ECEN 667
Power System Stability

Lecture 3: Electromagnetic Transients

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Announcements

- RSVP to Alex at zandra23@ece.tamu.edu for the TAMU ECE Energy and Power Group (EPG) picnic. It starts at 5pm on September 27, 2019
- Be reading Chapters 1 and 2
- Homework 1 is assigned today. It is due on Thursday September 12
- Classic reference paper on EMTP is H.W. Dommel, "Digital Computer Solution of Electromagnetic Transients in Single- and Multiphase Networks," *IEEE Trans. Power App. and Syst.*, vol. PAS-88, pp. 388-399, April 1969

Electromagnetic Transients

- The modeling of very fast power system dynamics (much less than one cycle) is known as electromagnetics transients program (EMTP) analysis
  - Covers issues such as lightning propagation and switching surges
- Concept originally developed by Prof. Hermann Dommel for his PhD in the 1960's (now emeritus at Univ. British Columbia)
  - After his PhD work Dr. Dommel worked at BPA where he was joined by Scott Meyer in the early 1970's
  - Alternative Transients Program (ATP) developed in response to commercialization of the BPA code

Power System Time Frames

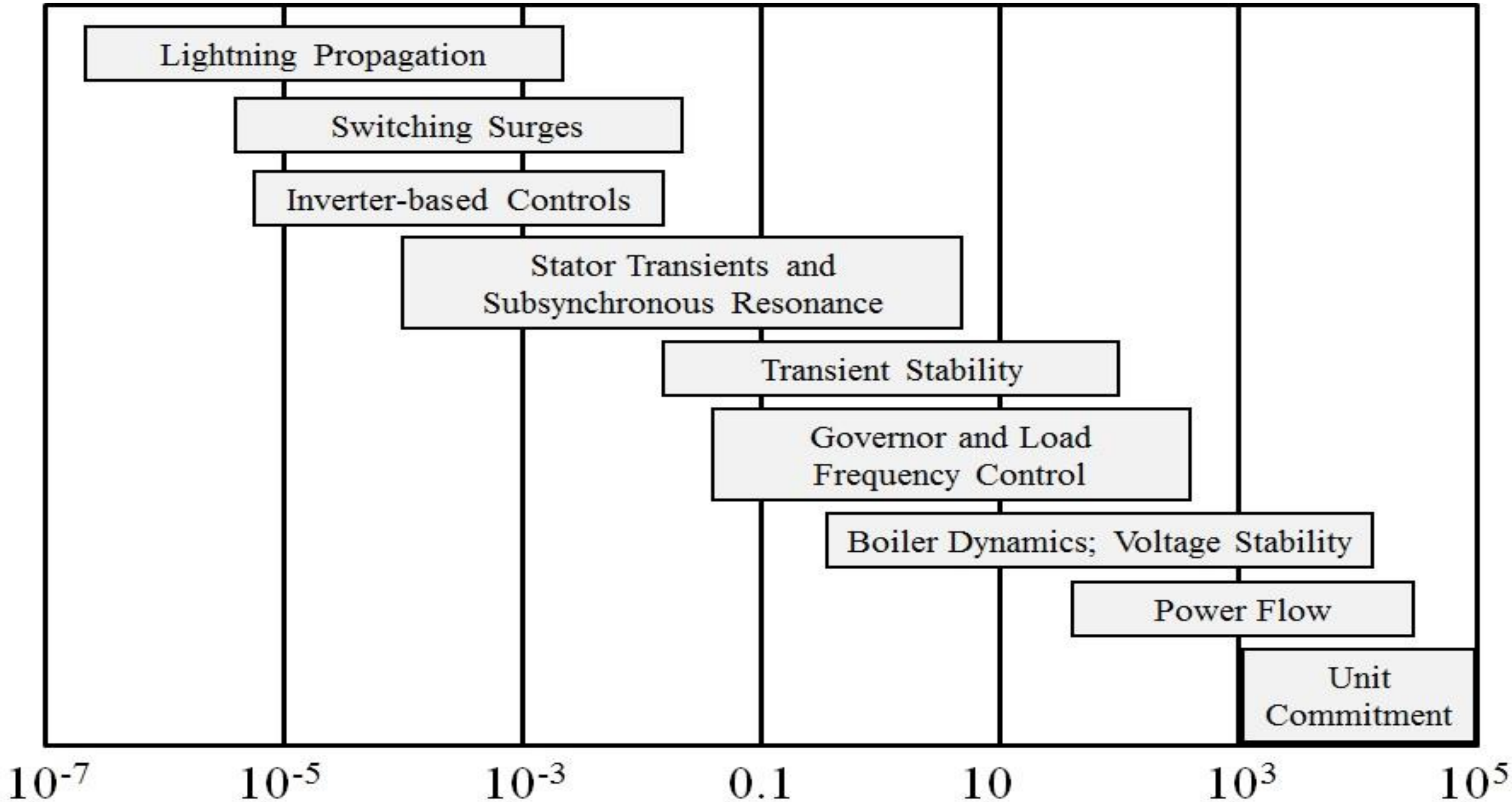


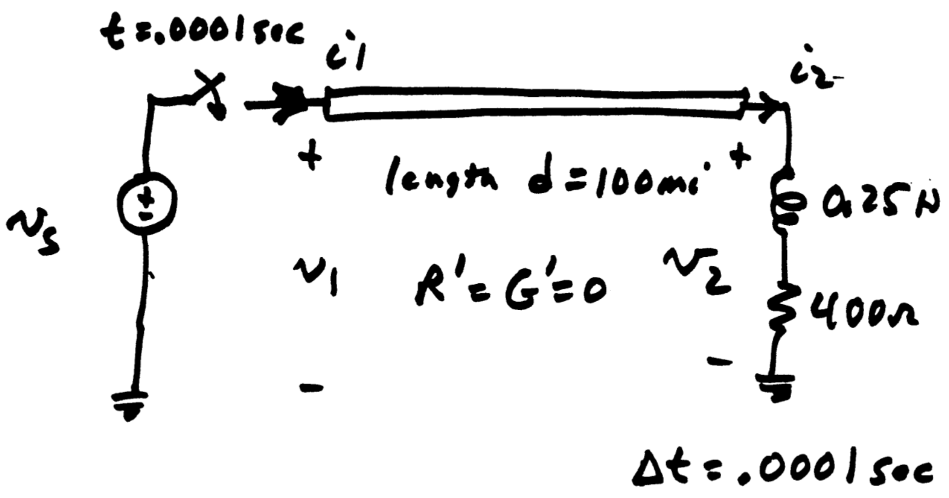
Image source: P.W. Sauer, M.A. Pai, Power System Dynamics and Stability, 1997, Fig 1.2, modified

Transmission Line Modeling

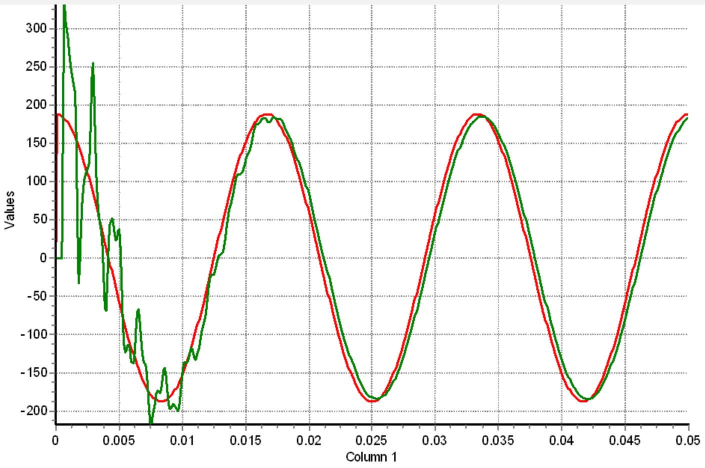
- In power flow and transient stability transmission lines are modeled using a lumped parameter approach
  - Changes in voltages and current in the line are assumed to occur instantaneously
  - Transient stability time steps are usually a few ms (1/4 cycle is common, equal to 4.167ms for 60Hz)
- In EMTP time-frame this is no longer the case; speed of light is 300,000km/sec or 300km/ms or 300m/μs
  - Change in voltage and/or current at one end of a transmission cannot instantaneously affect the other end

