ECEN 615
Methods of Electric Power Systems Analysis

Lecture 3: Per Unit, Ybus, Power Flow

Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

Announcements

- RSVP to Alex at zandra23@ece.tamu.edu for the TAMU ECE Energy and Power Group (EPG) picnic. It starts at 5pm on September 27, 2019
- Read Chapters 1 to 3 from the book (more as background material); starting reading Chapter 6
- Homework 1 is assigned today. It is due on Thursday September 12

Load Models (Omitted from Lecture 2)

- Ultimate goal is to supply loads with electricity at constant frequency and voltage
- Electrical characteristics of individual loads matter, but usually they can only be estimated
  - actual loads are constantly changing, consisting of a large number of individual devices
  - only limited network observability of load characteristics
- Aggregate models are typically used for analysis
- Two common models
  - constant power: $S_i = P_i + jQ_i$
  - constant impedance: $S_i = |V|^2 / Z_i$

The ZIP model combines constant impedance, current and power (P)

Reading Technical Papers

- As a graduate student you should get in the habit of reading many technical papers, including all the ones mentioned in these notes
- Papers are divided into 1) journal papers and 2) conference papers, with journal papers usually undergoing more review and of high quality
  - There are LOTs of exceptions
- Key journals in our area are from IEEE Power and Energy Society (PES); PSCC is a top conference
- I read papers by looking at 1) title, 2) abstract, 3) summary, 4) results, 5) intro, 6) the rest; many papers never make it beyond step 1 or 2.

Learn to Write Well

- Writing is a key skill for engineers, especially for students with advanced degrees
- If you are not currently a good technical writer, use your time at TAMU to learn how to write well!
- There are lots of good resources available to help you improve your writing. Books I've found helpful are
  - Zinsser, "On Writing Well: The Classic Guide to Writing Nonfiction"
  - Strunk and White, "The Elements of Style"
  - Alred, Oliu, Brusaw, "The Handbook of Technical Writing"
- TAMU Writing Center, writingcenter.tamu.edu/

Three-Phase Per Unit

Procedure is very similar to 1ϕ except we use a 3ϕ VA base, and use line to line voltage bases

1. Pick a 3ϕ VA base for the entire system,
2. Pick a voltage base for each different voltage level, V_B. Voltages are line to line.
3. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\,V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

Three-Phase Per Unit, cont'd

4. Calculate the current base, I_B

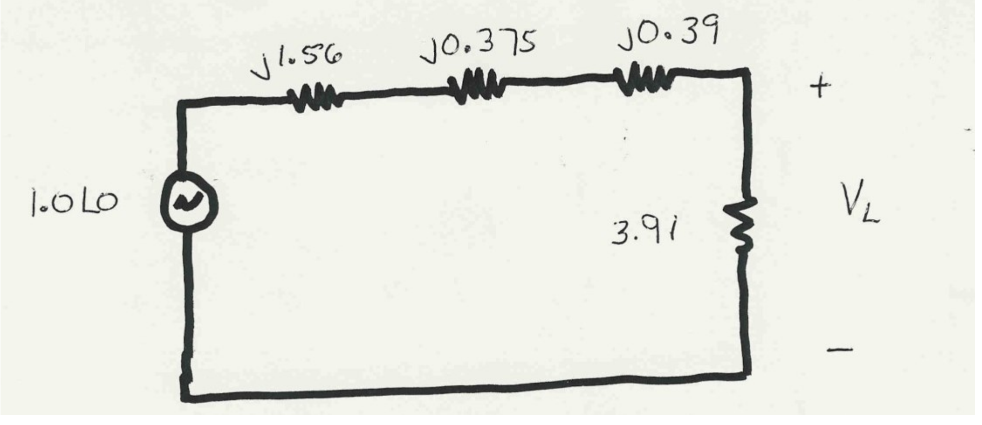
$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\, V_{B,LL}} = \frac{3\, S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\, V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_B^{1\phi}$$

Exactly the same current bases as with single-phase!

5. Convert actual values to per unit

Three-Phase Per Unit Example

Solve for the current, load voltage and load power in the previous circuit, assuming a 3ϕ power base of **300 MVA**, and line to line voltage bases of 13.8 kV, 138 kV and 27.6 kV (square root of 3 larger than the 1ϕ example voltages). Also assume the generator is Y-connected so its line to line voltage is 13.8 kV.



Convert to per unit as before. Note the system is exactly the same!

Three-Phase Per Unit Example, cont.

$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ \text{ p.u. (not amps)}$$

$$\mathrm{V_L} = 1.0\angle 0^\circ - 0.22\angle -30.8^\circ \times 2.327\angle 90^\circ$$

$$= 0.859\angle -30.8^\circ \text{ p.u.}$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

Again, analysis is exactly the same!

Three-Phase Per Unit Example, cont'd

Differences appear when we convert back to actual values

$$V_L^{\text{Actual}} = 0.859\angle -30.8° \times 27.6 \text{ kV} = 23.8\angle -30.8° \text{ kV}$$

$$S_L^{\text{Actual}} = 0.189\angle 0° \times 300 \text{ MVA} = 56.7\angle 0° \text{ MVA}$$

