ECEN 615
Methods of Electric Power Systems Analysis

Lecture 23: Optimal Power Flow

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TEXAS A&M UNIVERSITY

Announcements

- Read Chapters 3 and 8 from the book
- Homework 5 is due today
- Second exam is in class on November 21
  - Same format as with the first exam

LP Optimal Power Flow

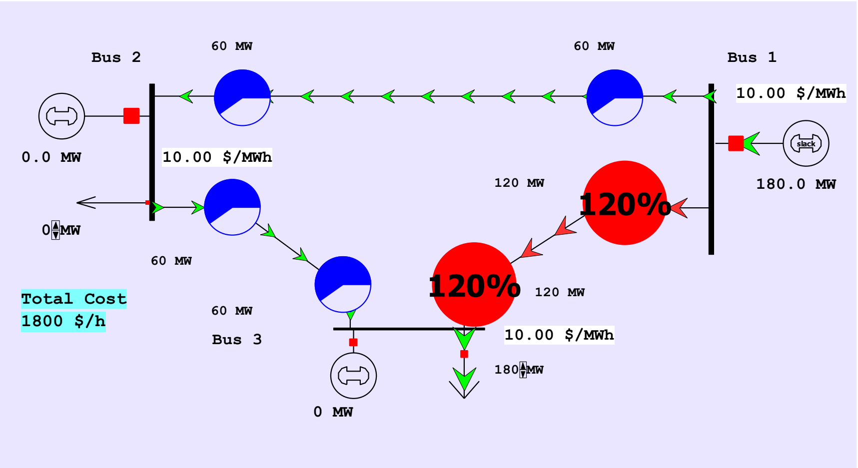
- LP OPF was introduced in
  - B. Stott, E. Hobson, “Power System Security Control Calculations using Linear Programming,” (Parts 1 and 2) *IEEE Trans. Power App and Syst.*, Sept/Oct 1978
  - O. Alsac, J. Bright, M. Prais, B. Stott, “Further Developments in LP-based Optimal Power Flow,” *IEEE Trans. Power Systems*, August 1990
- It is a widely used technique, particularly for real power optimization; it is the technique used in PowerWorld

LP Optimal Power Flow

- Idea is to iterate between solving the power flow, and solving an LP with just a selected number of constraints enforced
- The power flow (which could be ac or dc) enforces the standard power flow constraints
- The LP equality constraints include enforcing area interchange, while the inequality constraints include enforcing line limits; controls include changes in generator outputs
- LP results are transferred to the power flow, which is then resolved

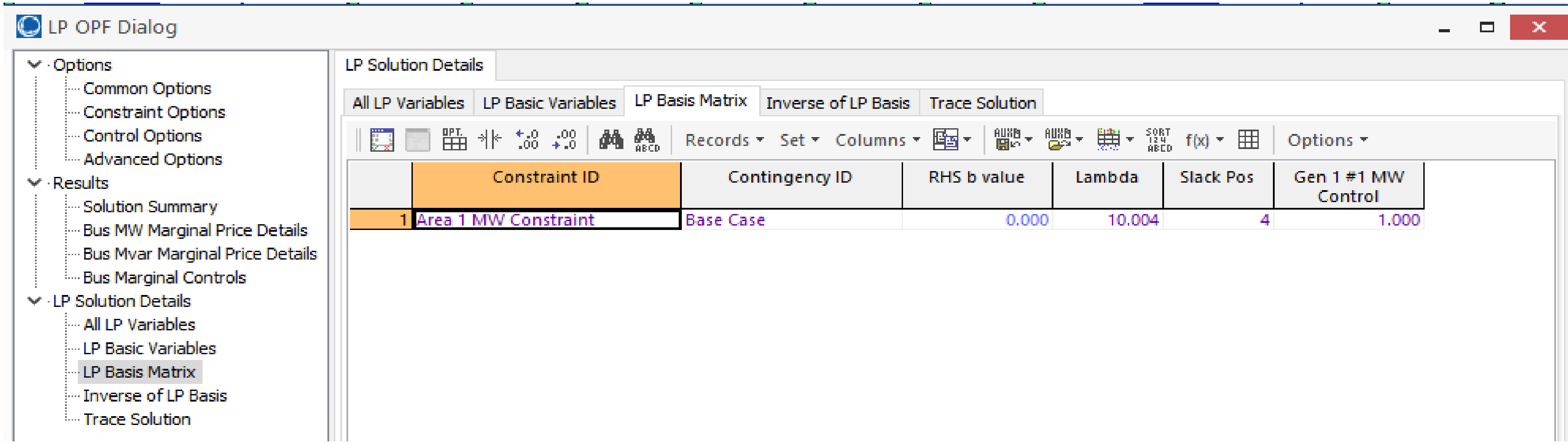
LP OPF Introductory Example

- In PowerWorld load the **B3LP** case and then display the LP OPF Dialog (select **Add-Ons, OPF Options and Results)**
- Use **Solve LP OPF** to solve the OPF, initially with no line limits enforced; this is similar to economic dispatch with a single power balance equality constraint