Need for EMTP

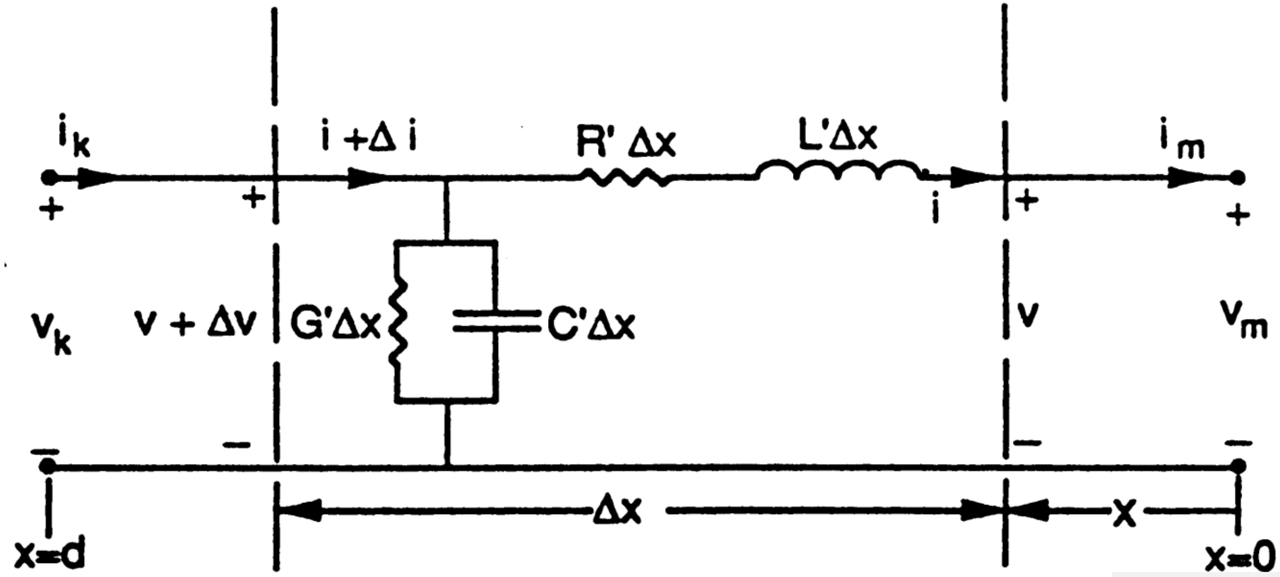
- The change isn't really instantaneous because of propagation delays, which are near the speed of light; there also wave reflection issues



Red is the v_s end, green the v_2 end



Incremental Transmission Line Modeling



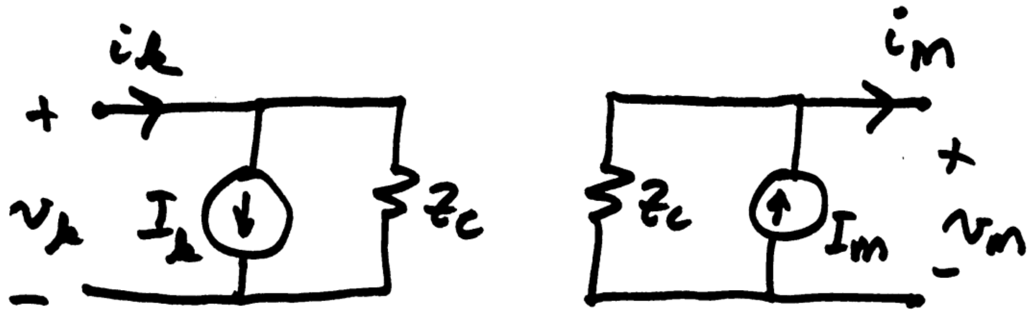
$$\Delta v = R'\Delta x i + L'\Delta x \frac{\partial i}{\partial t}$$

$$\Delta i = G'\Delta x\left(v + \Delta v\right) + C'\Delta x \frac{\partial}{\partial t}\left(v + \Delta v\right)$$

Define the receiving end as bus m (x=0) and the sending end as bus k (x=d)

Where We Will End Up

- Goal is to come up with model of transmission line suitable for numeric studies on this time frame



$$I_k = i_m\left(t - \frac{d}{v_p}\right) - \frac{1}{z_c} v_m\left(t - \frac{d}{v_p}\right)$$

$$I_m = i_k\left(t - \frac{d}{v_p}\right) + \frac{1}{z_c} v_k\left(t - \frac{d}{v_p}\right)$$

Both ends of the line are represented by Norton equivalents

Assumption is we don't care about what occurs along the line

Incremental Transmission Line Modeling

We are looking to determine v(x,t) and i(x,t)

Substitute $\Delta v = \Delta x\left(R'i + L'\frac{\partial i}{\partial t}\right)$

Into the equation for Δi and divide both by Δx

$$\frac{\Delta i}{\Delta x} = G'v + G'\left(R'\Delta x i + L'\Delta x\frac{\partial i}{\partial t}\right) + C'\frac{\partial v}{\partial t}$$

$$+C'\left[R'\Delta x\frac{\partial i}{\partial t} + L'\Delta x\frac{\partial^2 i}{\partial t^2}\right]$$

Incremental Transmission Line Modeling

Taking the limit we get

$$\lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} = \boxed{\frac{\partial v}{\partial x} = R' i + L' \frac{\partial i}{\partial t}}$$

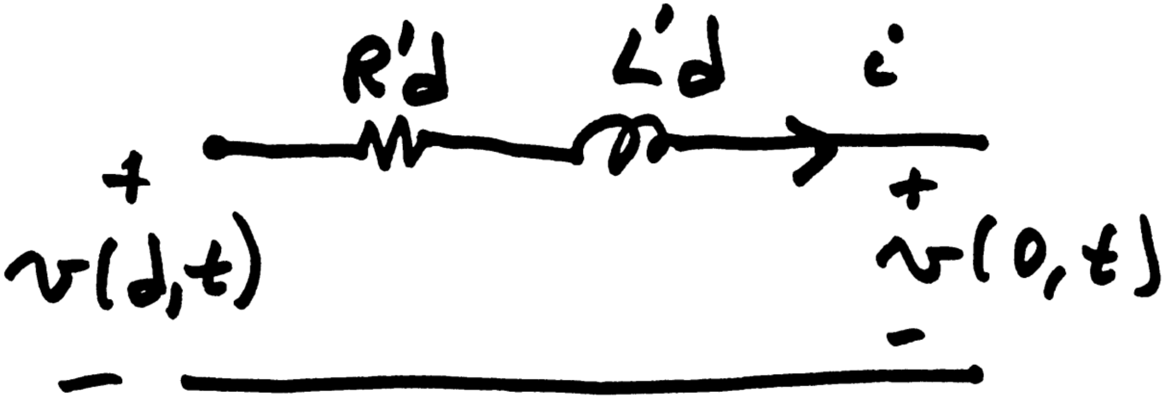
$$\lim_{\Delta x \to 0} \frac{\Delta i}{\Delta x} = \boxed{\frac{\partial i}{\partial x} = G' v + C' \frac{\partial v}{\partial t}}$$

Some authors have a negative sign with these equations; it just depends on the direction of increasing x; note values are function of both x and t

Special Case 1

C' = G' = 0 (neglect shunts)

$$v(x,t) = v(0,t) + R'x_i + L'x\frac{di}{dt}$$



This just gives a lumped parameter model, with all electric field effects neglected

Special Case 2: Wave Equation

The lossless line (R'=0, G'=0), which gives

$$\frac{\partial v}{\partial x} = L'\frac{\partial i}{\partial t}, \quad \frac{\partial i}{\partial x} = C'\frac{\partial v}{\partial t}$$

This is the wave equation with a general solution of

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$z_c = \sqrt{L'/C'}\ , \quad v_p = \frac{1}{\sqrt{L'C'}}$$

z_c is the characteristic impedance and v_p is the velocity of propagation

Special Case 2: Wave Equation

- This can be thought of as two waves, one traveling in the positive x direction with velocity v_p, and one in the opposite direction
- The values of f_1 and f_2 depend upon the boundary (terminal) conditions

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$z_c = \sqrt{L'/C'}\ , \quad v_p = \frac{1}{\sqrt{L'C'}}$$

Boundaries are receiving end with x=0 and the sending end with x=d

Calculating v_p

- To calculate v_p for a line in air we go back to the definition of L' and C'

$$L' = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{r'}\right), \quad C' = \frac{2\pi\varepsilon_0}{\ln D/r}$$

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu_0\varepsilon_0 \frac{\ln D/r'}{\ln D/r}}} = c\frac{1}{\sqrt{\frac{\ln D/r'}{\ln D/r}}}$$

With r'=0.78r this is very close to the speed of light

Important Insight

- The amount of time for the wave to go between the terminals is $d/v_p = \tau$ seconds
  - To an observer traveling along the line with the wave, $x+v_p t$, will appear constant
- What appears at one end of the line impacts the other end τ seconds later

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$v(x,t) + z_c i(x,t) = -2 z_c f_2(x + v_p t)$$

Both sides of the bottom equation are constant when $x+v_p t$ is constant

Determining the Constants

- If just the terminal characteristics are desired, then an approach known as Bergeron's method can be used.
- Knowing the values at the receiving end m (x=0) we get

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$i_m(t) = i(0,t) = -f_1(-v_p\, t) - f_2(v_p\, t)$$

$$v_m(t) = z_c f_1(-v_p\, t) - z_c f_2(v_p\, t)$$

This can be used to eliminate f_1

Determining the Constants

- Eliminating f_1 we get

$$v_m(t) = z_c f_1\left(-v_p\, t\right) - z_c f_2\left(v_p\, t\right)$$

$$f_1\left(-v_p\, t\right) = \frac{v_{m(t)}}{z_c} + f_2\left(v_p\, t\right)$$

$$i_m(t) = -\frac{v_m}{z_c} - 2 f_2\left(v_p\, t\right)$$

Determining the Constants

- To solve for f_2 we need to look at what is going on at the sending end (i.e., k at which x=d) $\tau = d/v_p$ seconds in the past

$$i_k\left(t-\frac{d}{v_p}\right)=-f_1\left(d-v_p\left(t-\frac{d}{v_p}\right)\right)-f_2\left(d+v_p\left(t-\frac{d}{v_p}\right)\right)$$

$$i_k\left(t-\frac{d}{v_p}\right)=-f_1\left(2d-v_pt\right)-f_2\left(v_pt\right)$$

$$v_k\left(t-\frac{d}{v_p}\right)=z_c f_1\left(2d-v_pt\right)-z_c f_2\left(v_pt\right)$$

