# ECEN 615
# Methods of Electric Power Systems Analysis

## Lecture 20: Voltage Stability

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# Announcements

- Read Chapters 3 and 8 from the book
- Homework 5 is due on Thursday November 14

# Power System Voltage Stability

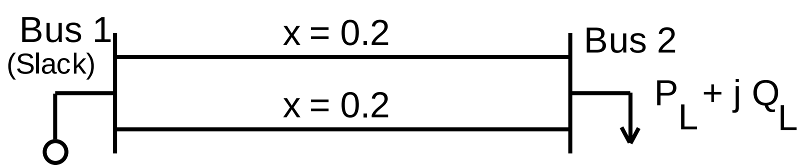
- **Voltage Stability**: The ability to maintain system voltage so that both power and voltage are controllable. System voltage responds as expected (i.e., an increase in load causes proportional decrease in voltage).
- **Voltage Instability**: Inability to maintain system voltage. System voltage and/or power become uncontrollable. System voltage does not respond as expected.
- **Voltage Collapse**: Process by which voltage instability leads to unacceptably low voltages in a significant portion of the system. Typically results in loss of system load.

# Voltage Stability

- Two good references are
  - P. Kundur, et. al., "Definitions and Classification of Power System Stability," *IEEE Trans. on Power Systems*, pp. 1387-1401, August 2004.
  - T. Van Cutsem, "Voltage Instability: Phenomena, Countermeasures, and Analysis Methods," *Proc. IEEE*, February 2000, pp. 208-227.
- Classified by either size of disturbance or duration
  - Small or large disturbance: small disturbance is just perturbations about an equilibrium point (power flow)
  - Short-term (several seconds) or long-term (many seconds to minutes) (covered in ECEN 667)

# Small Disturbance Voltage Stability

- Small disturbance voltage stability can be assessed using a power flow (maximum loadability)
- Depending on the assumed load model, the power flow can have multiple (or no) solutions
- PV curve is created by plotting power versus voltage




Assume $V_{slack}$=1.0

$$P_L - BV\sin\theta = 0$$

$$Q_L + BV\cos\theta - BV^2 = 0$$

Where B is the line susceptance =-10,
$V\angle\theta$ is the load voltage

# Small Disturbance Voltage Stability

- Question: how do the power flow solutions vary as the load is changed?
- A Solution: Calculate a series of power flow solutions for various load levels and see how they change
- Power flow Jacobian

$$\mathbf{J}(\theta,V)=\begin{bmatrix}-BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta-2BV\end{bmatrix}$$

$$\det\mathbf{J}(\theta,V)=VB^2\left(2V\cos\theta-\cos^2\theta-\sin^2\theta\right)$$

Singular when $\left(2V\cos\theta-1\right)=0$

# Maximum Loadability When Power Flow Jacobian is Singular

- An important paper considering this was by Sauer and Pai from IEEE Trans. Power Systems in Nov 1990, "Power system steady-state stability and the load-flow Jacobian"
- Other earlier papers were looking at the characteristics of multiple power flow solutions
- Work with the power flow optimal multiplier around the same time had shown that optimal multiplier goes to zero as the power flow Jacobian becomes singular
- The power flow Jacobian depends on the assumed load model (we'll see the impact in a few slides)

# Relationship Between Stability and Power Flow Jacobian

- The Sauer/Pai paper related system stability to the power flow Jacobian by noting the system dynamics could be written as a set of differential algebraic equations

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{p})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{p})$$

Linearing about and equilibrium gives

$$\begin{bmatrix} \Delta\dot{\mathbf{x}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} & \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{x}} & \frac{\partial \mathbf{g}}{\partial \mathbf{y}} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{y} \end{bmatrix}$$

# Relationship Between Stability and Power Flow Jacobian

- Then

Assuming $\frac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is nonsingular then

$$\Delta\dot{\mathbf{x}} = \left[ \frac{\partial \mathbf{f}}{\partial \mathbf{x}} - \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \left[ \frac{\partial \mathbf{g}}{\partial \mathbf{y}} \right]^{-1} \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \right] \Delta\mathbf{x}$$

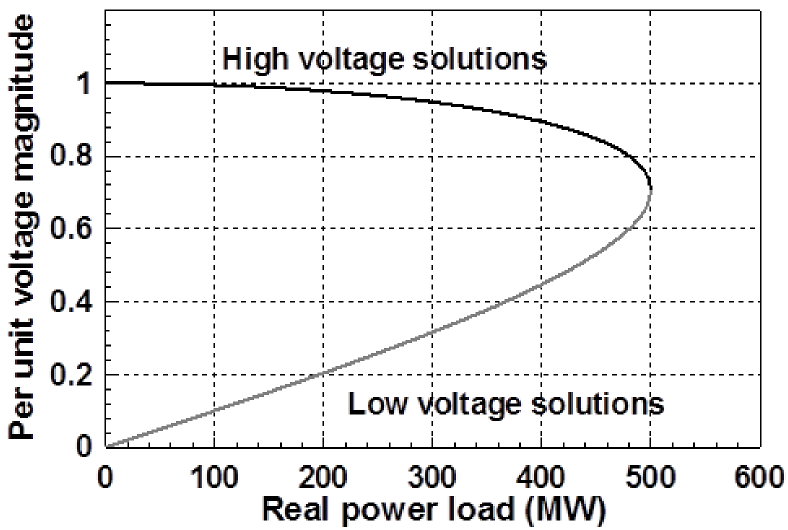
- What Sauer and Pai show is if $\partial \mathbf{g} / \partial \mathbf{y}$ is singular then the system is unstable; if $\partial \mathbf{g} / \partial \mathbf{y}$ is nonsingular then the system may or may not be stable
- Hence it provides an upper bound on stability

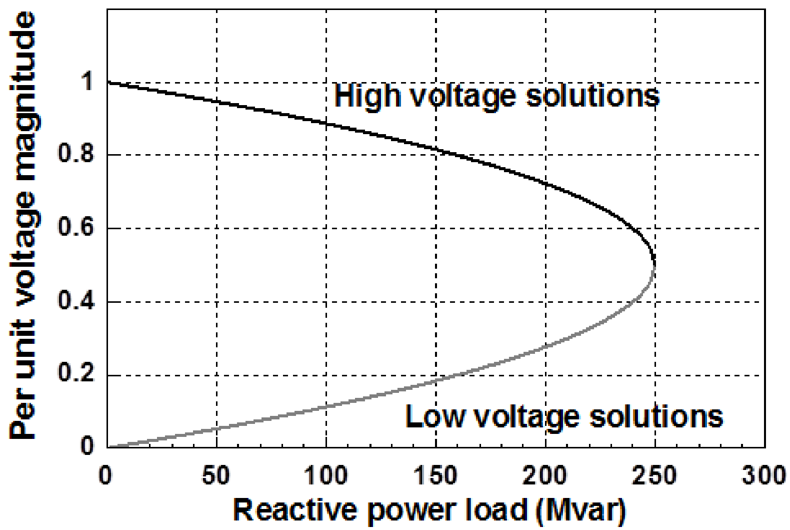
# Bifurcations

- In general, bifurcation is the division of something into two branches or parts
- For a dynamic system, a bifurcation occurs when small changes in a parameter cause a new quality of motion of the dynamic system
- Two types of bifurcation are considered for voltage stability
  - Saddle node bifurcation is the disappearance of an equilibrium point for parameter variation; for voltage stability it is two power flow solutions coalescing with parameter variation
  - Hopf bifurcation is cause by two eigenvalues crossing into the right-half plane

