# ECEN 615
# Methods of Electric Power Systems Analysis

## Lecture 19: Equivalents, Voltage Stability

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# Announcements

- Read Chapters 3 and 8 from the book
- Homework 4 is due today
- Homework 5 is due on Tuesday November 12

# QR Factorization

- Used in in SE since it handles ill-conditioned m by n matrices (with m >= n)
- Can be used with sparse matrices
- As before we will first split the $\mathbf{R}^{-1}$ matrix

$$\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} = \mathbf{H}^T \mathbf{R}^{-1/2} \mathbf{R}^{-1/2} \mathbf{H} = \mathbf{H}'^T \mathbf{H}'$$

- QR factorization represents the m by n **H'** matrix as

$$\mathbf{H}' = \mathbf{Q}\,\mathbf{U}$$

with **Q** an m by m orthonormal matrix and **U** an upper triangular matrix (most books use **Q R** but we use **U** to avoid confusion with the previous **R**)

# QR Factorization

- We then have $\mathbf{H'}^T\mathbf{H'} = \mathbf{U}^T\mathbf{Q}^T\mathbf{Q}\,\mathbf{U}$
- But since **Q** is an orthonormal matrix, $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$
- Hence we have $\mathbf{H'}^T\mathbf{H'} = \mathbf{U}^T\,\mathbf{U}$

Originally $\Delta\mathbf{x} = \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right]$

With $\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} = \mathbf{H'}^T\mathbf{H'} = \mathbf{H'}^T\mathbf{H'} = \mathbf{U}^T\mathbf{U}$

**Q** is an m by m matrix

Let $\mathbf{z'} = \mathbf{Q}^T\mathbf{R}^{-1/2}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right]$

$$\Delta\mathbf{x} = \left[\mathbf{U}^T\mathbf{U}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1/2}\mathbf{R}^{-1/2}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right] = \left[\mathbf{U}^T\mathbf{U}\right]^{-1}\mathbf{U}^T\mathbf{Q}^T\mathbf{R}^{-1/2}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right]$$

$$\mathbf{U}^T\mathbf{U}\Delta\mathbf{x} = \mathbf{U}^T\mathbf{z'} \rightarrow \Delta\mathbf{x} = \mathbf{U}^{-1}\mathbf{z'}$$

# QR Factorization

- Next we'll briefly discuss the QR factorization algorithm
- When factored the **U** matrix (i.e., what most call the **R** matrix ) will be an m by n upper triangular matrix
- Several methods are available including the Householder method and the Givens Method
- Givens is preferred when dealing with sparse matrices
- All good reference is Gene H. Golub and Charles F. Van Loan, "Matrix Computations," second edition, Johns Hopkins University Press, 1989.

# Givens Algorithm for Factoring a Matrix A

- The Givens algorithm works by pre-multiplying the initial matrix, **A**, by a series of matrices and their transposes, starting with $\mathbf{G}_1\mathbf{G}_1^T$
  - If **A** is m by n, then each **G** is an m by m matrix
- Algorithm proceeds column by column, sequentially zeroing out elements in the lower triangle of **A**, starting at the bottom of each column

$$\mathbf{G}_1 \ldots \mathbf{G}_p \mathbf{G}_p^T \ldots \mathbf{G}_1^T \mathbf{A} = \mathbf{QU}$$

$$\mathbf{G}_1 \ldots \mathbf{G}_p = \mathbf{Q}$$

$$\mathbf{G}_p^T \ldots \mathbf{G}_1^T \mathbf{A} = \mathbf{U}$$

If **A** is sparse, then we can take advantage of sparsity going up the column

# Givens Algorithm

- To zero out element **A**[i,j], with i > j we first solve with a=A[k,j], b= A[i,j]

$$\begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

$$r = \sqrt{a^2 + b^2}$$

To zero out an element we need a non-zero pivot element in column j; assume this row as k.

$r =$

- A numerically safe algorithm is

If b=0 then c=1, s=0 // i.e, no rotation is needed

Else If |b|>|a| then $\tau = -a/b; \quad s = 1/\sqrt{1+\tau^2}; c = s\tau$

Else $\tau = -b/a; \quad c = 1/\sqrt{1+\tau^2}; s = c\tau$

# Givens G Matrix

- The orthogonal **G**(i,k,θ) matrix is then

$$\mathbf{G}(i,\mathrm{k},\theta)=\begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & c & \cdots & s & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -s & \cdots & c & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

To zero out an element we need a non-zero pivot element in column j; assume this row as k. Row k here is the first non-zero above row i.

- Premultiplication by **G**(i,k,θ)$^T$ is a rotation by θ radians in the (i,k) coordinate plane

# Small Givens Example

- Let

$$\mathbf{A} = \begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 0 & 5 \\ 2 & 1 \end{bmatrix}$$

First start in column j=1; we will zero out A[4,1] with i=4, k=2

- First we zero out **A**[4,1], a=1, b=2 giving s= 0.8944, c=**-**0.4472

$$\mathbf{G}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.4472 & 0 & 0.8944 \\ 0 & 0 & 1 & 0 \\ 0 & -0.8944 & 0 & -0.4472 \end{bmatrix} \qquad \mathbf{G}_1^T\mathbf{A} = \begin{bmatrix} 4 & 2 \\ -2.236 & -0.8944 \\ 0 & 5 \\ 0 & -0.4472 \end{bmatrix}$$

# Small Givens Example

- Next zero out A[2,1] with a=4, b=-2.236, giving c= -0.8729, s=0.4880

$$\mathbf{G}_2 = \begin{bmatrix} 0.873 & 0.488 & 0 & 0 \\ -0.488 & 0.873 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{G}_2^T \mathbf{G}_1^T \mathbf{A} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5 \\ 0 & -0.447 \end{bmatrix}$$

- Next zero out A[4,2] with a=5, b=-0.447, c=0.996, s=0.089

$$\mathbf{G}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.996 & 0.089 \\ 0 & 0 & -0.089 & 0.996 \end{bmatrix} \quad \mathbf{G}_3^T \mathbf{G}_2^T \mathbf{G}_1^T \mathbf{A} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5.020 \\ 0 & 0 \end{bmatrix}$$