Determining the Constants

- Dividing v_k by z_c, and then adding it with i_k gives

$$i_k\left(t-\frac{d}{v_p}\right)+\frac{v_k}{z_c}\left(t-\frac{d}{v_p}\right)=-2f_2\left(v_p t\right)$$

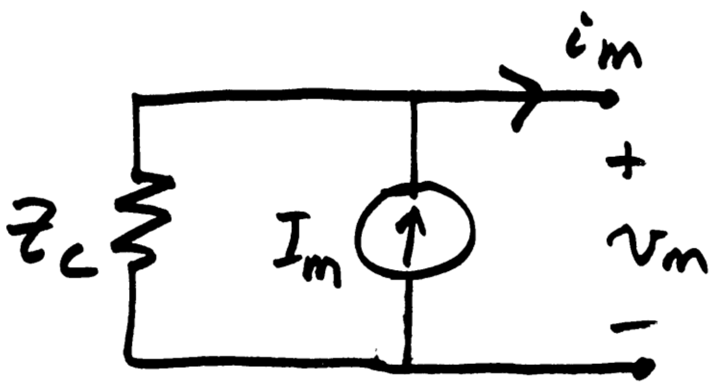
- Then substituting for f_2 in i_m gives

$$i_m\left(t\right)=-\frac{v_m\left(t\right)}{z_c}+i_k\left(t-\frac{d}{v_p}\right)+\frac{1}{z_c}v_k\left(t-\frac{d}{v_p}\right)$$

Equivalent Circuit Representation

- The receiving end can be represented in circuit form as

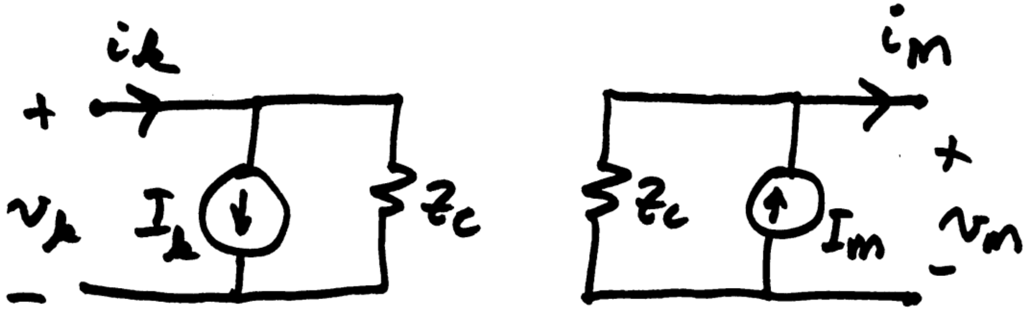
$$i_m(t) = -\frac{v_m(t)}{z_c} + \boxed{i_k\left(t-\frac{d}{v_p}\right)+\frac{1}{z_c}v_k\left(t-\frac{d}{v_p}\right)} \longrightarrow I_m$$



Since $\tau = d/v_p$, I_m just depends on the voltage and current at the other end of the line from τ seconds in the past. Since these are known values, it looks like a time-varying current source

Repeating for the Sending End

- The sending end has a similar representation



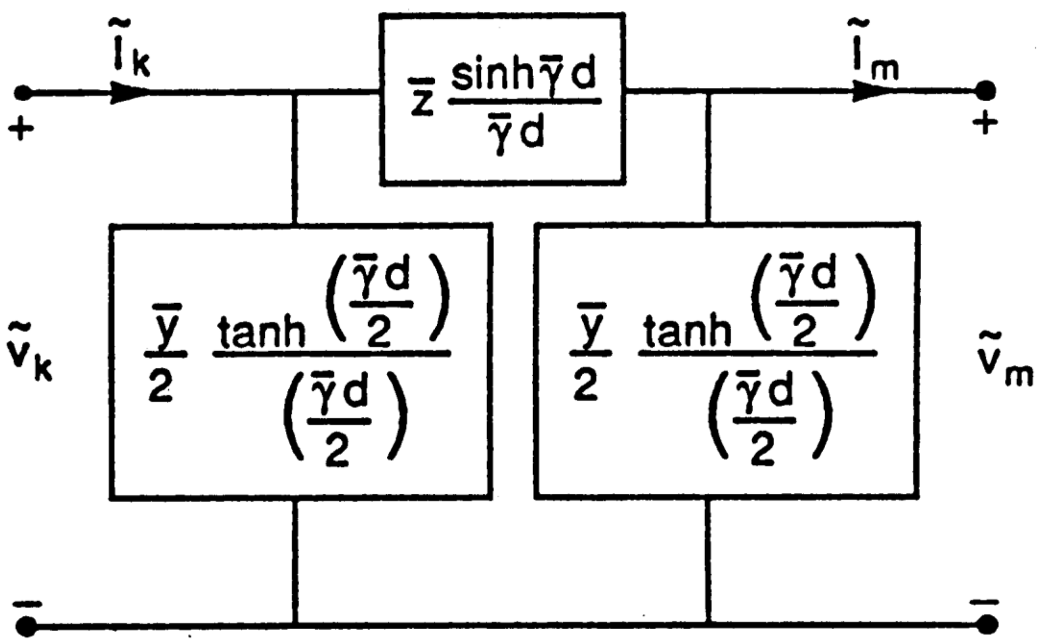
Both ends of the line are represented by Norton equivalents

$$I_k = i_m\left(t - \frac{d}{v_p}\right) - \frac{1}{z_c} v_m\left(t - \frac{d}{v_p}\right)$$

$$I_m = i_k\left(t - \frac{d}{v_p}\right) + \frac{1}{z_c} v_k\left(t - \frac{d}{v_p}\right)$$

Lumped Parameter Model

- In the special case of constant frequency, book shows the derivation of the common lumped parameter model



This is used in power flow and transient stability; in EMTP the frequency is not constant

Including Line Resistance

- An approach for adding line resistance, while keeping the simplicity of the lossless line model, is to just to place ½ of the resistance at each end of the line
  - Another, more accurate approach, is to place ¼ at each end, and ½ in the middle
- Standalone resistance, such as modeling the resistance of a switch, is just represented as an algebraic equation

$$i_{k,m} = \frac{1}{R}\left(v_k - v_m\right)$$

Numerical Integration with Trapezoidal Method

- Numerical integration is often done using the trapezoidal method discussed last time
  - Here we show how it can be applied to inductors and capacitors
- For a general function the trapezoidal approach is

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t))$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\left[f(\mathbf{x}(t)) + f(\mathbf{x}(t+\Delta t))\right]$$

- Trapezoidal integration introduces error on the order of Δt^3, but it is numerically stable

Trapezoidal Applied to Inductor with Resistance

- For a lossless inductor,

$$v = L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = \frac{v}{L} \qquad i(0) = i^0$$

$$i(t+\Delta t) = i(t) + \frac{\Delta t}{2L}\left(v(t) + v\left(t+\Delta t\right)\right)$$

- This can be represented as a Norton equivalent with current into the equivalent defined as positive (the last two terms are the current source)

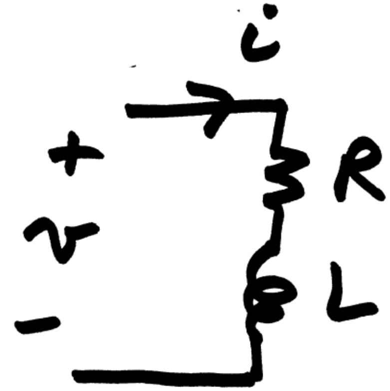
$$i(t+\Delta t) = \frac{v\left(t+\Delta t\right)}{2L/\Delta t} + i(t) + \frac{v(t)}{2L/\Delta t}$$

Trapezoidal Applied to Inductor with Resistance

- For an inductor in series with a resistance we have

$$v = iR + L\frac{di}{dt}$$

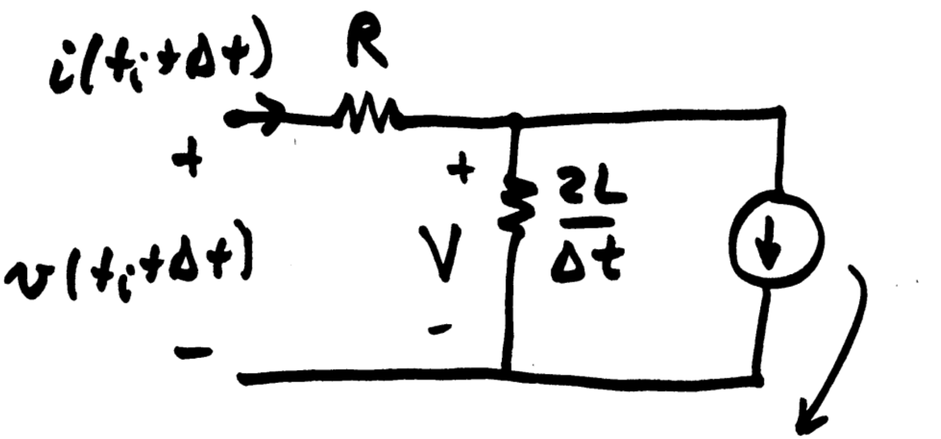
$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v \qquad i(0) = i^0$$