# PV and QV Curves

- PV curves can be traced by plotting the voltage as the real power is increased; QV curves as reactive power is increased
  - At least for the upper portion of the curve
- Two bus example PV and QV curves

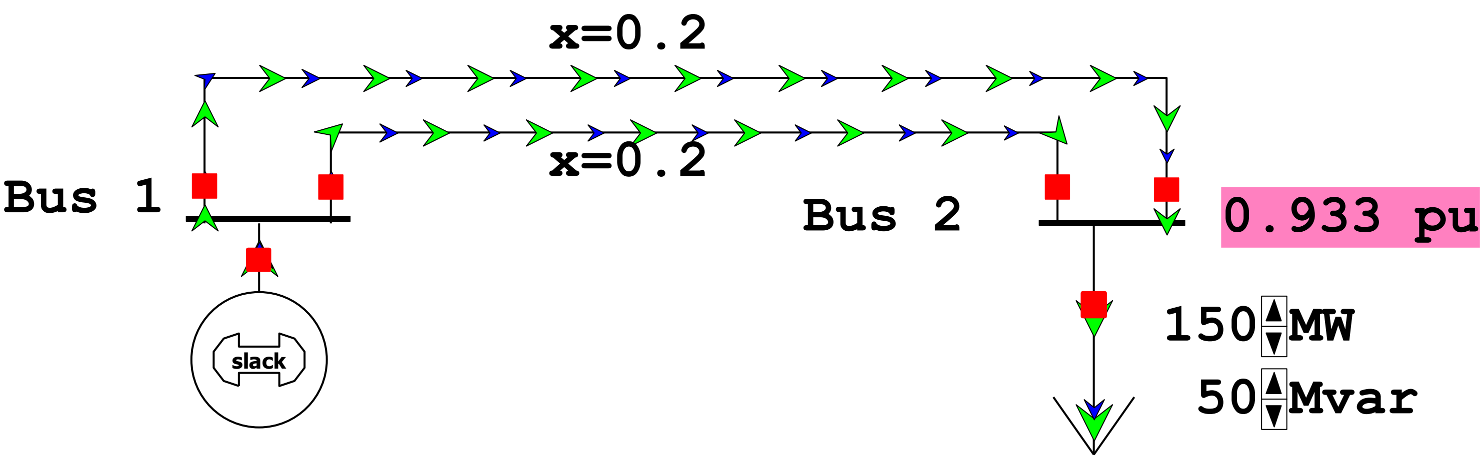


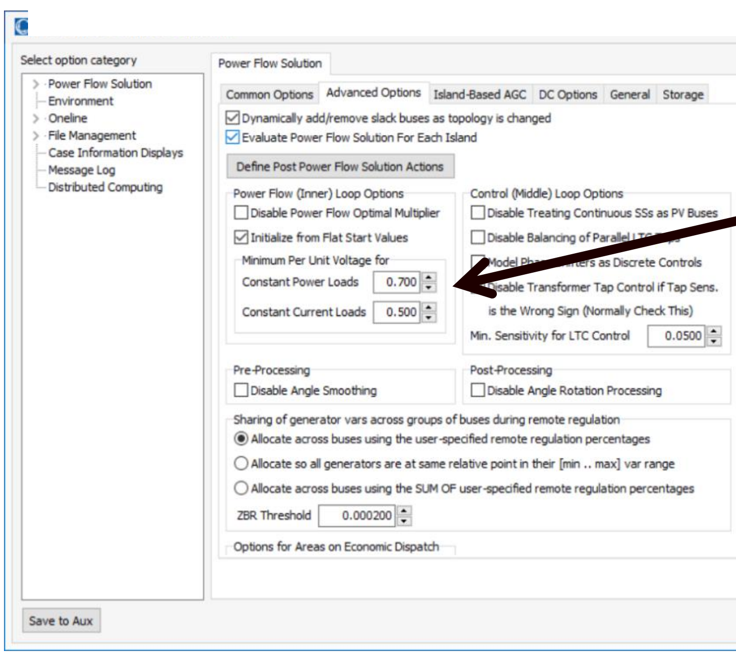



# Small Disturbance Voltage Collapse

- At constant frequency (e.g., 60 Hz) the complex power transferred down a transmission line is $S=VI^*$
  - V is phasor voltage, I is phasor current
  - This is the reason for using a high voltage grid
- Line real power losses are given by $RI^2$ and reactive power losses by $XI^2$
  - R is the line's resistance, and X its reactance; for a high voltage line X >> R
- Increased reactive power tends to drive down the voltage, which increases the current, which further increases the reactive power losses

