ECEN 615
Methods of Electric Power Systems Analysis

Lecture 21: Equivalencing,Voltage Stability, PV and QV Curves

Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

TEXAS A&M UNIVERSITY

Announcements

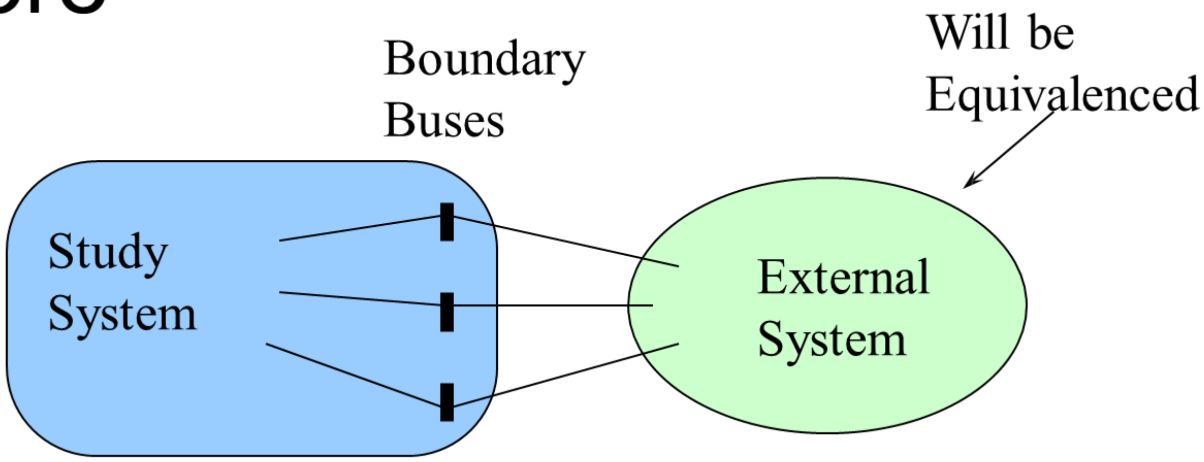
- Homework 5 is due on Tuesday Nov 13

Study vs External System

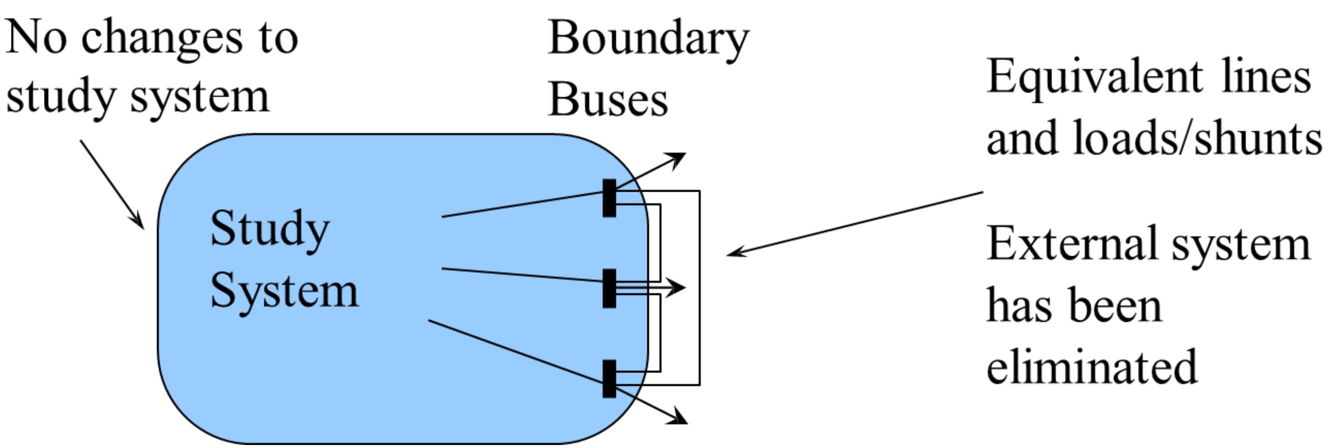
- The key decision in creating an equivalent is to divide the system into a study portion that is represented in detail, and an external portion that is represented by the equivalent
- The two systems are joined at boundary buses, which are part of the study subsystem
- How this is done is application specific; for example:
  - for real-time use it does not make sense to retain significant portions of the grid for which there is no real-time information
  - for contingency analysis the impact of the contingency is localized
  - for planning the new system additions have localized impacts

Ward Type Equivalencing

Before



After



Ward Type Equivalencing Considerations

- The Ward equivalent is calculated by doing a partial factorization of the $\mathbf{Y}_{\text{bus}}$
  - The equivalent buses are numbered after the study buses
  - As the equivalent buses are eliminated their first neighbors are joined together
  - At the end, many of the boundary buses are connected
  - This can GREATLY decrease the sparsity of the system
  - Buses with different voltages can be directly connected

$$\begin{bmatrix} I_s \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ss} & Y_{se} \\ Y_{es} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_s \\ V_e \end{bmatrix}$$