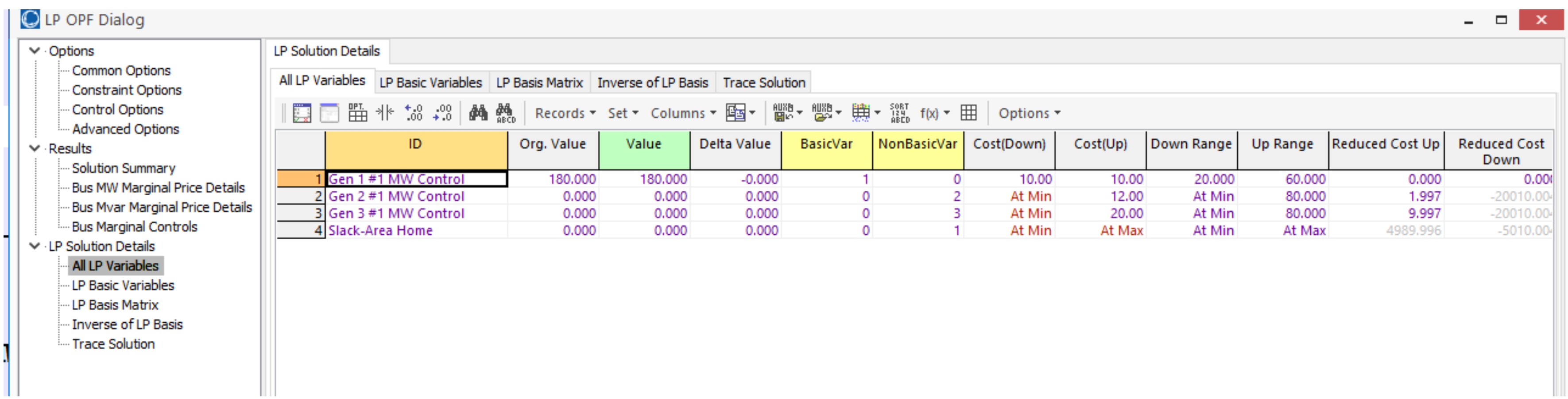


- The LP results are available from various pages on the dialog

LP OPF Introductory Example, cont



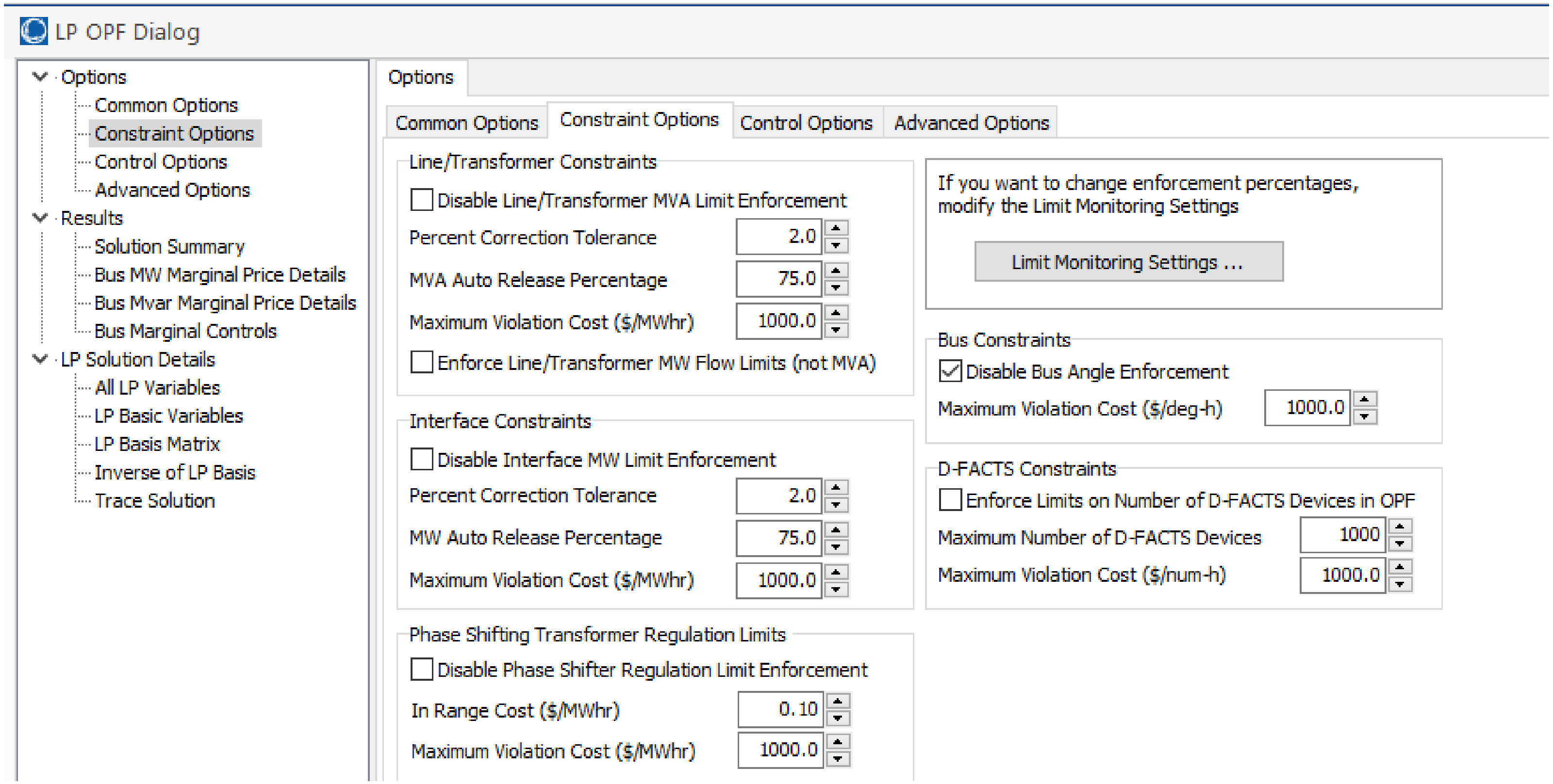
	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 1 #1 MW Control
1	Area 1 MW Constraint	Base Case	0.000	10.004	4	1.000