# Small Givens Example

- Next zero out A[3,2] with a=0.195, b=5.02, c=0.996, s=0.089

$$\mathbf{G}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.039 & 0.999 & 0 \\ 0 & -0.999 & -0.039 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{G}_4^T \mathbf{G}_3^T \mathbf{G}_2^T \mathbf{G}_1^T \mathbf{A} = \mathbf{U} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & -5.023 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Also we have

$$\mathbf{Q} = \mathbf{G}_1 \mathbf{G}_2 \mathbf{G}_3 \mathbf{G}_4 = \begin{bmatrix} 0.872 & -0.019 & 0.487 & 0 \\ 0.218 & 0.094 & -0.387 & 0.891 \\ 0 & -0.995 & -0.039 & 0.089 \\ 0.436 & -0.009 & -0.782 & -0.445 \end{bmatrix}$$

# Start of Givens for SE Example

- Starting with the **H** matrix we get

$$\mathbf{H}' = \mathbf{R}^{-1/2}\mathbf{H} = 100 \times \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ -10 & 0 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- To zero out **H**'[5,1]=1 we have b=100, a=-1000, giving c=0.995, s=0.0995

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.995 & 0.0995 & 0 \\ 0 & 0 & 0 & -0.0995 & 0.995 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Here the column (j) is 1, while i is 5 and k is 4.

# Start of Givens for SE Example

- Which gives

$$\mathbf{G}_1^T \mathbf{H}' = 100 \times \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ 10.049 & 0 & -9.95 \\ 0 & 0 & 0.995 \\ 0 & 0 & 1 \end{bmatrix}$$

- The next rotation would be to zero out element [4,1], continuing until all the elements in the lower triangle have been reduced

# Givens Comments

- For a full matrix, Givens is $O(mn^2)$ since each element in the lower triangle needs to be zeroed $O(nm)$, and each operation is $O(n)$
- Computation can be drastically reduced for a sparse matrix since we only need to zero out the elements that are initially non-zero, and any that become non-zero (i.e., the fills)
  - Also, for each multiply we only need to deal with the nonzeros in the impacted row
- Givens rotation is commonly used to solve the SE

# Power System Equivalents

- No electric grid model is ever going to completely represent a real electric grid
  - "All models are wrong but some models are useful"
- A key modeling consideration is how much of the electric grid to represent
  - For large-scale systems the distribution system is usually equivalenced at some point; this has few system level ramifications if it is radial; if it is networked then there are potential issues
  - At the transmission level either the full interconnect is represented or it is equivalenced
  - In an SE model in large grids (like the Eastern Interconnect) it is always an electrical equivalent

# Kron Reduction, Ward Equivalents

- For decades power system network models have been equivalenced using the approach originally presented by J.B. Ward in 1949 AIEE paper "Equivalent Circuits for Power-Flow Studies"
  - Paper's single reference is to 1939 book by Gabriel Kron, so this is also known as Kron's reduction or a Ward equivalent
- System buses are partitioned into a study system (s) to be retained and an external system (e) to be eliminated; buses in study system that connect to the external are known as boundary buses

# Ward Equivalents

- The Ward approach is based on the below relationship

$$\begin{bmatrix} I_s \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ss} & Y_{se} \\ Y_{es} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_s \\ V_e \end{bmatrix}$$

$$\left( I_s - Y_{se} Y_{ee}^{-1} I_e \right) = \left( Y_{ss} - Y_{se} Y_{ee}^{-1} Y_{es} \right) V_s$$

- No impact on study, non-boundary buses
- Equivalent is created by doing a partial factorization of the $\mathbf{Y}_{\text{bus}}$
  - Computationally efficient

# Other Types of Equivalents

- There are many different methods available for creating power system equivalents
  - A classic paper is by S. Deckmann, et. al., "Studies on Power System Load Flow Equivalencing," *IEEE Transactions Power App. And Syst*., Nov/Dec 1980
  - Companion paper covers numerical testing of equivalents
- The major equivalencing types are
  - Ward-Type Equivalence: this is what we'll be covering, with the major differences associated with how the generator buses and equivalent loads are represented
  - REI Equivalents: All boundary buses connect to one "REI" bus
  - Linearized Methods: Linearize about an operating point
  - Others: PTDF-based, backbone type

# Equivalent System Properties

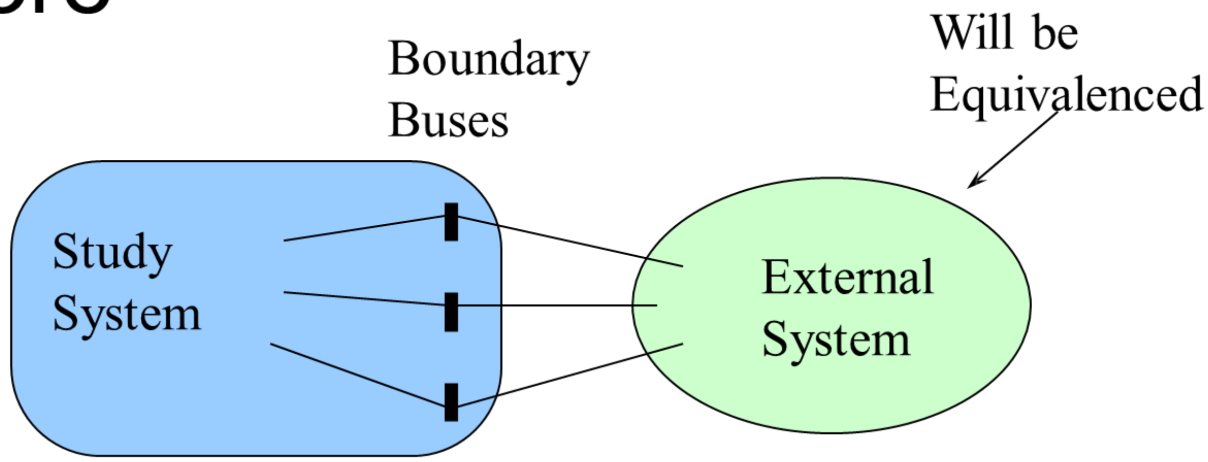
- An equivalent is usually created from a larger model
  - In the Eastern Interconnect there are full grid models that are used for wide-area planning, these are equivalenced for real-time usage or more specialized studies
- The equivalent is usually smaller and less detailed
  - Solves quicker
  - Requires less storage
  - Requires less up-to-date data
- Equivalences contain fictitious elements
  - This can make modeling/updating more difficult
- The equivalent only approximates the behavior of the original