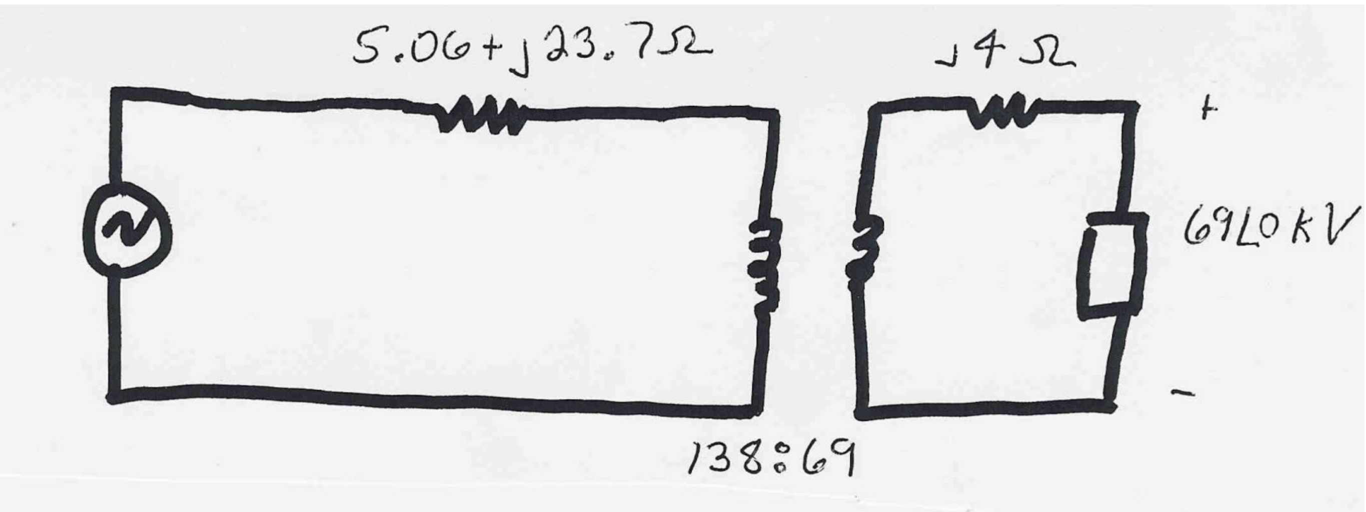
$$S_G^{\text{Actual}} = 0.22\angle 30.8° \times 300 \text{ MVA} = 66.0\angle 30.8° \text{ MVA}$$

$$\text{I}_B^{\text{Middle}} = \frac{300 \text{ MVA}}{\sqrt{3}\,138 \text{ kV}} = 1250 \text{ Amps} \quad \text{(same current!)}$$

$$\text{I}_{\text{Middle}}^{\text{Actual}} = 0.22\angle -30.8° \times 1250 \text{ Amps} = 275\angle -30.8° \text{ A}$$

Three-Phase Per Unit Example 2

- Assume a 3ϕ load of 100+j50 MVA with V_{LL} of 69 kV is connected to a source through the below network:



What is the supply current and complex power?

Answer: I=467 amps, S = 103.3 + j76.0 MVA

Power Flow Analysis

- We now have the necessary models to start to develop the power system analysis tools
- The most common power system analysis tool is the power flow (also known sometimes as the load flow)
  - power flow determines how the power flows in a network
  - also used to determine all bus voltages and all currents
  - because of constant power models, power flow is a nonlinear analysis technique
  - power flow is a steady-state analysis tool

Linear versus Nonlinear Systems

A function **H** is linear if

$$\mathbf{H}(\alpha_1\boldsymbol{\mu}_1 + \alpha_2\boldsymbol{\mu}_2) = \alpha_1\mathbf{H}(\boldsymbol{\mu}_1) + \alpha_2\mathbf{H}(\boldsymbol{\mu}_2)$$

That is

1) the output is proportional to the input
2) the principle of superposition holds

Linear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = \mathrm{c}\ \mathbf{x}$

$$\mathbf{y} = \mathrm{c}(\mathbf{x}_1+\mathbf{x}_2) = \mathrm{c}\mathbf{x}_1 + \mathrm{c}\ \mathbf{x}_2$$

Nonlinear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = \mathrm{c}\ \mathbf{x}^2$

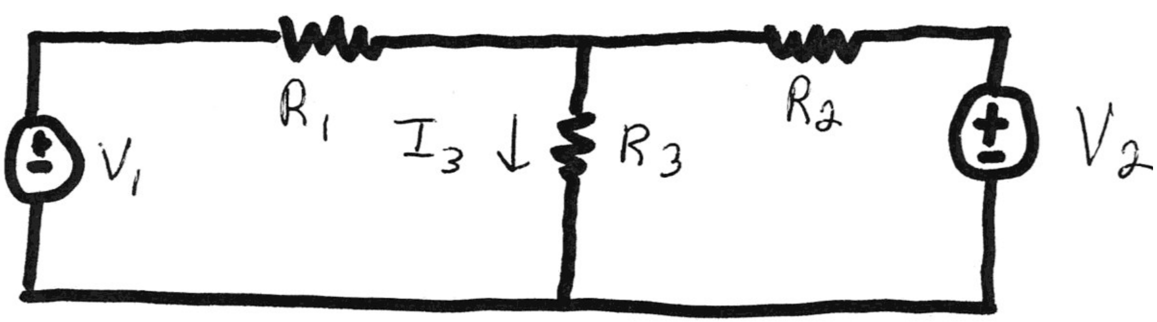
$$\mathbf{y} = \mathrm{c}(\mathbf{x}_1+\mathbf{x}_2)^2 \neq (\mathrm{c}\mathbf{x}_1)^2 + (\mathrm{c}\ \mathbf{x}_2)^2$$

Linear Power System Elements

Resistors, inductors, capacitors, independent voltage sources and current sources are linear circuit elements