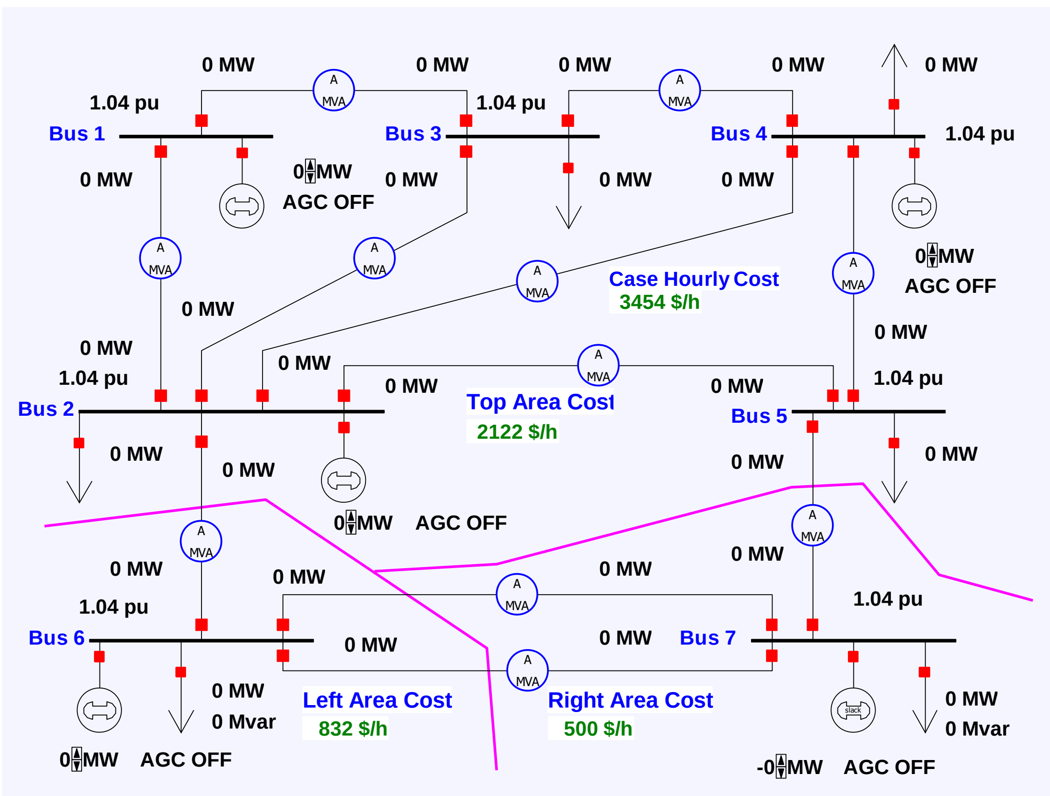
$$\left(I_s - Y_{se} Y_{ee}^{-1} I_e \right) = \left(Y_{ss} - Y_{se} Y_{ee}^{-1} Y_{es} \right) V_s$$

Ward Type Equivalencing Considerations

- At the end of the Ward process often many of the new equivalent lines have high impedances
  - Often there is an impedance threshold, and lines with impedances above this value are eliminated
- The equivalent lines may have unusual values, including negative resistances
- Load and generation is represented as equivalent current injections or shunts; sometimes these values are converted back to constant power
- Consideration needs to be given to loss of reactive support
- The equivalent embeds the present load and gen values

B7Flat_Eqv Example

- In this example the B7Flat_Eqv case is reduced, eliminating buses 1, 3 and 4. The study system is then 2, 5, 6, 7, with buses 2 and 5 the boundary buses



For ease of comparison system is modeled unloaded

B7Flat_Eqv Example

- Original $\mathbf{Y}_{\text{bus}}$

$$\mathbf{Y}_{bus} = j\begin{bmatrix} -20.83 & 16.67 & 4.17 & 0 & 0 & 0 & 0 \\ 16.67 & -52.78 & 5.56 & 5.56 & 8.33 & 16.67 & 0 \\ 4.17 & 5.56 & -43.1 & 33.3 & 0 & 0 & 0 \\ 0 & 5.56 & 33.3 & -43.1 & 4.17 & 0 & 0 \\ 0 & 8.33 & 0 & 4.17 & -29.17 & 0 & 16.67 \\ 0 & 16.67 & 0 & 0 & 0 & -25 & 8.33 \\ 0 & 0 & 0 & 0 & 16.67 & 8.33 & -25 \end{bmatrix}$$

$$\mathbf{Y}_{ee} = j\begin{bmatrix} -20.833 & 4.167 & 0 \\ 4.167 & -43.056 & 33.333 \\ 0 & 33.333 & -43.056 \end{bmatrix}$$

B7Flat_Eqv Example

$$\mathbf{Y}_{es} = j\begin{bmatrix} 16.667 & 0 & 0 & 0 \\ 5.556 & 0 & 0 & 0 \\ 5.556 & 4.167 & 0 & 0 \end{bmatrix} \qquad \mathbf{Y}_{se} = j\begin{bmatrix} 16.667 & 5.556 & 5.556 \\ 0 & 0 & 4.167 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

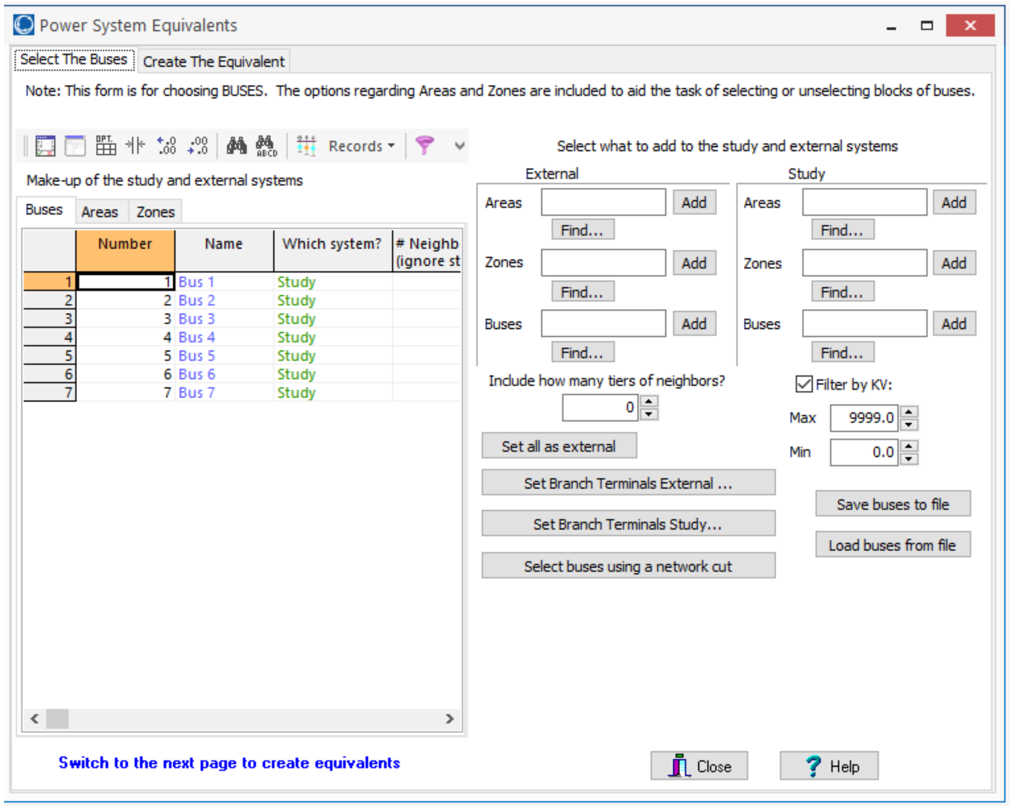
$$\mathbf{Y}_{ss} = j\begin{bmatrix} -52.778 & 8.333 & 16.667 & 0 \\ 8.333 & -29.167 & 0 & 16.667 \\ 16.667 & 0 & -25.0 & 8.333 \\ 0 & 16.667 & 8.333 & -25.0 \end{bmatrix}$$

