ECEN 615
Methods of Electric Power Systems Analysis

Lecture 5: Power Flow

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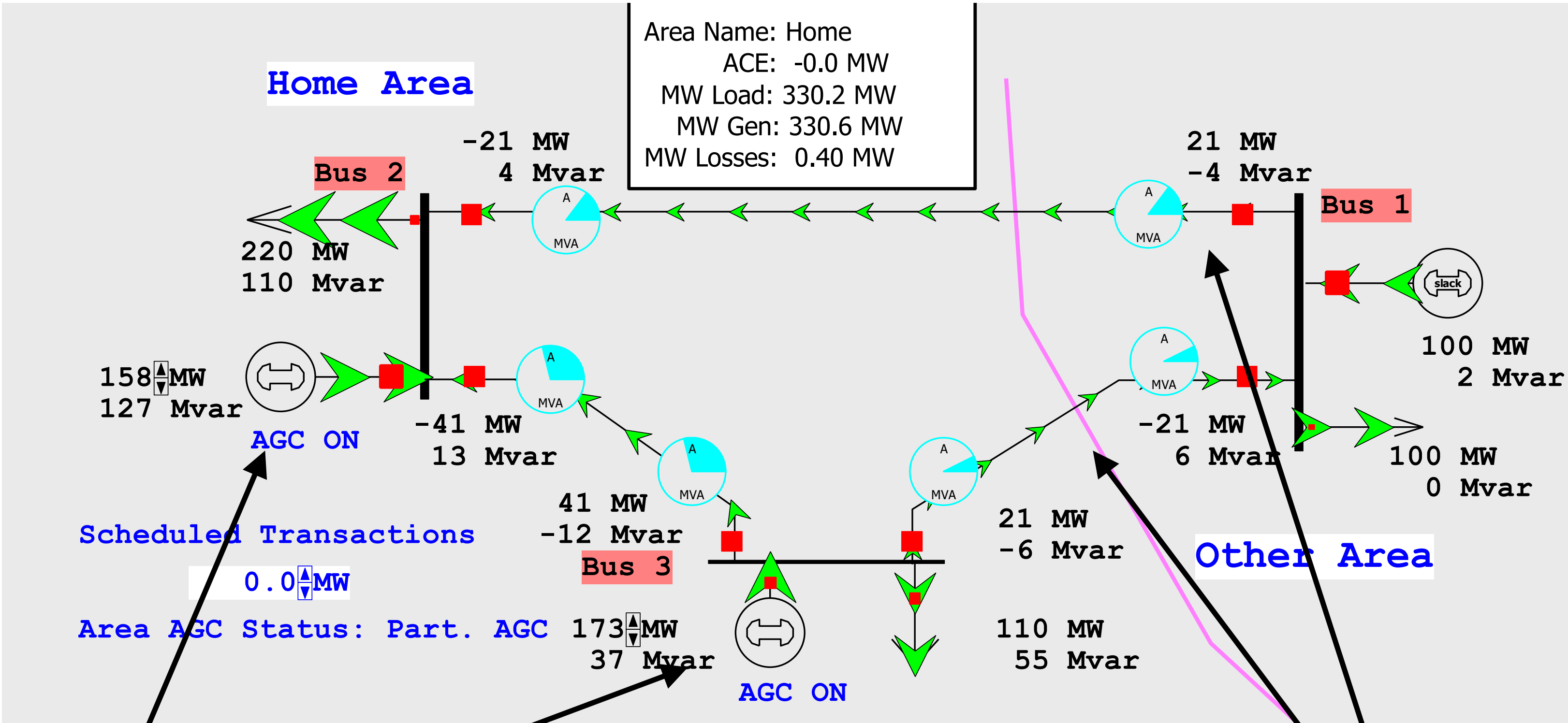
Announcements

- RSVP to Alex at zandra23@ece.tamu.edu for the TAMU ECE Energy and Power Group (EPG) picnic. It starts at 5pm on September 27, 2019
- Read Chapter 6 from the book
  - They formulate the power flow using the polar form for the Y_{bus} elements
- Homework 1 is due on Thursday September 12

Three Bus Case on AGC



Generation is automatically changed to match change in load

Net tie flow is close to zero

Generator Costs

- There are many fixed and variable costs associated with power system operation
- The major variable cost is associated with generation.
- Cost to generate a MWh can vary widely
- For some types of units (such as hydro and nuclear) it is difficult to quantify
- More others such as wind and solar the marginal cost of energy is essentially zero (actually negative for wind!)
- For thermal units it is straightforward to determine
- Many markets have moved from cost-based to price-based generator costs

Economic Dispatch

- Economic dispatch (ED) determines the least cost dispatch of generation for an area.
- For a lossless system, the ED occurs when all the generators have equal marginal costs.

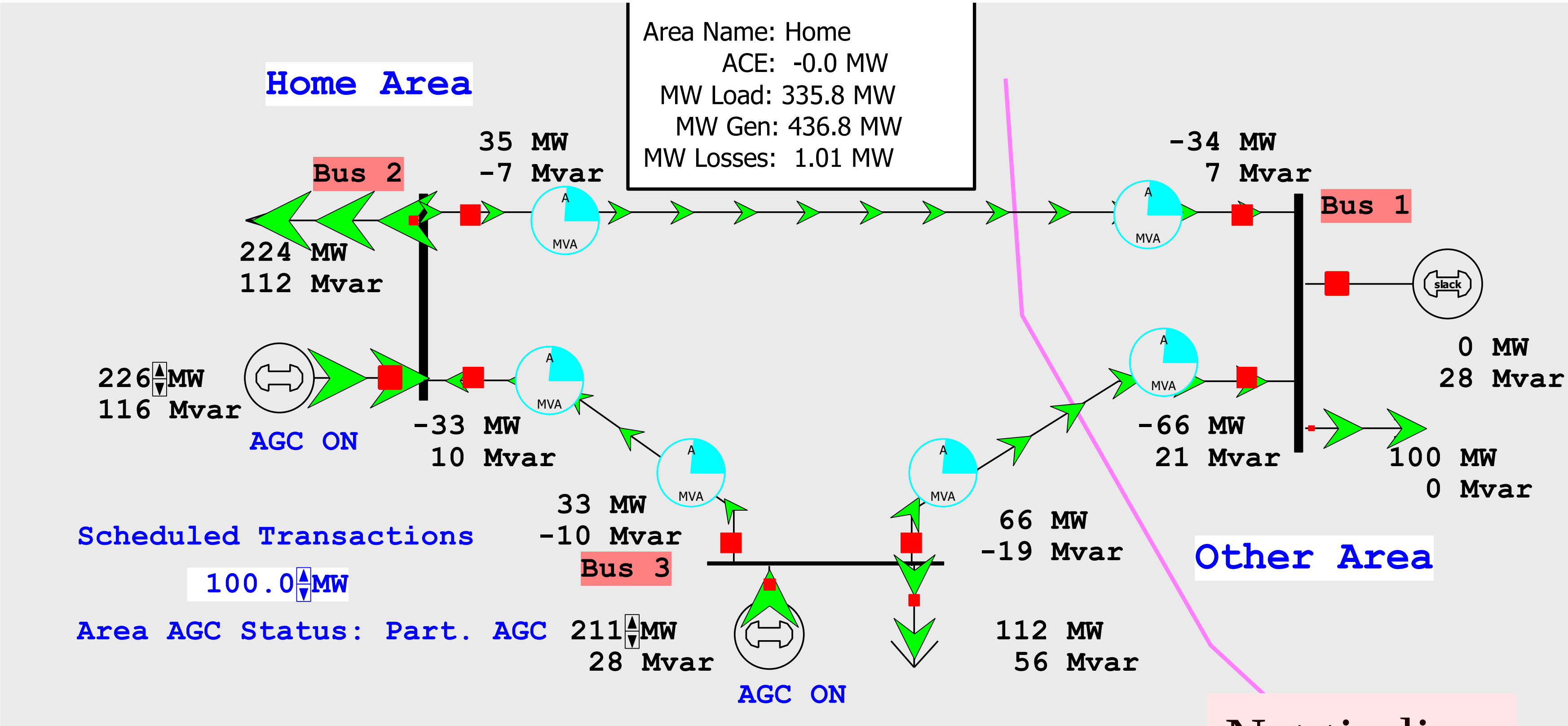
$$IC_1(P_{G,1}) = IC_2(P_{G,2}) = \ldots = IC_m(P_{G,m})$$

Power Transactions

- Power transactions are contracts between areas to do power transactions.
- Contracts can be for any amount of time at any price for any amount of power.
- Scheduled power transactions are implemented by modifying the area ACE:

$$ACE = P_{actual,tie\text{-}flow} - P_{sched}$$

100 MW Transaction



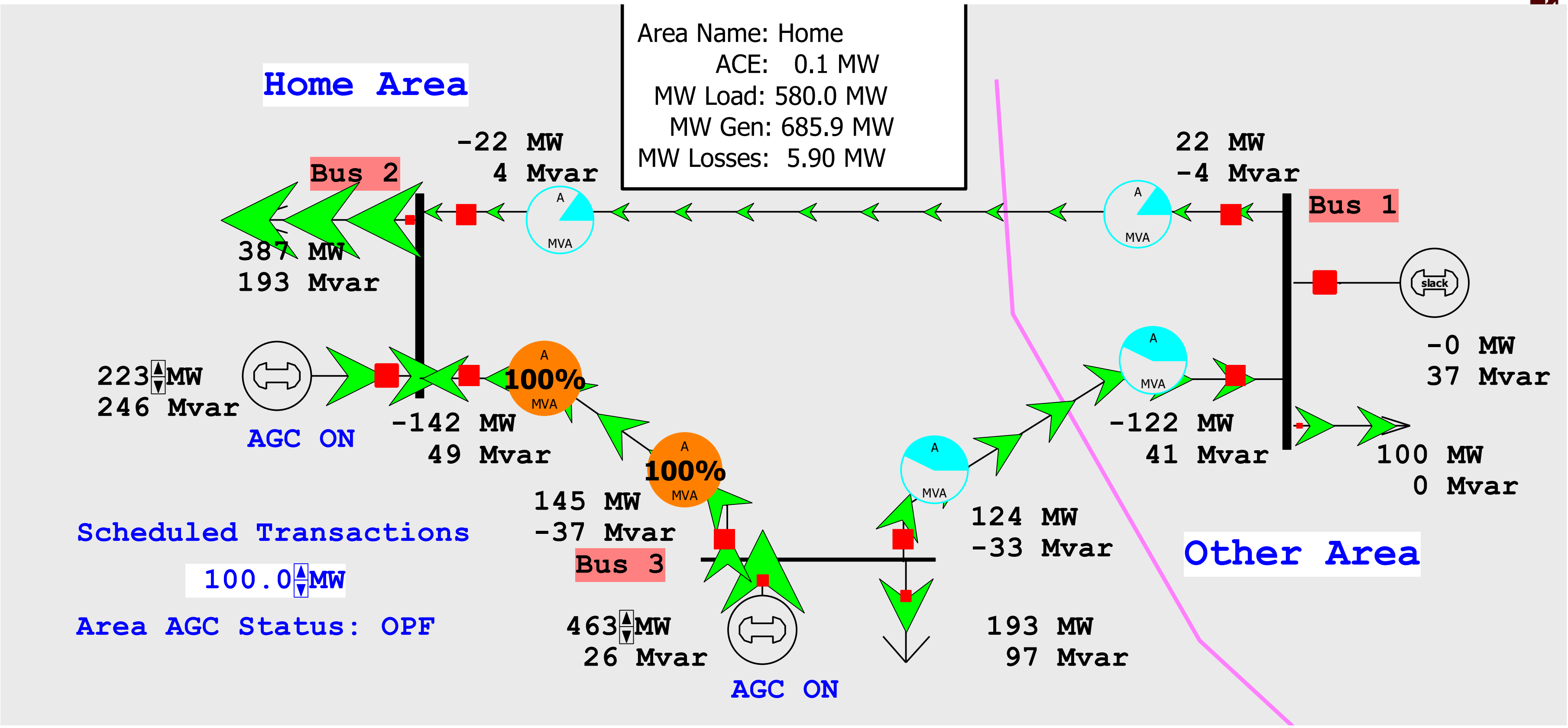
Scheduled 100 MW transaction from the Home Area to the Other Area

Net tie-line flow is now 100 MW

Security Constrained ED

- Transmission constraints often limit system economic operation.
- Such limits required a constrained dispatch in order to maintain system security.
- In the three bus case the generation at bus 3 must be constrained to avoid overloading the line from bus 2 to bus 3.