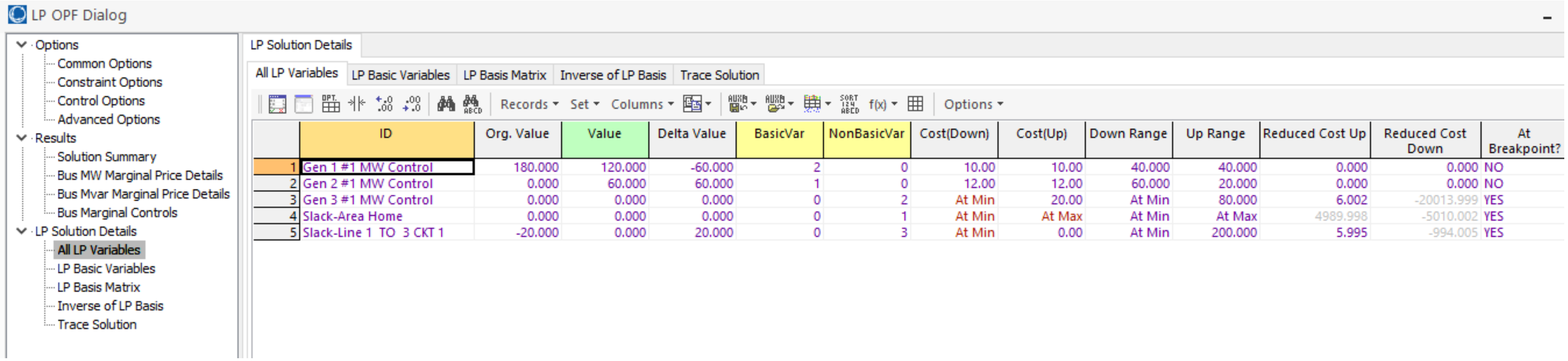
	ID	Org. Value	Value	Delta Value	BasicVar	NonBasicVar	Cost(Down)	Cost(Up)	Down Range	Up Range	Reduced Cost Up	Reduced Cost Down
1	Gen 1 #1 MW Control	180.000	180.000	-0.000	1	0	10.00	10.00	20.000	60.000	0.000	0.00
2	Gen 2 #1 MW Control	0.000	0.000	0.000	0	2	At Min	12.00	At Min	80.000	1.997	-20010.00
3	Gen 3 #1 MW Control	0.000	0.000	0.000	0	3	At Min	20.00	At Min	80.000	9.997	-20010.00
4	Slack-Area Home	0.000	0.000	0.000	0	1	At Min	At Max	At Min	At Max	4989.996	-5010.00