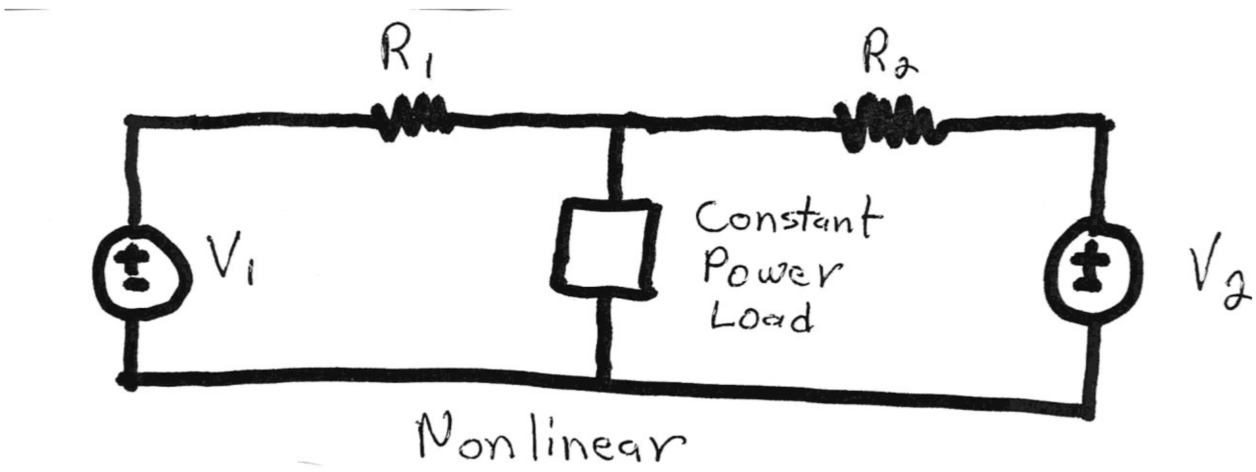
$$V = R\,I \qquad V = j\omega L\, I \qquad V = \frac{1}{j\omega C} I$$

Such systems may be analyzed by superposition



Nonlinear System Example

- Constant power loads and generator injections are nonlinear and hence systems with these elements can not be analyzed by superposition



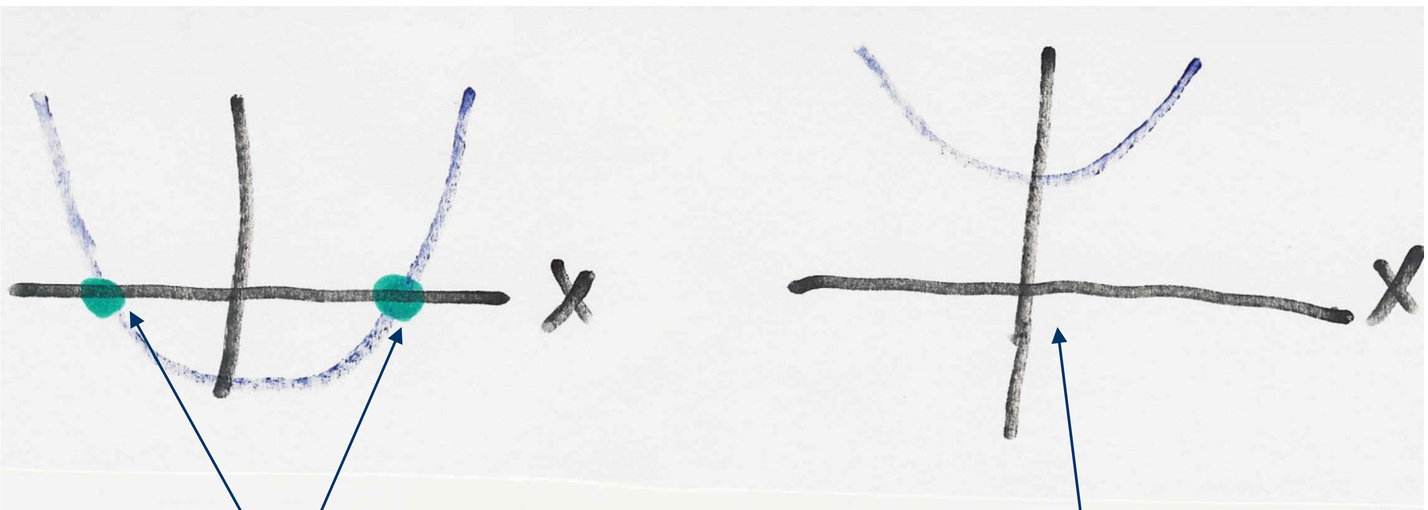
Nonlinear problems can be very difficult to solve, and usually require an iterative approach

Nonlinear Systems May Have Multiple Solutions or No Solution

Example 1: $x^2 - 2 = 0$ has solutions $x = \pm 1.414\ldots$

Example 2: $x^2 + 2 = 0$ has no real solution

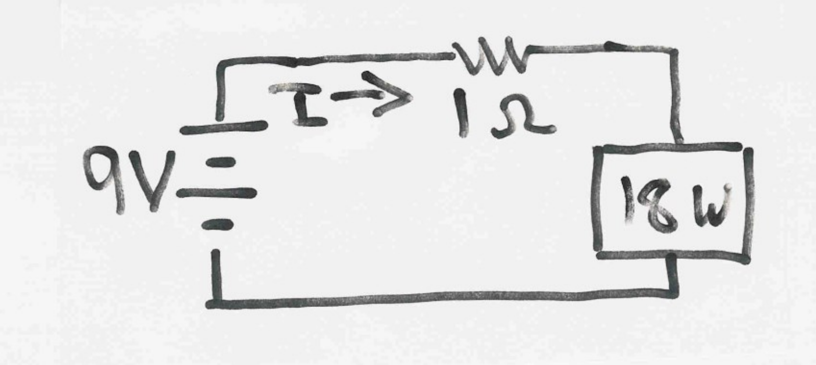
$f(x) = x^2 - 2$ $\qquad$ $f(x) = x^2 + 2$



two solutions where $f(x) = 0$ $\qquad$ no solution $f(x) = 0$

Multiple Solution Example

- The dc system shown below has two solutions:



where the 18 watt load is a resistive load

The equation we're solving is

$$I^2 R_{Load} = \left(\frac{9 \text{ volts}}{1\Omega + \text{R}_{\text{Load}}}\right)^2 R_{Load} = 18 \text{ watts}$$

One solution is $\text{R}_{\text{Load}} = 2\Omega$

Other solution is $\text{R}_{\text{Load}} = 0.5\Omega$

What is the maximum P_{Load}?