# Study vs External System

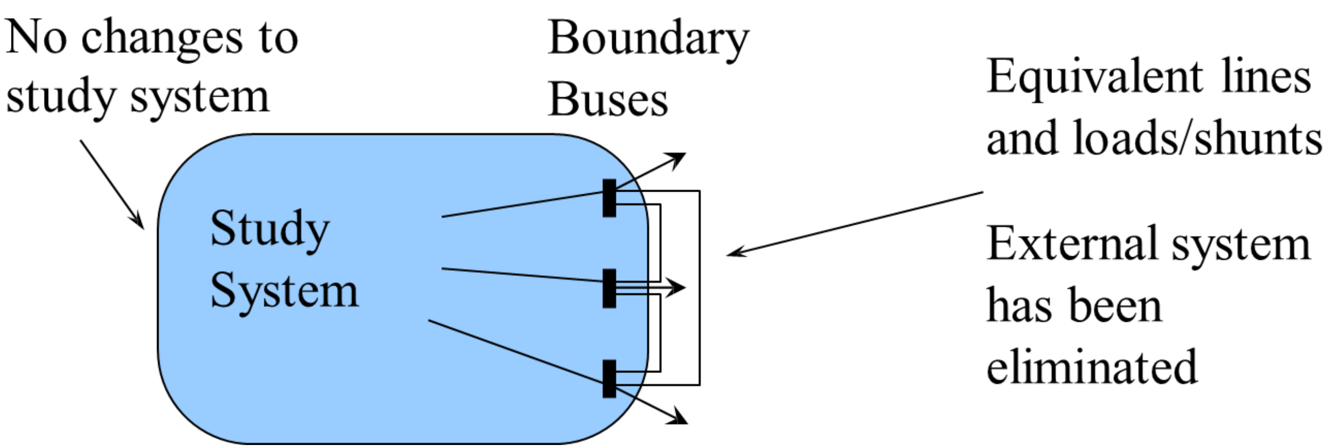
- The key decision in creating an equivalent is to divide the system into a study portion that is represented in detail, and an external portion that is represented by the equivalent
- The two systems are joined at boundary buses, which are part of the study subsystem
- How this is done is application specific; for example:
  - for real-time use it does not make sense to retain significant portions of the grid for which there is no real-time information
  - for contingency analysis the impact of the contingency is localized
  - for planning the new system additions have localized impacts

# Ward Type Equivalencing

## Before




## After



# Ward Type Equivalencing Considerations

- The Ward equivalent is calculated by doing a partial factorization of the $\mathbf{Y}_{bus}$
  - The equivalent buses are numbered before the study buses
  - As the equivalent buses are eliminated their first neighbors are joined together
  - At the end, many of the boundary buses are connected
  - This can GREATLY decrease the sparsity of the system
  - Buses with different voltages can be directly connected

$$\begin{bmatrix} I_s \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ss} & Y_{se} \\ Y_{es} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_s \\ V_e \end{bmatrix}$$

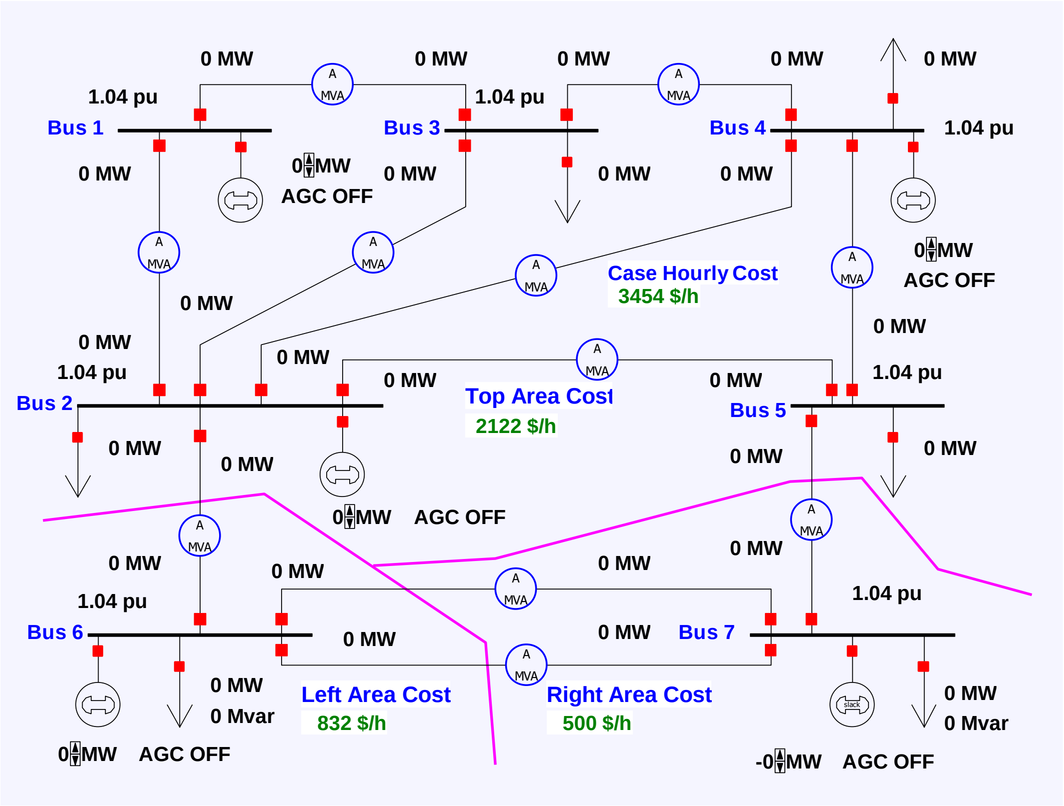
$$\left( I_s - Y_{se} Y_{ee}^{-1} I_e \right) = \left( Y_{ss} - Y_{se} Y_{ee}^{-1} Y_{es} \right) V_s$$