LP OPF Introductory Example, cont

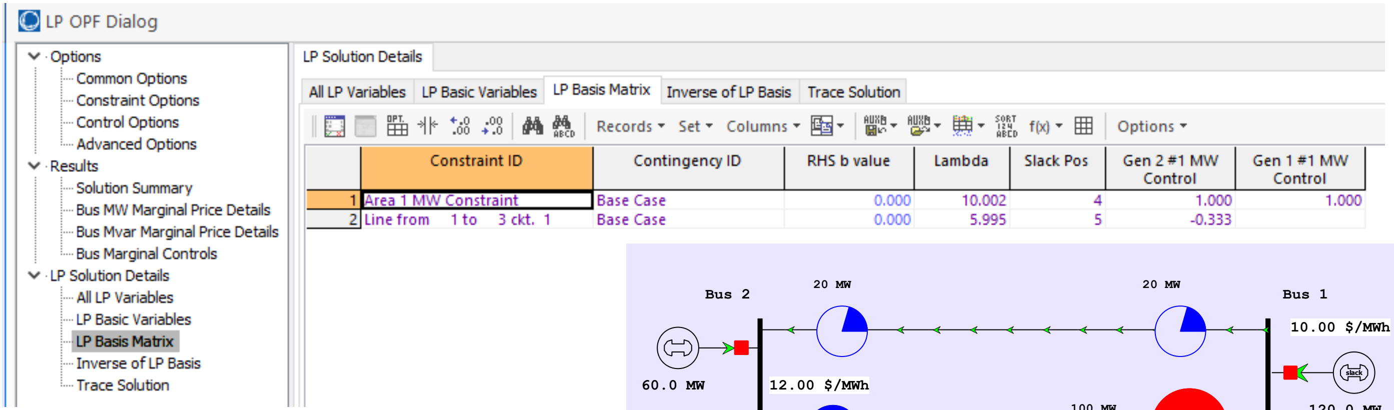
- On use **Options, Constraint Options** to enable the enforcement of the Line/Transformer MVA limits



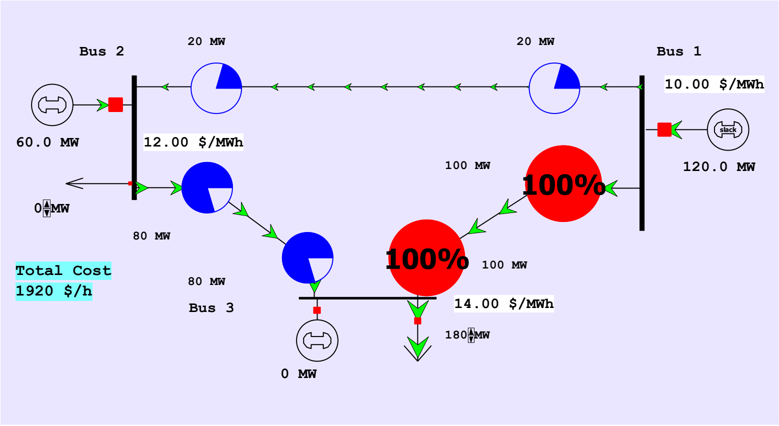
LP OPF Introductory Example, cont.