Security Constrained Dispatch



Dispatch is no longer optimal due to need to keep the line from bus 2 to bus 3 from overloading

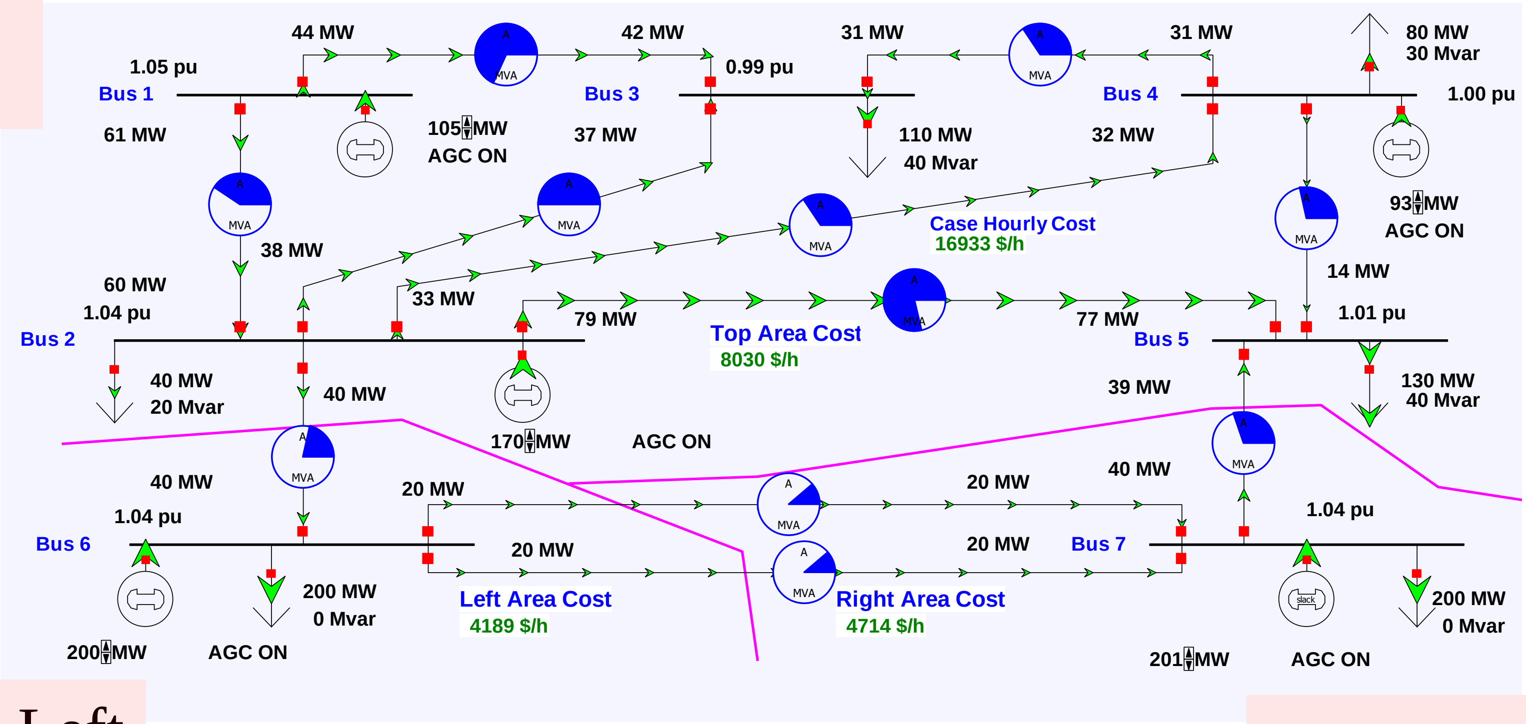
Multi-Area Operation

- If areas have direct interconnections then they may directly transact, up to the capacity of their tie-lines.
- Actual power flows through the entire network according to the impedance of the transmission lines.
- Flow through other areas is known as "parallel path" or "loop flow."

Seven Bus Case: One-line

Area Top has five buses

System has three areas



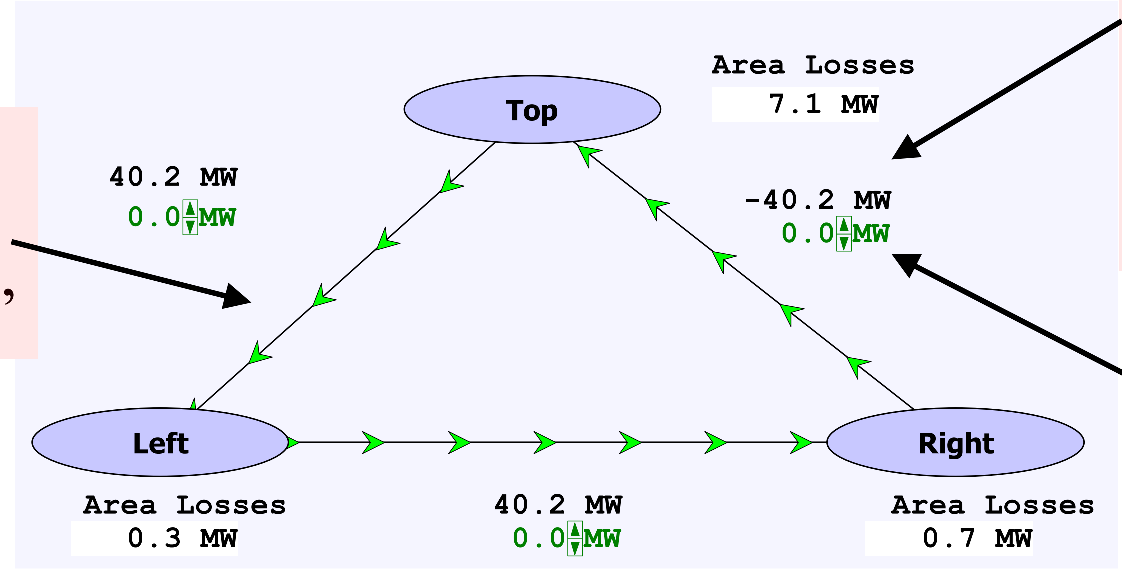
Area Left has one bus

Area Right has one bus

PowerWorld Case: **B7Flat**

Seven Bus Case: Area View

System has 40 MW of “Loop Flow”



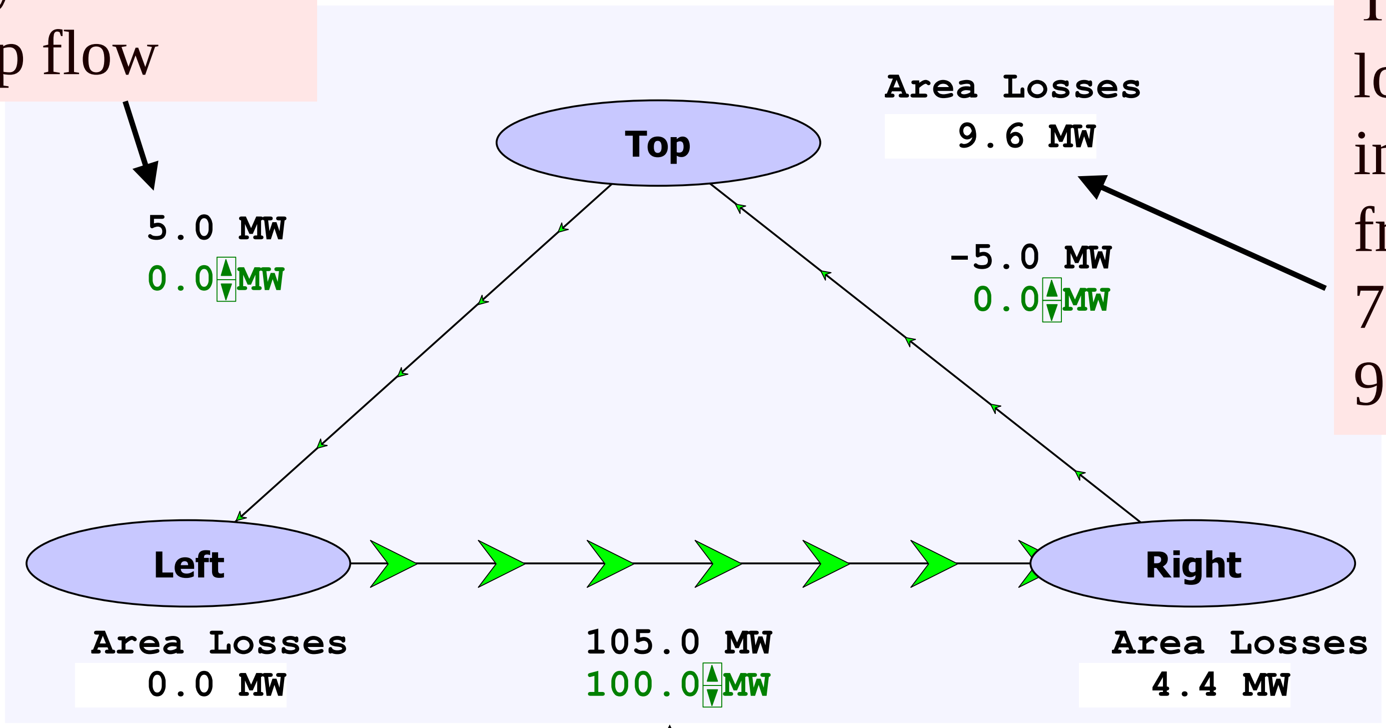
Actual flow between areas

Scheduled flow

Seven Bus - Loop Flow?