Note $\mathbf{Y}_{es}=\mathbf{Y}_{se}'$ if no phase shifters

$$\left(\mathbf{Y}_{ss} - \mathbf{Y}_{se}\mathbf{Y}_{ee}^{-1}\mathbf{Y}_{es}\right) = j\begin{bmatrix} -28.128 & 11.463 & 16.667 & 0 \\ 11.463 & -28.130 & 0 & 16.667 \\ 16.667 & 0 & -25.0 & 8.333 \\ 0 & 16.667 & 8.333 & -25.0 \end{bmatrix}$$

Equivalencing in PowerWorld

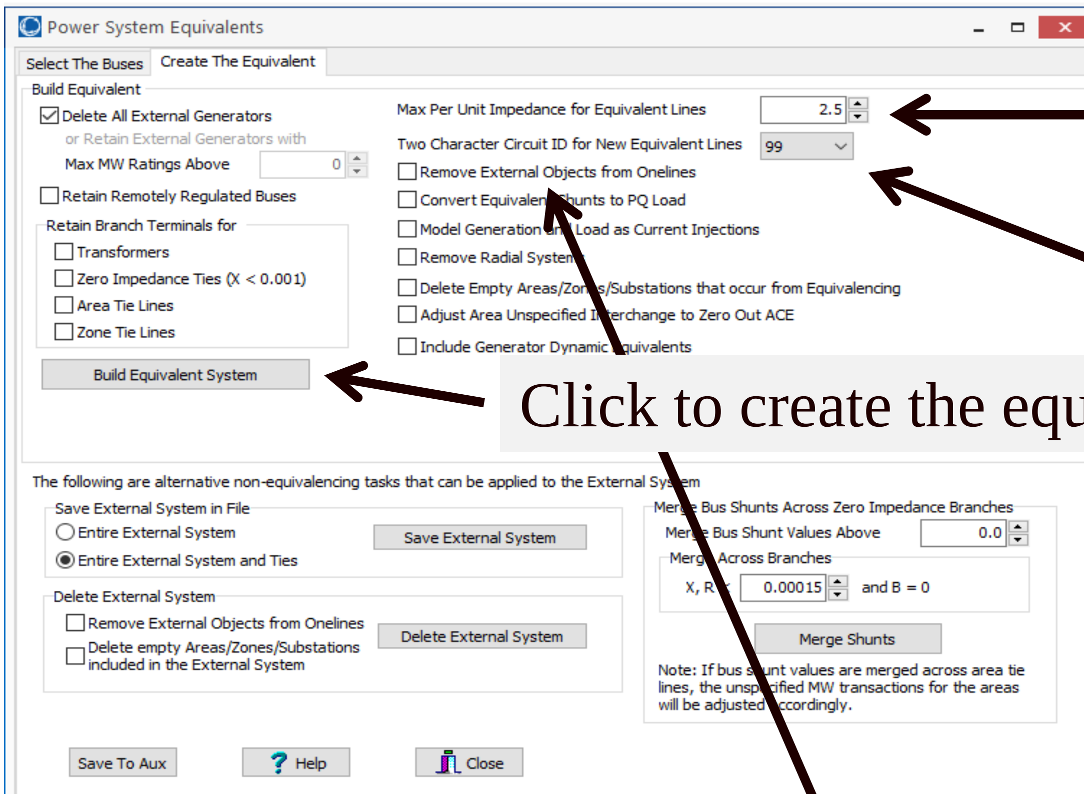
- Open a case and solve it; then select **Edit Mode**, **Tools**, **Equivalencing;** this displays the Power System Equivalents Form



Next step is then to divide the buses into the study system and the external system; buses can be loaded from a text file as well

Equivalencing in PowerWorld

- Then go to the **Create The Equivalent** page, select the desired options and select **Build Equivalent System**