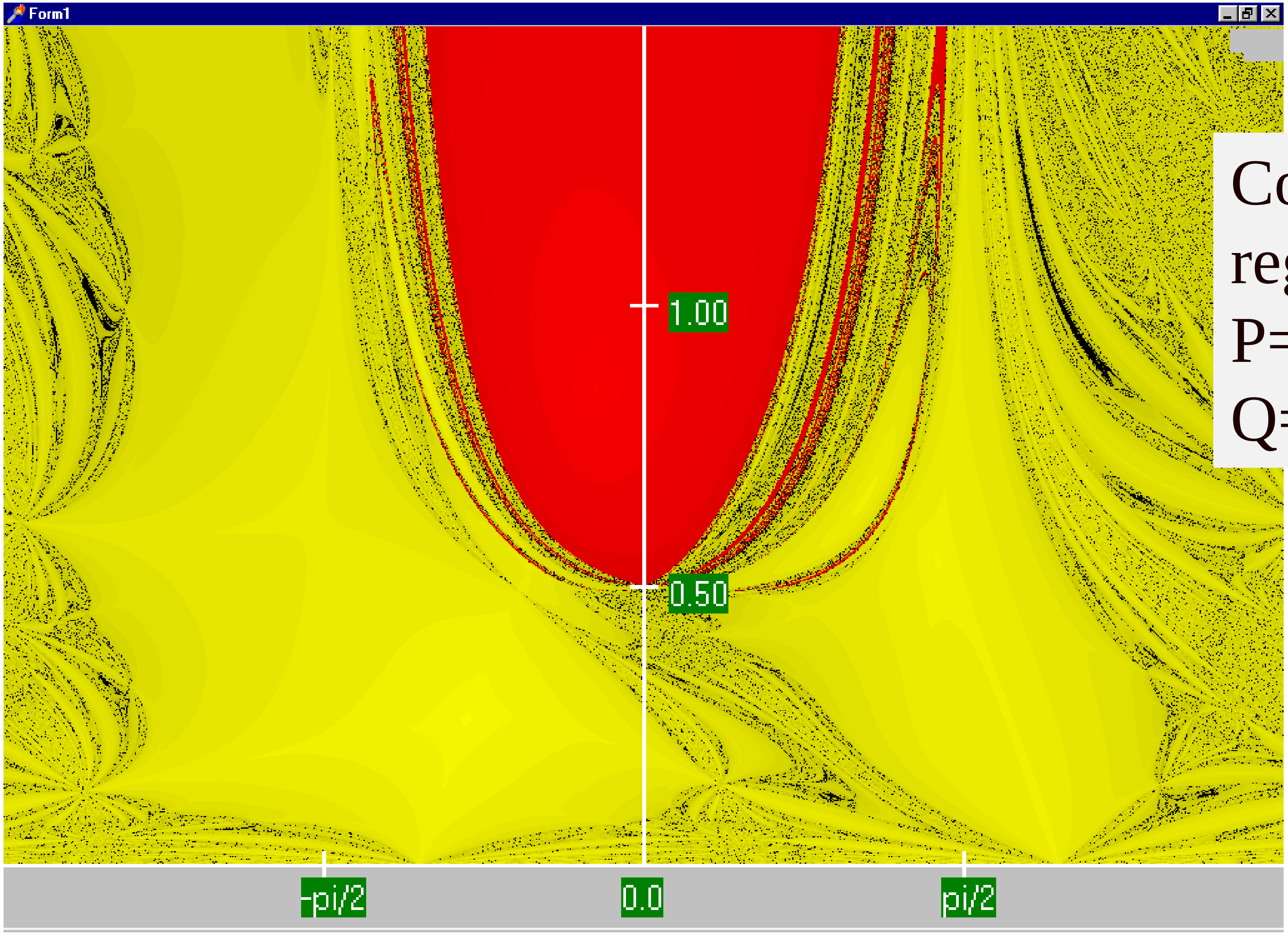
# PowerWorld Two Bus Example




Commercial power flow software usually auto converts constant power loads at low voltages; set these fields to zero to disable this conversion

# Power Flow Region of Convergence

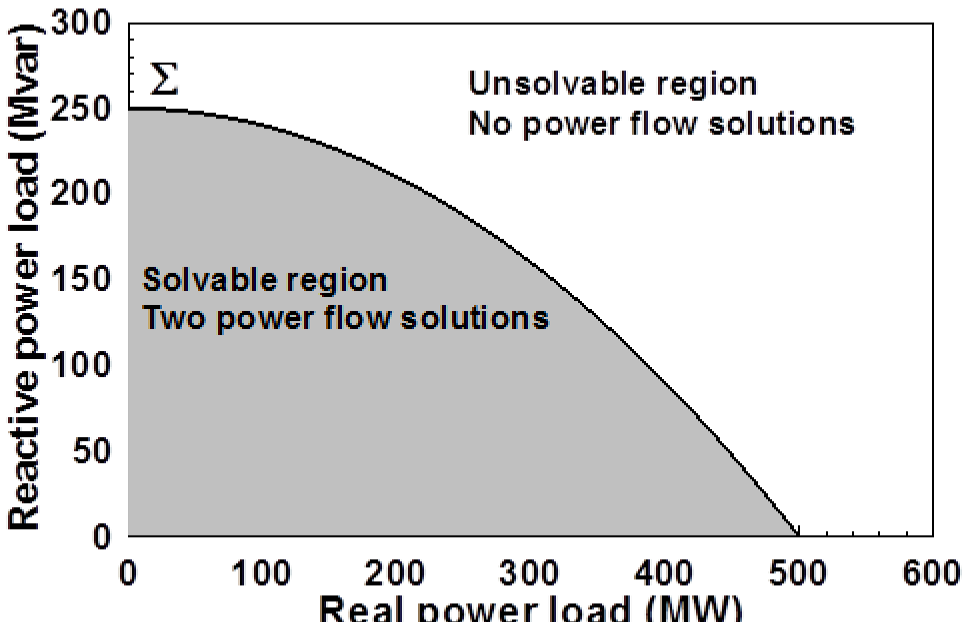



Convergence regions with P=100 MW, Q=0 Mvar

# Load Parameter Space Representation

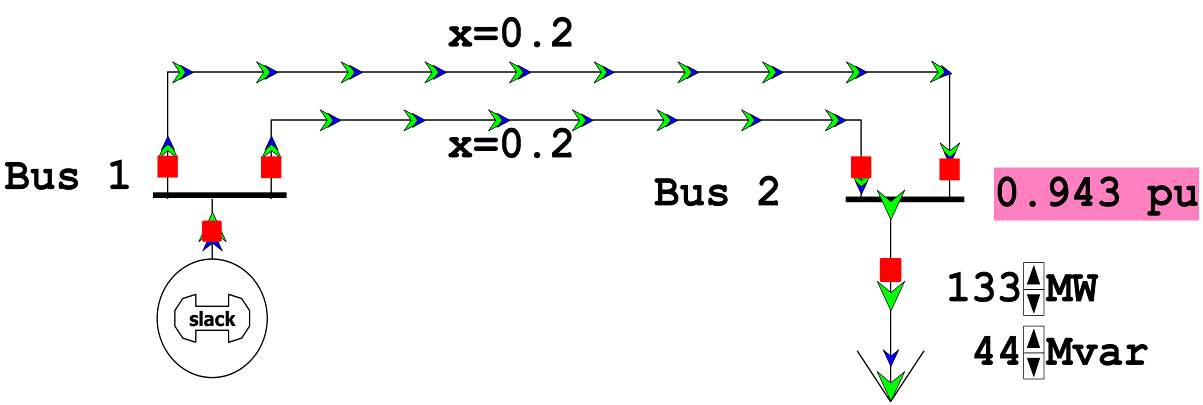
- With a constant power model there is a maximum loadability surface, Σ
  - Defined as point in which the power flow Jacobian is singular
  - For the lossless two bus system it can be determined as

$$-\frac{P_L^2}{B} + Q_L + \frac{1}{4}B = 0$$



# Load Model Impact

- With a static load model regardless of the voltage dependency the same PV curve is traced
  - But whether a point of maximum loadability exists depends on the assumed load model
    - If voltage exponent is > 1 then multiple solutions do not exist (see B.C. Lesieutre, P.W. Sauer and M.A. Pai "Sufficient conditions on static load models for network solvability," NAPS 1992, pp. 262-271)




Change load to constant impedance; hence it becomes a linear model

# ZIP Model Coefficients

- One popular static load model is the ZIP; lots of papers on the “correct” amount of each type

TABLE I
ZIP COEFFICIENTS FOR EACH CUSTOMER CLASS

| *Class* | $Z_p$ | $I_p$ | $P_p$ | $Z_q$ | $I_q$ | $P_q$ |
|---|---|---|---|---|---|---|
| Large commercial | 0.47 | -0.53 | 1.06 | 5.30 | -8.73 | 4.43 |
| Small commercial | 0.43 | -0.06 | 0.63 | 4.06 | -6.65 | 3.59 |
| Residential | 0.85 | -1.12 | 1.27 | 10.96 | -18.73 | 8.77 |
| Industrial | 0 | 0 | 1 | 0 | 0 | 1 |