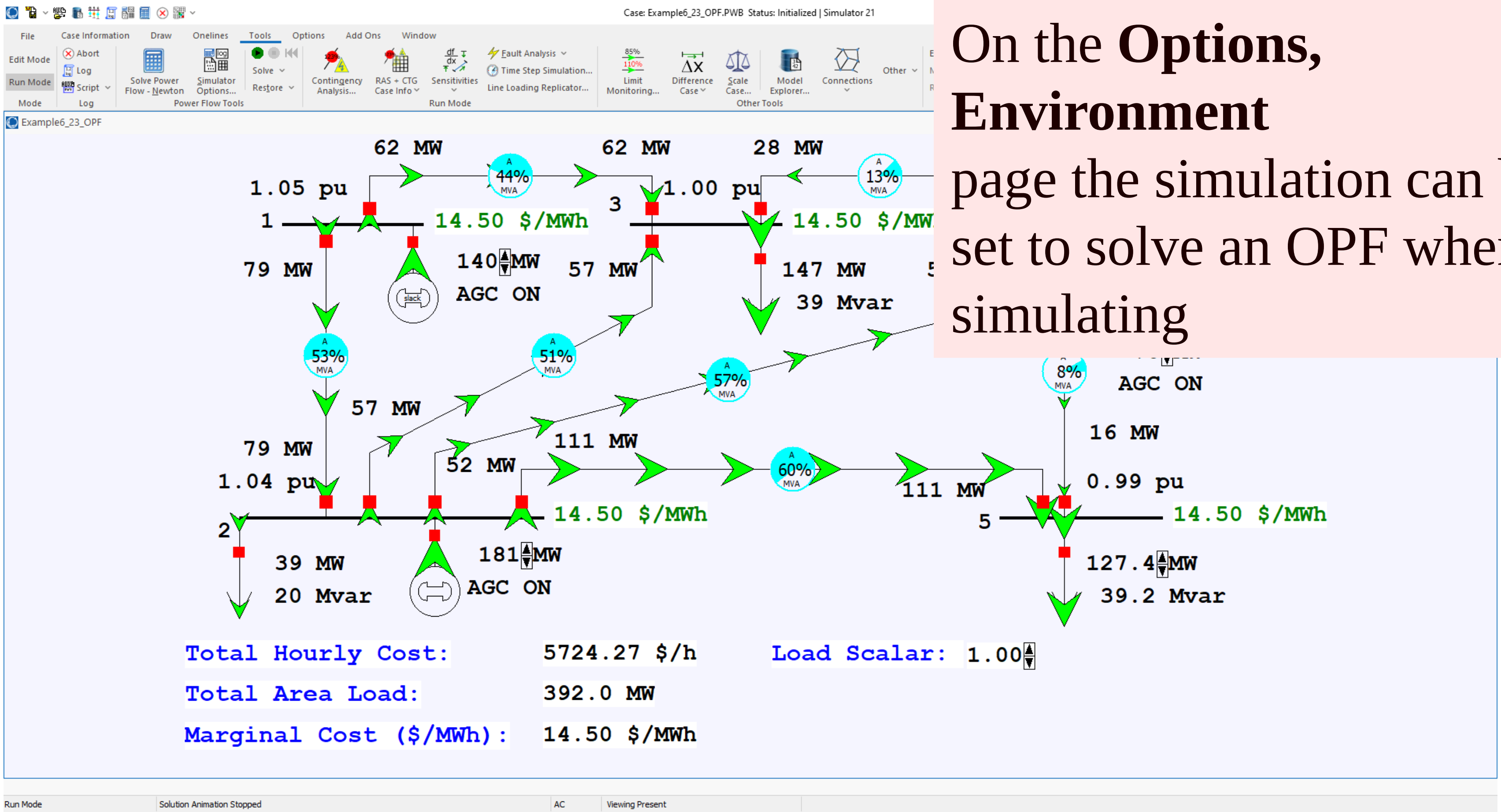
	ID	Org. Value	Value	Delta Value	BasicVar	NonBasicVar	Cost(Down)	Cost(Up)	Down Range	Up Range	Reduced Cost Up	Reduced Cost Down	At Breakpoint?
1	Gen 1 #1 MW Control	180.000	120.000	-60.000	2	0	10.00	10.00	40.000	40.000	0.000	0.000	NO
2	Gen 2 #1 MW Control	0.000	60.000	60.000	1	0	12.00	12.00	60.000	20.000	0.000	0.000	NO
3	Gen 3 #1 MW Control	0.000	0.000	0.000	0	2	At Min	20.00	At Min	80.000	6.002	-20013.999	YES
4	Slack-Area Home	0.000	0.000	0.000	0	1	At Min	At Max	At Min	At Max	4989.998	-5010.002	YES
5	Slack-Line 1 TO 3 CKT 1	-20.000	0.000	20.000	0	3	At Min	0.00	At Min	200.000	5.995	-994.005	YES



	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 2 #1 MW Control	Gen 1 #1 MW Control
1	Area 1 MW Constraint	Base Case	0.000	10.002	4	1.000	1.000
2	Line from 1 to 3 ckt. 1	Base Case	0.000	5.995	5	-0.333	