Bus Admittance Matrix or $\mathbf{Y}_{bus}$

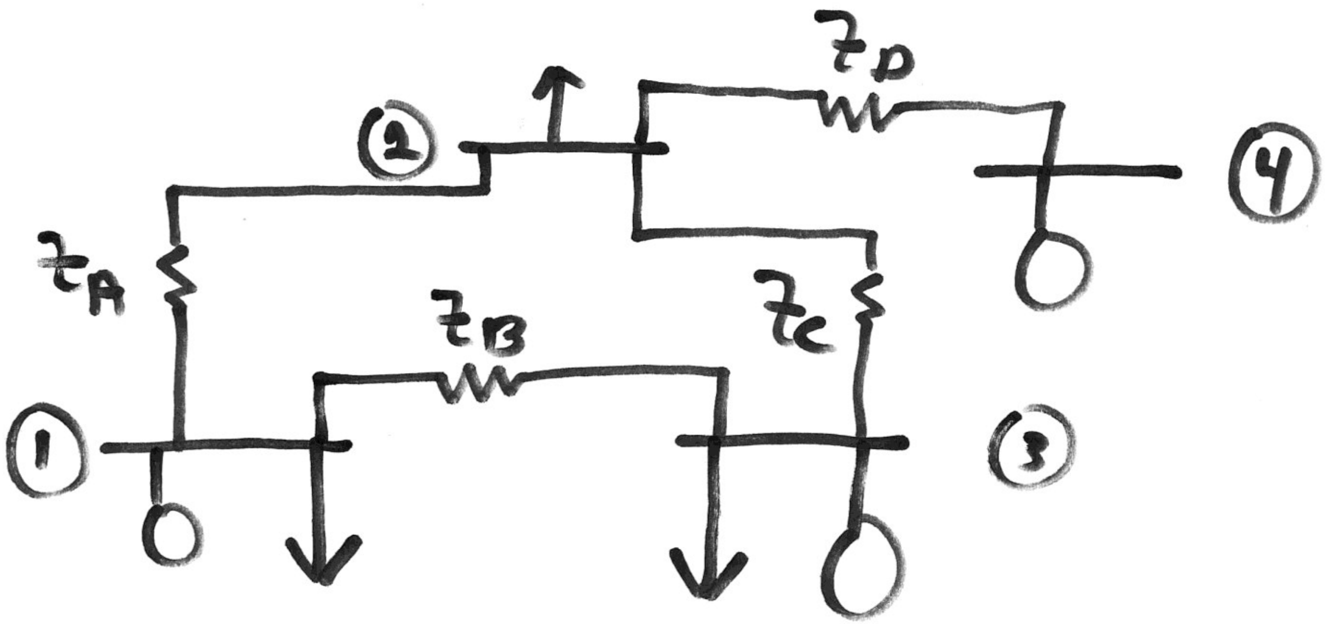
- First step in solving the power flow is to create what is known as the bus admittance matrix, often call the $\mathbf{Y}_{bus}$.
- The $\mathbf{Y}_{bus}$ gives the relationships between all the bus current injections, $\mathbf{I}$, and all the bus voltages, $\mathbf{V}$,

$$\mathbf{I} = \mathbf{Y}_{bus}\,\mathbf{V}$$

- The $\mathbf{Y}_{bus}$ is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances

Y_{bus} Example

Determine the bus admittance matrix for the network shown below, assuming the current injection at each bus i is $I_i = I_{Gi} - I_{Di}$ where I_{Gi} is the current injection into the bus from the generator and I_{Di} is the current flowing into the load



Y_{bus} Example, cont'd

By KCL at bus 1 we have

$$I_1 = I_{G1} - I_{D1}$$

$$I_1 = I_{12} + I_{13} = \frac{V_1 - V_2}{Z_A} + \frac{V_1 - V_3}{Z_B}$$

$$I_1 = (V_1 - V_2)Y_A + (V_1 - V_3)Y_B \qquad (\text{with } Y_j = \frac{1}{Z_j})$$

$$= (Y_A + Y_B)V_1 - Y_A V_2 - Y_B V_3$$

Similarly

$$I_2 = I_{21} + I_{23} + I_{24}$$

$$= -Y_A V_1 + (Y_A + Y_C + Y_D)V_2 - Y_C V_3 - Y_D V_4$$

Y_{bus} Example, cont'd

We can get similar relationships for buses 3 and 4

The results can then be expressed in matrix form

$$\mathbf{I} = \mathbf{Y}_{bus}\mathbf{V}$$

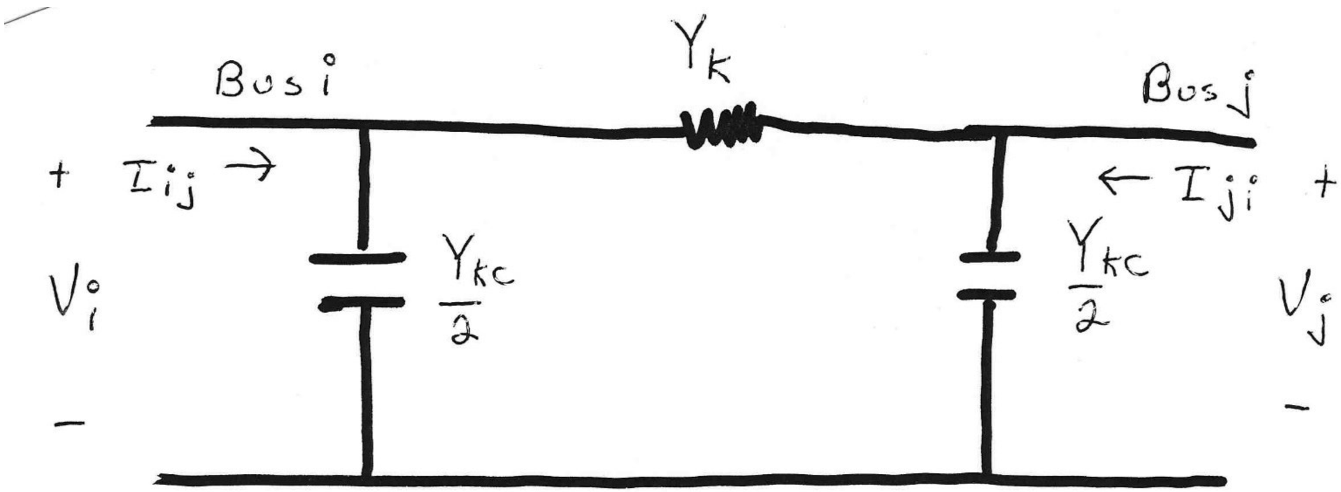
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_A & -Y_B & 0 \\ -Y_A & Y_A + Y_C + Y_D & -Y_C & -Y_D \\ -Y_B & -Y_C & Y_B + Y_C & 0 \\ 0 & -Y_D & 0 & Y_D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