TABLE VII
ACTIVE AND REACTIVE ZIP MODEL. FIRST HALF OF THE ZIPS WITH 100-V CUTOFF VOLTAGE. SECOND HALF REPORTS THE ZIPS WITH ACTUAL CUTOFF VOLTAGE

| Equipment/ component | No. tested | $V_{cut}$ | $V_o$ | $P_o$ | $Q_o$ | $Z_p$ | $I_p$ | $P_p$ | $Z_q$ | $I_q$ | $P_q$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Air compressor 1 Ph | 1 | 100 | 120 | 1109.01 | 487.08 | 0.71 | 0.46 | -0.17 | -1.33 | 4.04 | -1.71 |
| Air compressor 3 Ph | 1 | 174 | 208 | 1168.54 | 844.71 | 0.24 | -0.23 | 0.99 | 4.79 | -7.61 | 3.82 |
| Air conditioner | 2 | 100 | 120 | 496.33 | 125.94 | 1.17 | -1.83 | 1.66 | 15.68 | -27.15 | 12.47 |
| CFL bulb | 2 | 100 | 120 | 25.65 | 37.52 | 0.81 | -1.03 | 1.22 | 0.86 | -0.82 | 0.96 |
| Coffeemaker | 1 | 100 | 120 | 1413.04 | 13.32 | 0.13 | 1.62 | -0.75 | 3.89 | -6 | 3.11 |
| Copier | 1 | 100 | 120 | 944.23 | 84.57 | 0.87 | -0.21 | 0.34 | 2.14 | -3.67 | 2.53 |
| Electronic ballast | 3 | 100 | 120 | 59.02 | 5.06 | 0.22 | -0.5 | 1.28 | 9.64 | -21.59 | 12.95 |
| Elevator | 3 | 174 | 208 | 1381.17 | 1008.3 | 0.4 | -0.72 | 1.32 | 3.76 | -5.74 | 2.98 |
| Fan | 2 | 100 | 120 | 163.25 | 83.28 | -0.47 | 1.71 | -0.24 | 2.34 | -3.12 | 1.78 |
| Game consol | 3 | 100 | 120 | 60.65 | 67.61 | -0.63 | 1.23 | 0.4 | 0.76 | -0.93 | 1.17 |
| Halogen | 3 | 100 | 120 | 97.36 | 0.84 | 0.46 | 0.64 | -0.1 | 4.26 | -6.62 | 3.36 |
| High pressure sodium HID | 4 | 100 | 120 | 276.09 | 52.65 | 0.09 | 0.7 | 0.21 | 16.6 | -28.77 | 13.17 |
| Incandescent light | 2 | 100 | 120 | 87.16 | 0.85 | 0.47 | 0.63 | -0.1 | 0.55 | 0.38 | 0.07 |
| Induction light | 1 | 100 | 120 | 44.5 | 4.8 | 2.96 | -6.04 | 4.08 | 1.48 | -1.29 | 0.81 |
| Laptop charger | 1 | 100 | 120 | 35.94 | 71.64 | -0.28 | 0.5 | 0.78 | -0.37 | 1.24 | 0.13 |

Table 1 from M. Diaz-Aguilo, et. al., “Field-Validated Load Model for the Analysis of CVR in Distribution Secondary Networks: Energy Conservation,” IEEE Trans. Power Delivery, Oct. 2013

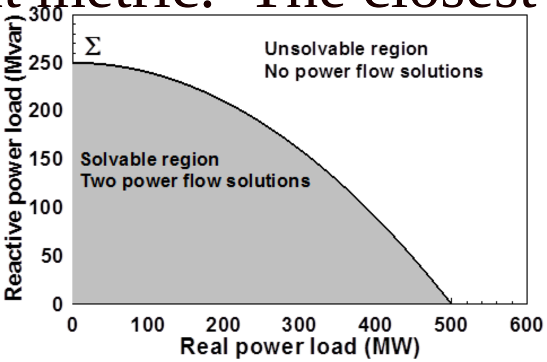
Table 7 from A, Bokhari, et. al., “Experimental Determination of the ZIP Coefficients for Modern Residential, Commercial, and Industrial Loads,” IEEE Trans. Power Delivery, June. 2014

# Application: Conservation Voltage Reduction (CVR)

- If the “steady-state” load has a true dependence on voltage, then a change (usually a reduction) in the voltage should result in a total decrease in energy consumption
- If an “optimal” voltage could be determined, then this could result in a net energy savings
- Some challenges are 1) the voltage profile across a feeder is not constant, 2) the load composition is constantly changing, 3) a decrease in power consumption might result in a decrease in useable output from the load, and 4) loads are dynamic and an initial decrease might be balanced by a later increase

# Determining a Metric to Voltage Collapse

- The goal of much of the voltage stability work was to determine an easy to calculate metric (or metrics) of the current operating point to voltage collapse
  - PV and QV curves (or some combination) can determine such a metric along a particular path
  - Goal was to have a path independent metric. The closest boundary point was considered, but this could be quite misleading if the system was not going to move in that direction




  - Any linearization about the current operating point (i.e., the Jacobian) does not consider important nonlinearities like generators hitting their reactive power limits

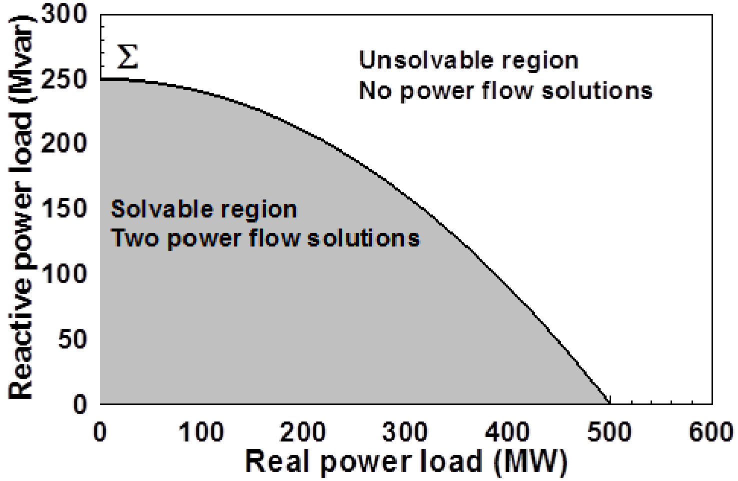
# Determining a Metric to Voltage Collapse

- A paper by Dobson in 1992 (see below) noted that at a saddle node bifurcation, in which the power flow Jacobian is singular, that
  - The right eigenvector associated with the Jacobian zero eigenvalue tells the direction in state space of the voltage collapse
  - The left eigenvector associated with the Jacobian zero eigenvalue gives the normal in parameter space to the boundary Σ. This can then be used to estimate the minimum distance in parameter space to bifurcation.

I. Dobson, "Observations on the Geometry of Saddle Node Bifurcation and Voltage Collapse in Electrical Power Systems," IEEE Trans. Circuits and Systems, March 1992

# Determining a Metric to Voltage Collapse Example

- For the previous two bus example we had




$$P_L - BV\sin\theta = 0$$

$$Q_L + BV\cos\theta - BV^2 = 0$$

$$\mathbf{J}(\theta, V) = \begin{bmatrix} -BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta - 2BV \end{bmatrix}$$

Singular when $(2V\cos\theta - 1) = 0$

So consider $B = -10,\ V = 0.6,\ \theta = -33.56°$, then $P_L = 3.317$, $Q_L = 1.400$

$$\mathbf{J} = \begin{bmatrix} 5 & -5.528 \\ -3.317 & 3.667 \end{bmatrix}$$

# Determining a Metric to Voltage Collapse Example

- Calculating the right and left eigenvectors associated with the zero eigenvalue we get

$$\mathbf{J} = \begin{bmatrix} 5 & -5.528 \\ -3.317 & 3.667 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} 0.742 \\ 0.671 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0.553 \\ 0.833 \end{bmatrix}$$