Example 6_23 Optimal Power Flow



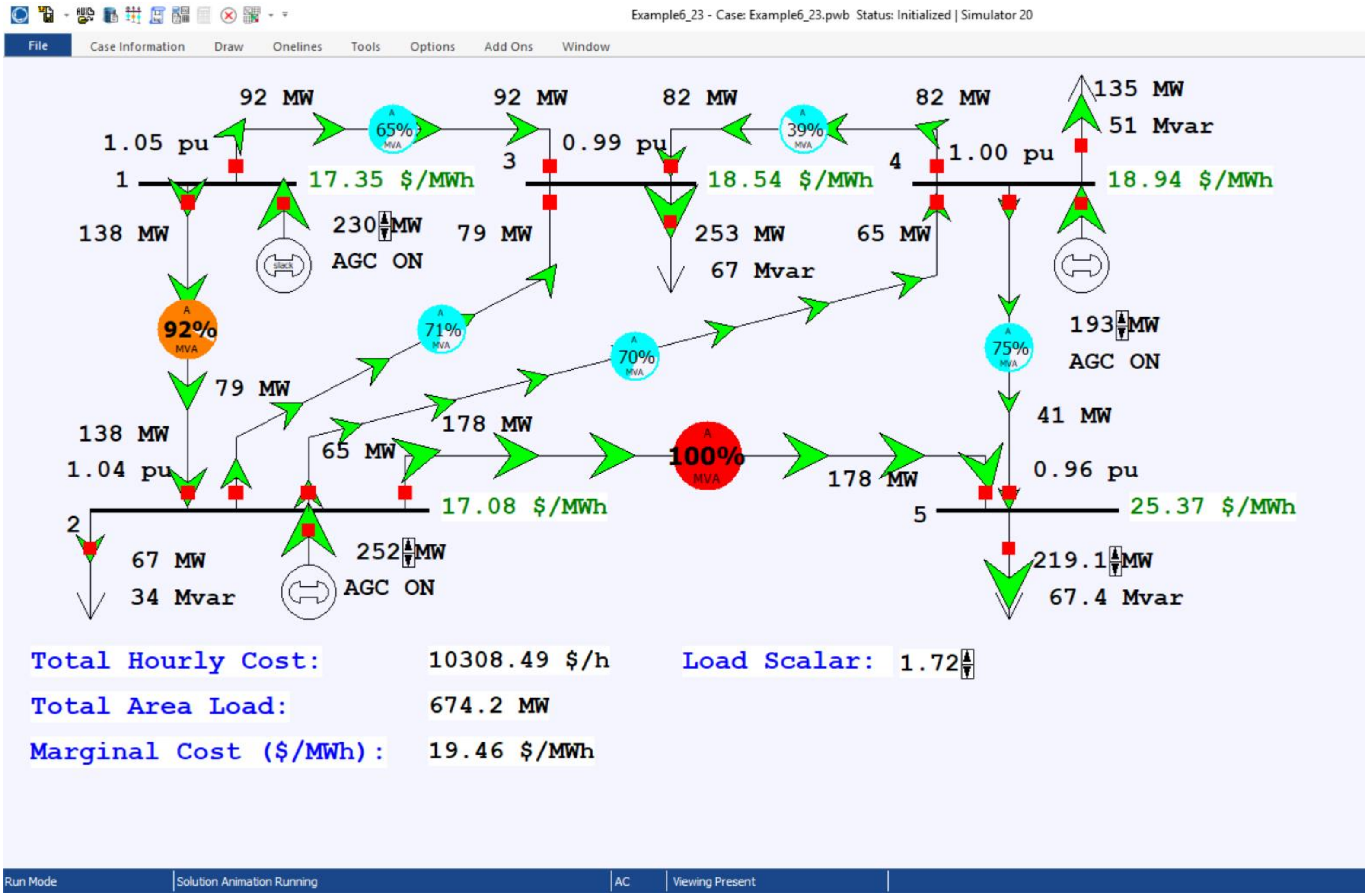
On the **Options, Environment** page the simulation can be set to solve an OPF when simulating

This is **Example6_23_OPF**. In this example the load is gradually increased

Locational Marginal Costs (LMPs)

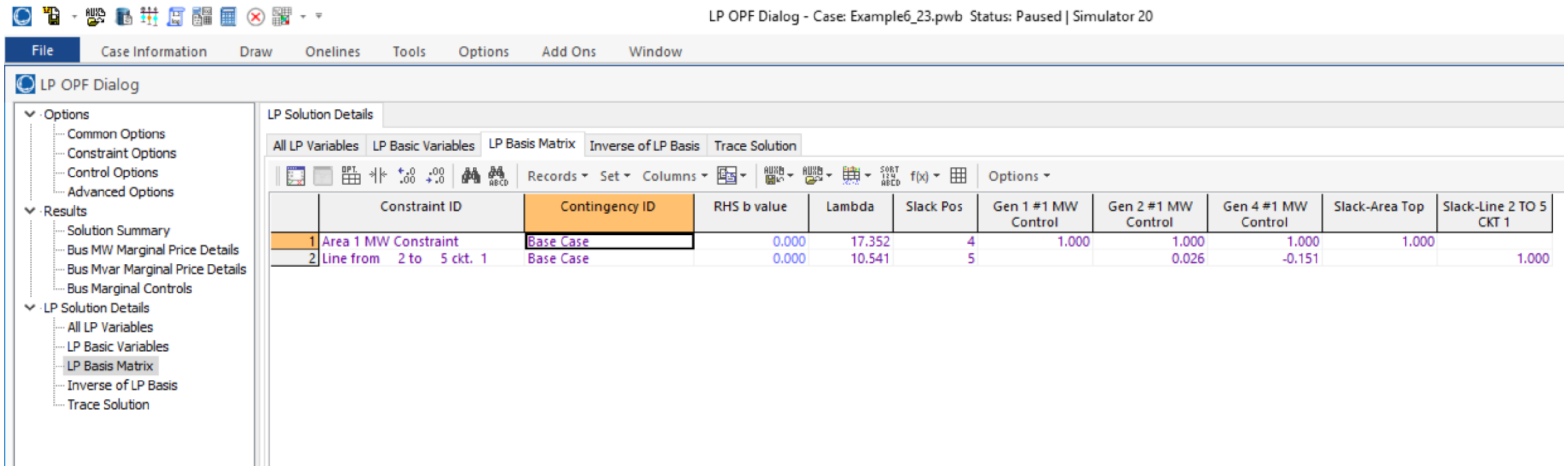
- In an OPF solution, the bus LMPs tell the marginal cost of supplying electricity to that bus
- The term "congestion" is used to indicate when there are elements (such as transmission lines or transformers) that are at their limits; that is, the constraint is binding
- Without losses and without congestion, all the LMPs would be the same
- Congestion or losses causes unequal LMPs
- LMPs are often shown using color contours; a challenge is to select the right color range!