Maximum impedance lines to retain

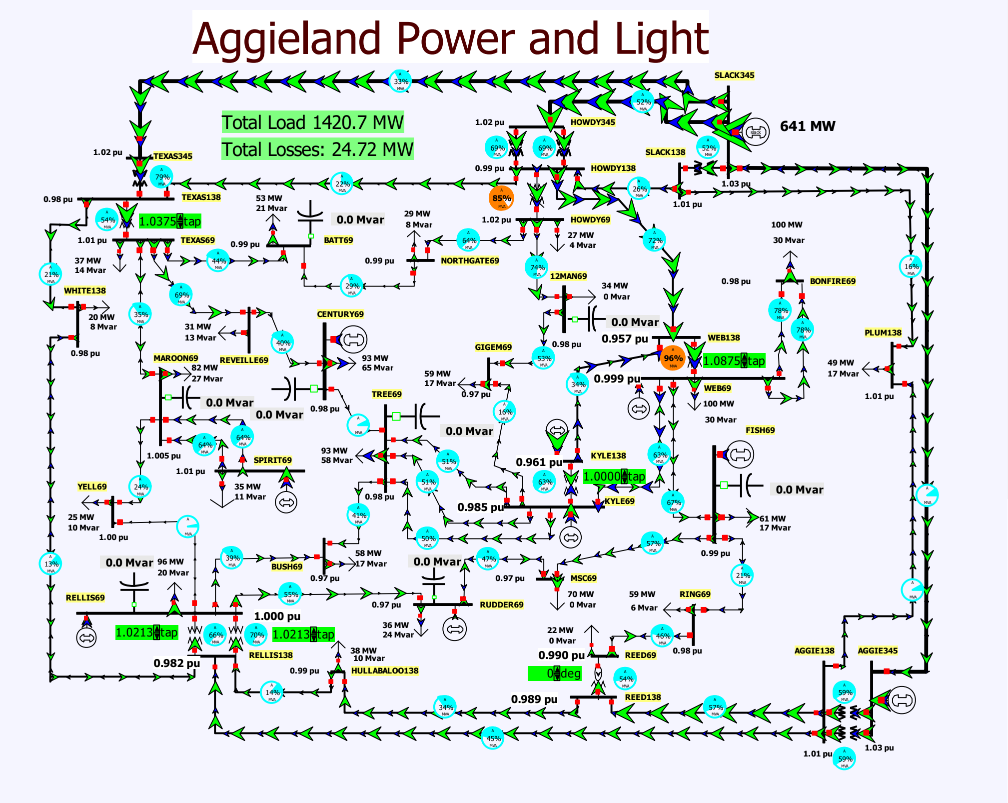
'99' or 'EQ' are common circuit values for equivalent lines

Click to create the equivalent

Removes equivalenced objects from the oneline

Small System Equivalent Example

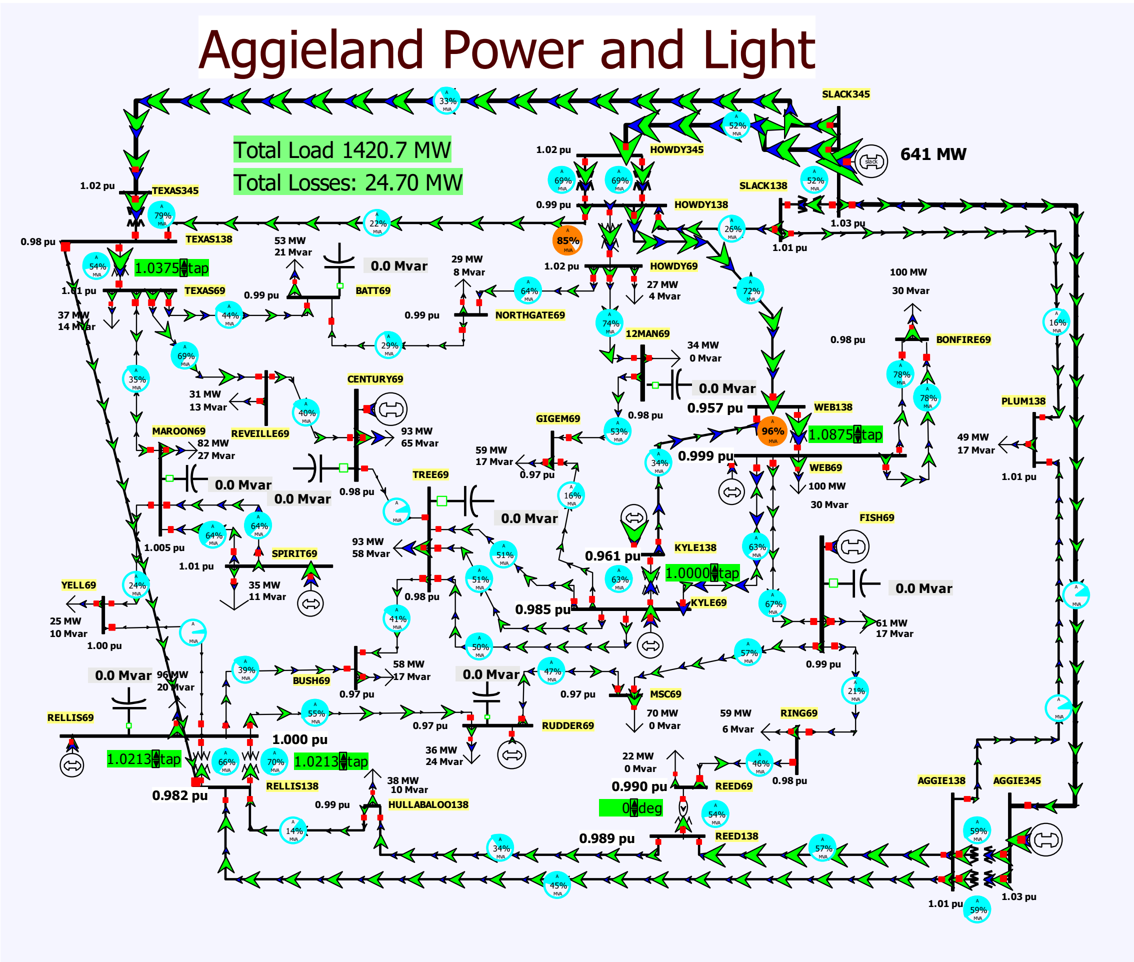
- Example shows the creation of an equivalent for Aggieland37 example



First example is simple, just removing WHITE138 (bus 3); note TEXAS138 is now directly joined to RELLIS138..