# Quantifying Power Flow Unsolvability

- Since lack of power flow convergence can be a major problem, it would be nice to have a measure to quantify the degree of unsolvability of a power flow
  - And then figure out the best way to restore solvabiblity
- T.J. Overbye, "A Power Flow Measure for Unsolvable Cases," IEEE Trans. Power Systems, August 1994

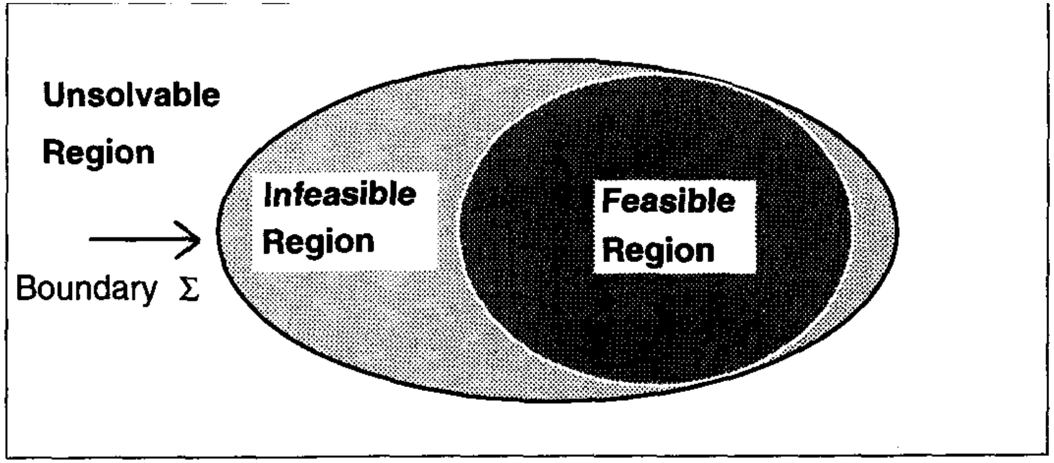



Figure 1 : Power Flow Security Regions

# Quantifying Power Flow Unsolvability

- To setup the problem, first consider the power flow iteration without and with the optimal multiplier

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}^k$$

$$\Delta\mathbf{x}^k = -\mathbf{J}(\mathbf{x}^k)^{-1}\left(\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right)$$

With the optimal multiplier we are minimizing

$$\mathrm{F}(\mathbf{x}^{k+1}) = \frac{1}{2}\left(\mathbf{f}(\mathbf{x}^k) + \mu\Delta\mathbf{x}^k - \mathbf{S}\right)^T\left(\mathbf{f}(\mathbf{x}^k) + \mu\Delta\mathbf{x}^k - \mathbf{S}\right)$$

When there is a solution $\mu \to 1$ and the cost function goes to zero

# Quantifying Power Flow Unsolvability

$$\det(\mathbf{J}) \quad = \quad B_{12}(B_{12} + 2eB_{22}) \quad = \quad 0 \qquad (12)$$

Here, where $B_{12} = -B_{22}$, the solution of (12) is e = 0.5. Substituting this solution for e into (10b) and using (10a) to solve for the f component of the bus 2 voltage, one gets $\Sigma$ to be the set of all points where

$$\frac{P^2}{B_{12}} + Q - \frac{1}{4}B_{12} = 0 \qquad (13)$$

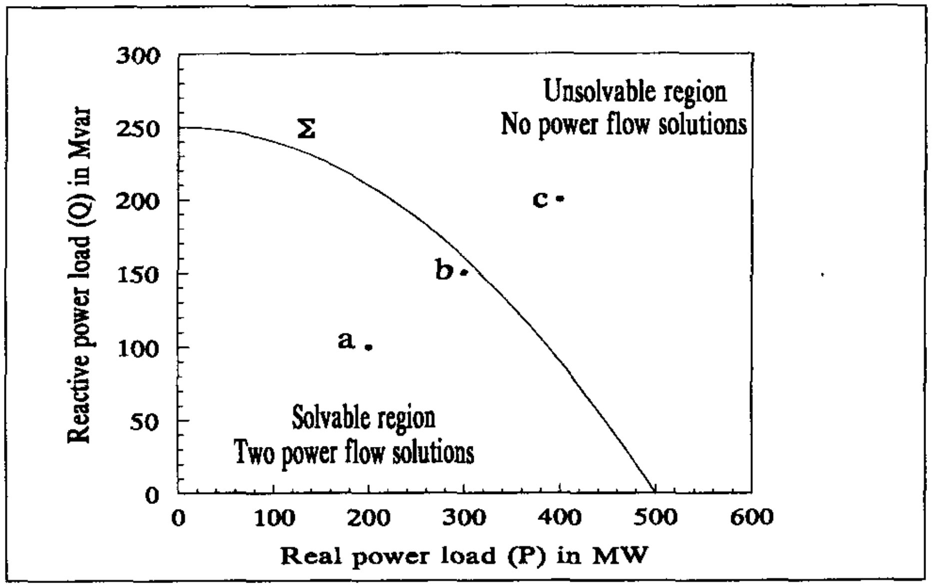



Figure 2 : Solvable and Unsolvable Regions in Parameter Space

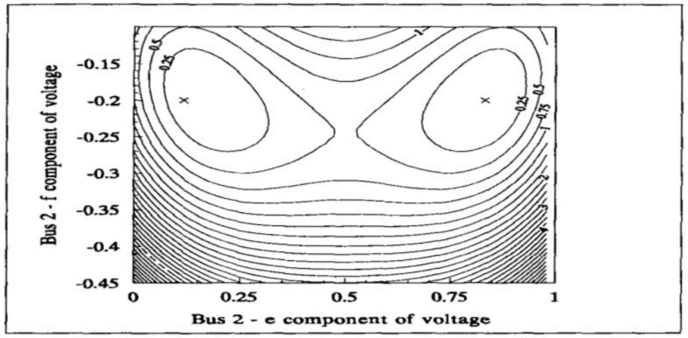



Figure 3a : Two Bus Cost Contours - Load of 200 MW and 100 Mvar

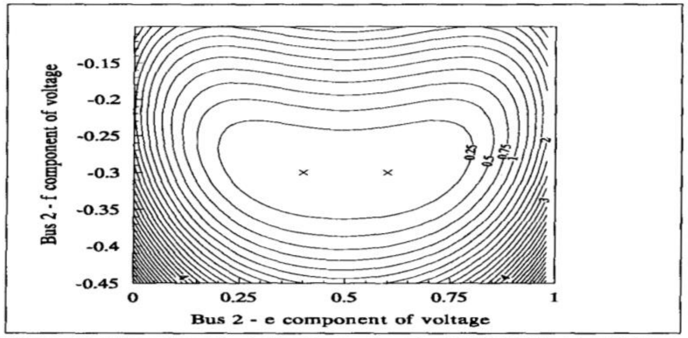



Figure 3b : Two Bus Cost Contours - Load of 300 MW and 150 Mvar

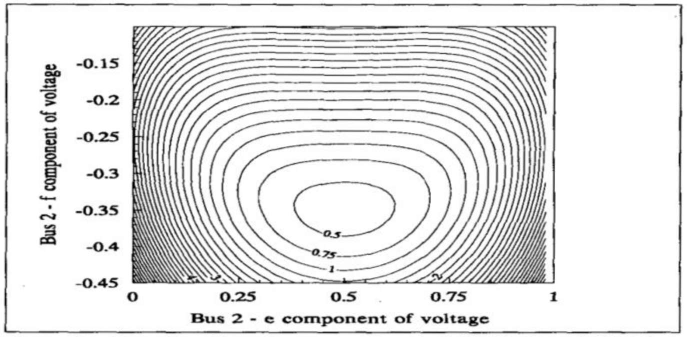



Figure 3c : Two Bus Cost Contours - Load of 400 MW and 200 Mvar

# Quantifying Power Flow Unsolvability

- However, when there is no solution the standard power flow would diverge. But the approach with the optimal multiplier tends to point in the direction of minimizing $F(x^{k+1})$. That is,

$$\nabla F(\mathbf{x}^k) = \left[\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right]^T \mathbf{J}(\mathbf{x}^k)$$

Also

$$\Delta \mathbf{x}^k = - \mathbf{J}(\mathbf{x}^k)^{-1}\left[\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right]$$

where how far to move in this direction is limited by $\mu$.