Trapezoidal Applied to Inductor with Resistance

$$i(t_i + \Delta t) \approx i(t_i) + \frac{\Delta t}{2}\Big[-\frac{R}{L}i(t_i) + \frac{1}{L}v(t_i)$$
$$-\frac{R}{L}i(t_i + \Delta t) + \frac{1}{L}v(t_i + \Delta t)\Big]$$

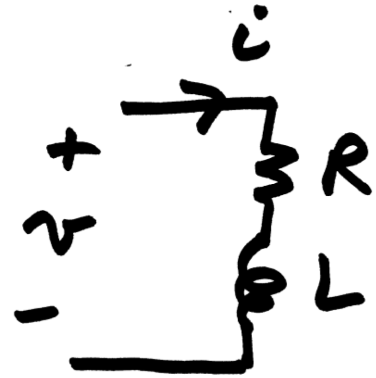
This also becomes a Norton equivalent. A similar expression will be developed for capacitors



$$i(t_i) + \frac{v(t_i) - R\,i(t_i)}{2L/\Delta t}$$

RL Example

- Assume a series RL circuit with an open switch with R= 200Ω and L = 0.3H, connected to a voltage source with $v = 133{,}000\sqrt{2}\cos(2\pi 60t)$
- Assume the switch is closed at t=0
- The exact solution is



$$i = -712.4e^{-667t} + 578.8\sqrt{2}\cos(2\pi 60t - 29.5^\circ)$$

$$v = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v \qquad i(0) = i^0$$

R/L=667, so the dc offset decays quickly

RL Example Trapezoidal Solution

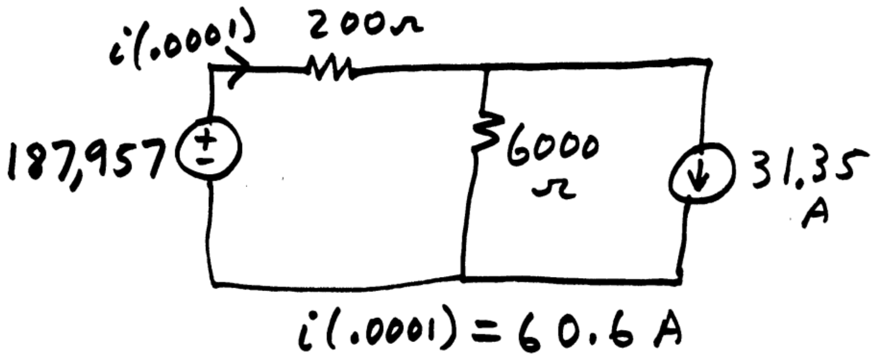
$$\frac{2L}{\Delta t} = \frac{2*0.3}{0.0001} = 6000$$

$$\Delta t = .0001 \text{sec}$$

$$t = 0 \quad i(0) = 0$$

$$t = .0001$$

$$i(0) + \frac{v(0) - Ri(0)}{6000} = 31.35\text{A}$$



Numeric solution: $i(.0001) = \dfrac{187,957}{6200} + \dfrac{31.35 \times 6000}{6200} = 60.65\text{A}$

Exact solution:

$$i(.0001) = -712.4e^{-.0677} + 578.8\sqrt{2}\cos\left(2\pi 60 \times .0001 - 29.5\frac{\pi}{180}\right)$$

$$= -666.4 + 727.0$$

$$= 60.6A$$

RL Example Trapezoidal Solution

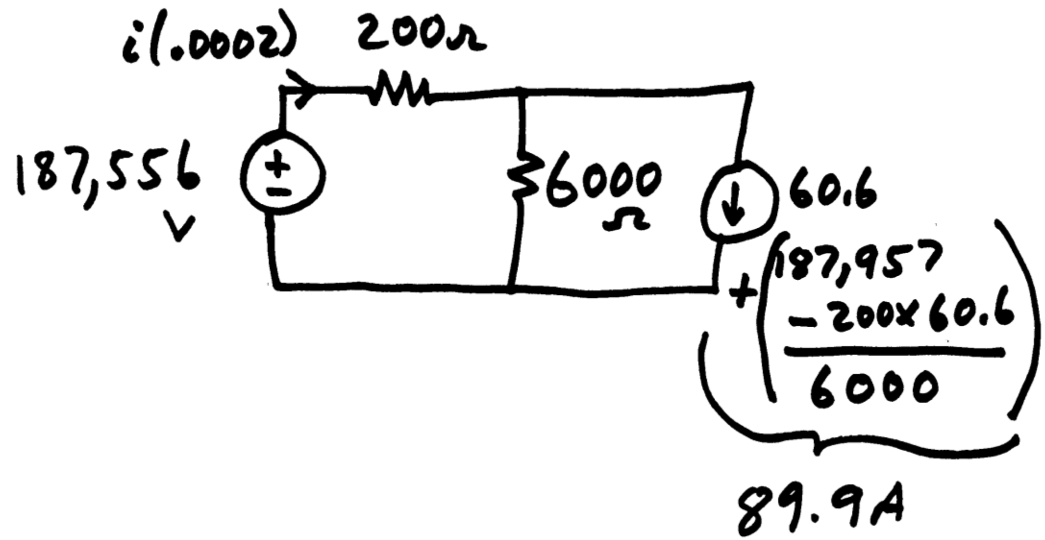
t = .0002

Solving for i(.0002)