# Ward Type Equivalencing Considerations

- At the end of the Ward process often many of the new equivalent lines have high impedances
  - Often there is an impedance threshold, and lines with impedances above this value are eliminated
- The equivalent lines may have unusual values, including negative resistances
- Load and generation is represented as equivalent current injections or shunts; sometimes these values are converted back to constant power
- Consideration needs to be given to loss of reactive support
- The equivalent embeds the present load and gen values

# B7Flat_Eqv Example

- In this example the B7Flat_Eqv case is reduced, eliminating buses 1, 3 and 4. The study system is then 2, 5, 6, 7, with buses 2 and 5 the boundary buses




For ease of comparison system is modeled unloaded

# B7Flat_Eqv Example

- Original $\mathbf{Y}_{\text{bus}}$

$$\mathbf{Y}_{bus} = j\begin{bmatrix} -20.83 & 16.67 & 4.17 & 0 & 0 & 0 & 0 \\ 16.67 & -52.78 & 5.56 & 5.56 & 8.33 & 16.67 & 0 \\ 4.17 & 5.56 & -43.1 & 33.3 & 0 & 0 & 0 \\ 0 & 5.56 & 33.3 & -43.1 & 4.17 & 0 & 0 \\ 0 & 8.33 & 0 & 4.17 & -29.17 & 0 & 16.67 \\ 0 & 16.67 & 0 & 0 & 0 & -25 & 8.33 \\ 0 & 0 & 0 & 0 & 16.67 & 8.33 & -25 \end{bmatrix}$$

$$\mathbf{Y}_{ee} = j\begin{bmatrix} -20.833 & 4.167 & 0 \\ 4.167 & -43.056 & 33.333 \\ 0 & 33.333 & -43.056 \end{bmatrix}$$

# B7Flat_Eqv Example

$$\mathbf{Y}_{es} = j\begin{bmatrix} 16.667 & 0 & 0 & 0 \\ 5.556 & 0 & 0 & 0 \\ 5.556 & 4.167 & 0 & 0 \end{bmatrix} \qquad \mathbf{Y}_{se} = j\begin{bmatrix} 16.667 & 5.556 & 5.556 \\ 0 & 0 & 4.167 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

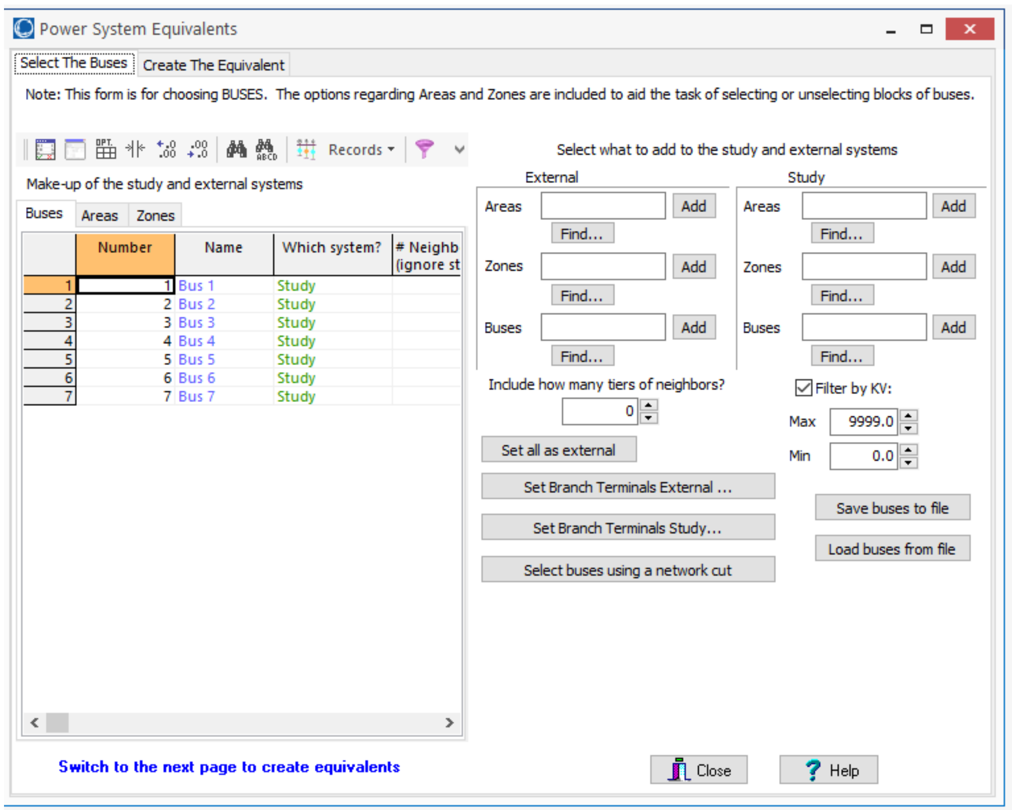
$$\mathbf{Y}_{ss} = j\begin{bmatrix} -52.778 & 8.333 & 16.667 & 0 \\ 8.333 & -29.167 & 0 & 16.667 \\ 16.667 & 0 & -25.0 & 8.333 \\ 0 & 16.667 & 8.333 & -25.0 \end{bmatrix}$$

Note $\mathbf{Y}_{es}=\mathbf{Y}_{se}'$ if no phase shifters

$$\left(\mathbf{Y}_{ss} - \mathbf{Y}_{se}\mathbf{Y}_{ee}^{-1}\mathbf{Y}_{es}\right) = j\begin{bmatrix} -28.128 & 11.463 & 16.667 & 0 \\ 11.463 & -28.130 & 0 & 16.667 \\ 16.667 & 0 & -25.0 & 8.333 \\ 0 & 16.667 & 8.333 & -25.0 \end{bmatrix}$$