For a system with n buses the Y_{bus} is an n by n symmetric matrix (i.e., one where $A_{ij} = A_{ji}$); however this will not be true in general when we consider phase shifting transformers

Y_{bus} General Form

- The diagonal terms, Y_{ii}, are the self admittance terms, equal to the sum of the admittances of all devices incident to bus i.
- The off-diagonal terms, Y_{ij}, are equal to the negative of the sum of the admittances joining the two buses.
- With large systems Y_{bus} is a sparse matrix (that is, most entries are zero)
- Shunt terms, such as with the π line model, only affect the diagonal terms.

Modeling Shunts in the Y_{bus}

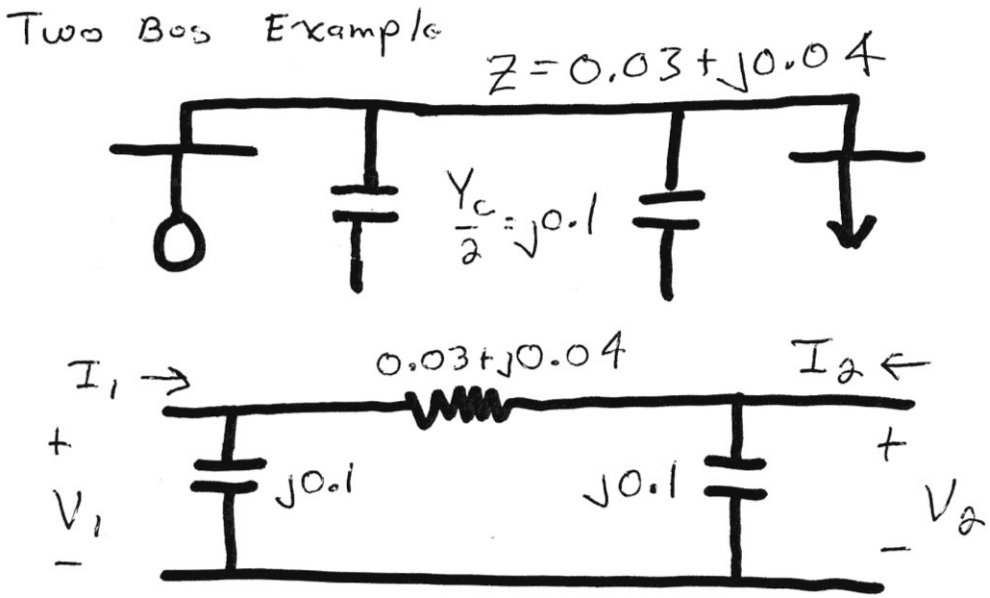


$$\text{Since} \quad I_{ij} = (V_i - V_j)Y_k + V_i \frac{Y_{kc}}{2}$$

$$Y_{ii} = Y_{ii}^{\text{from other lines}} + Y_k + \frac{Y_{kc}}{2}$$

$$\text{Note} \quad Y_k = \frac{1}{Z_k} = \frac{1}{R_k + jX_k} \frac{R_k - jX_k}{R_k - jX_k} = \frac{R_k - jX_k}{R_k^2 + X_k^2}$$

Two Bus System Example



$$I_1 = \frac{(V_1 - V_2)}{Z} + V_1 \frac{Y_c}{2} \qquad \frac{1}{0.03 + j0.04} = 12 - j16$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Using the Y_{bus}

If the voltages are known then we can solve for the current injections:

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

If the current injections are known then we can solve for the voltages:

$$\mathbf{Y}_{bus}^{-1}\mathbf{I} \;=\; \mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

where $\mathbf{Z}_{\text{bus}}$ is the bus impedance matrix

However, this requires that $\mathbf{Y}_{\text{bus}}$ not be singular; note it will be singular if there are no shunt connections!

Solving for Bus Currents

For example, in previous case assume

$$\mathbf{V} = \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is

$$\mathrm{S}_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$

Solving for Bus Voltages

For example, in previous case assume

$$\mathbf{I} = \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}^{-1} \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j1.098 \end{bmatrix}$$

Therefore the power injected is

$$\mathrm{S}_1 = V_1 I_1^* = (0.0738 - j0.902) \times 5 = 0.37 - j4.51$$

$$S_2 = V_2 I_2^* = (-0.0738 - j1.098) \times (-4.8) = 0.35 + j5.27$$

Power Flow Analysis

- When analyzing power systems we know neither the complex bus voltages nor the complex current injections
- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes
- Therefore we can not directly use the Y_{bus} equations, but rather must use the power balance equations

Power Balance Equations

From KCL we know at each bus i in an n bus system the current injection, I_i, must be equal to the current that flows into the network

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} I_{ik}$$

Since $\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V}$ we also know

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} Y_{ik} V_k$$

The network power injection is then $\mathrm{S_i} = V_i I_i^*$

Power Balance Equations, cont'd

$$S_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

This is an equation with complex numbers.