Transaction has actually decreased the loop flow

Note that Top's losses have increased from 7.1 MW to 9.6 MW

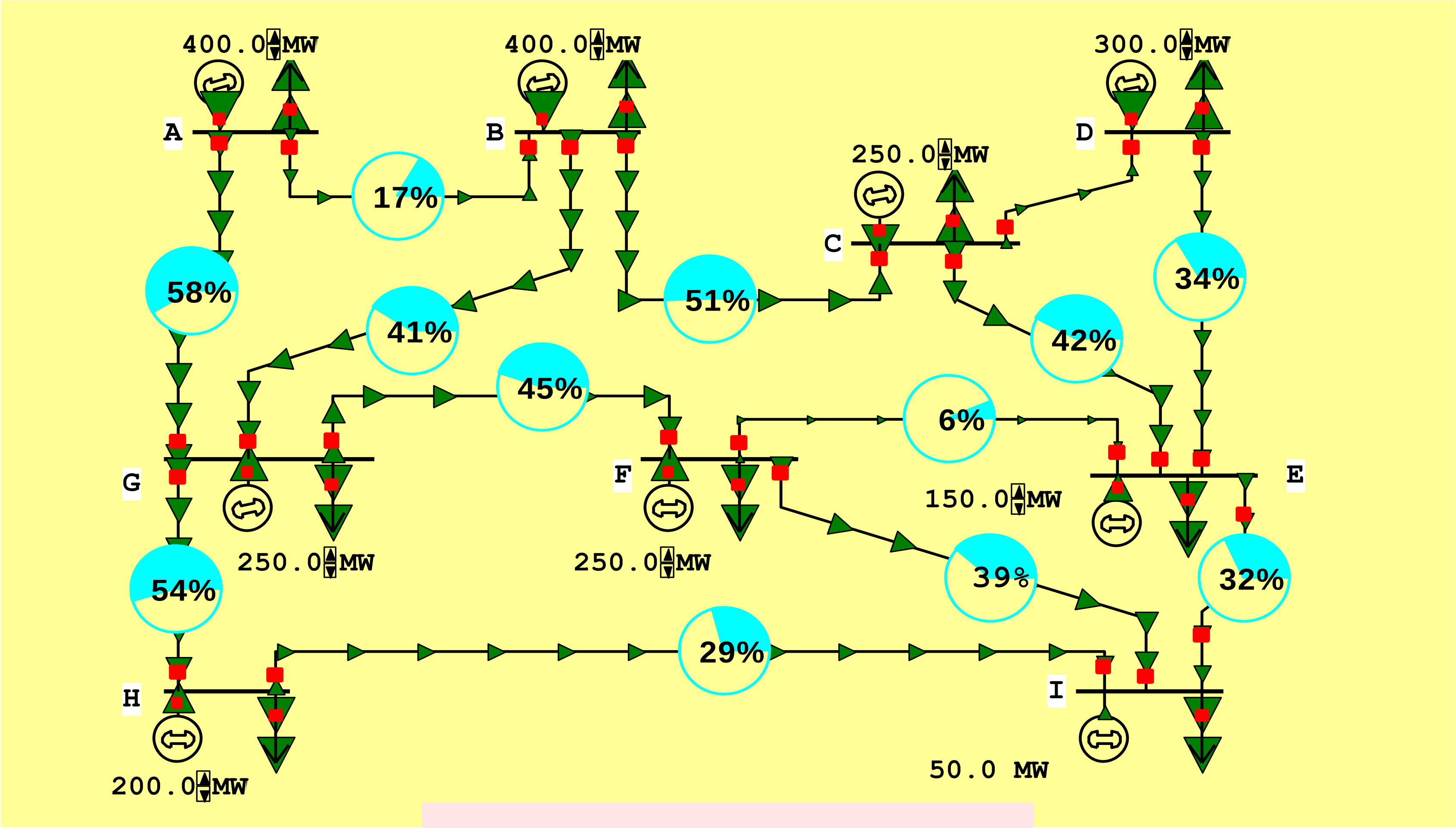


100 MW Transaction between Left and Right

Power Transfer Distribution Factors (PTDFS)

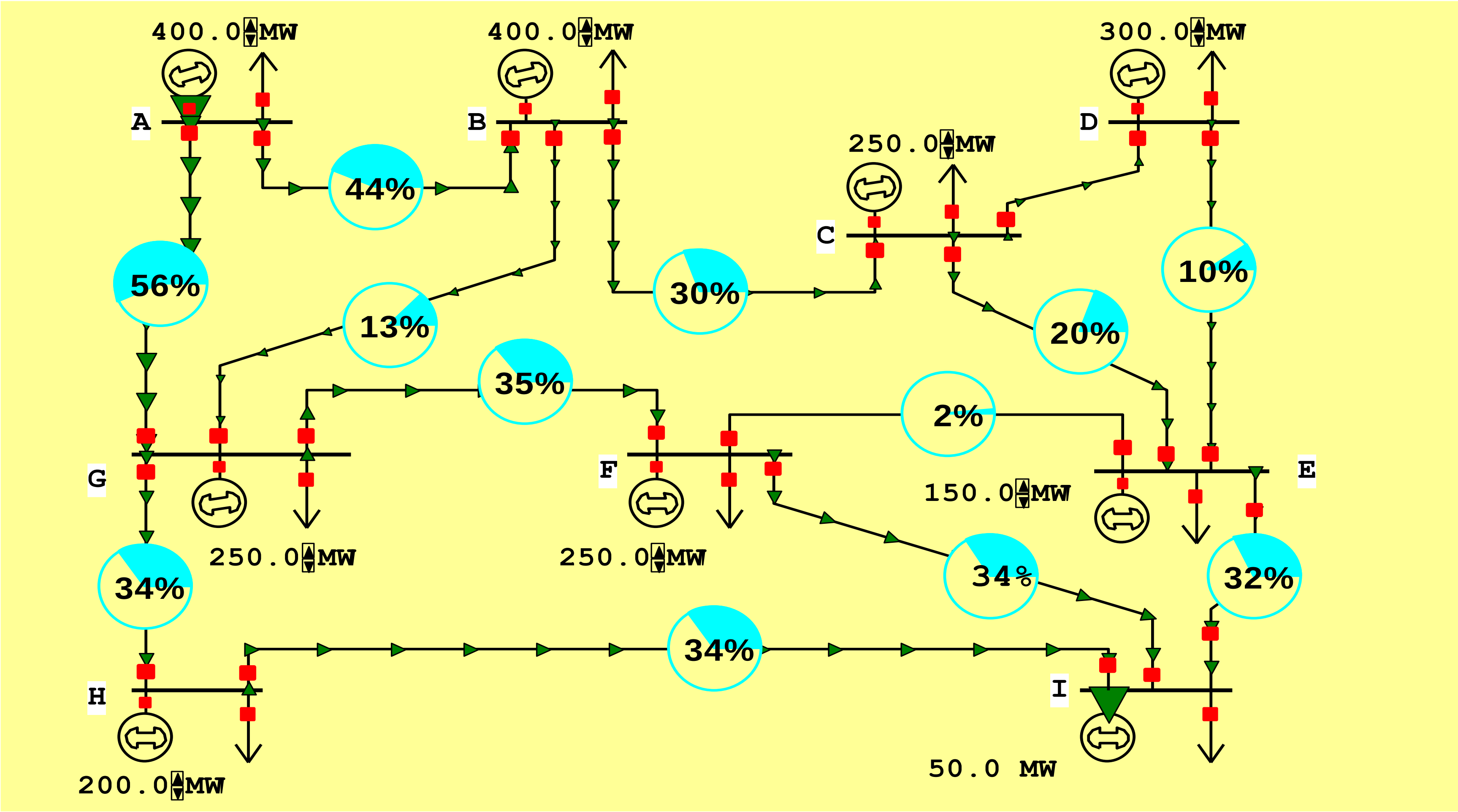
- PTDFs are used to show how a particular transaction will affect the system
- The power transfers through the system according to the impedances of the lines, without respect to ownership
- All transmission players in network could potentially be impacted (to a greater or lesser extent)
- Later in the semester we'll consider techniques for calculating PTDFs