# Equivalencing in PowerWorld

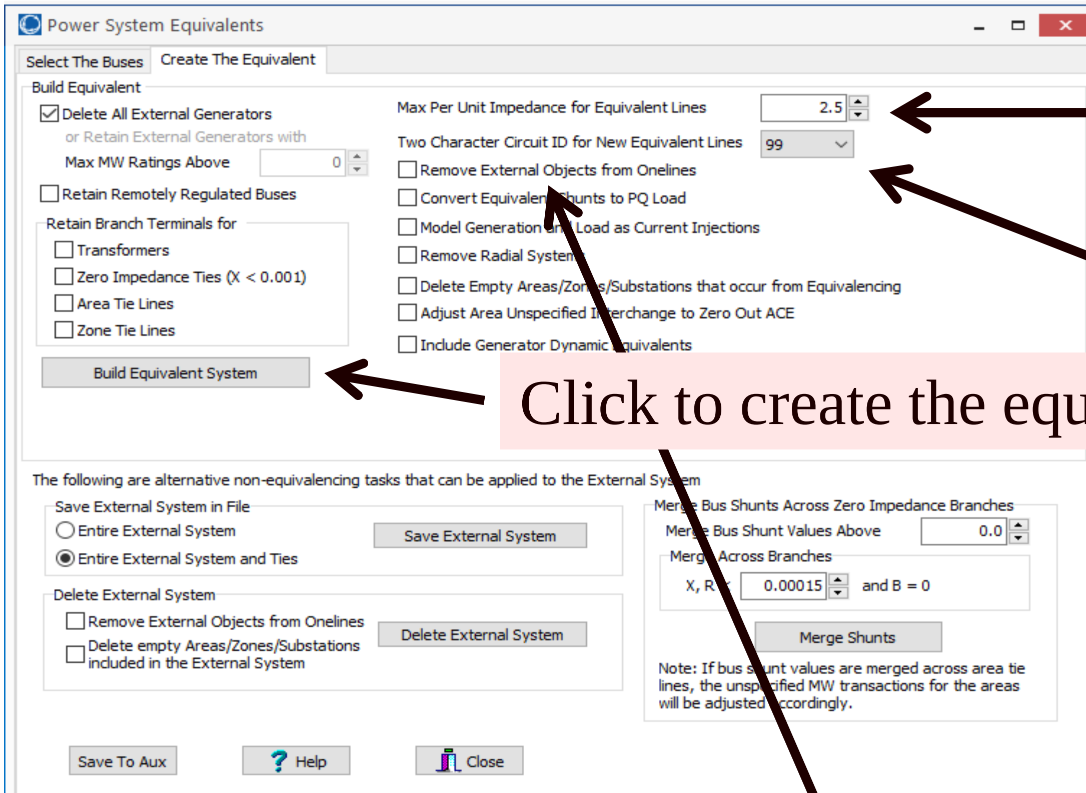
- Open a case and solve it; then select **Edit Mode**, **Tools**, **Equivalencing;** this displays the Power System Equivalents Form




Next step is then to divide the buses into the study system and the external system; buses can be loaded from a text file as well

# Equivalencing in PowerWorld

- Then go to the **Create The Equivalent** page, select the desired options and select **Build Equivalent System**




Maximum impedance lines to retain

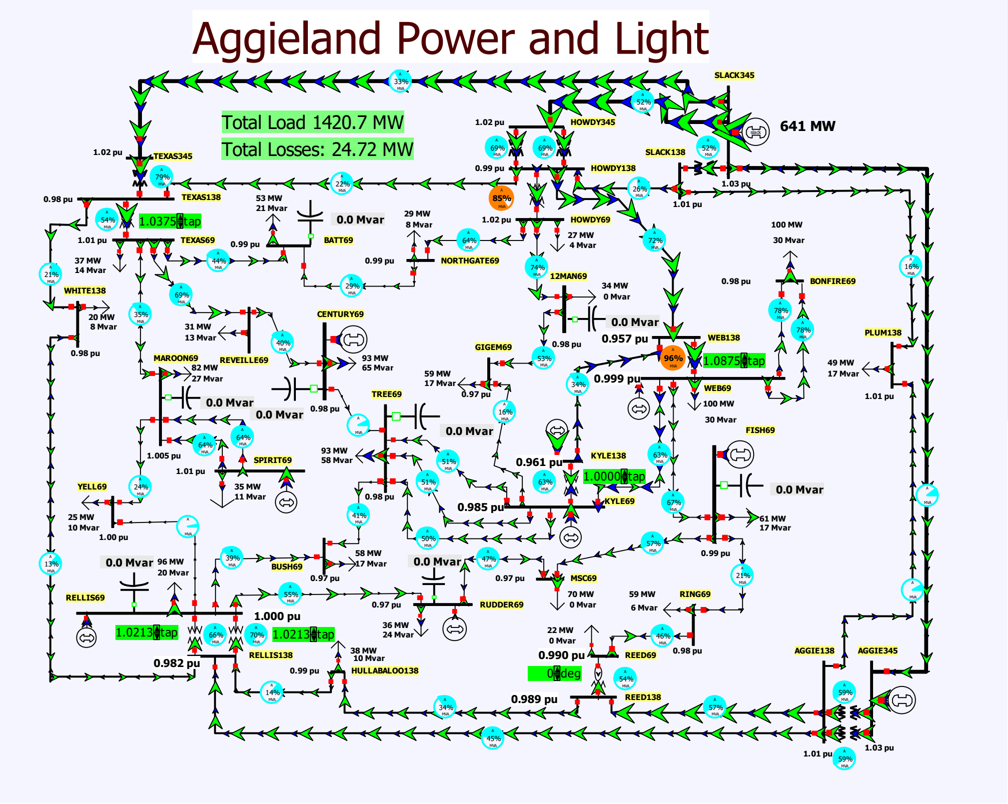
'99' or 'EQ' are common circuit values for equivalent lines

