ECEN 615 Problem Set #5 Fall 2018

Due 11/13/18

1. Book problem 9.1. Assume a linear (dc power flow approximation) system model. That is, **f(x)** = **Hx**.

a) $M_{12} = \frac{\theta_1 - \theta_2}{0.2}$; $M_{13} = \frac{\theta_1 - \theta_3}{0.4}$; $M_{32} = \frac{\theta_3 - \theta_2}{0.25}$; $\boldsymbol{H} = \begin{bmatrix} 5.0 & -5.0 \\ 2.5 & 0 \\ 0 & -4.0 \end{bmatrix}$; $\boldsymbol{z}^{meas} = \begin{bmatrix} 0.60 \\ 0.40 \\ 0.405 \end{bmatrix} p.u$; $\theta_3 = 0$

$$\boldsymbol{R} = \begin{bmatrix} 4 \times 10^{-4} & 0 & 0 \\ 0 & 1 \times 10^{-4} & 0 \\ 0 & 0 & 4 \times 10^{-6} \end{bmatrix}$$

$$\boldsymbol{x}^{est} = [\boldsymbol{H}'\boldsymbol{R}^{-1}\boldsymbol{H}]^{-1}\boldsymbol{H}'\boldsymbol{R}^{-1}\boldsymbol{z}^{meas}$$

$$\boldsymbol{x}^{est} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0.0174 \\ -0.1013 \end{bmatrix} radians$$

b) Residual, $J(x) = J(\theta_1, \theta_2) = \frac{[z_{12} - M_{12}(\theta_1,\theta_2)]^2}{\sigma_{12}{}^2} + \frac{[z_{13} - M_{13}(\theta_1,\theta_3)]^2}{\sigma_{13}{}^2} + \frac{[z_{32} - M_{32}(\theta_2,\theta_3)]^2}{\sigma_{32}{}^2}$ = 0.2345

Degrees of freedom (K) = Number of measurements – number of states = 3-2 = 1

Using a Chi distribution table, for a significant level ($\alpha = 0.01$) and $K = 1$, the threshold residual, $t_j = 6.635$

Since J(x) $<<$ t_j, it is safe to assume the likely absence of bad data in the measurements

2. Book problem 9.3. Again assume a linear system model.

a) The network is unobservable since there are no known measurements that pertain to bus 4

$$M_{13} = \frac{\theta_1 - \theta_3}{0.5} \; ; \; M_{31} = \frac{\theta_3 - \theta_1}{0.5} \; ; \; M_{12} = \frac{\theta_1 - \theta_2}{0.25}; \; \boldsymbol{H} = \begin{bmatrix} 2 & 0 & -2 \\ -2 & 0 & 2 \\ 4 & -4 & 0 \end{bmatrix}; \; \boldsymbol{z}^{meas} = \begin{bmatrix} -0.705 \\ 0.721 \\ 0.212 \end{bmatrix} p.u\, ;$$

$$\theta_3 = 0$$

$$\boldsymbol{R} = \begin{bmatrix} 1 \times 10^{-4} & 0 & 0 \\ 0 & 1 \times 10^{-4} & 0 \\ 0 & 0 & 4 \times 10^{-4} \end{bmatrix}$$

$$\boldsymbol{H}'\boldsymbol{R}^{-1}\boldsymbol{H} = \begin{bmatrix} 12000 & 40000 & -80000 \\ -4000 & 40000 & 0 \\ -8000 & 0 & 80000 \end{bmatrix}$$

$\boldsymbol{H}'\boldsymbol{R}^{-1}\boldsymbol{H}$ is singular (hence, invertible) because the system is unobservable

b) If $M_{3,gen}$ is available, $M_{34} = P_{31} + P_{34} = \frac{\theta_3 - \theta_1}{0.5} + \frac{\theta_3 - \theta_4}{x_{34}}$

$$\boldsymbol{H} = \begin{bmatrix} 2 & 0 & -2 \\ -2 & 0 & 2 \\ 4 & -4 & 0 \\ -2 & 0 & 12 \end{bmatrix}; \boldsymbol{R} = \begin{bmatrix} 1\times10^{-4} & 0 & 0 & 0 \\ 0 & 1\times10^{-4} & 0 & 0 \\ 0 & 0 & 4\times10^{-4} & 0 \\ 0 & 0 & 0 & 2.25\times10^{-4} \end{bmatrix}; \ \boldsymbol{z}^{\boldsymbol{meas}} = \begin{bmatrix} -0.705 \\ 0.721 \\ 0.212 \\ 0.920 \end{bmatrix} p.u$$

$$\boldsymbol{H}'\boldsymbol{R}^{-1}\boldsymbol{H} = \begin{bmatrix} 137780 & -40000 & -186670 \\ -40000 & 40000 & 0 \\ -186670 & 0 & 720000 \end{bmatrix}$$