Example 6_23 Optimal Power Flow with Load Scale = 1.72



Example 6_23 Optimal Power Flow with Load Scale = 1.72

- LP Sensitivity Matrix (**A** Matrix)



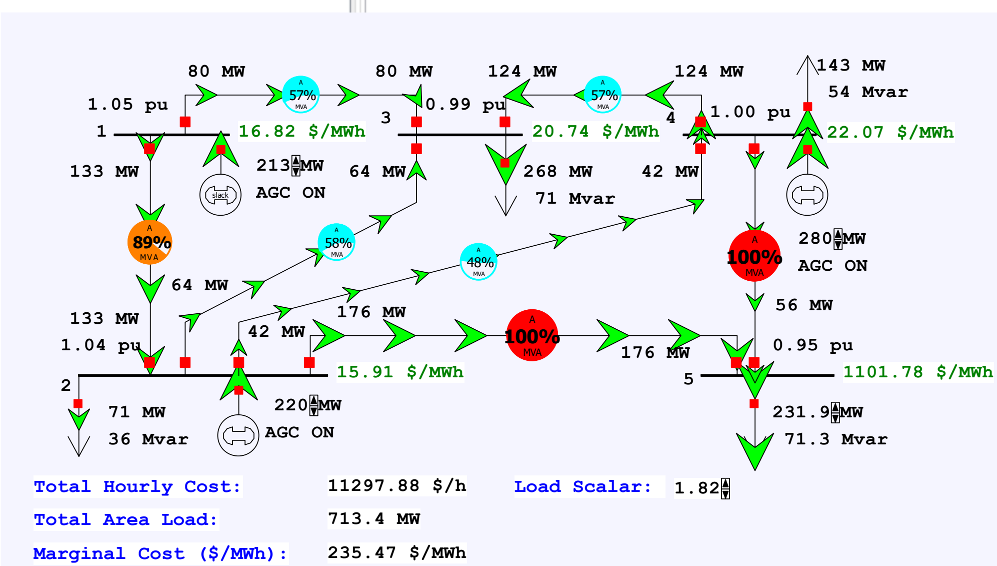
	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 1 #1 MW Control	Gen 2 #1 MW Control	Gen 4 #1 MW Control	Slack-Area Top	Slack-Line 2 TO 5 CKT 1
1	Area 1 MW Constraint	Base Case	0.000	17.352	4	1.000	1.000	1.000	1.000	
2	Line from 2 to 5 ckt. 1	Base Case	0.000	10.541	5		0.026	-0.151		1.000

The first row is the power balance constraint, while the second row is the line flow constraint. The matrix only has the line flows that are being enforced.

Example 6_23 Optimal Power Flow with Load Scale = 1.82

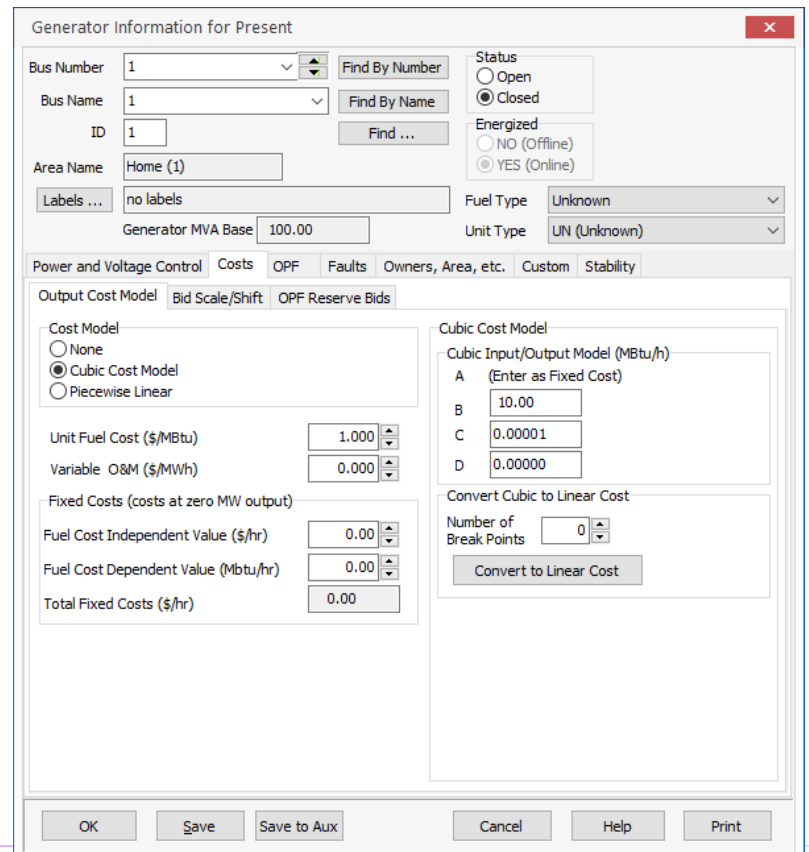
- This situation is infeasible, at least with available controls. There is a solution because the OPF is allowing one of the constraints to violate (at high cost)