Click to create the equivalent

Removes equivalenced objects from the oneline

# Small System Equivalent Example

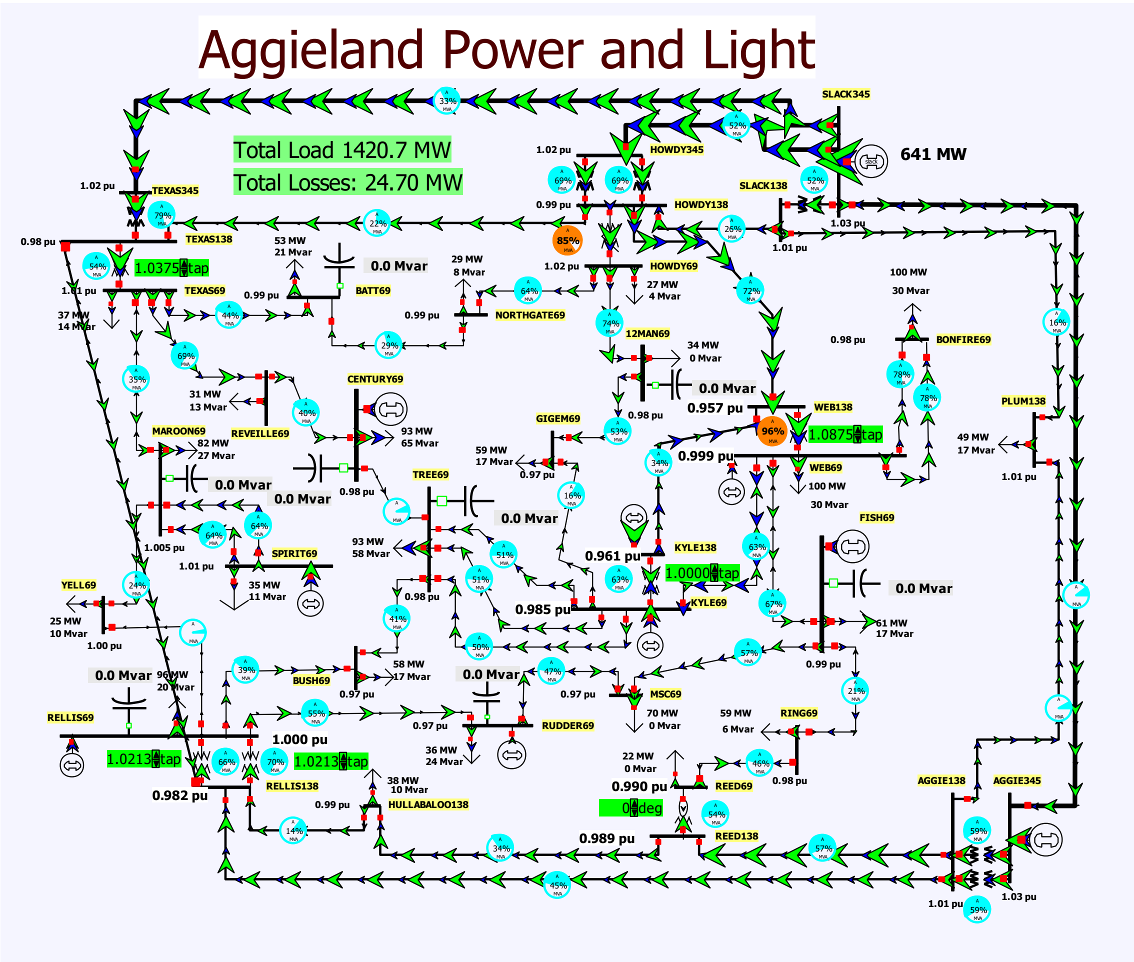
- Example shows the creation of an equivalent for Aggieland37 example




First example is simple, just removing WHITE138 (bus 3); note TEXAS138 is now directly joined to RELLIS138..

Case is Aggieland37_HW5

# Small System Equivalent Example

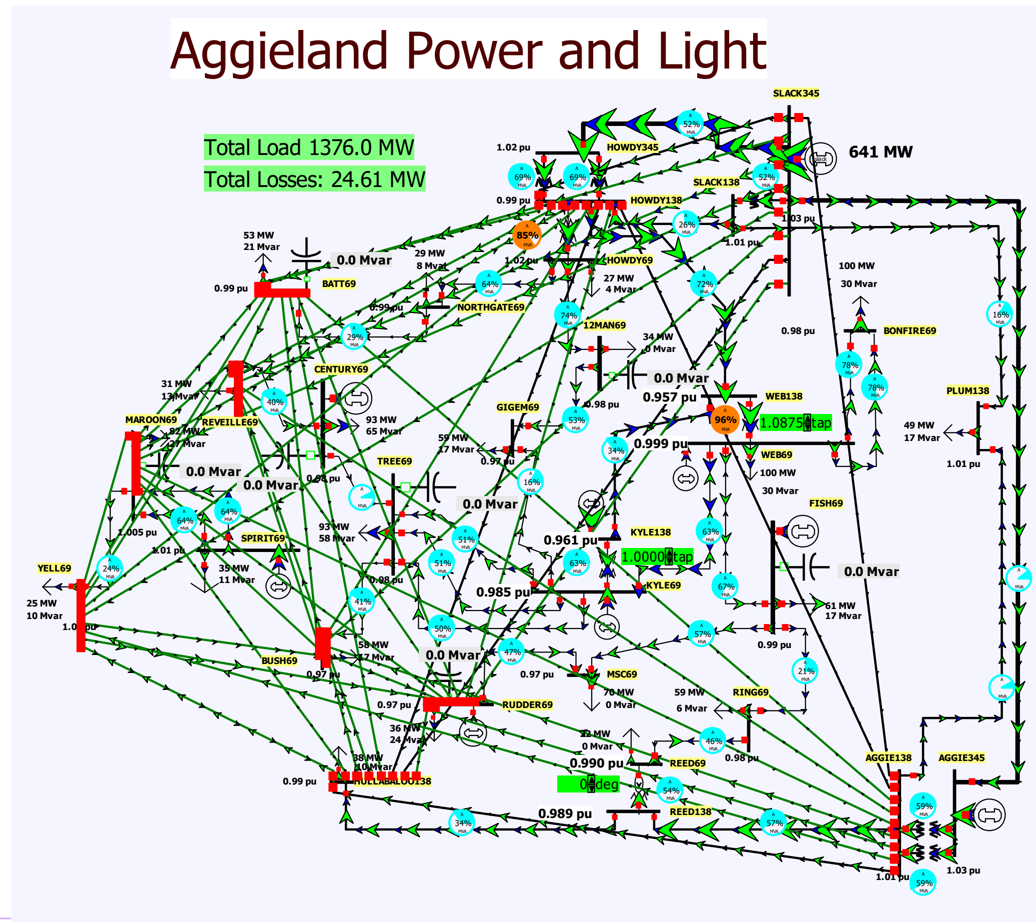



Only bus 3 was removed; the new equivalent line was auto-inserted.