# Quantifying Power Flow Unsolvability

- The only way we cannot reduce the cost function some would be if the two directions were perpendicular, hence with a zero dot product. So

$$\frac{\nabla F\left(\mathbf{x}^k\right)\cdot\Delta\mathbf{x}^k}{\left\|\mathbf{x}^k\right\|} = \frac{= \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]^T \mathbf{J}(\mathbf{x}^{\mathrm{k}})\mathbf{J}(\mathbf{x}^{\mathrm{k}})^{-1}\left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]}{\left\|\mathbf{x}^k\right\|}$$

$$= \frac{= \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]^T \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]}{\left\|\mathbf{x}^k\right\|}$$

(provided the Jacobian is not singular). As we approach singularity this goes to zero. Hence we converge to a point on the boundary $\Sigma$, but not necessarilty at the closest boundary point.

# Quantifying Power Flow Unsolvability

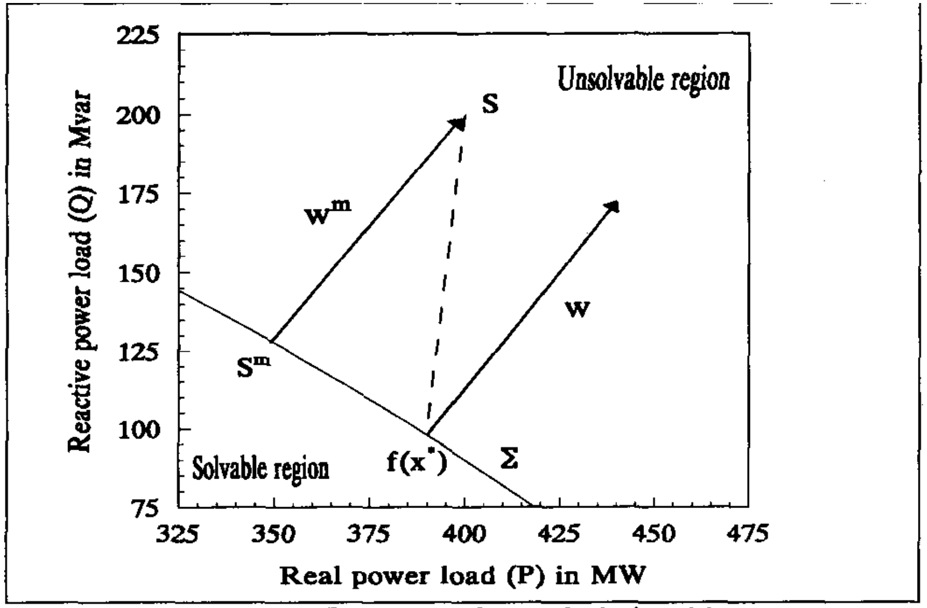



Figure 5 : Parameter Space Relationships

If Σ were flat then **w** is parallel to **w**$^m$

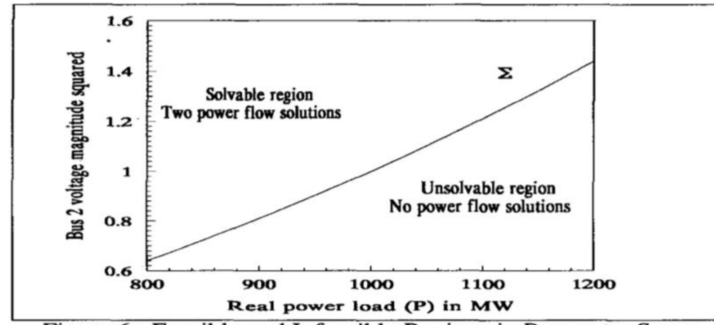



Figure 6 : Feasible and Infeasible Regions in Parameter Space

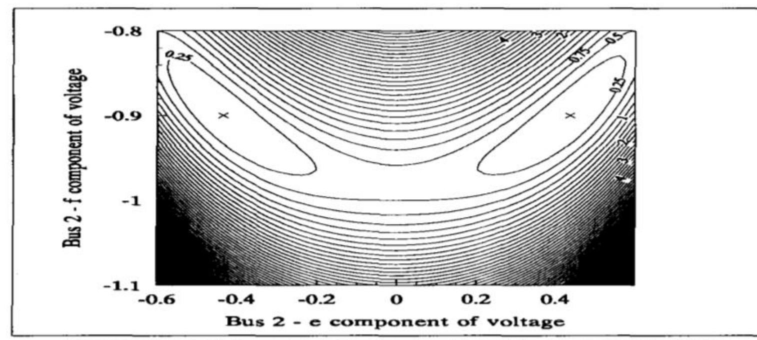



Figure 7a : PV Bus Cost Contours - Feasible load of 900 MW

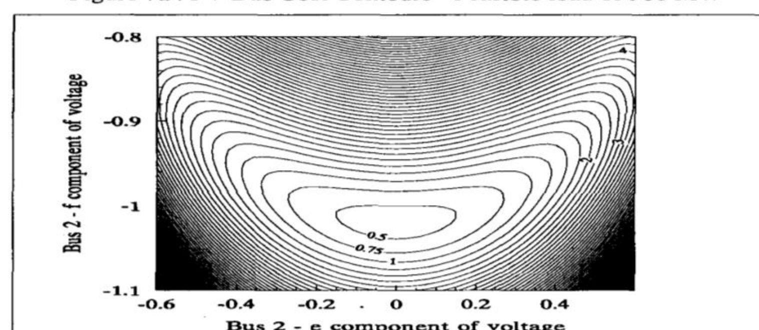



Figure 7b : PV Bus Cost Contours - Infeasible load of 1100 MW

# Quantifying Power Flow Unsolvability

- The left eigenvector associated with the zero eigenvalue of the Jacobian (defined as $\mathbf{w}^{\mathrm{i}*}$) is perpendicular to Σ (as noted in the early 1992 Dobson paper)
- We can get the closest point on the Σ just by iterating, updating the **S** Vector as

$$\mathbf{S}^{i+1} = \mathbf{S} + [(\mathbf{f}(\mathbf{x}^{\mathrm{i}*}) - \mathbf{S}) \cdot \mathbf{w}^{\mathrm{i}*}]\, \mathbf{w}^{\mathrm{i}*}$$

(here **S** is the initial power injection, $\mathbf{x}^{\mathrm{i}*}$ a boundary solution)

- Converges when $\left\| (\mathbf{f}(\mathbf{x}^{\mathrm{i}*}) - \mathbf{S}^{i}) \right\| < \varepsilon$