$\boldsymbol{H}'\boldsymbol{R}^{-1}\boldsymbol{H}$ is now full-rank (hence, invertible) and thus observable

$$\boldsymbol{x}^{\boldsymbol{est}} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -0.3358 \\ -0.3888 \\ 0.0207 \end{bmatrix} radians$$

3. Do the next two iterations of the two bus, ac (i.e., nonlinear) state estimation example from lecture 19.

$$\boldsymbol{X}^{\boldsymbol{1}} = \begin{bmatrix} 1.003 \\ -0.2 \\ 0.8775 \end{bmatrix}; \boldsymbol{X}^{\boldsymbol{2}} = \begin{bmatrix} 1.030 \\ -0.217 \\ 0.901 \end{bmatrix}; \ \boldsymbol{X}^{\boldsymbol{3}} = \begin{bmatrix} 1.033 \\ -0.214 \\ 0.905 \end{bmatrix}$$

4. Using the Givens Rotation algorithm, manually perform a QR factorization of the matrix given below. Show your work at each step.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Eliminate A(3,1): b =5, a = 3

$$\boldsymbol{G_1} = \begin{bmatrix} 1.000 & 0 & 0 \\ 0 & -0.515 & 0.858 \\ 0 & -0.858 & -0.515 \end{bmatrix}; \boldsymbol{G}_{\boldsymbol{1}}'\boldsymbol{A} = \begin{bmatrix} 1.000 & 2.000 \\ -5.831 & -7.2029 \\ 0 & 0.343 \end{bmatrix}$$

Eliminate A(2,1): b =-5.831, a = 1

$$\boldsymbol{G_2} = \begin{bmatrix} 0.169 & 0.986 & 0 \\ -0.986 & 0.169 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \boldsymbol{G}_{\boldsymbol{2}}'\boldsymbol{G}_{\boldsymbol{1}}'\boldsymbol{A} = \begin{bmatrix} 5.916 & 7.437 \\ 0 & 0.754 \\ 0 & 0.343 \end{bmatrix}$$

Eliminate A(3,2): b =0.343, a = 0.754

$$\boldsymbol{G_3} = \begin{bmatrix} 1.000 & 0 & 0 \\ 0 & 0.910 & -0.414 \\ 0 & 0.414 & 0.910 \end{bmatrix}; \boldsymbol{G}_{\boldsymbol{3}}'\boldsymbol{G}_{\boldsymbol{2}}'\boldsymbol{G}_{\boldsymbol{1}}'\boldsymbol{A} = \begin{bmatrix} 5.916 & 7.437 \\ 0 & 0.828 \\ 0 & 0 \end{bmatrix}$$

Thus, $\boldsymbol{G_1G_2G_3G_3'G_2'G_1'A} = \boldsymbol{QU} = \begin{bmatrix} 0.169 & 0.897 & -0.408 \\ 0.507 & 0.276 & 0.817 \\ 0.845 & -0.345 & -0.408 \end{bmatrix} \begin{bmatrix} 5.916 & 7.437 \\ 0 & 0.828 \\ 0 & 0 \end{bmatrix}$

Depending on the order of zero-ing the lower, non-diagonal, matrix entry, other approximate QU matrices will include:

$$\boldsymbol{QU} = \begin{bmatrix} -0.169 & 0.897 & 0.408 \\ -0.507 & 0.276 & -0.817 \\ -0.845 & -0.345 & 0.408 \end{bmatrix} \begin{bmatrix} -5.916 & -7.437 \\ 0 & 0.828 \\ 0 & 0 \end{bmatrix}$$

$$\boldsymbol{QU} = \begin{bmatrix} 0.169 & -0.897 & 0.408 \\ 0.507 & -0.276 & -0.817 \\ 0.845 & 0.345 & 0.408 \end{bmatrix} \begin{bmatrix} 5.916 & 7.437 \\ 0 & -0.828 \\ 0 & 0 \end{bmatrix}$$

5. Not graded

6. Using the data for the B7Flat_DC PowerWorld case from Problem Set 4, manually create an equivalent eliminating buses 2, 3, and 6. Give the bus admittance matrix for the modified system, and the impedance of the new equivalent lines.