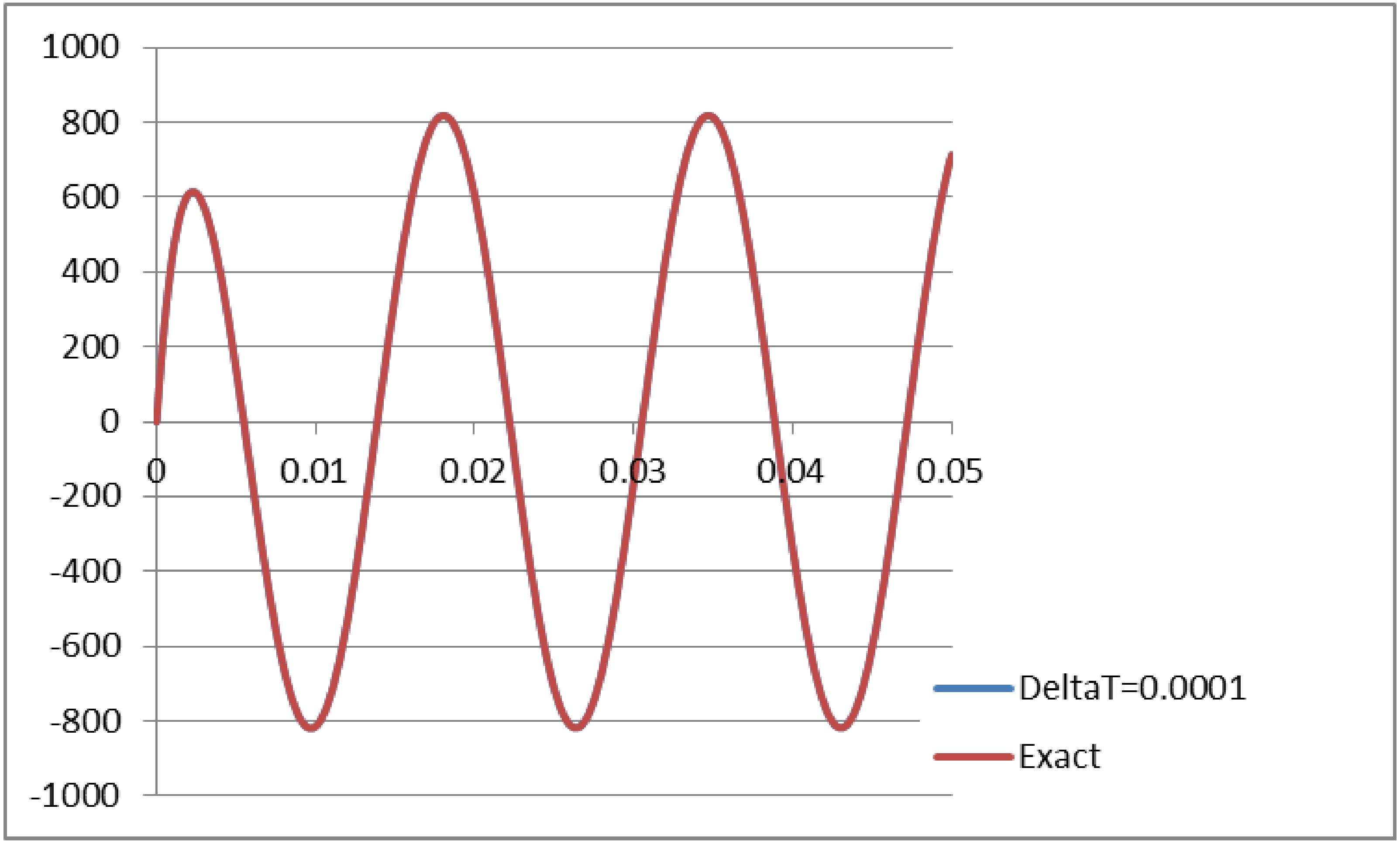
i(.0002) = 117.3A

Exact solution

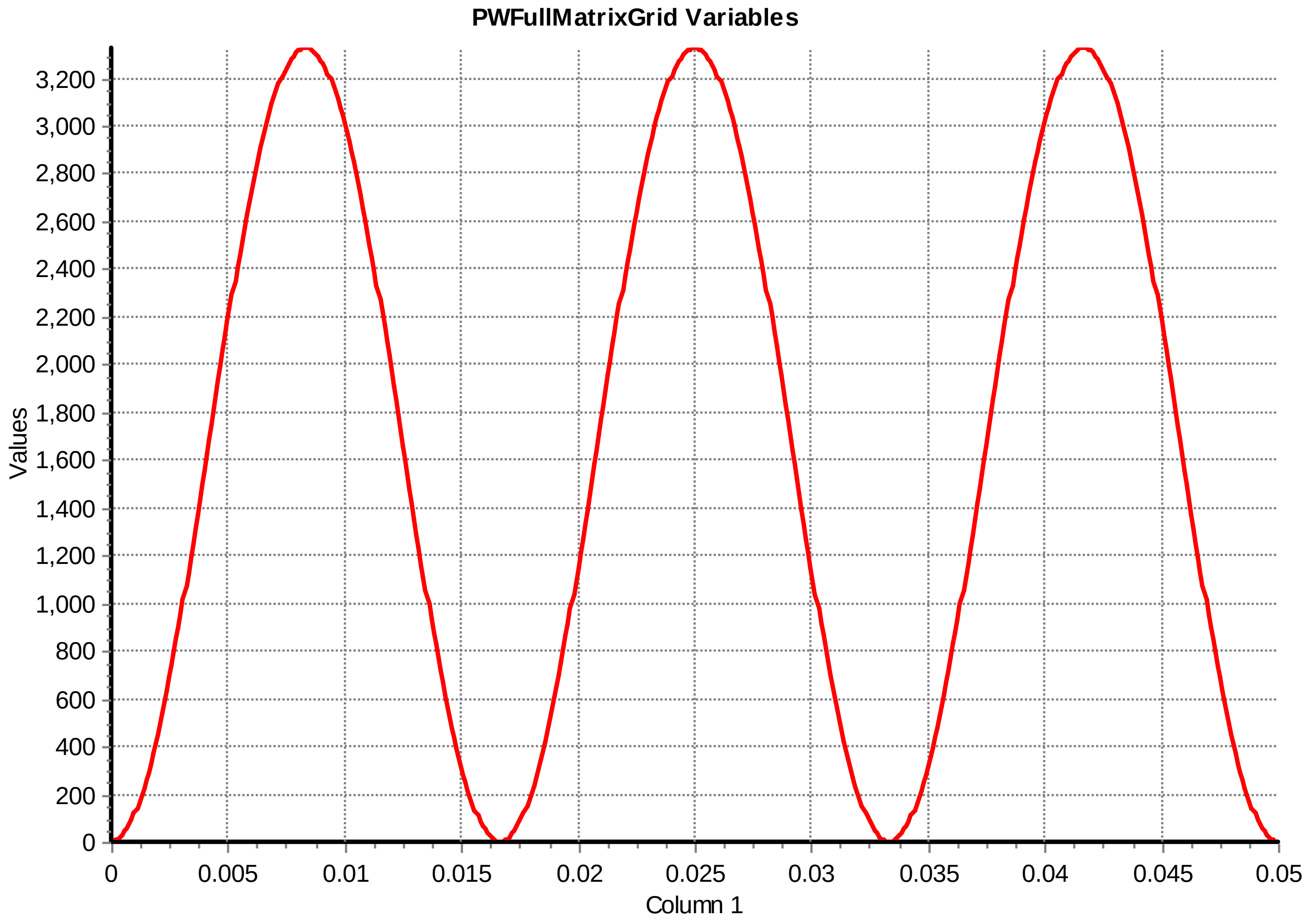
i(.0002) = 117.3A



Full Solution Over Three Cycles



A Favorite Problem: R=0 Case, with v(t) = Sin(2*pi*60)



Lumped Capacitance Model

- The trapezoidal approach can also be applied to model lumped capacitors

$$i(t) = C\frac{dv(t)}{dt}$$

- Integrating over a time step gives

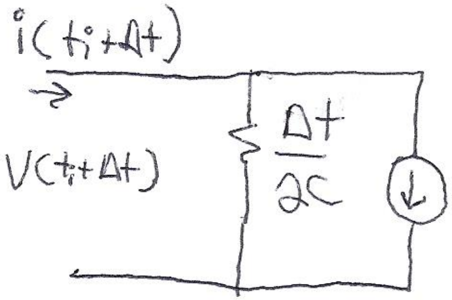
$$v(t+\Delta t) = v(t) + \frac{1}{C}\int_{t}^{t+\Delta t} i(t)$$

- Which can be approximated by the trapezoidal as

$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2C}\left(i(t+\Delta t) + i(t)\right)$$