Case is Aggieland37_HW5

Small System Equivalent Example

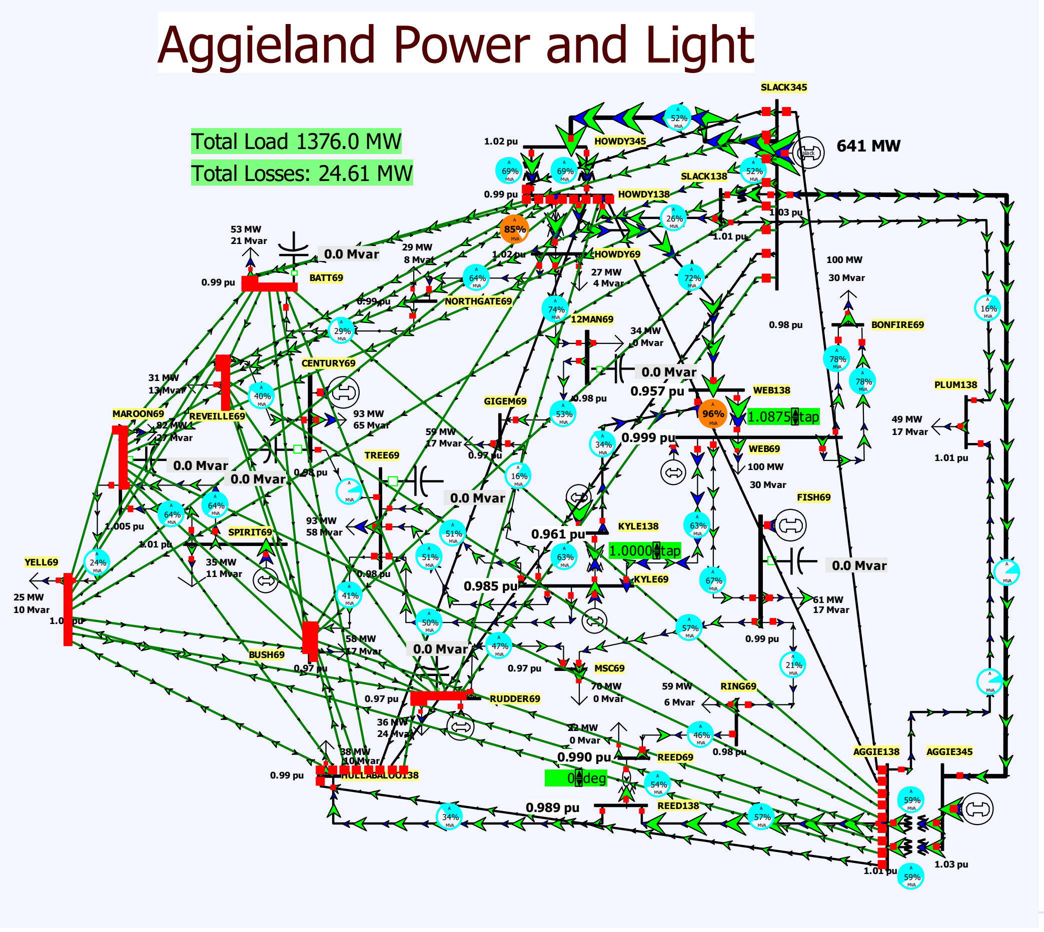


Only bus 3 was removed; the new equivalent line was auto-inserted.

Don’t save the equivalent with the same name as the original, unless you want to lose the original

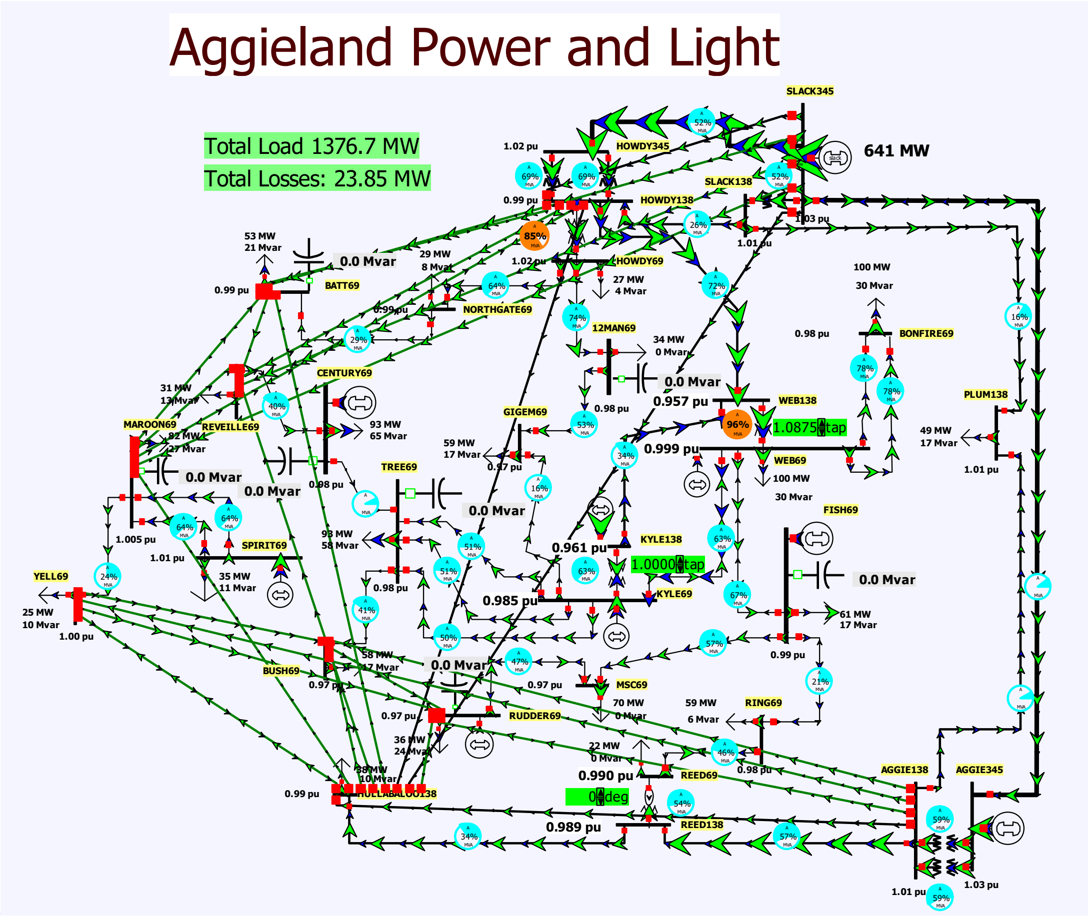
Small System Equivalent Example

- Now remove buses at WHITE138 and TEXAS and RELLIS (1, 3, 12, 40, 41, 44); set **Max Per Unit Impedance for Equivalent Lines** to 99 (per unit) to retain all lines. Again to an auto-insert to show the equivalent lines.