PTDF Example: Nine Bus System, Actual Flows



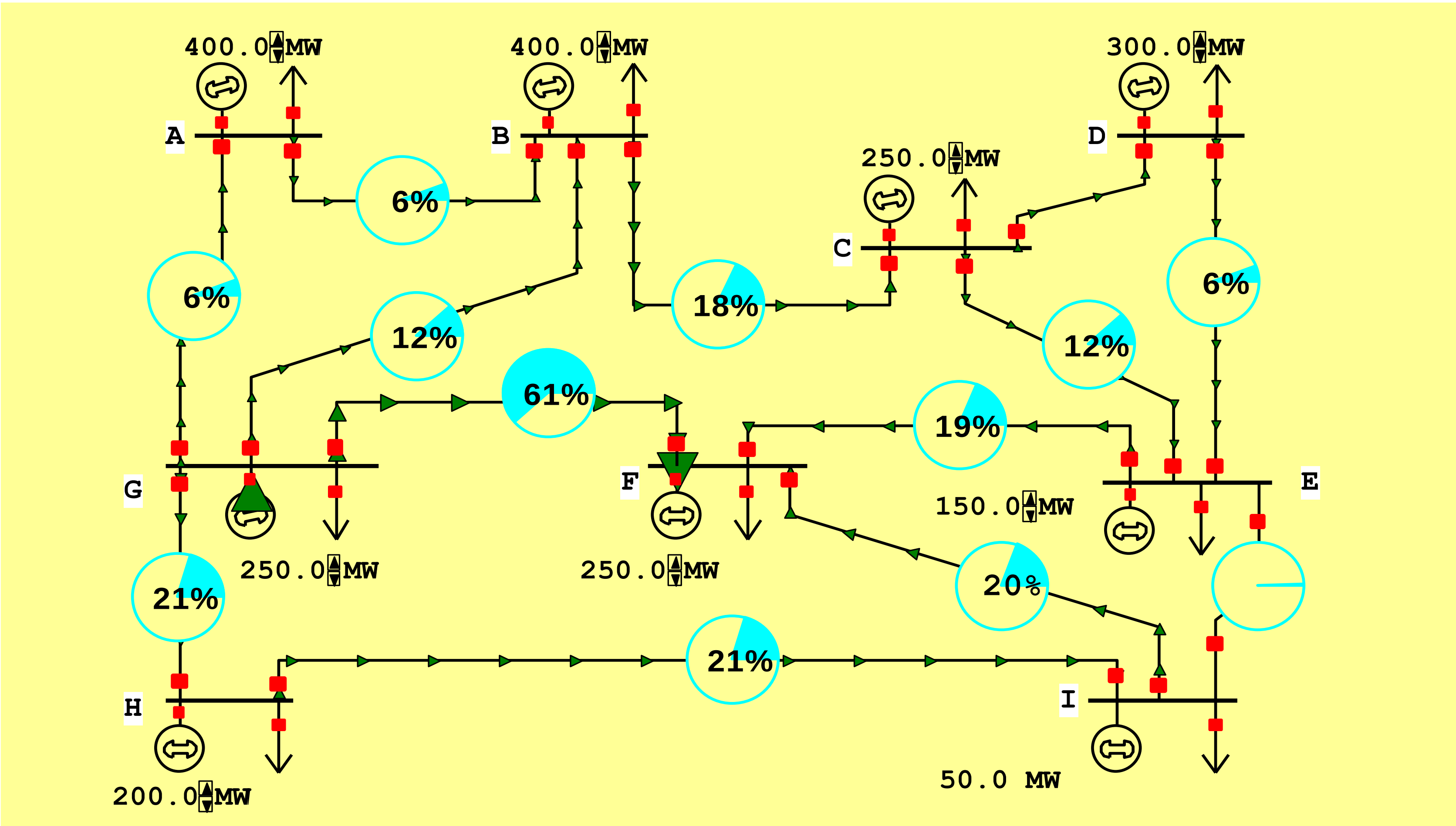
PowerWorld Case: **B9**

PTDF Example: Nine Bus System, Transfer from A to I



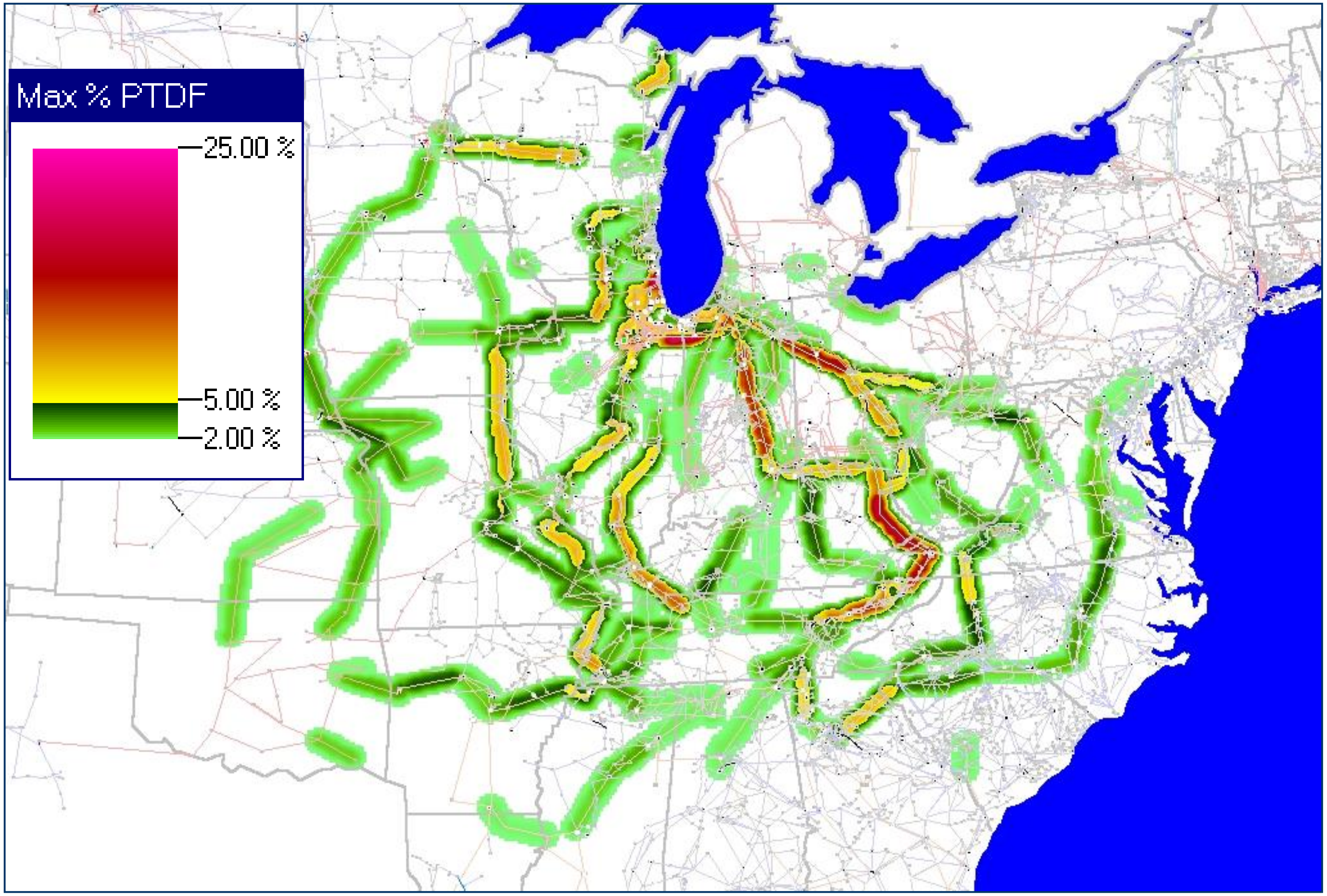
Values now tell percentage of flow that will go on line

PTDF Example: Nine Bus System, Transfer From G to F



Values now tell percentage of flow that will go on line

Wisconsin to TVA Line PTDF Contour



Contours show lines that would carry at least 2% of a power transfer from Wisconsin to TVA

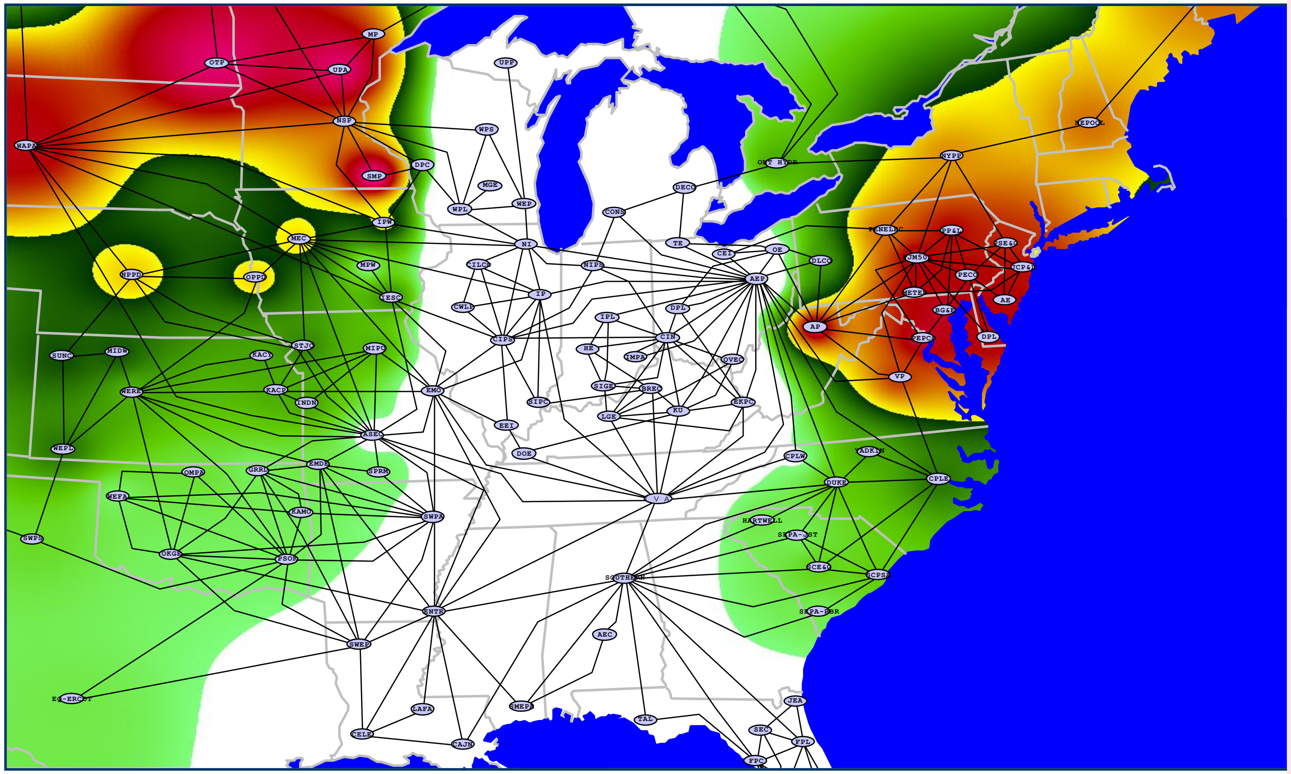
NERC Flowgates

- A convenient glossary of terms used for power system operations in North America is available at http://www.nerc.com/files/glossary_of_terms.pdf
- One common term is a “flowgate,” which is a mathematical construct to measure the MW flow on one or more elements in the bulk transmission system
  - Sometimes they include the impact of contingencies, something we will consider later in the semester
- A simple flowgate would be the MW flow through a single transmission line or transformer

NERC TLRs

- In the North American transmission loading relief procedures (TLRs) are used to mitigate the overloads on the bulk transmission system
  - https://www.nerc.com/pa/Stand/Reliability%20Standards/IRO-006-5.pdf
  - Called TLR in the East, WECC Unscheduled Flow Mitigation or Congestion Management Procedures (ERCOT)
- In the Eastern Interconnect TLRs consider the PTDFs associated with transactions on flowgates if there is a flowgate violation

Loop Flow Impact: Market Segmentation



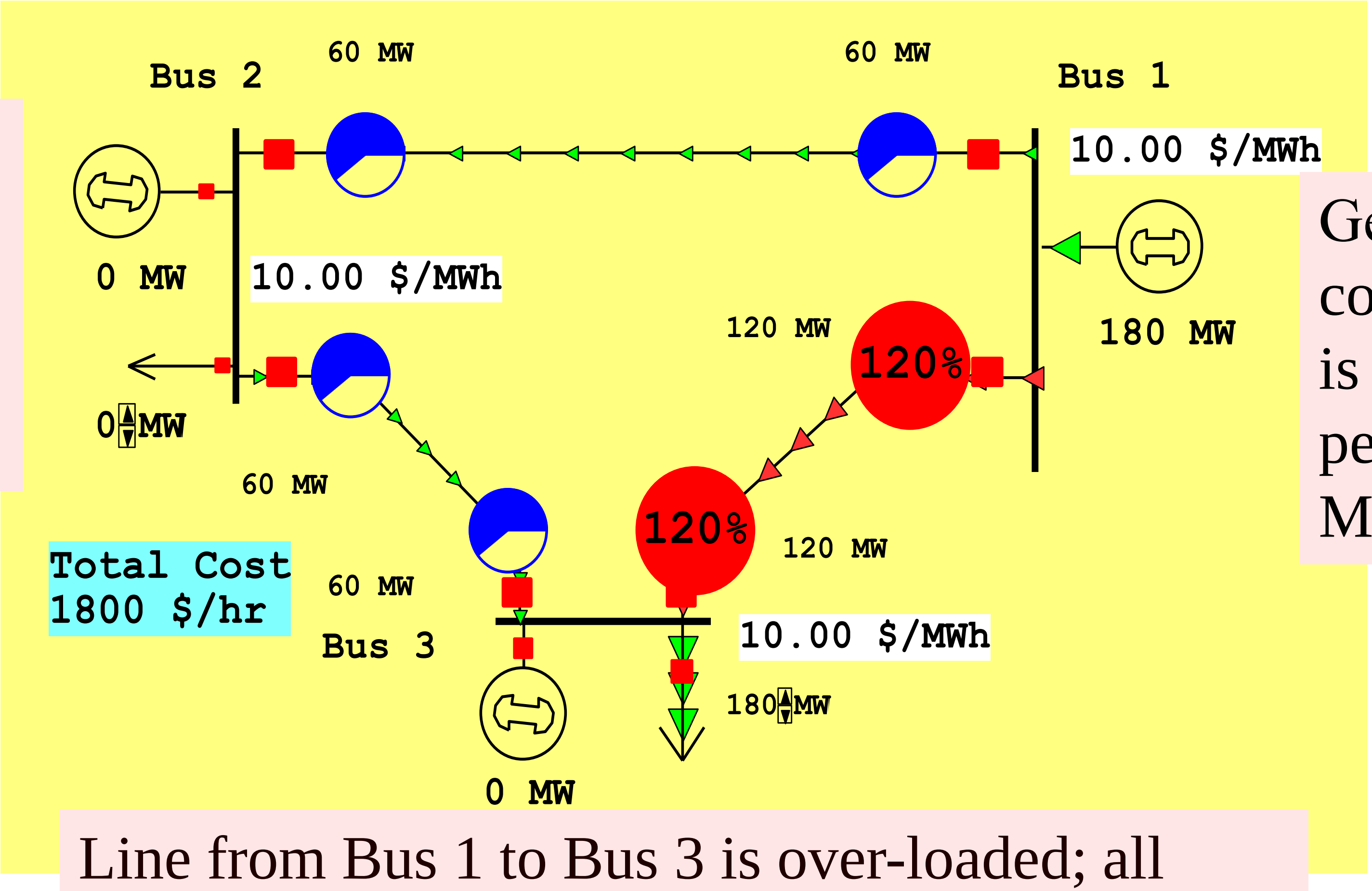
During summer of 1998 con-gestion on just two elements pushed Midwest spot market prices up by a factor of 200: from \$ 20/MWh to \$ 7500/MWh!