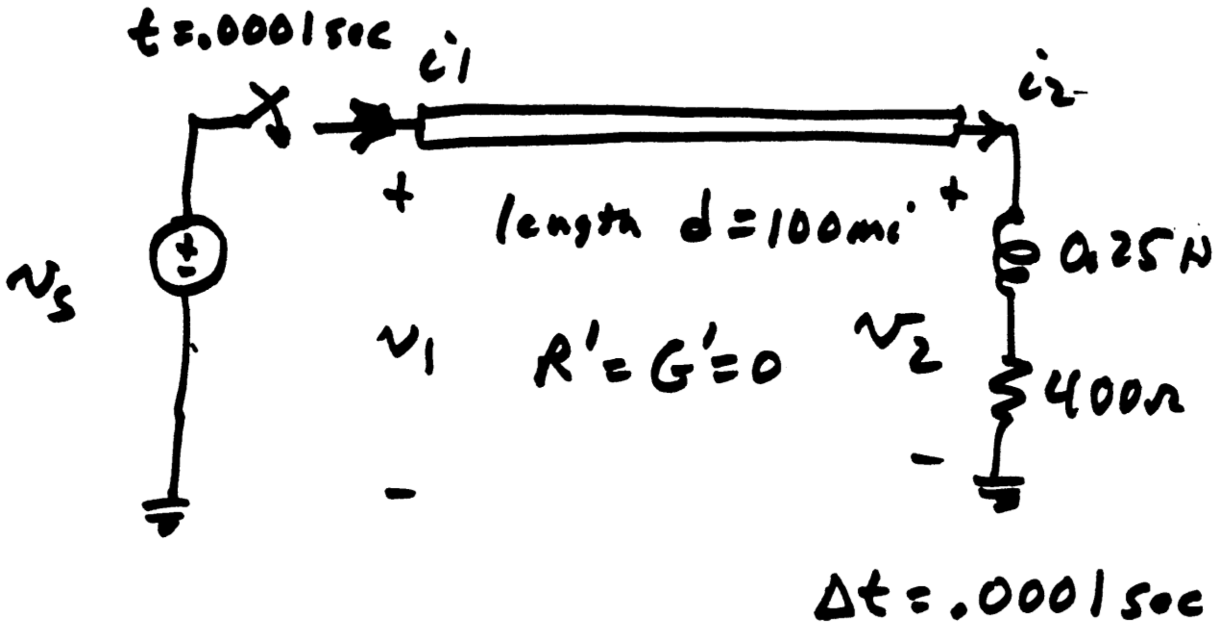
Lumped Capacitance Model

- Hence we can derive a circuit model similar to what was done for the inductor



$$-v(t)-\frac{2C}{\Delta t}i(t)$$

Example 2.1: Line Closing



$$L' = 1.5 \times 10^{-3} H/mi$$

$$C' = 0.02 \times 10^{-6} F/mi$$

Switch is closed at time t = 0.0001 sec

Example 2.1: Line Closing

Initial conditions: $i_1 = i_2 = v_1 = v_2 = 0$

for $t < .0001$ sec

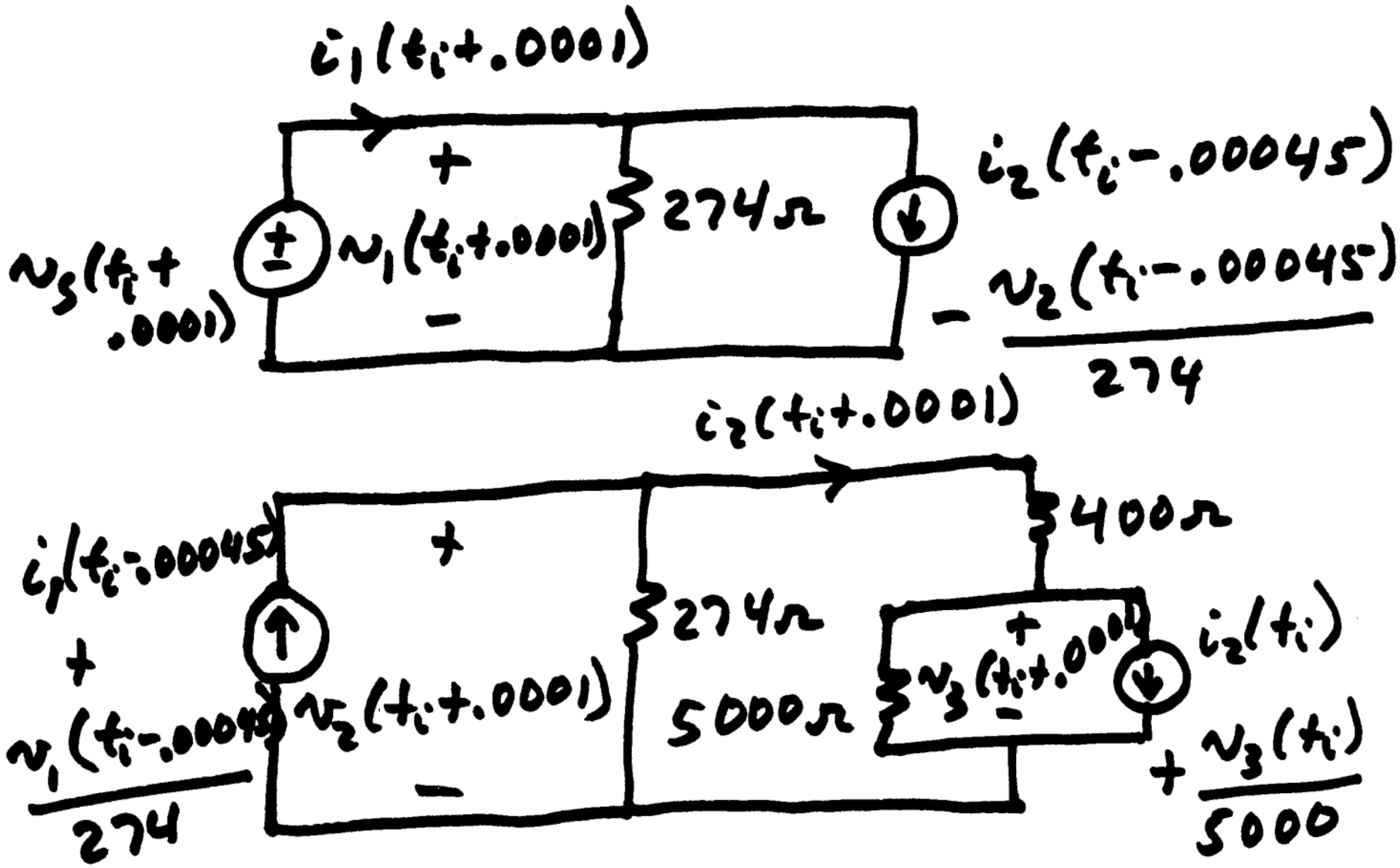
$$z_c = \sqrt{\frac{L'}{C'}} = 274\Omega \qquad v_p = \frac{1}{\sqrt{L'C'}} = 182{,}574 mi/\sec$$

$$\frac{d}{v_p} = .00055\sec$$

$$\frac{2L}{\Delta t} = 5000\Omega$$

Because of finite propagation speed, the receiving end of the line will not respond to energizing the sending end for at least 0.00055 seconds

Example 2.1: Line Closing



Note we have two separate circuits, coupled together only by past values

Example 2.1: t=0.0001

Need: $i_1(-.00045),\ v_1(-.00045),\ i_2(-.00045),$

$v_2(-.00045),\ i_2(0),\ v_3(0),\ v_s(.0001)$

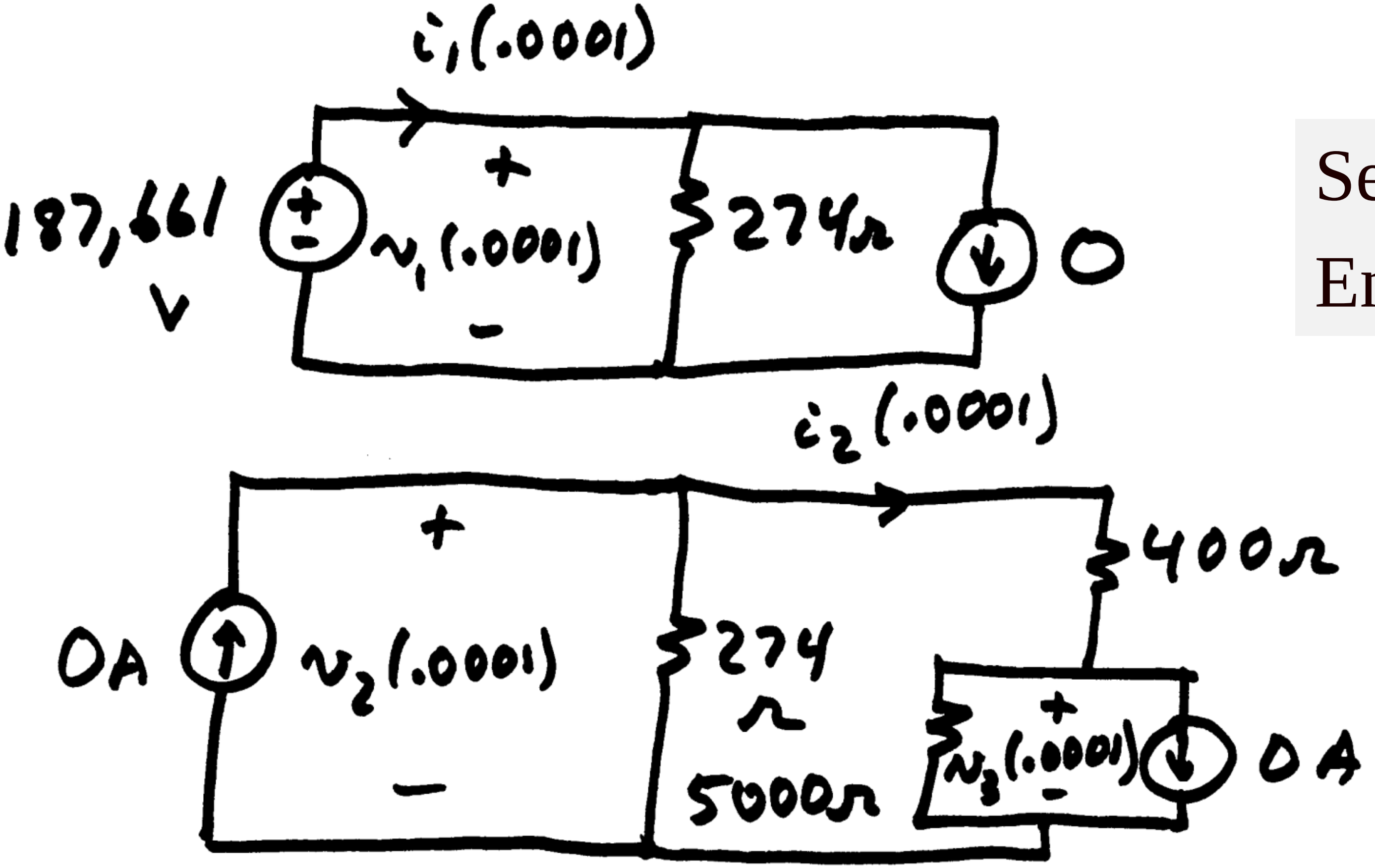
$$i_1(-.00045)=0 \quad i_2(0)=0$$

$$v_1(-.00045)=0 \quad v_3(0)=0$$

$$i_2(-.00045)=0 \quad v_2(-.00045)=0$$

$$v_s(.0001)=230{,}000\sqrt{\frac{2}{3}}\cos(2\pi 60\times .0001)=187{,}661\,V$$

Example 2.1: t=0.0001



Sending End

Receiving End

Example 2.1: t=0.0001

$$i_1(.0001) = 685A$$

$$v_1(.0001) = 187{,}661V$$

$$i_2(.0001) = 0$$

$$v_2(.0001) = 0$$

$$v_3(.0001) = 0$$

Instantaneously changed from zero at t = .0001 sec.

