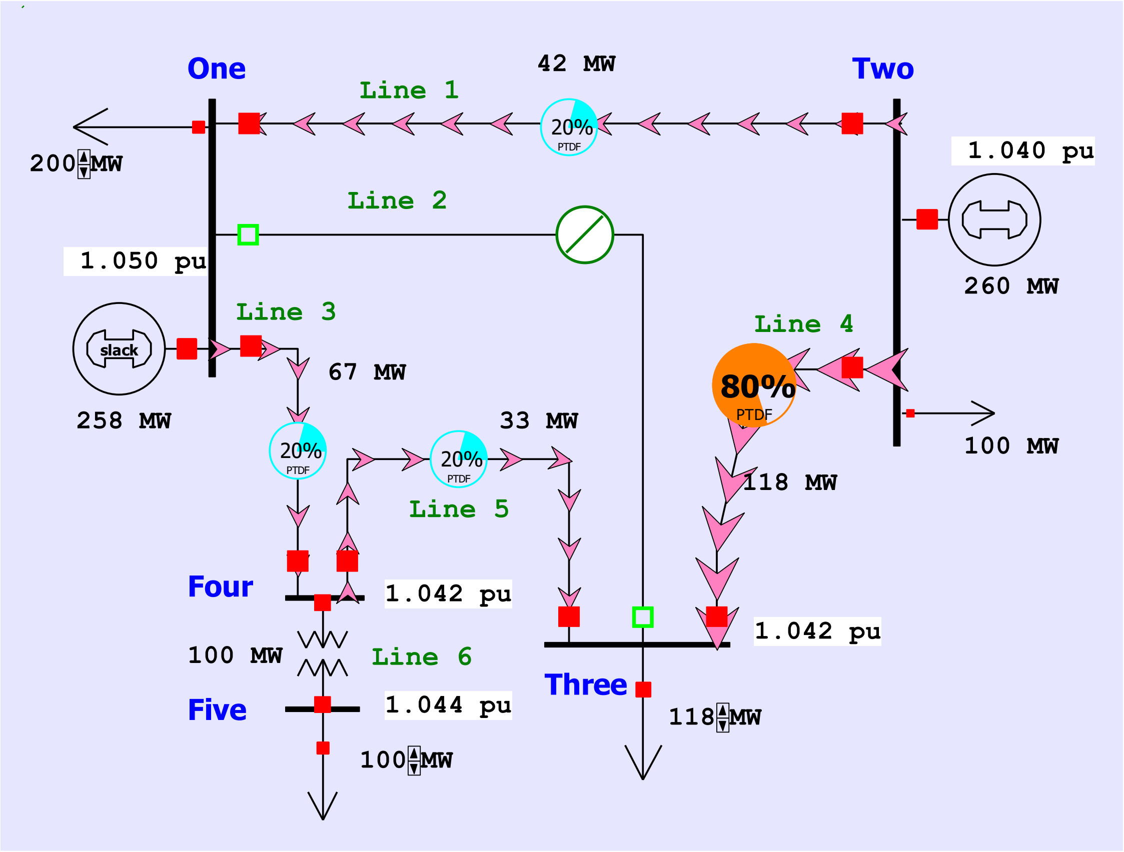
Five Bus Example

- Say we would like to know the PTDF on line 1 for a transaction between buses 2 and 3 with line 2 out



Five Bus Example

- Hence we want to calculate these values without having to explicitly outage line 2



Hence the value we are looking for is 0.2 (20%)

Five Bus Example

- Evaluating: the PTDF for the bus 2 to 3 transaction on line 1 is 0.2727; it is 0.1818 on line 2 (from buses 1 to 3); the LODF is on line 1 for the outage of line 2 is -0.4
- Hence $\left(\varphi_{\ell}^{(w)}\right)^{k} = \varphi_{\ell}^{(w)} + d_{\ell}^{k}\varphi_{k}^{(w)}$

$$\mathbf{0.2727 + (-0.4) \times (0.1818) = 0.200}$$

- For line 4 (buses 2 to 3) the value is

$$\mathbf{0.7273 + (0.4) \times (0.1818) = 0.800}$$

August 14, 2003 OTDF Example

- Flowgate 2264 monitored the flow on Star-Juniper 345 kV line for contingent loss of Hanna-Juniper 345 kV normally the LODF for this flowgate is 0.361
  - flowgate had a limit of 1080 MW
  - at 15:05 EDT the flow as 517 MW on Star-Juniper, 1004 MW on Hanna-Juniper, giving a flowgate value of 520+0.361*1007=884 (82%)
  - Chamberlin-Harding 345 opened at 15:05, but was missed
  - At 15:06 EDT (after loss of Chamberlin-Harding 345) #2265 had an incorrect value because its LODF was not updated.
  - Value should be 633+0.463*1174=1176 (109%)
  - Value was 633 + 0.361*1174=1057 (98%)

UTC Revisited

- We can now revisit the uncommitted transfer capability (UTC) calculation using PTDFs and LODFs
- Recall trying to determine maximum transfer between two areas (or buses in our example)
- For base case maximums are quickly determined with PTDFs

$$u_{m,n}^{(0)} = \min_{\varphi_{\ell}^{(w)}>0} \left\{ \frac{f_{\ell}^{max} - f_{\ell}^{(0)}}{\varphi_{\ell}^{(w)}} \right\}$$

Note we are ignoring zero (or small) PTDFs; would also need to consider flow reversal