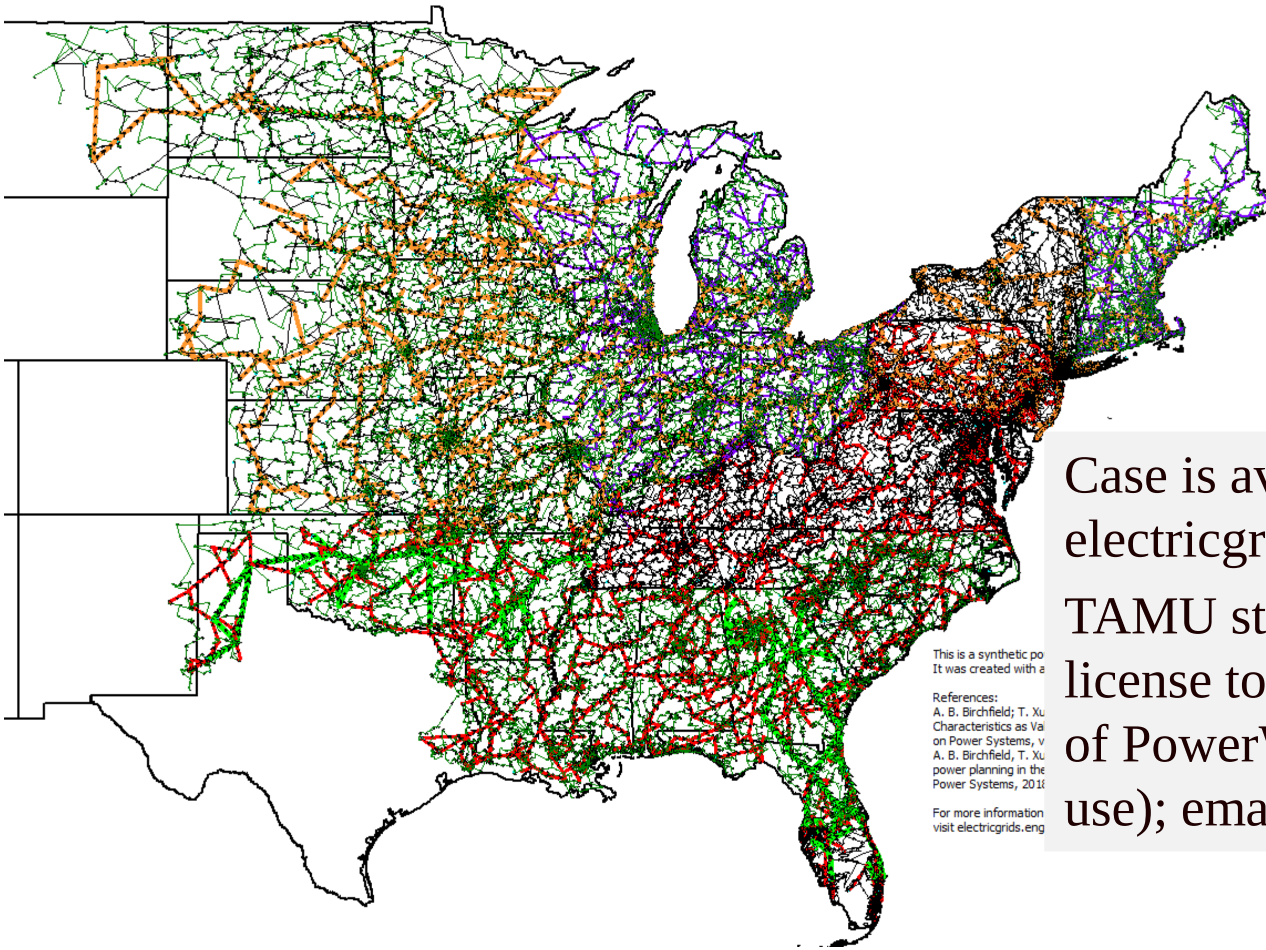
Small System Equivalent Example

- Now set the **Max Per Unit Impedance for Equivalent Lines to 2.5.**



Large System Example: 70K Case

- Original System has 70,000 buses and 71,343 lines

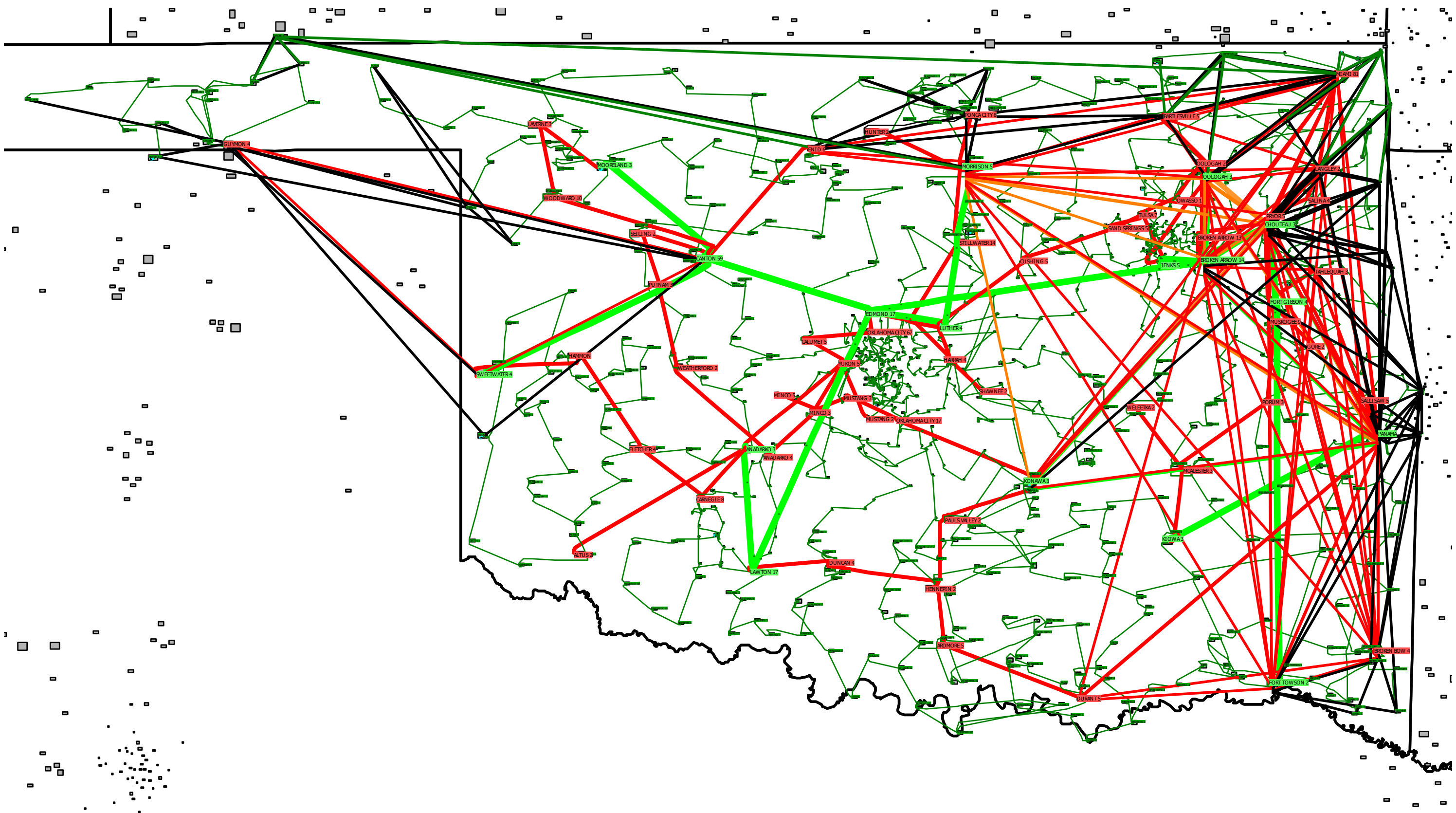


Case is available at electricgrids.engr.tamu.edu;

TAMU students can get a license to the full version of PowerWorld (for educational use); email Iyke for details

Large System Example: 70K Case

- Just retain the Oklahoma Area; now 1591 buses and 1745 lines (deleting ones above 2.5 pu impedance)



Grid Equivalent Examples

- A 2016 EI case had about 350 lines with a circuit ID of '99' and about 60 with 'EQ' (out of a total of 102,000)
  - Both WECC and the EI use '99' or 'EQ' circuit IDs to indicate equivalent lines
  - One would expect few equivalent lines in interconnect wide models
- A ten year old EI case had about 1633 lines with a circuit ID of '99' and 400 with 'EQ' (out of a total of 65673)
- A ten year old case with about 5000 buses and 5000 lines had 600 equivalent lines

Power System Voltage Stability

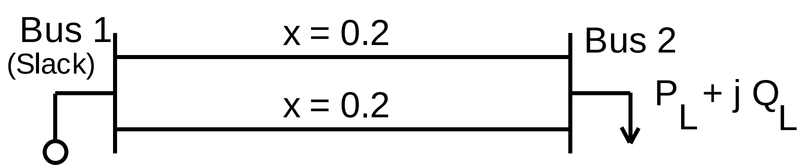
- **Voltage Stability**: The ability to maintain system voltage so that both power and voltage are controllable. System voltage responds as expected (i.e., an increase in load causes proportional decrease in voltage).
- **Voltage Instability**: Inability to maintain system voltage. System voltage and/or power become uncontrollable. System voltage does not respond as expected.