Pricing Electricity

- Cost to supply electricity to bus is called the locational marginal price (LMP)
- Presently some electric markets post LMPs on the web
- In an ideal electricity market with no transmission limitations the LMPs are equal
- Transmission constraints can segment a market, resulting in differing LMP
- Determination of LMPs requires the solution on an Optimal Power Flow (OPF), which will be covered later in the semester

Three Bus LMPs – Constraints Ignored

Gen 2's cost is $12 per MWh

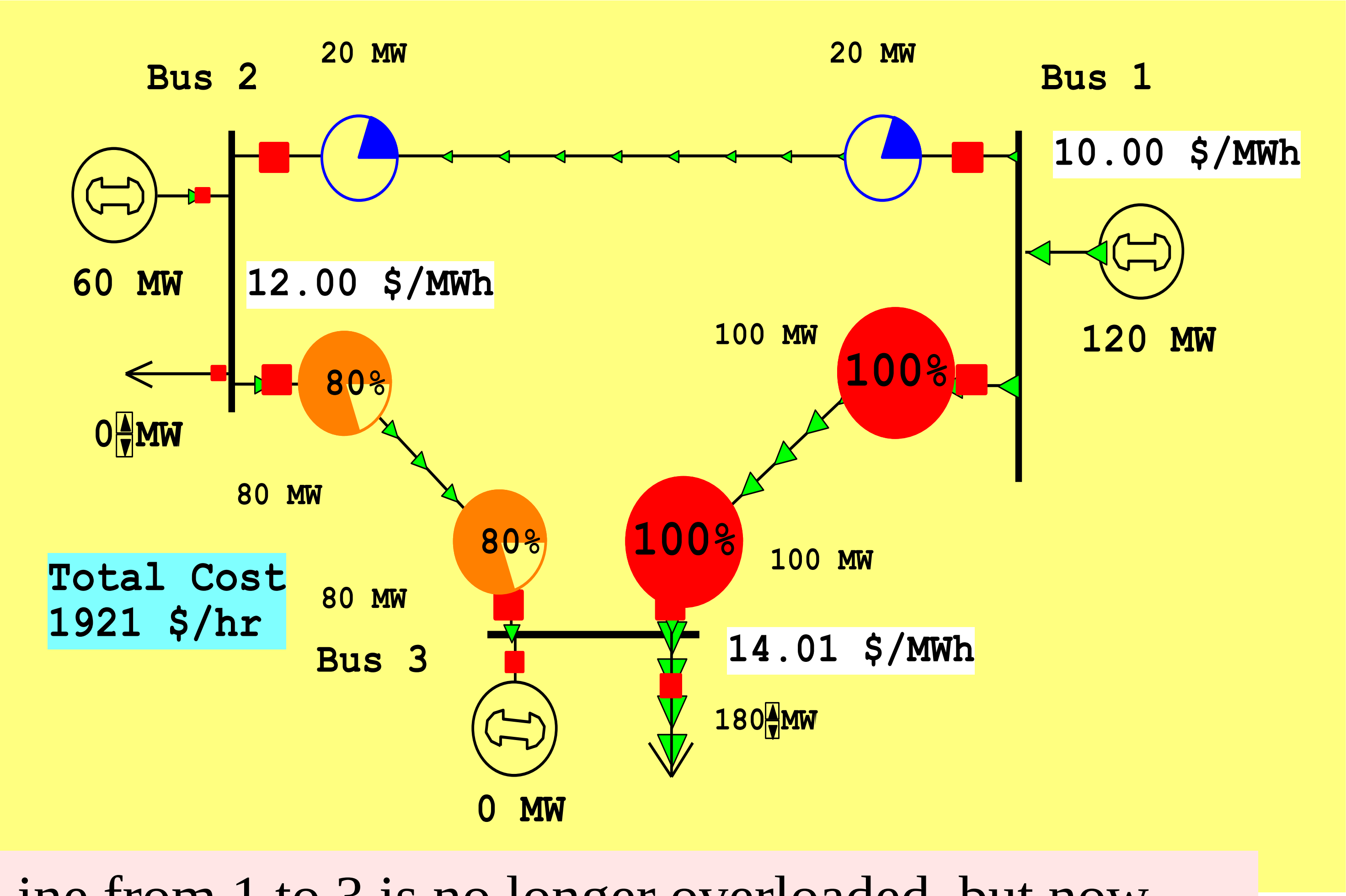


Gen 1's cost is $10 per MWh

Line from Bus 1 to Bus 3 is over-loaded; all buses have same marginal cost

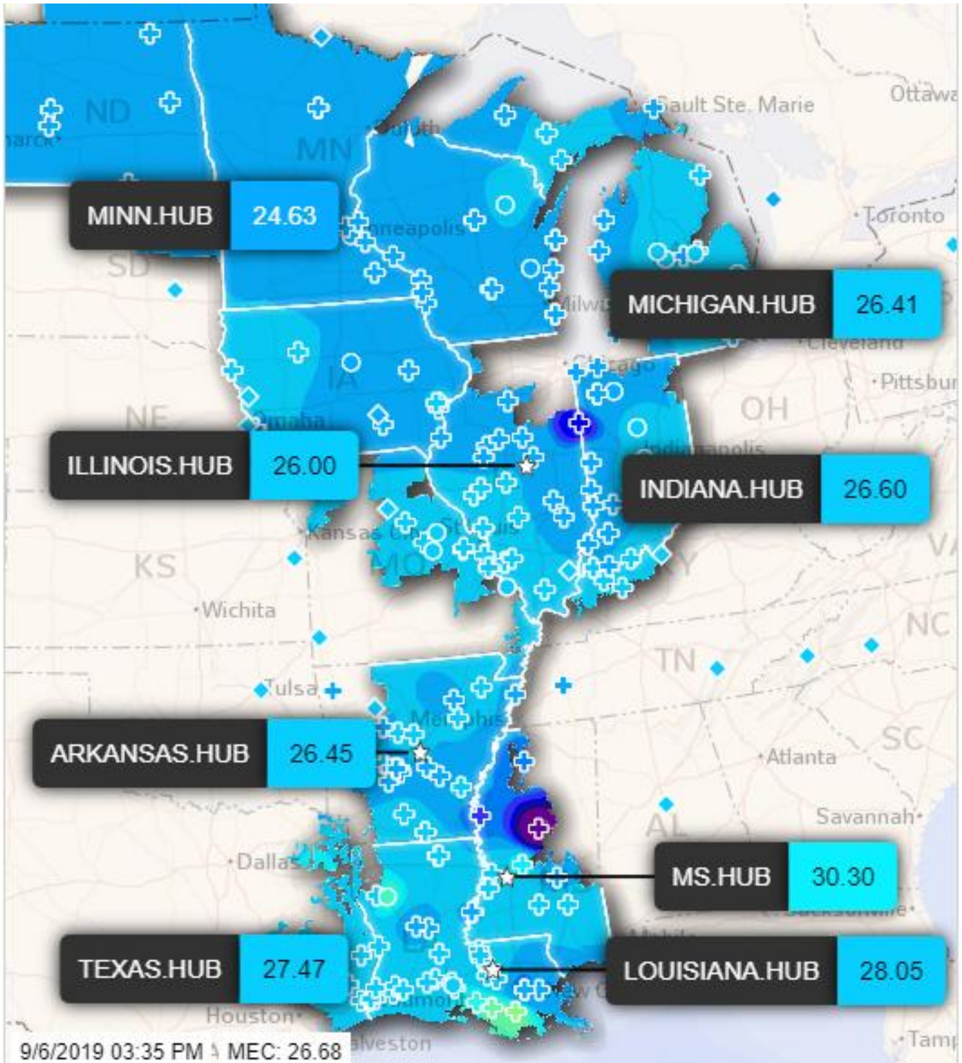
PowerWorld Case: **B3LP**

Three Bus LMPs – Constraint Unforced



Line from 1 to 3 is no longer overloaded, but now the marginal cost of electricity at bus 3 is $14 / MWh