Sometimes we would like an equivalent set of real power equations. These can be derived by defining

$$Y_{ik} = G_{ik} + jB_{ik}$$

$$V_i = |V_i| e^{j\theta_i} = |V_i| \angle \theta_i$$

$$\theta_{ik} = \theta_i - \theta_k$$

Recall $e^{j\theta} = \cos\theta + j\sin\theta$

Real Power Balance Equations

$$S_i = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} = \sum_{k=1}^{n} |V_i||V_k| e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts

$$P_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Power Flow Analysis

- When analyzing power systems we know neither the complex bus voltages nor the complex current injections
- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes
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Power Flow Analysis

- Classic paper for this lecture is W.F. Tinney and C.E. Hart, "Power Flow Solution by Newton's Method," *IEEE Power App System*, Nov 1967
- Basic power flow is also covered in essentially any power system analysis textbooks.
- We use the term "power flow" not "load flow" since power flows not load. Also, the power flow usage is not new (see title of Tinney's 1967 paper, and note Tinney references Ward's 1956 power flow paper)
  - A nice history of the power flow is given in an insert by Alvarado and Thomas in T.J. Overbye, J.D. Weber, "Visualizing the Electric Grid," *IEEE Spectrum*, Feb 2001.

Early Power Flow System Size

- In 1957 Bill Tinney, in a paper titled "Digital Solutions for Large Power Networks," studied a 100 bus, 200 branch system (with 2 KB of memory)!
- In Tinney's 1963 "Techniques for Exploiting Sparsity of the Network Admittance Matrix" paper (which gave us the Tinney Schemes 1, 2, and 3), uses 32 kB for 1000 nodes.
- In Tinney's classic 1967 "Power Flow Solution by Newton's Method" paper he applies his method to systems with up to about 1000 buses (with 32 kB of memory) and provides a solution time of 51 seconds for a 487 bus system.

Slack Bus

- We can not arbitrarily specify S at all buses because total generation must equal total load + total losses
- We also need an angle reference bus.
- To solve these problems we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, and a varying real/reactive power injection.
- In an actual power system the slack bus does not really exist; frequency changes locally when the power supplied does not match the power consumed

Three Types of Power Flow Buses

- There are three main types of power flow buses
  - Load (PQ) at which P/Q are fixed; iteration solves for voltage magnitude and angle.
  - Slack at which the voltage magnitude and angle are fixed; iteration solves for P/Q injections
  - Generator (PV) at which P and |V| are fixed; iteration solves for voltage angle and Q injection

Newton-Raphson Algorithm

- Most common technique for solving the power flow problem is to use the Newton-Raphson algorithm
- Key idea behind Newton-Raphson is to use sequential linearization

General form of problem: Find an $\mathbf{x}$ such that

$$\mathbf{f}(\hat{\mathbf{x}}) = 0$$

Newton-Raphson Power Flow

In the Newton-Raphson power flow we use Newton's method to determine the voltage magnitude and angle at each bus in the power system.

We need to solve the power balance equations

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Power Flow Variables

Assume the slack bus is the first bus (with a fixed voltage angle/magnitude). We then need to determine the voltage angle/magnitude at the other buses.

$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_n \\ |V_2| \\ \vdots \\ |V_n| \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_{G2} + P_{D2} \\ \vdots \\ P_n(\mathbf{x}) - P_{Gn} + P_{Dn} \\ Q_2(\mathbf{x}) - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n(\mathbf{x}) - Q_{Gn} + Q_{Dn} \end{bmatrix}$$

N-R Power Flow Solution

The power flow is solved using the same procedure discussed with the general Newton-Raphson:

Set $v = 0$; make an initial guess of $\mathbf{x}$, $\mathbf{x}^{(v)}$

While $\left\| \mathbf{f}(\mathbf{x}^{(v)}) \right\| > \varepsilon$ Do

$$\mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(v)})^{-1}\mathbf{f}(\mathbf{x}^{(v)})$$

$$v = v + 1$$

End While

Power Flow Jacobian Matrix

The most difficult part of the algorithm is determining and inverting the n by n Jacobian matrix, $\mathbf{J}(\mathbf{x})$

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \frac{\partial f_n(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}$$

Power Flow Jacobian Matrix, cont'd

Jacobian elements are calculated by differentiating each function, $\mathrm{f_i}(\mathbf{x})$, with respect to each variable.

For example, if $\mathrm{f_i}(\mathbf{x})$ is the bus i real power equation

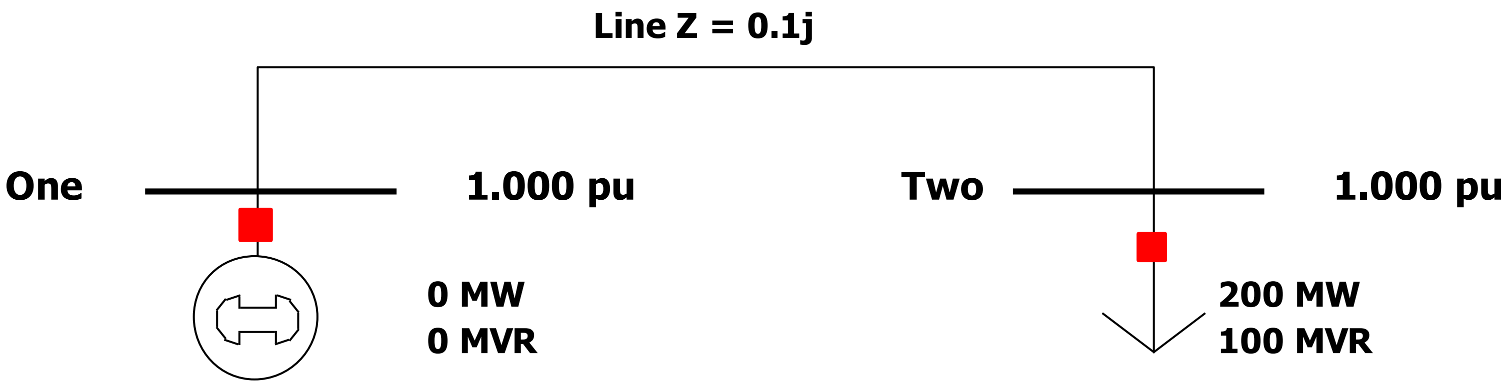
$$\mathrm{f_i}(x) \;=\; \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) - P_{Gi} + P_{Di}$$