- **Voltage Collapse**: Process by which voltage instability leads to unacceptably low voltages in a significant portion of the system. Typically results in loss of system load.

Voltage Stability

- Two good references are
  - P. Kundur, et. al., "Definitions and Classification of Power System Stability," *IEEE Trans. on Power Systems*, pp. 1387-1401, August 2004.
  - T. Van Cutsem, "Voltage Instability: Phenomena, Countermeasures, and Analysis Methods," *Proc. IEEE*, February 2000, pp. 208-227.
- Classified by either size of disturbance or duration
  - Small or large disturbance: small disturbance is just perturbations about an equilibrium point (power flow)
  - Short-term (several seconds) or long-term (many seconds to minutes) (covered in ECEN 667)

Small Disturbance Voltage Stability

- Small disturbance voltage stability can be assessed using a power flow (maximum loadability)
- Depending on the assumed load model, the power flow can have multiple (or now solutions)
- PV curve is created by plotting power versus voltage



Assume V_{slack}=1.0

$$P_L - BV\sin\theta = 0$$

$$Q_L + BV\cos\theta - BV^2 = 0$$

Where B is the line susceptance =-10,
$V\angle\theta$ is the load voltage

Small Disturbance Voltage Stability

- Question: how do the power flow solutions vary as the load is changed?
- A Solution: Calculate a series of power flow solutions for various load levels and see how they change
- Power flow Jacobian

$$\mathbf{J}(\theta,V)=\begin{bmatrix} -BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta-2BV \end{bmatrix}$$

$$\det\mathbf{J}(\theta,V)=VB^2\left(2V\cos\theta-\cos^2\theta-\sin^2\theta\right)$$