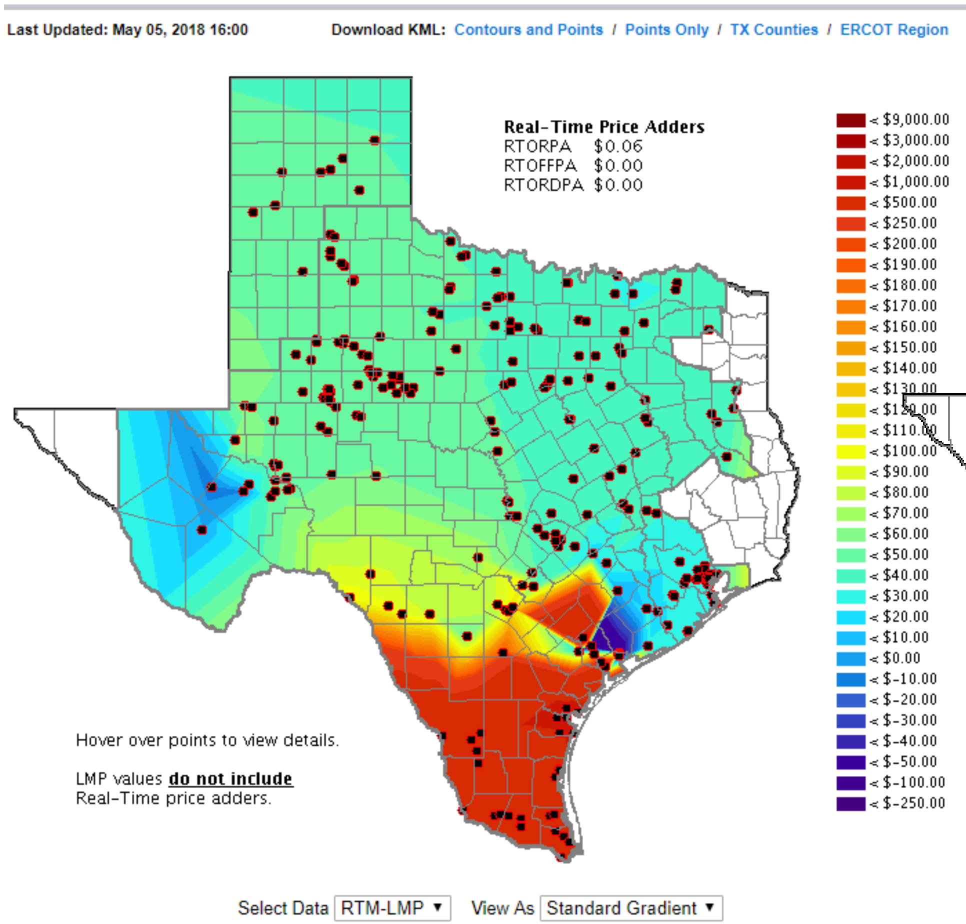
MISO LMPs on 9/6/2019, 3:35 PM



Five minute LMPs are posted online for the MISO footprint

Source: https://www.misoenergy.org/markets-and-operations/real-time--market-data/real-time-displays/

ERCOT LMPS: 5/5/18 and 9/6/19



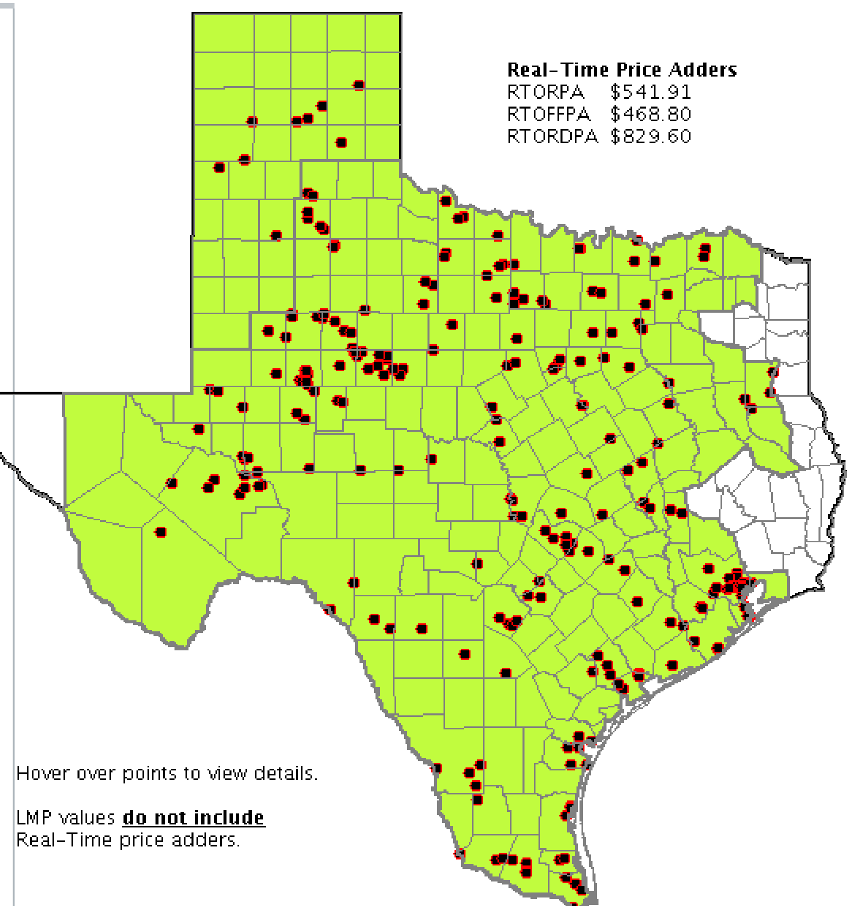


Image Source: http://www.ercot.com/content/cdr/contours/rtmLmp.html

Advanced Power Flow

- Next slides cover some more advanced power flow topics that need to be considered in many commercial power flow studies
- An important consideration in the power flow is the assumed time scale of the response, and the assumed model of operator actions
  - Planning power flow studies usually assume automatic modeling of operator actions and a longer time frame of response (controls have time to reach steady-state)
    - For example, who is actually doing the volt/var control
  - In real-time applications operator actions are usually not automated and controls may be more limited in time

Power Flow Optimal Multiplier

- Classic reference on power flow optimal multiplier is S. Iwamoto, Y. Tamura, “A Load Flow Calculation Method for Ill-Conditioned Power Systems,” *IEEE Trans. Power App. and Syst.*, April 1981
- Another paper is J.E. Tate, T.J. Overbye, “A Comparison of the Optimal Multiplier in Power and Rectangular Coordinates,” *IEEE Trans. Power Systems*, Nov. 2005
- Key idea is once NR method has selected a direction, we can analytically determine the distance to move in that direction to minimize the norm of the mismatch
  - Goal is to help with stressed power systems

Power Flow with Optimal Multiplier

- Consider an n bus power system with **f(x) = S** where **S** is the vector of the constant real and reactive power load minus generation at all buses except the slack, **x** is the vector of the bus voltages in rectangular coordinates: $V_i = e_i + jf_i,$
  and **f** is the function of the power balance constraints

$$f_{pi} = \sum_{j=1}^{n}\left(e_i\left(e_j G_{ij} - f_j B_{ij}\right) + f_i\left(f_j G_{ij} + e_j B_{ij}\right)\right)$$

$$f_{qi} = \sum_{j=1}^{n}\left(f_i\left(e_j G_{ij} - f_j B_{ij}\right) - e_i\left(f_j G_{ij} + e_j B_{ij}\right)\right)$$

G + j**B** is the bus admittance matrix

Power Flow with Optimal Multiplier

- With a standard NR approach we would get

$$\mathbf{x}^{k+1} = \mathbf{x}^{k} + \Delta\mathbf{x}^{k}$$

$$\Delta\mathbf{x}^{k} = -\mathbf{J}(\mathbf{x}^{k})^{-1}\left(\mathbf{f}(\mathbf{x}^{k}) - \mathbf{S}\right)$$

- If we are close enough to the solution the iteration converges quickly, but if the system is heavily loaded it can diverge

Power Flow with Optimal Multiplier

- Optimal multiplier approach modifies the iteration as

$$\mathbf{x}^{k+1} = \mathbf{x}^{k} + \mu \Delta \mathbf{x}^{k}$$

$$\Delta \mathbf{x}^{k} = -\mathbf{J}(\mathbf{x}^{k})^{-1}\left(\mathbf{f}(\mathbf{x}^{k}) - \mathbf{S}\right)$$

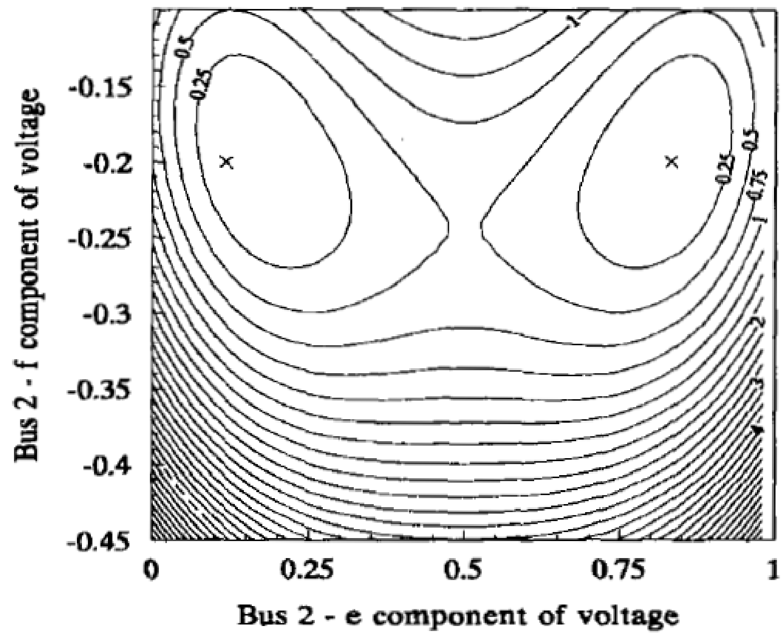
- Scalar μ is chosen to minimize the norm of the mismatch F in direction $\Delta\mathbf{x}$

$$F\left(\mathbf{x}^{k+1}\right) = \frac{1}{2}\left[\mathbf{f}\left(\mathbf{x}^{k} + \mu \Delta \mathbf{x}^{k}\right) - \mathbf{S}\right]^{T}\left[\mathbf{f}\left(\mathbf{x}^{k} + \mu \Delta \mathbf{x}^{k}\right) - \mathbf{S}\right]$$

- Paper by Iwamoto, Y. Tamura from 1981 shows μ can be computed analytically with little additional calculation when rectangular voltages are used

Power Flow with Optimal Multiplier

- Determination of μ involves solving a cubic equation, which gives either three real solutions, or one real and two imaginary solutions
- 1989 PICA paper by Iba ("A Method for Finding a Pair of Multiple Load Flow Solutions in Bulk Power Systems") showed that NR tends to converge along line joining the high and a low voltage solution



However, there are some model restrictions

Quasi-Newton Power Flow Methods

- First we consider some modified versions of the Newton power flow (NPF)
- Since most of the computation in the NPF is associated with building and factoring the Jacobian matrix, **J**, the focus is on trying to reduce this computation
- In a pure NPF **J** is build and factored each iteration
- Over the years pretty much every variation of the NPF has been tried; here we just touch on the most common
- Whether a method is effective can be application dependent
  - For example, in contingency analysis we are usually just resolving a solved case with an often small perturbation

Quasi-Newton Power Flow Methods

- The simplest modification of the NPF results when **J** is kept constant for a number of iterations, say k iterations
  - Sometimes known as the Dishonest Newton
- The approach balances increased speed per iteration, with potentially more iterations to perform
- There is also an increased possibility for divergence
- Since the mismatch equations are not modified, if it converges it should converge to the same solution as the NPF
- These methods are not commonly used, except in very short duration, sequential power flows with small mismatches