Example 2.1: t=0.0002

Need:

$$i_1(-.00035) = 0$$

$$v_1(-.00035) = 0$$

$$i_2(-.00035) = 0$$

$$v_2(-.00035) = 0$$

$$i_2(.0001) = 0$$

$$v_3(.0001) = 0$$

$$v_s(.0002) = 187,261V$$

Circuit is essentially the same

$$i_1(.0002) = 683A$$

$$v_1(.0002) = 187,261V$$

$$i_2(.0002) = 0.$$

$$v_2(.0002) = 0.$$

$$v_3(.0002) = 0.$$

Wave is traveling down the line

Example 2.1: t=0.0002 to 0.006

$$\frac{d}{v_p} = .00055 \qquad \Delta t = .0001$$

$$\begin{aligned} t_i &= 0 \\ t_i &= .0001 \\ &= .0002 \\ &= .0003 \\ &= .0004 \\ &= .0005 \\ &= .0006 \end{aligned}$$

$$\begin{aligned} t &= .0001 \leftarrow \\ t &= .0002 \\ &= .0003 \\ &= .0004 \\ &= .0005 \\ &= .0006 \leftarrow \\ &= .0007 \leftarrow \end{aligned}$$

switch closed

With interpolation receiving end will see wave

Example 2.1: t=0.0007

Need: $i_1(.00015)$

$v_1(.00015),\ v_2(.00015)$

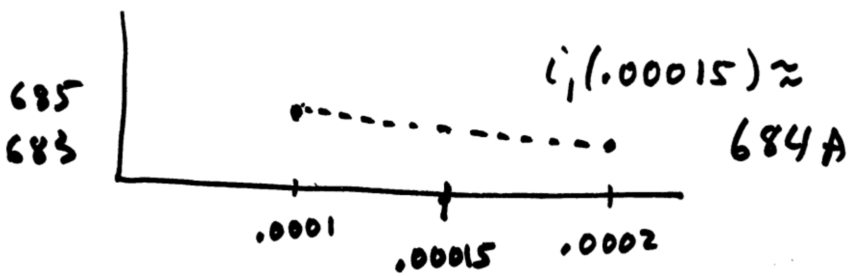
$i_2(.0006),\ v_3(.0006),\ v_s(.0007)$

$$i_1(.0001) = 685A$$

$$i_1(.0002) = 683A$$

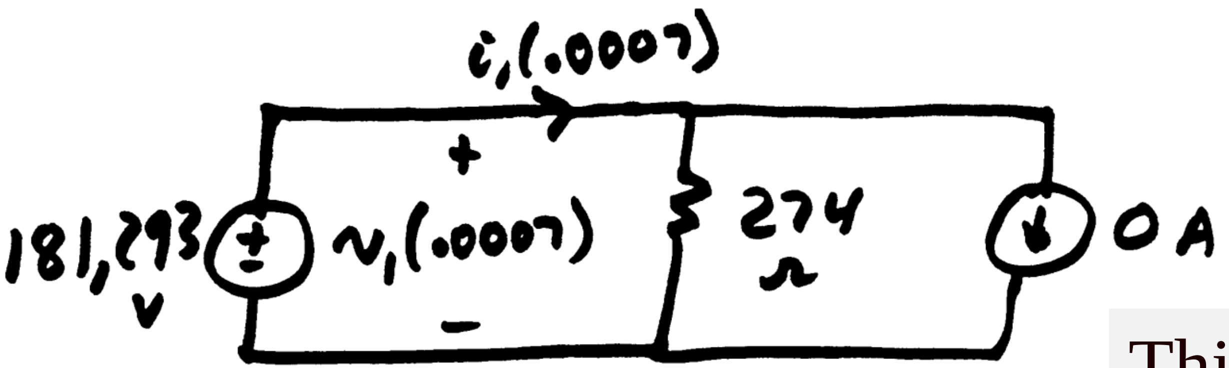
(linear interpolation)

$$i_1(.00015) \approx i_1(.0001) + \frac{.00015 - .0001}{.0002 - .0001} \times \left(i_1(.0002) - i_1(.0001)\right)$$



Example 2.1: t=0.0007

For t_i = .0006 (t = .0007 sec) at the sending end



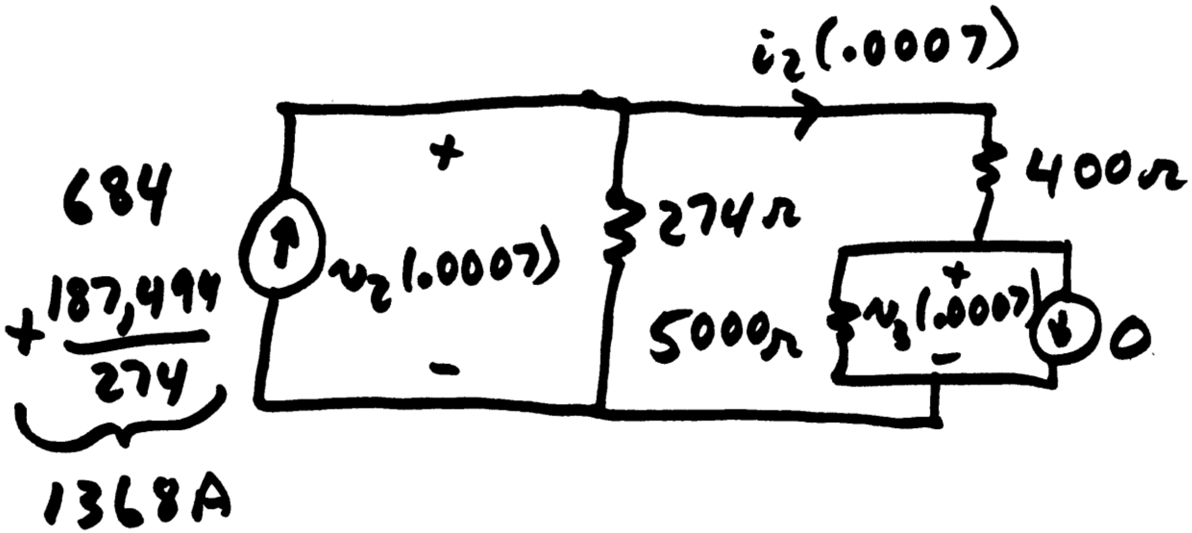
$$i_1(.0007) = 662A$$

$$v_1(.0007) = 181{,}293V$$

This current source will stay zero until we get a response from the receiving end, at about 2τ seconds