Singular when $\left(2V\cos\theta-1\right)=0$

Maximum Loadability When Power Flow Jacobian is Singular

- An important paper considering this was by Sauer and Pai from IEEE Trans. Power Systems in Nov 1990, "Power system steady-state stability and the load-flow Jacobian"
- Other earlier papers were looking at the characteristics of multiple power flow solutions
- Work with the power flow optimal multiplier around the same time had shown that optimal multiplier goes to zero as the power flow Jacobian becomes singular
- The power flow Jacobian depends on the assumed load model (we'll see the impact in a few slides)

Relationship Between Stability and Power Flow Jacobian

- The Sauer/Pai paper related system stability to the power flow Jacobian by noting the system dynamics could be written as a set of differential algebraic equations

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{p})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{p})$$

Linearing about and equilibrium gives

$$\begin{bmatrix} \Delta\dot{\mathbf{x}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} & \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{x}} & \frac{\partial \mathbf{g}}{\partial \mathbf{y}} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{y} \end{bmatrix}$$

Relationship Between Stability and Power Flow Jacobian

- Then

Assuming $\dfrac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is nonsingular then

$$\Delta \dot{\mathbf{x}} = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} - \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \left[\frac{\partial \mathbf{g}}{\partial \mathbf{y}} \right]^{-1} \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \right] \Delta \mathbf{x}$$

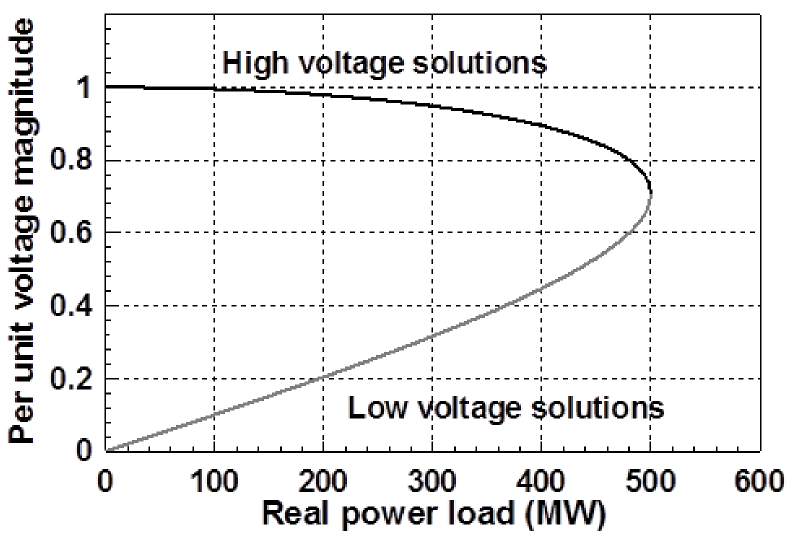
- What Sauer and Pai show is if $\partial \mathbf{g} / \partial \mathbf{y}$ is singular then the system is unstable; if $\partial \mathbf{g} / \partial \mathbf{y}$ is nonsingular then the system may or may not be stable
- Hence it provides an upper bound on stability

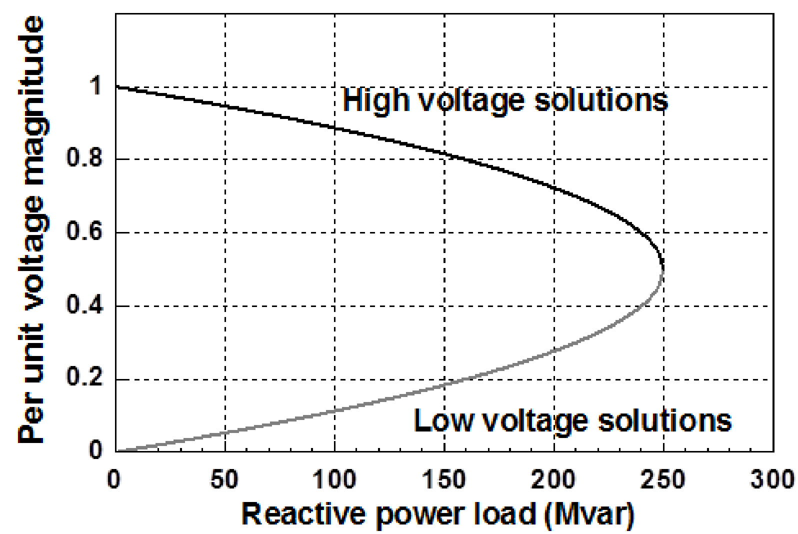
Bifurcations

- In general, bifurcation is the division of something into two branches or parts
- For a dynamic system, a bifurcation occurs when small changes in a parameter cause a new quality of motion of the dynamic system
- Two types of bifurcation are considered for voltage stability
  - Saddle node bifurcation is the disappearance of an equilibrium point for parameter variation; for voltage stability it is two power flow solutions coalescing with parameter variation
  - Hopf bifurcation is cause by two eigenvalues crossing into the right-half plane

PV and QV Curves

- PV curves can be traced by plotting the voltage as the real power is increased; QV curves as reactive power is increased
  – At least for the upper portion of the curve
- Two bus example PV and QV curves





Small Disturbance Voltage Collapse

- At constant frequency (e.g., 60 Hz) the complex power transferred down a transmission line is $S=VI^*$
  - V is phasor voltage, I is phasor current
  - This is the reason for using a high voltage grid
- Line real power losses are given by RI^2 and reactive power losses by XI^2
  - R is the line's resistance, and X its reactance; for a high voltage line X >> R
- Increased reactive power tends to drive down the voltage, which increases the current, which further increases the reactive power losses