Dishonest N-R Example

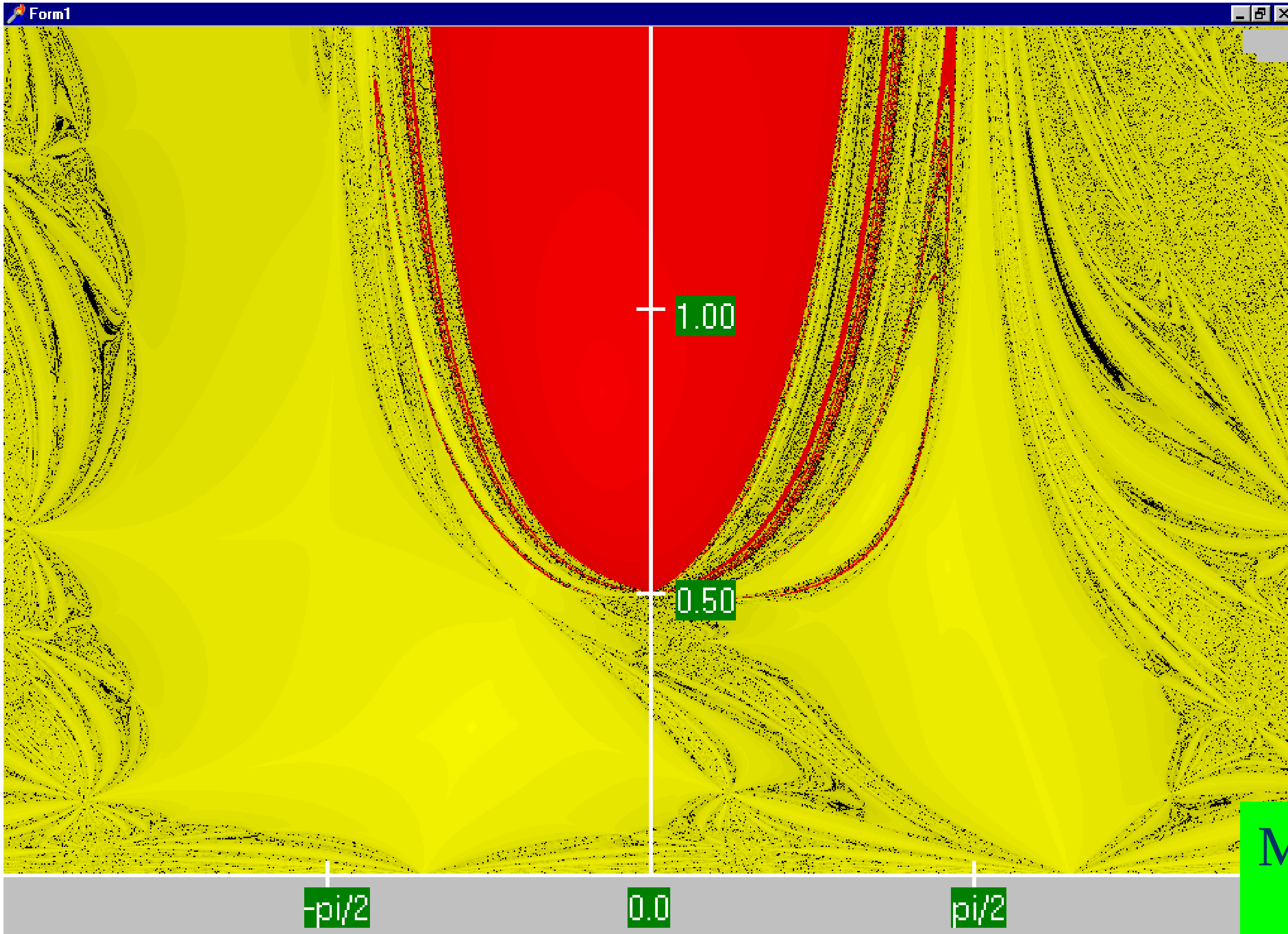
$$x^{(v+1)} = x^{(v)} - \left[\frac{1}{2x^{(0)}}\right]((x^{(v)})^2 - 2)$$

Guess $x^{(0)} = 1$. Iteratively solving we get

v	$x^{(v)}$ (honest)	$x^{(v)}$ (dishonest)
0	1	1
1	1.5	1.5
2	1.41667	1.375
3	1.41422	1.429
4	1.41422	1.408

We pay a price in increased iterations, but with decreased computation per iteration; that price is too high in this example

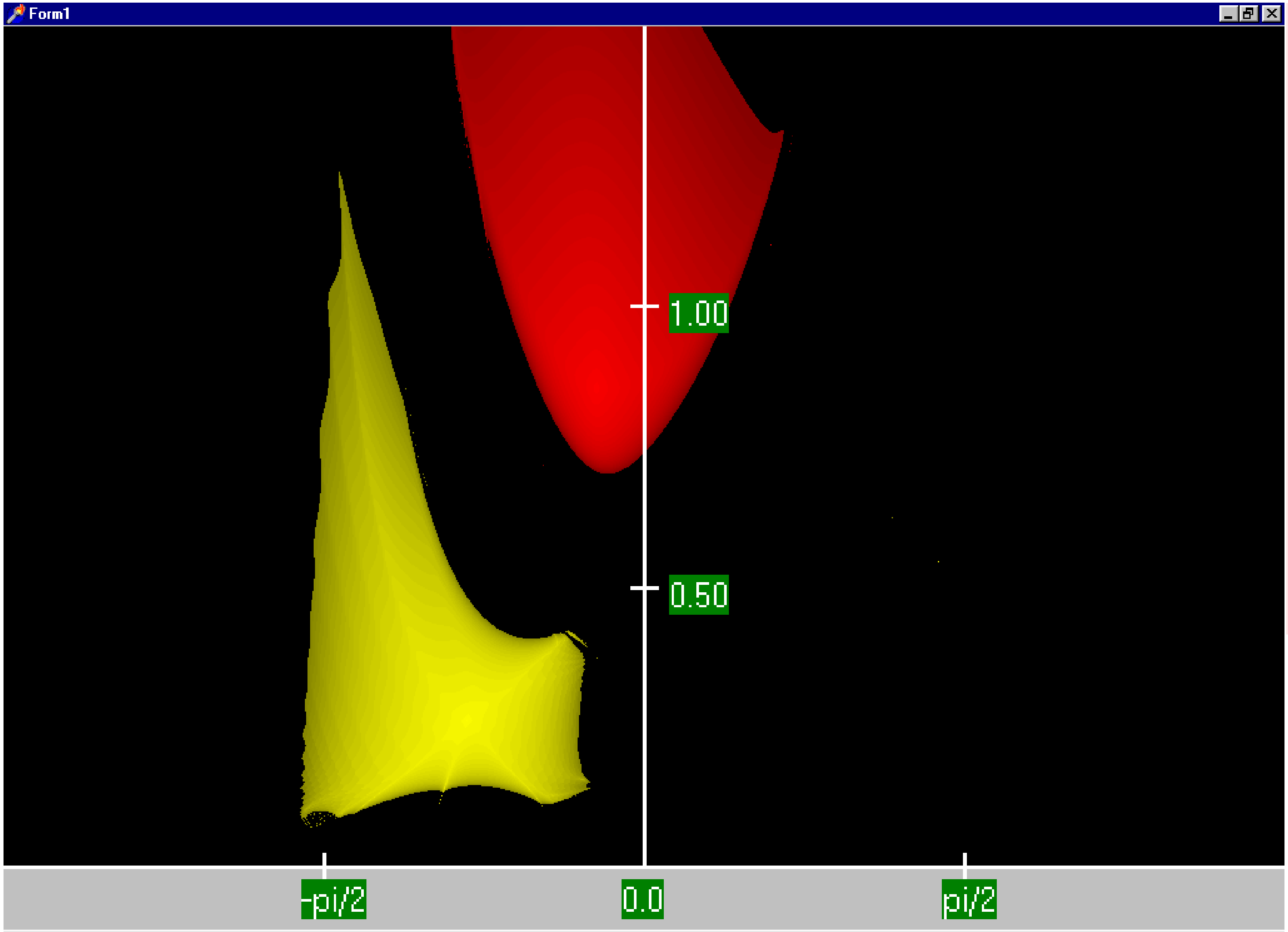
NPF (Honest) Region of for Two Bus Example Convergence



Red region converges to the high voltage solution, while the yellow region converges to the low voltage solution

Maximum of 15 iterations

Two Bus Dishonest ROC



In this case being honest pays! At least with respect to the region of convergence (ROC)

Quasi-Newton Power Flow Methods

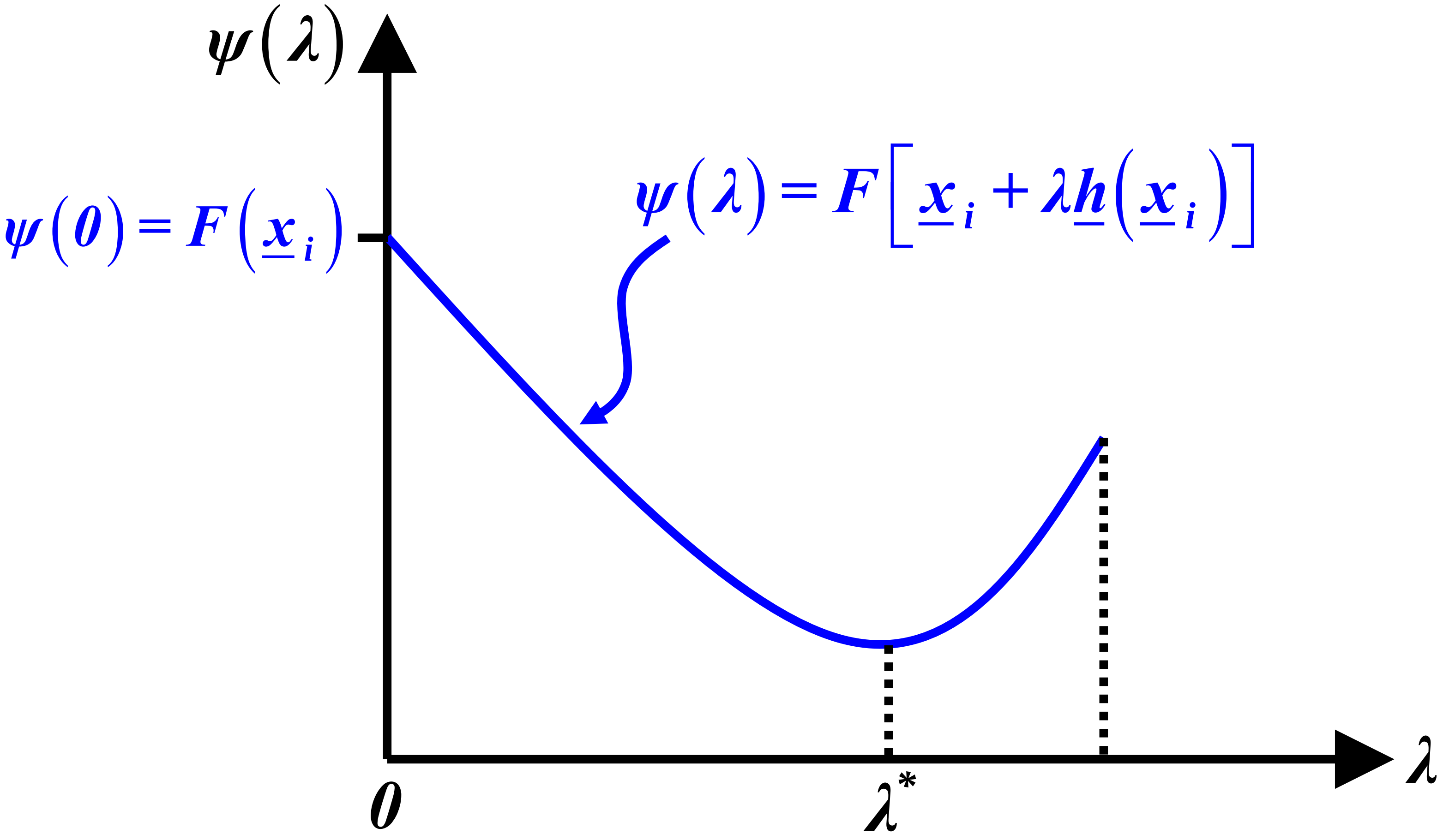
- A second modification is to modify the step size in the direction given by the NPF
  - This is one we've already considered with the optimal multiplier approach

$$\Delta \mathbf{x}^k = -\mathbf{J}(\mathbf{x}^k)^{-1}\left(\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right)$$