# Challenges

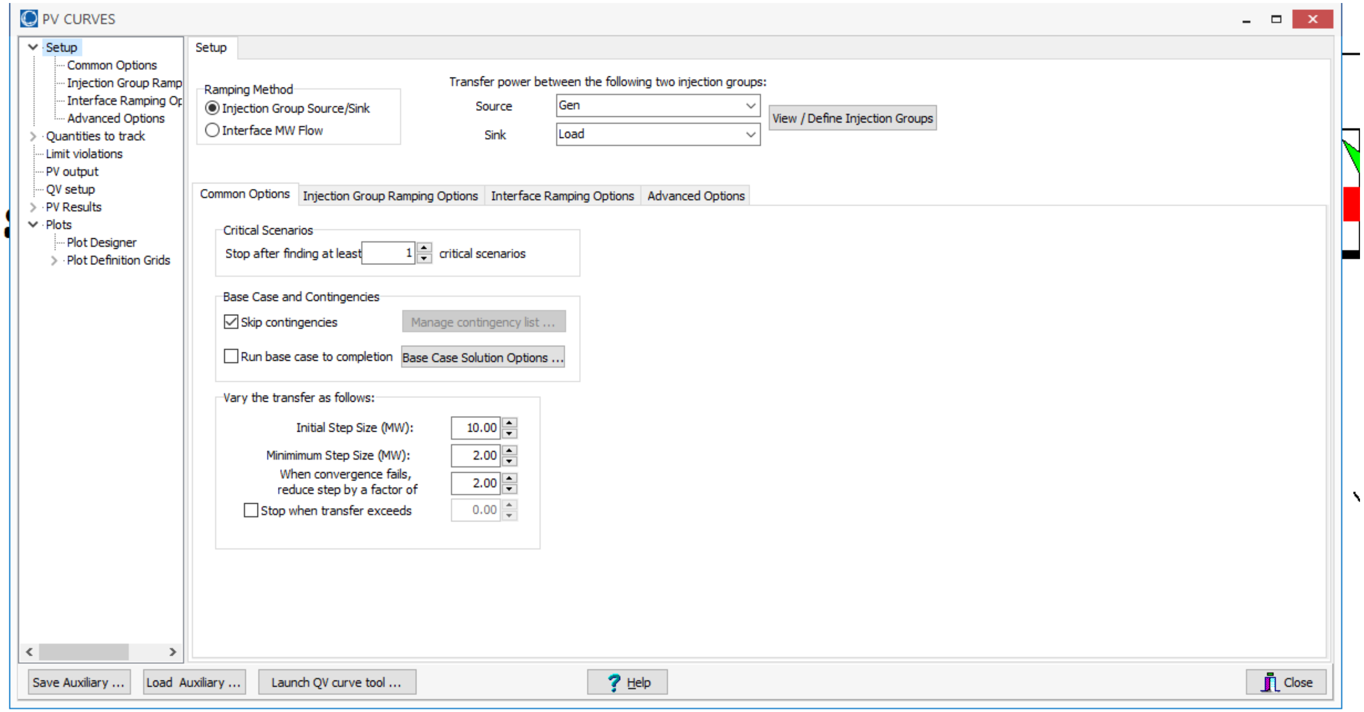
- The key issues is actual power systems are quite complex, with many nonlinearities. For example, generators hitting reactive power limits, switched shunts, LTCs, phase shifters, etc.
- Practically people would like to know how far some system parameters can be changed before running into some sort of limit violation, or maximum loadability.
  - The system is changing in a particular direction, such as a power transfer; this often includes contingency analysis
- Line limits and voltage magnitudes are considered
  - Lower voltage lines tend to be thermally constrained
- Solution is to just to trace out the PV or QV curves

# PV and QV Analysis in PowerWorld

- Requires setting up what is known in PowerWorld as an injection group
  - An injection group specifies a set of objects, such as generators and loads, that can inject or absorb power
  - Injection groups can be defined by selecting **Case Information, Aggregation, Injection Groups**
- The PV and/or QV analysis then varies the injections in the injection group, tracing out the PV curve
- This allows optional consideration of contingencies
- The PV tool can be displayed by selecting **Add-Ons, PV**

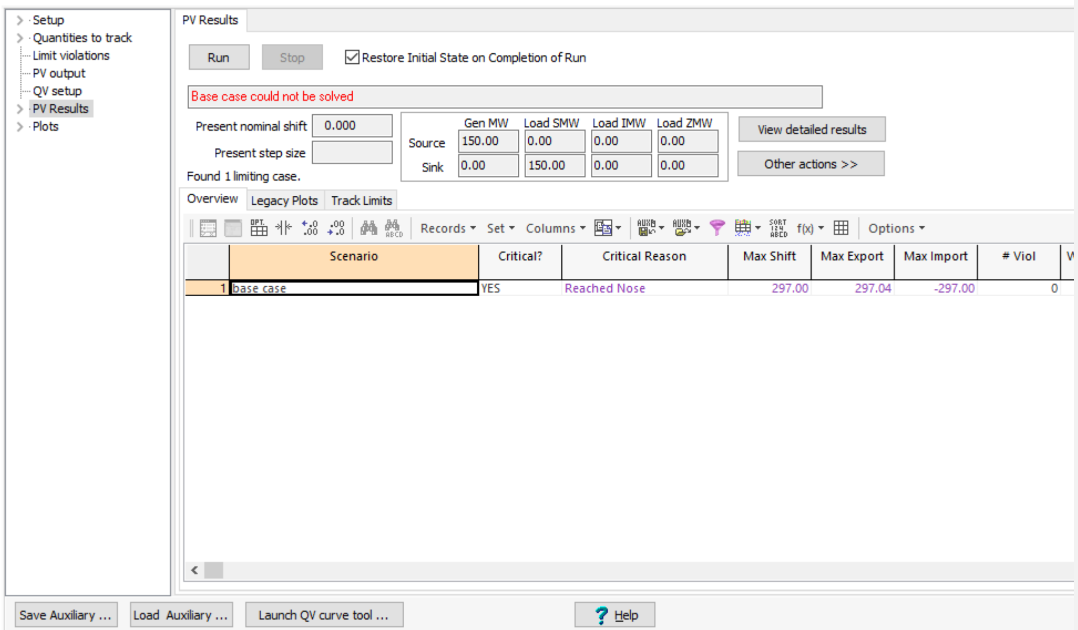
# PV and QV Analysis in PowerWorld: Two Bus Example