$$\frac{\partial \mathrm{f_i}(x)}{\partial \theta_i} = \sum_{\substack{k=1\\ k\neq i}}^{n} |V_i||V_k|(-G_{ik}\sin\theta_{ik} + B_{ik}\cos\theta_{ik})$$

$$\frac{\partial \mathrm{f_i}(x)}{\partial \theta_j} = |V_i||V_j|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \qquad (j \neq i)$$

Two Bus Newton-Raphson Example

- For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and S_{Base} = 100 MVA.



$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix} \qquad \mathbf{Y}_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

Two Bus Example, cont'd

General power balance equations

$$\mathrm{P_i} \;=\; \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} \;=\; \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Bus two power balance equations

$$|V_2||V_1|(10\sin\theta_2) + 2.0 \;=\; 0$$

$$|V_2||V_1|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 = 0$$

Two Bus Example, cont'd

$$\mathrm{P}_2(\mathbf{x}) \;=\; |V_2|(10\sin\theta_2) + 2.0 \;=\; 0$$

$$Q_2(\mathbf{x}) = |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 = 0$$

Now calculate the power flow Jacobian

$$J(\mathbf{x}) \;=\; \begin{bmatrix} \dfrac{\partial \mathrm{P}_2(\mathbf{x})}{\partial \theta_2} & \dfrac{\partial \mathrm{P}_2(\mathbf{x})}{\partial |V|_2} \\[2ex] \dfrac{\partial \mathrm{Q}_2(\mathbf{x})}{\partial \theta_2} & \dfrac{\partial \mathrm{Q}_2(\mathbf{x})}{\partial |V|_2} \end{bmatrix}$$

$$= \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| \end{bmatrix}$$

Two Bus Example, First Iteration

Set $v = 0$, guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Calculate

$$\mathrm{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\theta_2) + 2.0 \\ |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

Solve $\mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$

Two Bus Example, Next Iterations

$$\mathrm{f}(\mathbf{x}^{(1)}) = \begin{bmatrix} 0.9(10\sin(-0.2)) + 2.0 \\ 0.9(-10\cos(-0.2)) + 0.9^2 \times 10 + 1.0 \end{bmatrix} = \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(1)}) = \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}$$

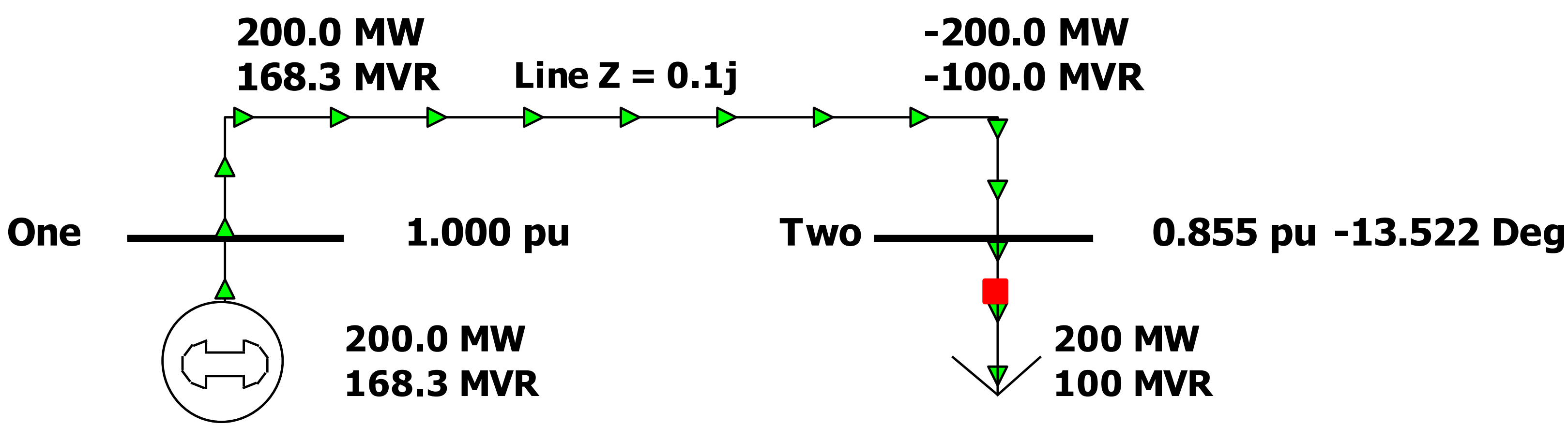
$$\mathbf{x}^{(2)} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}^{-1} \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix} = \begin{bmatrix} -0.233 \\ 0.8586 \end{bmatrix}$$

$$\mathrm{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.0145 \\ 0.0190 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} -0.236 \\ 0.8554 \end{bmatrix}$$

$$\mathrm{f}(\mathbf{x}^{(3)}) = \begin{bmatrix} 0.0000906 \\ 0.0001175 \end{bmatrix} \qquad \text{Done!} \qquad \mathrm{V}_2 = 0.8554\angle -13.52°$$

Two Bus Solved Values

- Once the voltage angle and magnitude at bus 2 are known we can calculate all the other system values, such as the line flows and the generator reactive power



PowerWorld Case Name: Bus2_Intro
Note, most PowerWorld cases will be available on the course website

Two Bus Case Low Voltage Solution

This case actually has two solutions! The second "low voltage" is found by using a low initial guess.