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \mu \Delta \mathbf{x}^k$$

- The generalized approach is to solve what is known as the line search (i.e., a one-dimensional optimization) to determine μ

The Single Dimensional $\psi(\lambda)$



Line Search

- We need a cost function, which is usually the Euclidean norm of the mismatch vector
- The line search is a general optimization problem for which there are many potential solution approaches
  - Determines a local optimum within some search boundaries
  - Approaches depend on whether there is gradient information available
- Aside from the optimal multiplier approach, which can be quite helpful with little additional computation, the convergence gain from determining the "optimal" μ is usually more than offset by the line search computation

Decoupled Power Flow

- Rather than not updating the Jacobian, the decoupled power flow takes advantage of characteristics of the power grid in order to decouple the real and reactive power balance equations
  - There is a strong coupling between real power and voltage angle, and reactive power and voltage magnitude
  - There is a much weaker coupling between real power and voltage angle, and reactive power and voltage angle
- Key reference is B. Stott, "Decoupled Newton Load Flow," *IEEE Trans. Power. App and Syst*., Sept/Oct. 1972, pp. 1955-1959

Decoupled Power Flow Formulation

General form of the power flow problem

$$-\begin{bmatrix} \dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} & \dfrac{\partial \mathbf{P}}{\partial |\mathbf{V}|}^{(v)} \\ \dfrac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}^{(v)} & \dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}^{(v)} \\ \Delta |\mathbf{V}|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \\ \Delta \mathbf{Q}(\mathbf{x}^{(v)}) \end{bmatrix} = \mathbf{f}(\mathbf{x}^{(v)})$$

where

$$\Delta \mathbf{P}(\mathbf{x}^{(v)}) = \begin{bmatrix} P_2(\mathbf{x}^{(v)}) + P_{D2} - P_{G2} \\ \vdots \\ P_n(\mathbf{x}^{(v)}) + P_{Dn} - P_{Gn} \end{bmatrix}$$

Decoupling Approximation

Usually the off-diagonal matrices, $\dfrac{\partial \mathbf{P}}{\partial |\mathbf{V}|}^{(v)}$ and $\dfrac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}^{(v)}$

are small. Therefore we approximate them as zero:

$$-\begin{bmatrix} \dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} & \mathbf{0} \\ \mathbf{0} & \dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}^{(v)} \\ \Delta |\mathbf{V}|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \\ \Delta \mathbf{Q}(\mathbf{x}^{(v)}) \end{bmatrix} = \mathbf{f}(\mathbf{x}^{(v)})$$

Then the problem can be decoupled

$$\Delta \boldsymbol{\theta}^{(v)} = -\left[\dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)}\right]^{-1} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \quad \Delta |\mathbf{V}|^{(v)} = -\left[\dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)}\right]^{-1} \Delta \mathbf{Q}(\mathbf{x}^{(v)})$$

Off-diagonal Jacobian Terms

Justification for Jacobian approximations:

1. Usually r << x, therefore $|G_{ij}| << |B_{ij}|$
2. Usually θ_{ij} is small so $\sin\theta_{ij} \approx 0$

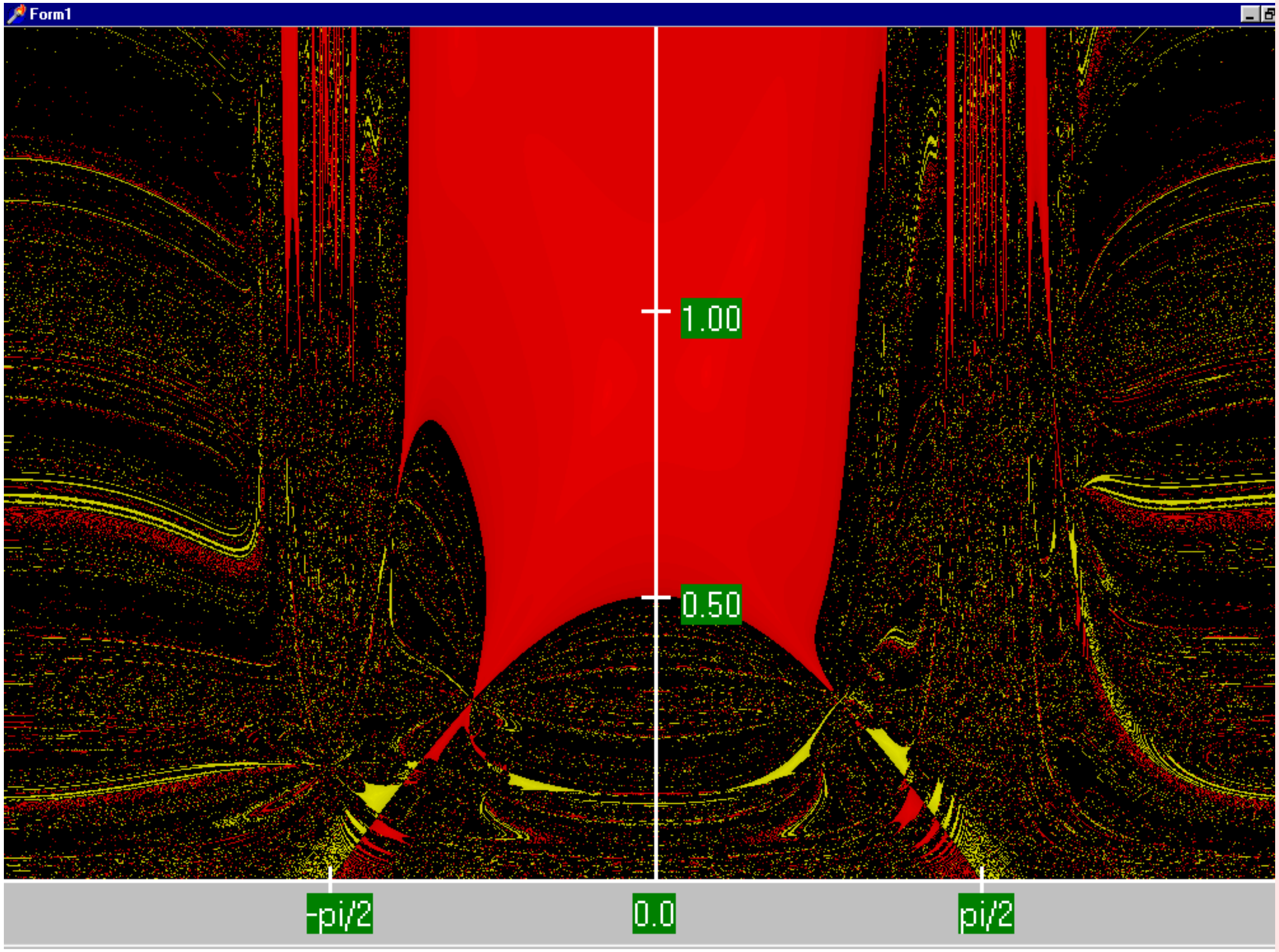
Therefore

$$\frac{\partial \mathbf{P}_i}{\partial |\mathbf{V}_j|} = |V_i|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \approx 0$$

$$\frac{\partial \mathbf{Q}_i}{\partial \boldsymbol{\theta}_j} = -|V_i||V_j|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \approx 0$$

By assuming ½ the elements are zero, we only have to do ½ the computations

Decoupled N-R Region of Convergence



The high solution ROC is actually larger than with the standard NPF. Obviously this is not a good a way to get the low solution

Fast Decoupled Power Flow

- By continuing with our Jacobian approximations we can actually obtain a reasonable approximation that is independent of the voltage magnitudes/angles.
- This means the Jacobian need only be built/inverted once per power flow solution
- This approach is known as the fast decoupled power flow (FDPF)

Fast Decoupled Power Flow, cont.

- FDPF uses the same mismatch equations as standard power flow (just scaled) so it should have same solution
- The FDPF is widely used, though usually only when we only need an approximate solution
- Key fast decoupled power flow reference is B. Stott, O. Alsac, "Fast Decoupled Load Flow," *IEEE Trans. Power App. and Syst.*, May 1974, pp. 859-869
- Modified versions also exist, such as D. Jajicic and A. Bose, "A Modification to the Fast Decoupled Power Flow for Networks with High R/X Ratios, "*IEEE Transactions on Power Sys.*, May 1988, pp. 743-746

FDPF Approximations

The FDPF makes the following approximations:

1. $\left|G_{ij}\right| = 0$
2. $\left|V_i\right| \quad = \quad 1$
3. $\sin\theta_{ij} = 0 \qquad \cos\theta_{ij} = 1$

To see the impact on the real power equations recall

$$\mathrm{P_i} \quad = \quad \sum_{k=1}^{n} V_i V_k (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

Which can also be written as

$$\frac{\mathrm{P_i}}{V_i} \quad = \quad \sum_{k=1}^{n} V_k (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = \frac{P_{Gi} - P_{Di}}{V_i}$$

FDPF Approximations

- With the approximations for the diagonal term we get

$$\frac{\partial \mathrm{P_i}}{\partial \theta_i} \approx \sum_{\substack{k=1 \\ k \neq i}}^{n} B_{ik} = -B_{ii}$$

The for the off-diagonal terms (k $\neq$ i) with **G=0** and **V=1**

$$\frac{\partial P_i}{\partial \theta_k} = -B_{ik} \cos \theta_{ik} \approx -B_{ik}$$

- Hence the Jacobian for the real equations can be approximated as –**B**

FPDF Approximations

- For the reactive power equations we also scale by V_i

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

$$\frac{\mathrm{Q_i}}{V_i} = \sum_{k=1}^{n} V_k(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = \frac{Q_{Gi} - Q_{Di}}{V_i}$$

- For the Jacobian off-diagonals we get

$$\frac{\partial Q_i}{\partial V_k} = -B_{ik}\cos\theta_{ik} \approx -B_{ik}$$

FDPF Approximations

- And for the reactive power Jacobian diagonal we get

$$\frac{\partial Q_{\mathrm{i}}}{\partial V_i} \approx -2B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} B_{ik} = -B_{ii}$$

- As derived the real and reactive equations have a constant Jacobian equal to –**B**
  - Usually modifications are made to omit from the real power matrix elements that affect reactive flow (like shunts) and from the reactive power matrix elements that affect real power flow, like phase shifters
  - We'll call the real power matrix **B**' and the reactive **B**"