- Setup page defines the source and sink and step size



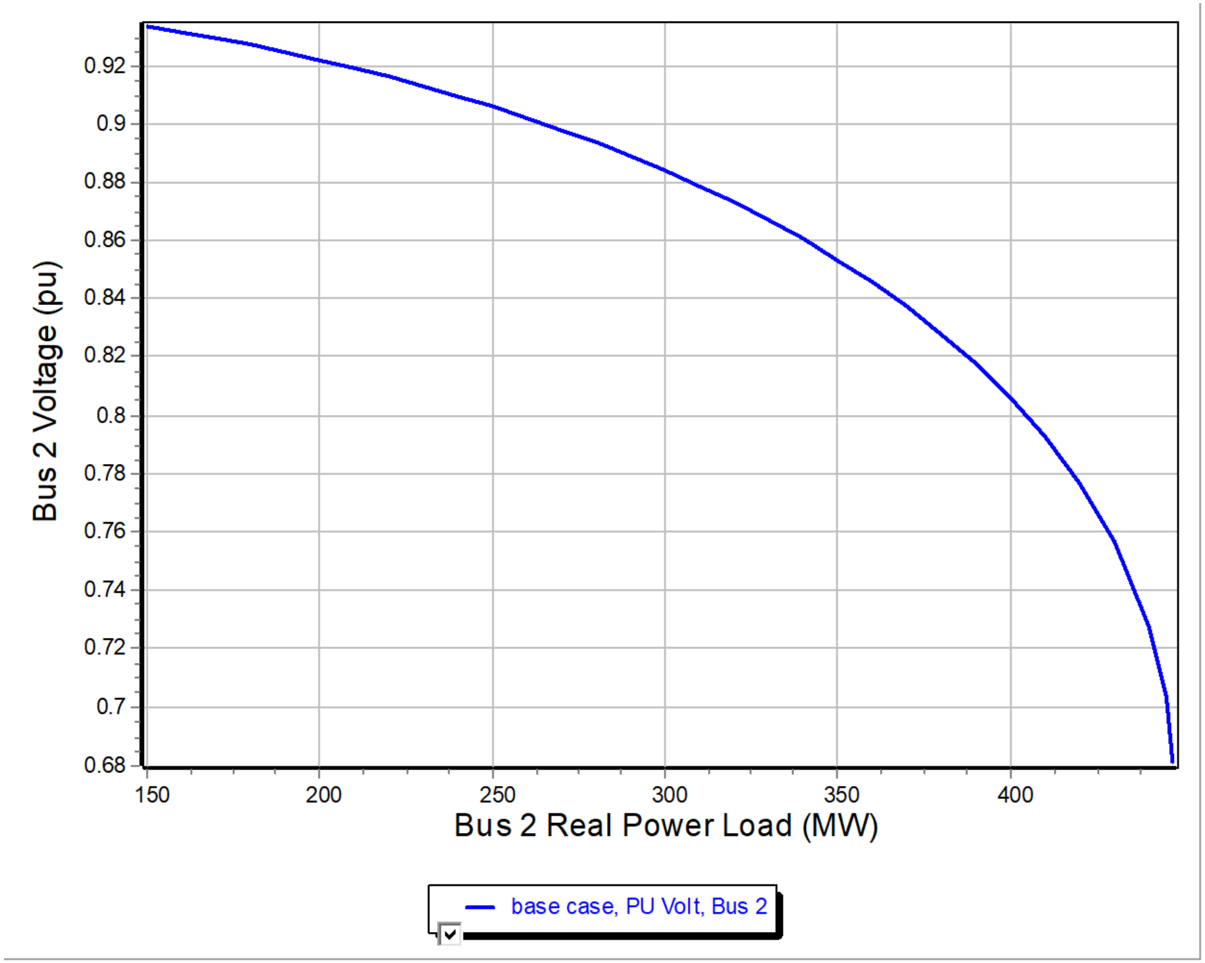
# PV and QV Analysis in PowerWorld: Two Bus Example

- The PV Results Page does the actual solution
  - Plots can be defined to show the results
  - **Other Actions, Restore initial state** restores the pre-study state

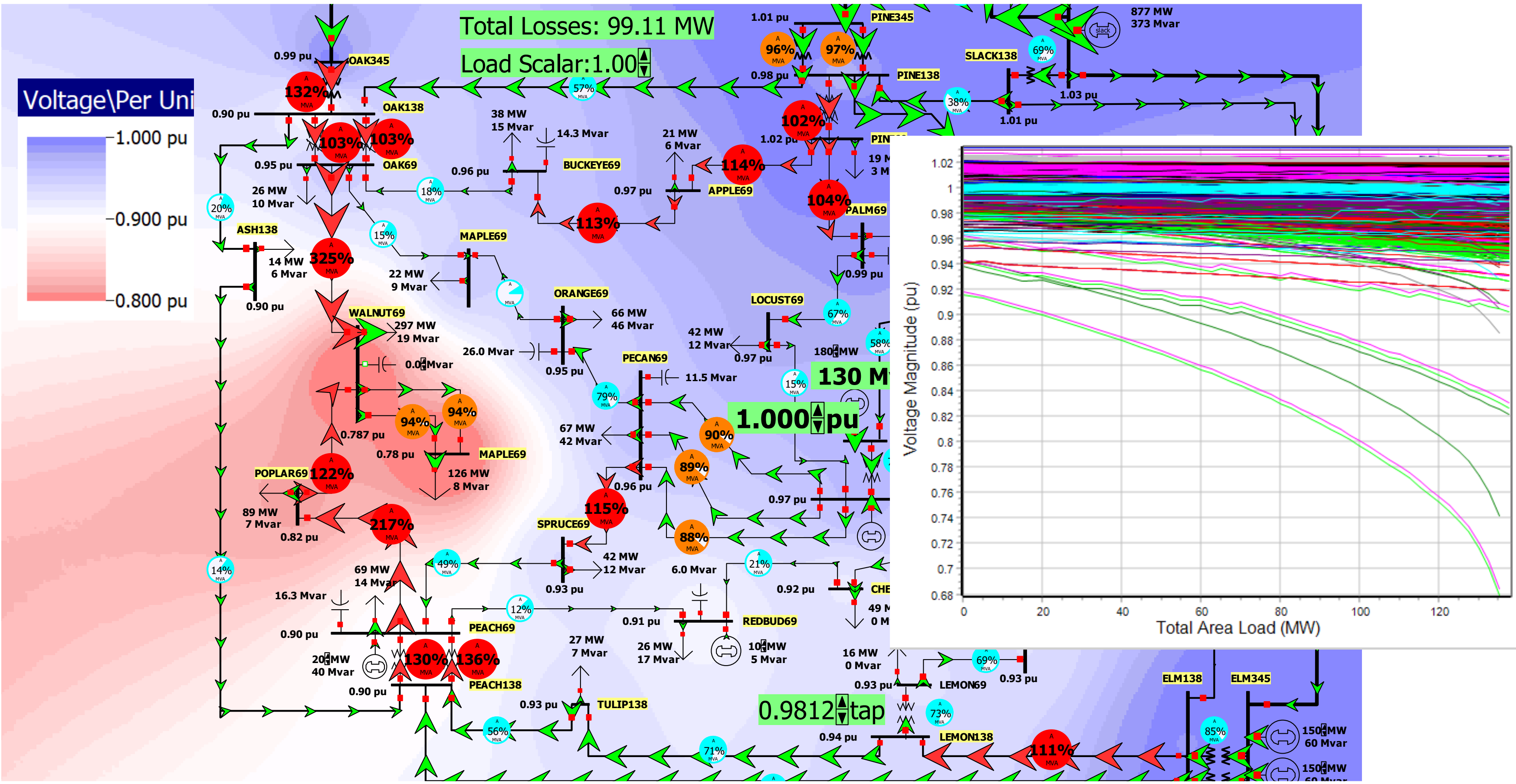



Click the Run button to run the PV analysis; Check the **Restore Initial State on Completion of Run** to restore the pre-PV state (by default it is not restored)

# PV and QV Analysis in PowerWorld: Two Bus Example



# PV and QV Analysis in PowerWorld: 37 Bus Example




Usually other limits also need to be considered in doing a realistic PV analysis