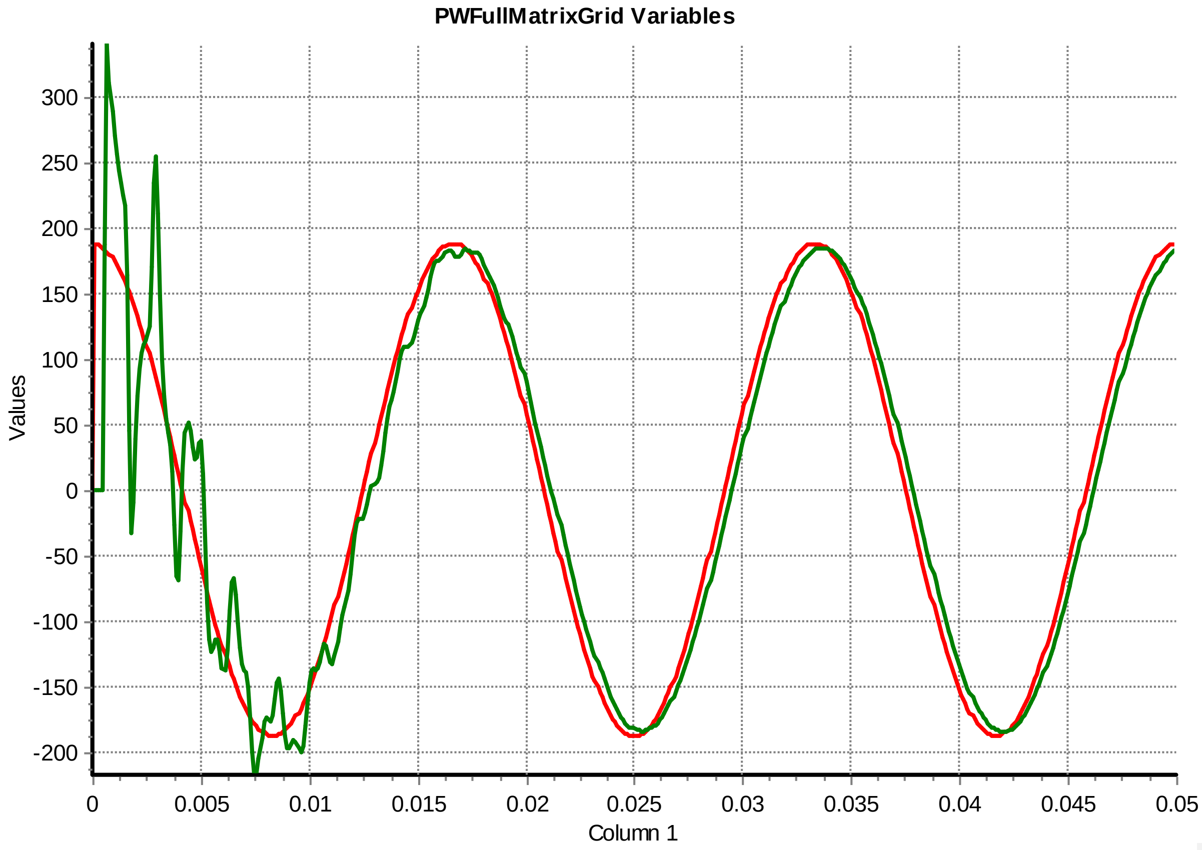
Example 2.1: t=0.0007

For t_i = .0006 (t = .0007 sec) at the receiving end



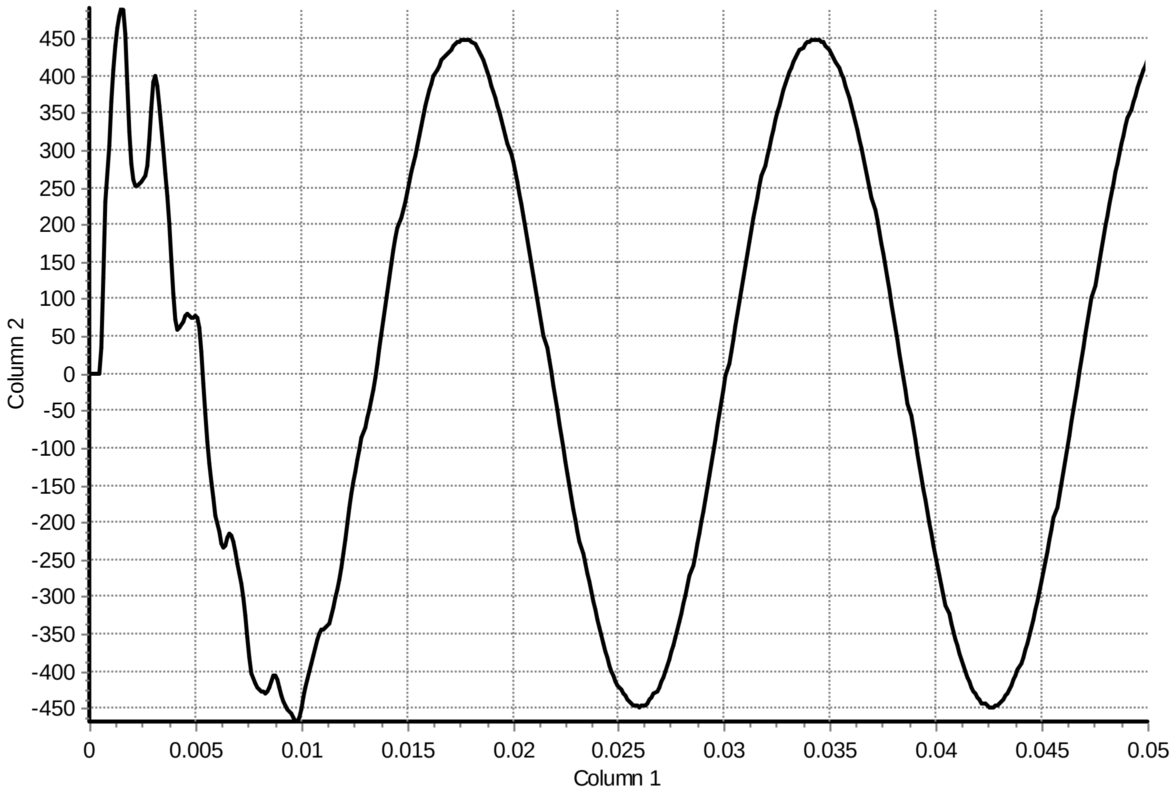
$$v_2(.0007) = 356{,}731V$$
$$i_2(.0007) = 66A$$

Example 2.1: First Three Cycles



Red is the sending end voltage (in kv), while green is the receiving end voltage. Note the approximate voltage doubling at the receiving end

Example 2.1: First Three Cycles

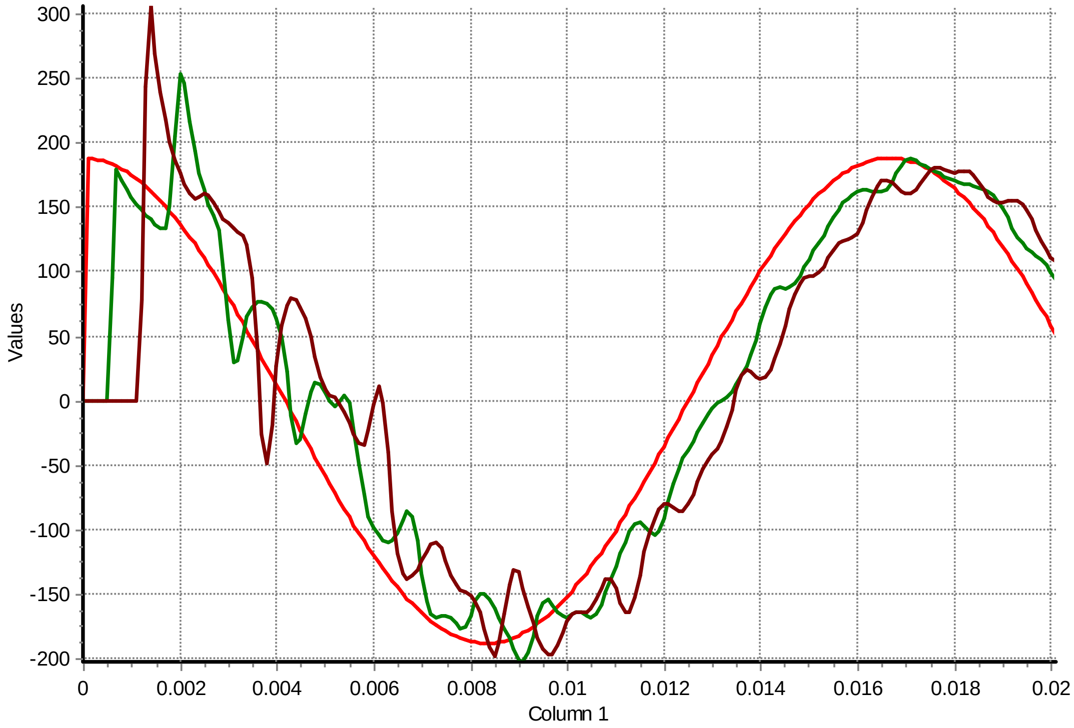


Graph shows the current (in amps) into the RL load over the first three cycles.

To get a **ballpark** value on the expected current, solve the simple circuit assuming the transmission line is just an inductor

$$I_{load,rms} = \frac{230,000/\sqrt{3}}{400 + j94.2 + j56.5} = 311\angle -20.6°,$$ hence a peak value of 439 amps

Three Node, Two Line Example

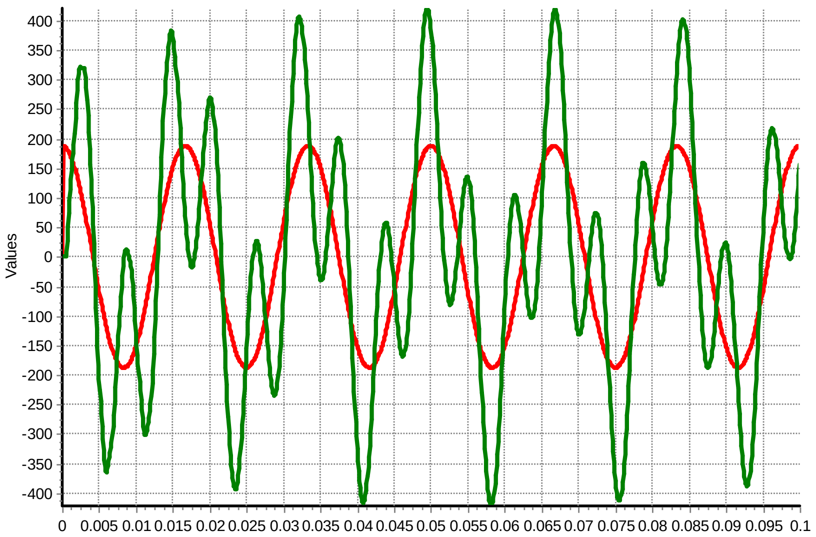


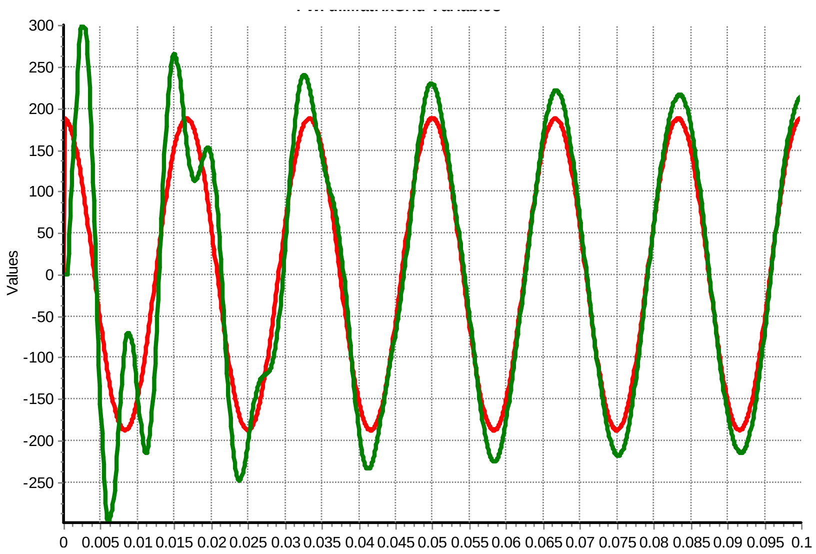
Graph shows the voltages for 0.02 seconds for the Example 2.1 case extended to connect another 120 mile line to the receiving end with an identical load

Note that there is no longer an initial overshoot for the receiving (green) end since wave continues into the second line

Example 2.1 with Capacitance

- Below graph shows example 2.1 except the RL load is replaced by a 5 μF capacitor (about 100 Mvar)
- Graph on left is unrealistic case of no line resistance
- Graph on right has R=0.1 Ω/mile





EMTP Network Solution

- The EMTP network is represented in a manner quite similar to what is done in the dc power flow or the transient stability network power balance equations or geomagnetic disturbance modeling (GMD)
- Solving set of dc equations for the nodal voltage vector **V** with

  $$\mathbf{V} = \mathbf{G}^{-1}\mathbf{I}$$

  where **G** is the bus conductance matrix and **I** is a vector of the Norton current injections