$$\boldsymbol{Y_{ee}} = \begin{bmatrix} -52.778 & 5.556 & 16.667 \\ 5.556 & -43.056 & 0 \\ 16.667 & 0 & -25.000 \end{bmatrix}$$

$$\boldsymbol{Y_{es}} = \begin{bmatrix} 16.667 & 5.556 & 8.333 & 0.0 \\ 4.1667 & 33.333 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 8.333 \end{bmatrix}$$

$$\boldsymbol{Y_{se}} = \boldsymbol{Y_{es}}'$$

$$\boldsymbol{Y_{ss}} = \begin{bmatrix} -20.833 & 0.0 & 0.0 & 0.0 \\ 0.0 & -43.056 & 4.167 & 0.0 \\ 0.0 & 4.167 & -29.167 & 16.667 \\ 0.0 & 0.0 & 16.667 & -25.000 \end{bmatrix}$$

$$\boldsymbol{Y_{eq}} = \boldsymbol{Y_{ss}} - \boldsymbol{Y_{se}}\boldsymbol{Y_{ee}}^{-1}\boldsymbol{Y_{es}}$$

$$\boldsymbol{Y_{eq}} = j\begin{bmatrix} -13.202 & 7.367 & 3.501 & 2.334 \\ 7.367 & -14.877 & 6.173 & 1.337 \\ 3.501 & 6.172 & -27.471 & 17.797 \\ 2.334 & 1.337 & 17.797 & -21.469 \end{bmatrix}$$

New bus index for equivalent system: {**1,4,5,7**} = {**1’,2’,3’4’**}, thus

Line	$\boldsymbol{Y_{eq}(i,j)}$	$\boldsymbol{X_{eq}(i,j) = -1/\,Y_{eq}(i,j)}$
1'-2'	$j\,7.367$	$j\,0.136$
1'-3'	$j\,3.501$	$j\,0.286$
1'-4'	$j\,2.334$	$j\,0.428$
2'-3'	$j\,6.173$	$j\,0.162$
2'-4'	$j\,1.337$	$j\,0.748$
3'-4'	$j\,17.797$	$j\,0.056$

7. In PowerWorld Simulator using the Aggieland37_HW5 case, first calculate the line flows and bus voltage magnitudes for the contingent opening of both of the transformers between

buses 41 and 44. You may wish to store these results in a spreadsheet. Then, reopen the case (i.e., without the contingency) and in PowerWorld create an equivalent eliminating all the buses with bus numbers less than 21. Then, repeat the previous contingency, and compare the results with the full system (obviously only comparing for the retained buses and lines).