Don’t save the equivalent with the same name as the original, unless you want to lose the original

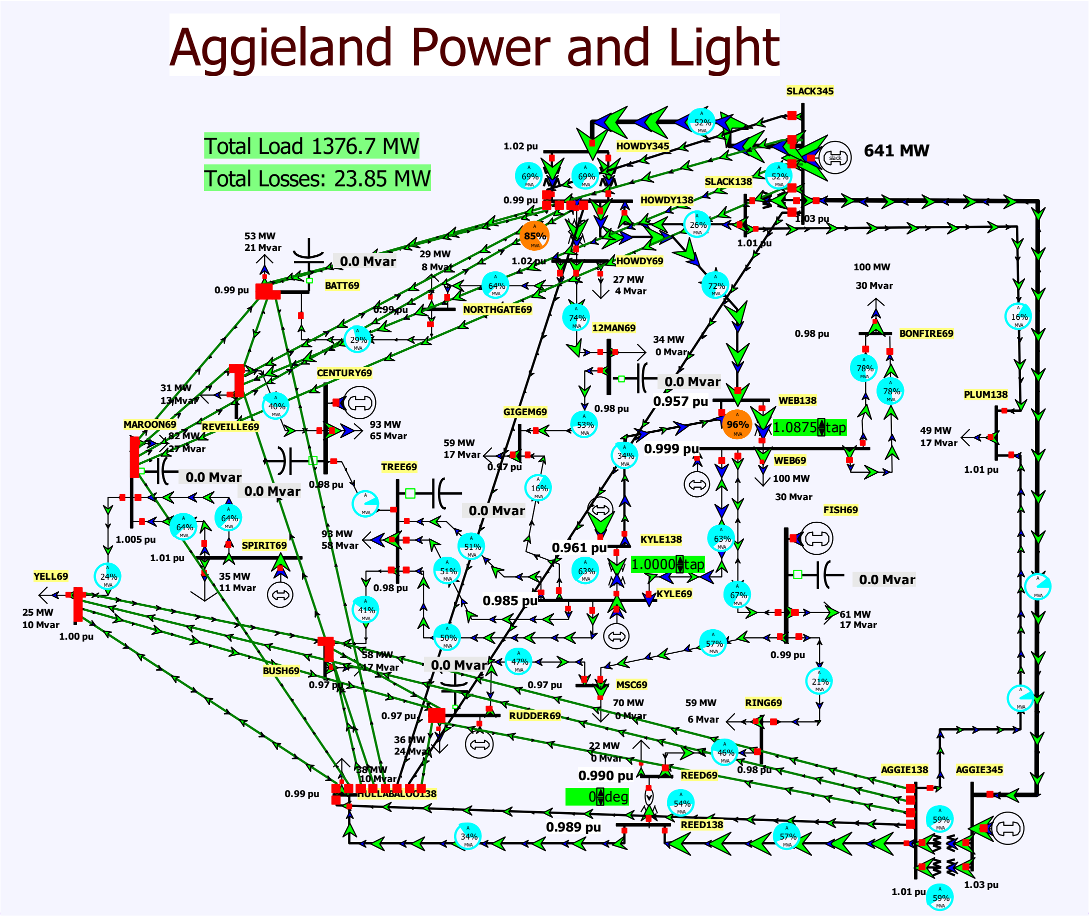
# Small System Equivalent Example

- Now remove buses at WHITE138 and TEXAS and RELLIS (1, 3, 12, 40, 41, 44); set **Max Per Unit Impedance for Equivalent Lines** to 99 (per unit) to retain all lines. Again to an auto-insert to show the equivalent lines.