Set $v = 0$, guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$

Calculate

$$\mathrm{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\theta_2) + 2.0 \\ |V_2|(-10\cos\theta_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2 \\ -0.875 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos\theta_2 & 10\sin\theta_2 \\ 10|V_2|\sin\theta_2 & -10\cos\theta_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 2.5 & 0 \\ 0 & -5 \end{bmatrix}$$

Low Voltage Solution, cont'd

$$\text{Solve} \quad \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix} - \begin{bmatrix} 2.5 & 0 \\ 0 & -5 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -0.875 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0.075 \end{bmatrix}$$

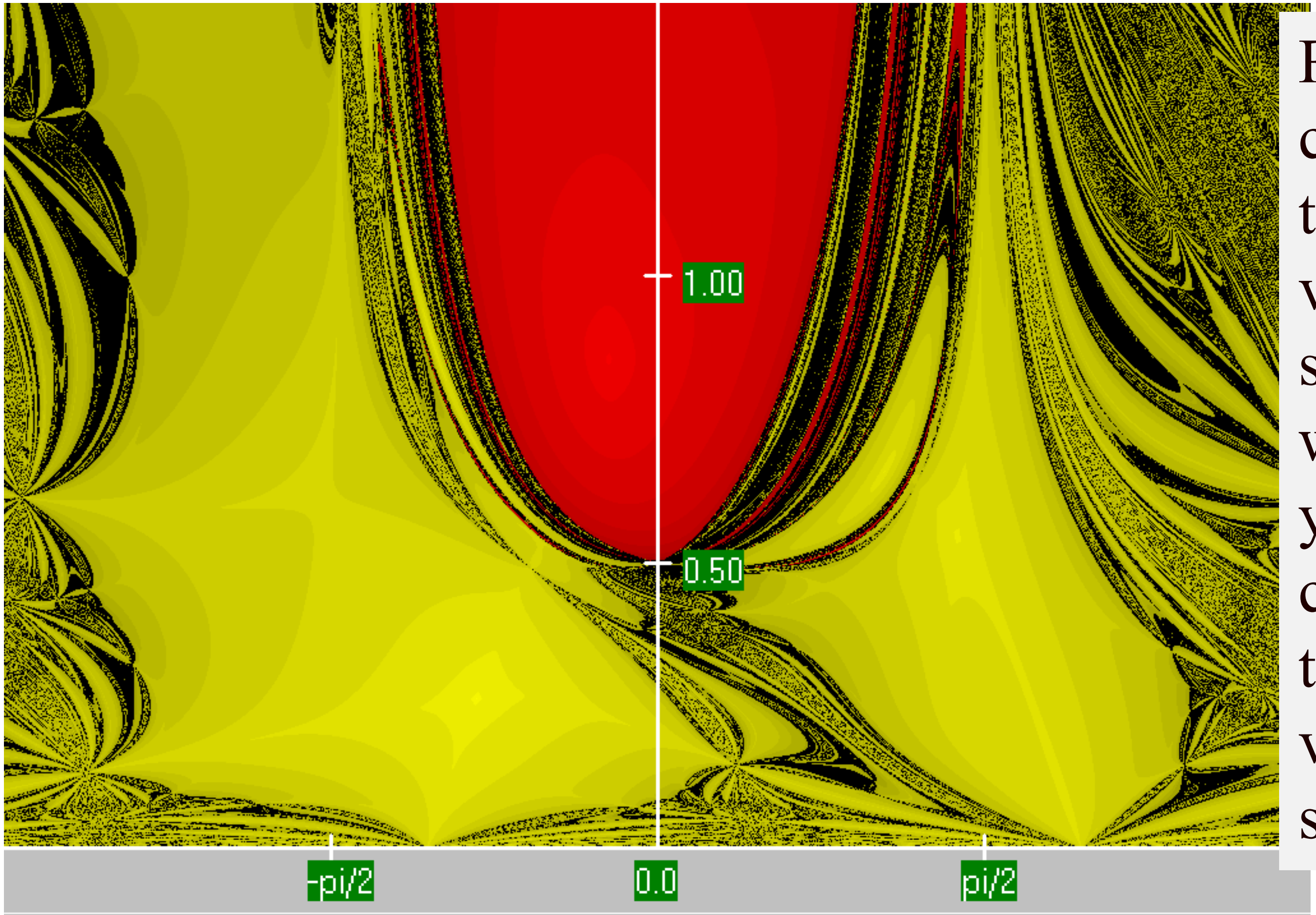
$$\mathbf{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 1.462 \\ 0.534 \end{bmatrix} \quad \mathbf{x}^{(2)} = \begin{bmatrix} -1.42 \\ 0.2336 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} -0.921 \\ 0.220 \end{bmatrix}$$

Practical Power Flow Software Note

- Most commercial software packages have built in defaults to prevent convergence to low voltage solutions.
  - One approach is to automatically change the load model from constant power to constant current or constant impedance when the load bus voltage gets too low
  - In PowerWorld these defaults can be modified on the Tools, Simulator Options, Advanced Options page; note you also need to disable the "Initialize from Flat Start Values" option
  - The PowerWorld case Bus2_Intro_Low is set solved to the low voltage solution
  - Initial bus voltages can be set using the Bus Information Dialog

Two Bus Region of Convergence

Slide shows the region of convergence for different initial guesses of bus 2 angle (x-axis) and magnitude (y-axis)



Red region converges to the high voltage solution, while the yellow region converges to the low voltage solution

Power Flow Fractal Region of Convergence

- Earliest paper showing fractal power flow regions of convergence is by C.L DeMarco and T.J. Overbye, "Low Voltage Power Flow Solutions and Their Role in Exit Time Bases Security Measures for Voltage Collapse," *Proc. 27th IEEE CDC*, December 1988
- A more widely known paper is J.S. Thorp, S.A. Naqavi, "Load-Flow Fractals Draw Clues to Erratic Behavior," *IEEE Computer Applications in Power*, January 1997