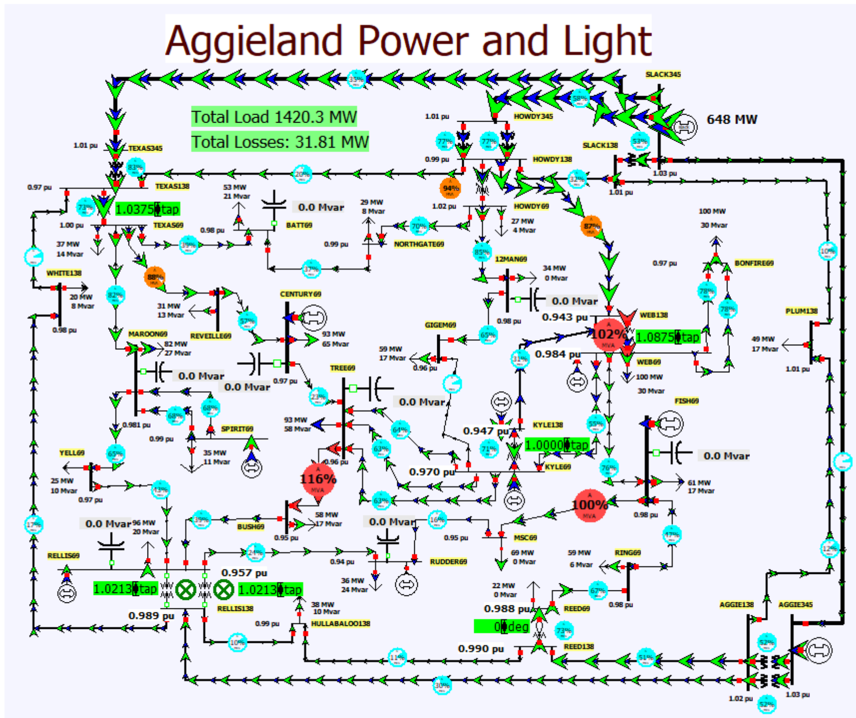


Fig. 1. Base Case

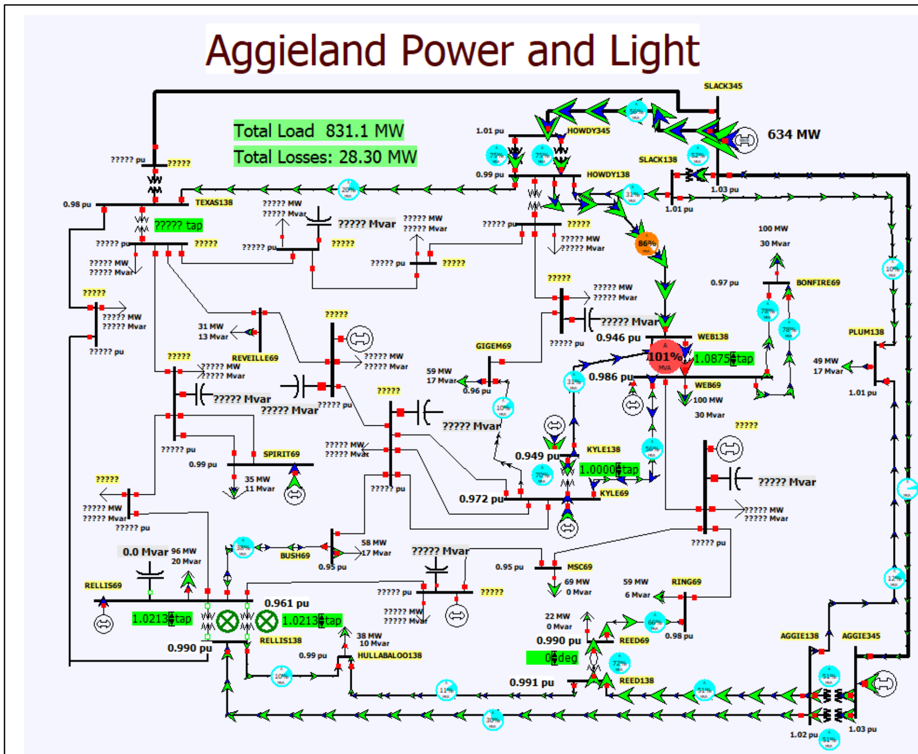
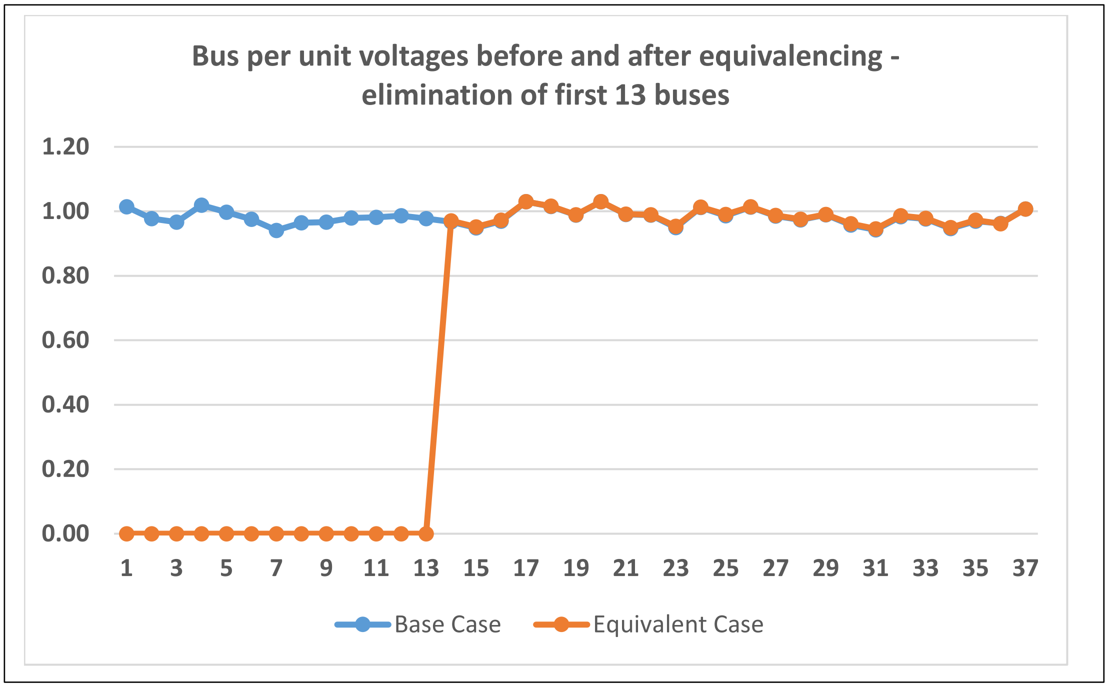


Fig. 2. Equivalenced Case



Barring small differences, the values for the power system states- bus voltages and line MVA percentages - in both figures are very similar. It shows the close similarity of the equivalenced, 4-bus case and the actual, 7-bus case. Also, the line overload between WEB138-WEB69 is retained in the smaller, 4-bus case. Selected states of large-scale systems can be easily and quickly analyzed if smaller, equivalent systems (which retains original grid dynamics and containing the region of interest) are used.