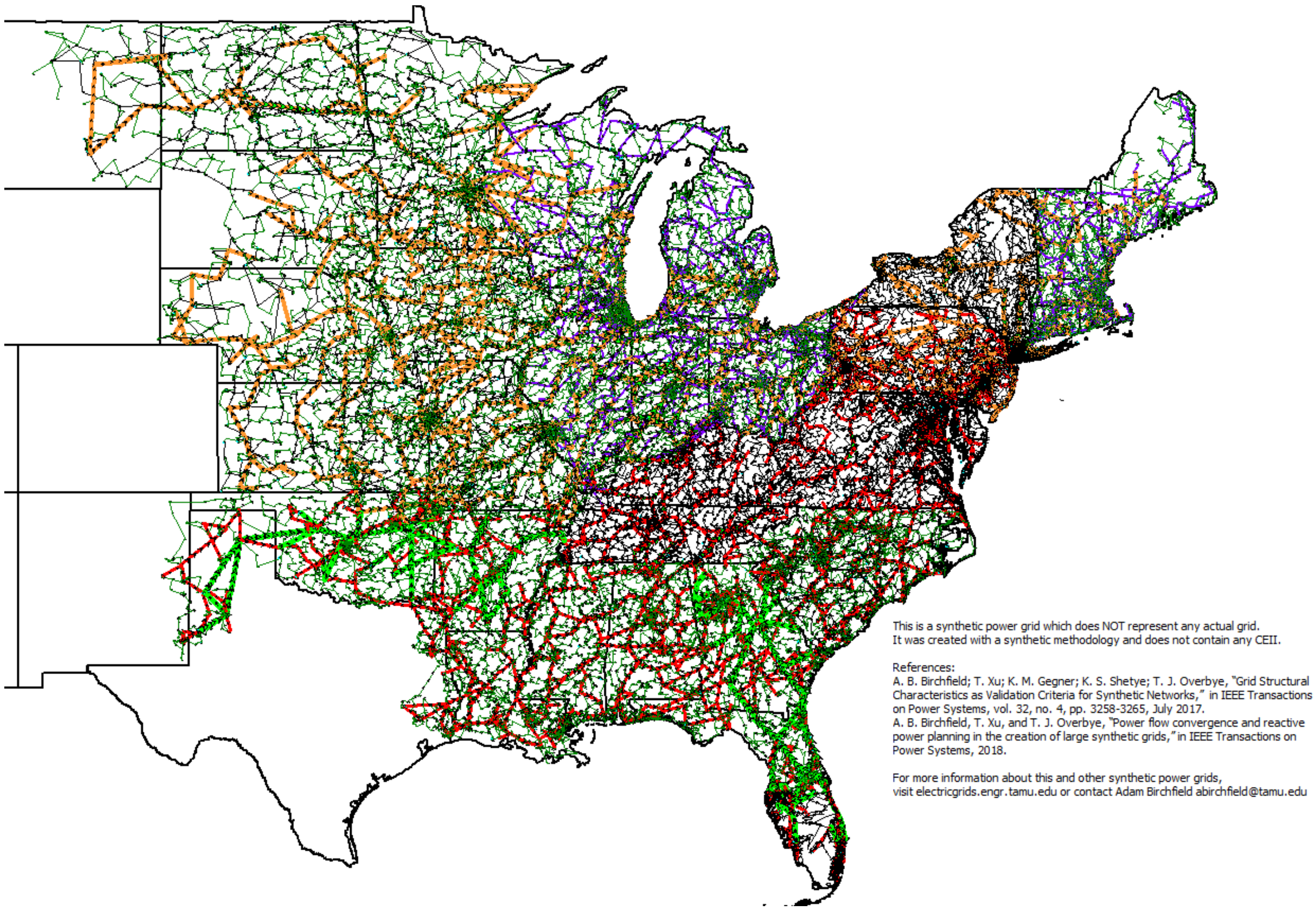
# Small System Equivalent Example

- Now set the **Max Per Unit Impedance for Equivalent Lines to 2.5.**



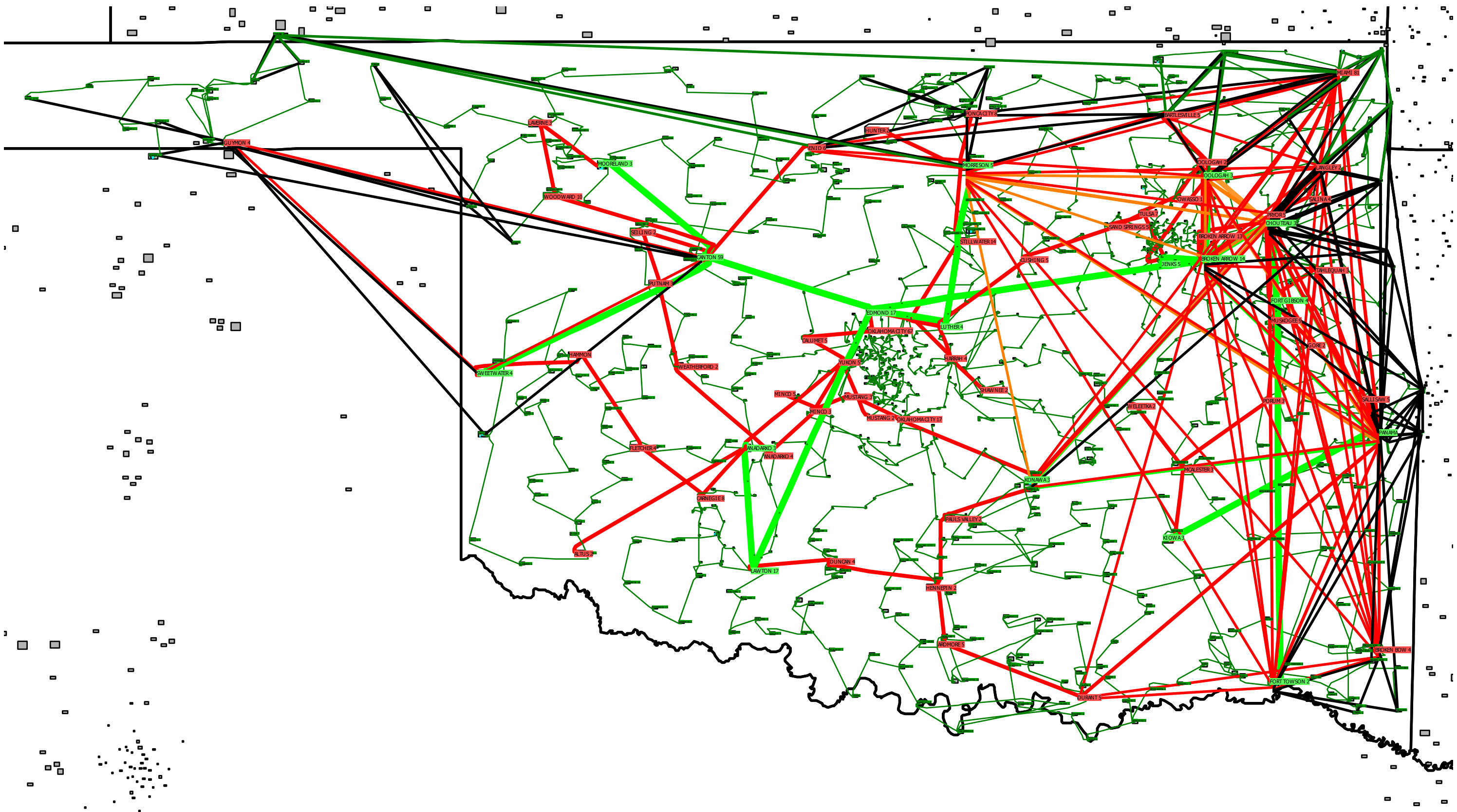
# Large System Example: 70K Case

- Original System has 70,000 buses and 71,343 lines



# Large System Example: 70K Case

- Just retain the Oklahoma Area; now 1591 buses and 1745 lines (deleting ones above 2.5 pu impedance)



# Grid Equivalent Examples

- A 2016 EI case had about 350 lines with a circuit ID of '99' and about 60 with 'EQ' (out of a total of 102,000)
  - Both WECC and the EI use '99' or 'EQ' circuit IDs to indicate equivalent lines
  - One would expect few equivalent lines in interconnect wide models
- A ten year old EI case had about 1633 lines with a circuit ID of '99' and 400 with 'EQ' (out of a total of 65673)
- A ten year old case with about 5000 buses and 5000 lines had 600 equivalent lines