UTC Revisited

- For the contingencies we use

$$u_{m,n}^{(1)} = \min_{\left(\varphi_\ell^{(w)}\right)^k > 0} \left\{ \frac{f_\ell^{max} - f_\ell^{(0)} - d_\ell^k f_k^{(0)}}{\left(\varphi_\ell^{(w)}\right)^k} \right\}$$

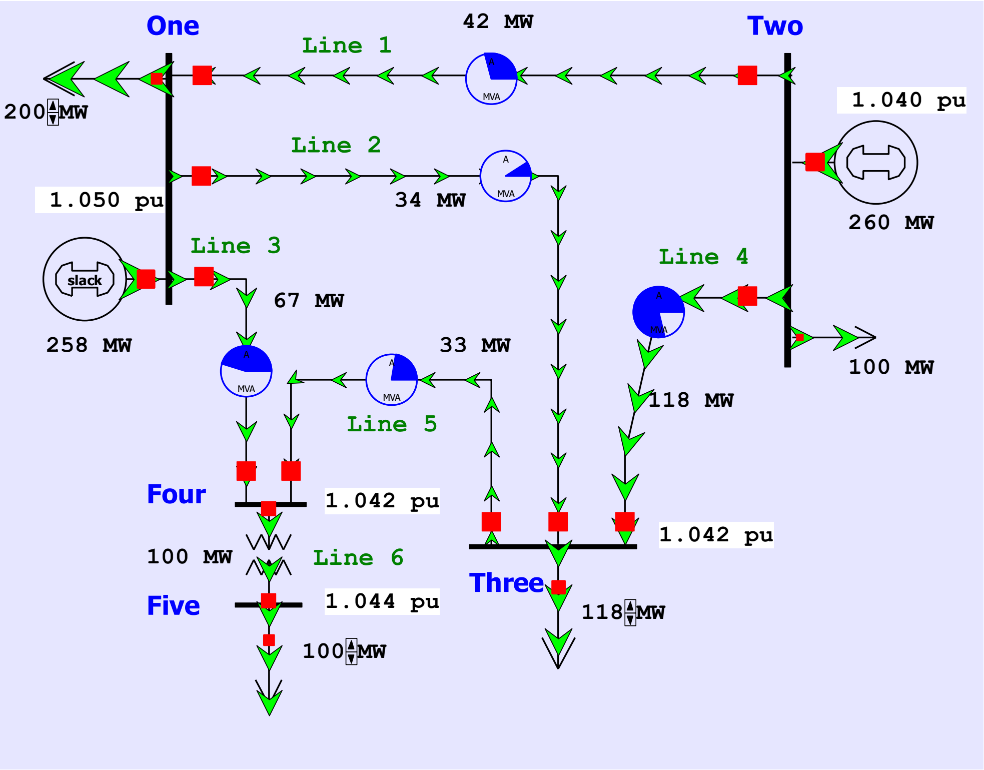
- Then as before $u_{m,n} = \min\left\{u_{m,n}^{(0)},\ u_{m,n}^{(1)}\right\}$

We would need to check all contingencies! Also, this is just a linear estimate and is not considering voltage violations.

Five Bus Example

$$w = \{2, 3, \Delta t\} \qquad \mathbf{f}^{(0)} = [42\,, 34\,, 67\,, 118\,, 33\,, 100]^T$$

$$\mathbf{f}^{max} = [150\,, 400\,, 150\,, 150\,, 150\,, 1{,}000]^T$$



Five Bus Example

Therefore, for the base case

$$u_{2,2}^{(0)} = \min_{\varphi_\ell^{(w)}>0} \left\{ \frac{f_\ell^{max} - f_\ell^{(0)}}{\varphi_\ell^{(w)}} \right\}$$

$$= \min \left\{ \frac{150-42}{0.2727}, \frac{400-34}{0.1818}, \frac{150-67}{0.0909}, \frac{150-118}{0.7273}, \frac{150-33}{0.0909} \right\}$$

$$= 44.0$$

Five Bus Example

- For the contingency case corresponding to the outage of the line 2

$$u_{2,3}^{(1)} = \min_{\left(\varphi_{\ell}^{(w)}\right)^2 > 0} \left\{ \frac{f_{\ell}^{max} - f_{\ell}^{(0)} - d_{\ell}^{2} f_{2}^{(0)}}{\left(\varphi_{\ell}^{(w)}\right)^2} \right\}$$

The limiting value is line 4

$$\frac{f_{\ell}^{max} - f_{\ell}^{(0)} - d_{\ell}^{2} f_{2}^{(0)}}{\left(\varphi_{\ell}^{(w)}\right)^2} = \frac{150 - 118 - 0.4 \times 34}{0.8}$$

Hence the UTC is limited by the contingency to 23.0

Additional Comments

- Distribution factors are defined as small signal sensitivities, but in practice, they are also used for simulating large signal cases
- Distribution factors are widely used in the operation of the electricity markets where the rapid evaluation of the impacts of each transaction on the line flows is required
- Applications to actual system show that the distribution factors provide satisfactory results in terms of accuracy
- For multiple applications that require fast turn around time, distribution factors are used very widely, particularly, in the market environment
- They do not work well with reactive power!

Least Squares

- So far we have considered the solution of $\mathbf{Ax} = \mathbf{b}$ in which $\mathbf{A}$ is a square matrix; as long as $\mathbf{A}$ is nonsingular there is a single solution
  - That is, we have the same number of equations (m) as unknowns (n)
- Many problems are overdetermined in which there more equations than unknowns ($m > n$)
  - Overdetermined systems are usually inconsistent, in which no value of $\mathbf{x}$ exactly solves all the equations
- Underdetermined systems have more unknowns than equations ($m < n$); they never have a unique solution but are usually consistent