						Control	Control	Control		CKT 1
1	Area 1 MW Constraint	Base Case	0.000	16.824	4	1.000	1.000	1.000	1.000	
2	Line from 2 to 5 ckt. 1	Base Case	0.000	993.664	5		0.026	-0.146		1.000
3	Line from 4 to 5 ckt. 1	Base Case	-0.002	1000.000	6		-0.024	0.140		



Generator Cost Curve Modeling

- LP algorithms require linear cost curves, with piecewise linear curves used to approximate a nonlinear cost function
- Two common ways of entering cost information are
  - Quadratic function
  - Piecewise linear curve
- The PowerWorld OPF supports both types



Security Constrained OPF

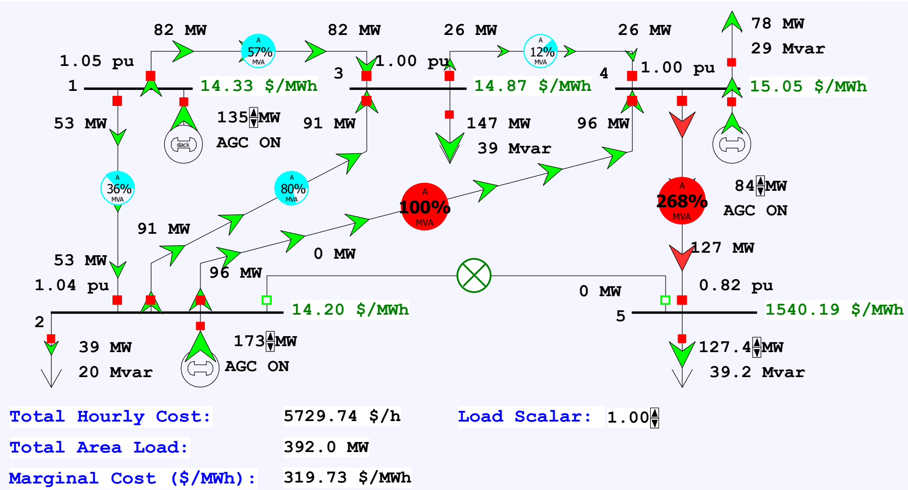
- Security constrained optimal power flow (SCOPF) is similar to OPF except it also includes contingency constraints
  - Again the goal is to minimize some objective function, usually the current system cost, subject to a variety of equality and inequality constraints
  - This adds significantly more computation, but is required to simulate how the system is actually operated (with N-1 reliability)
- A common solution is to alternate between solving a power flow and contingency analysis, and an LP

Security Constrained OPF, cont.

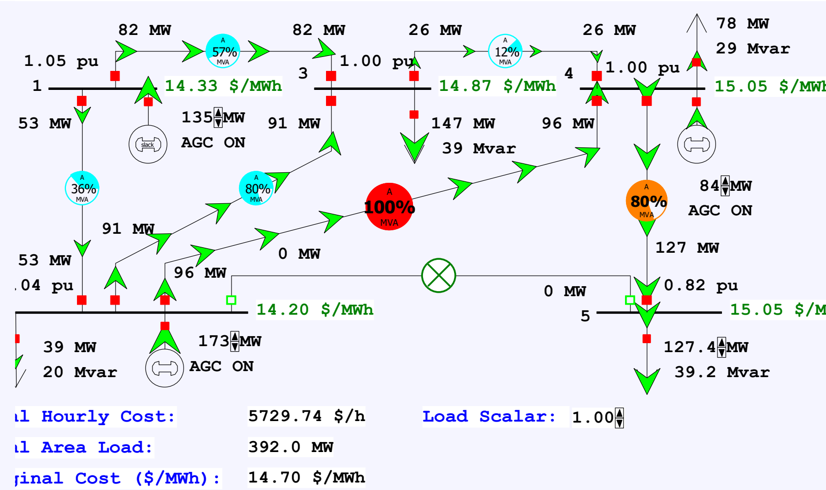
- With the inclusion of contingencies, there needs to be a distinction between what control actions must be done pre-contingent, and which ones can be done post-contingent
  - The advantage of post-contingent control actions is they would only need to be done in the unlikely event the contingency actually occurs
- Pre-contingent control actions are usually done for line overloads, while post-contingent control actions are done for most reactive power control and generator outage re-dispatch

SCOPF Example

- We'll again consider Example 6_23, except now it has been enhanced to include contingencies and we've also greatly increased the capacity on the line between buses 4 and 5



Original with line 4-5 limit of 60 MW with 2-5 out

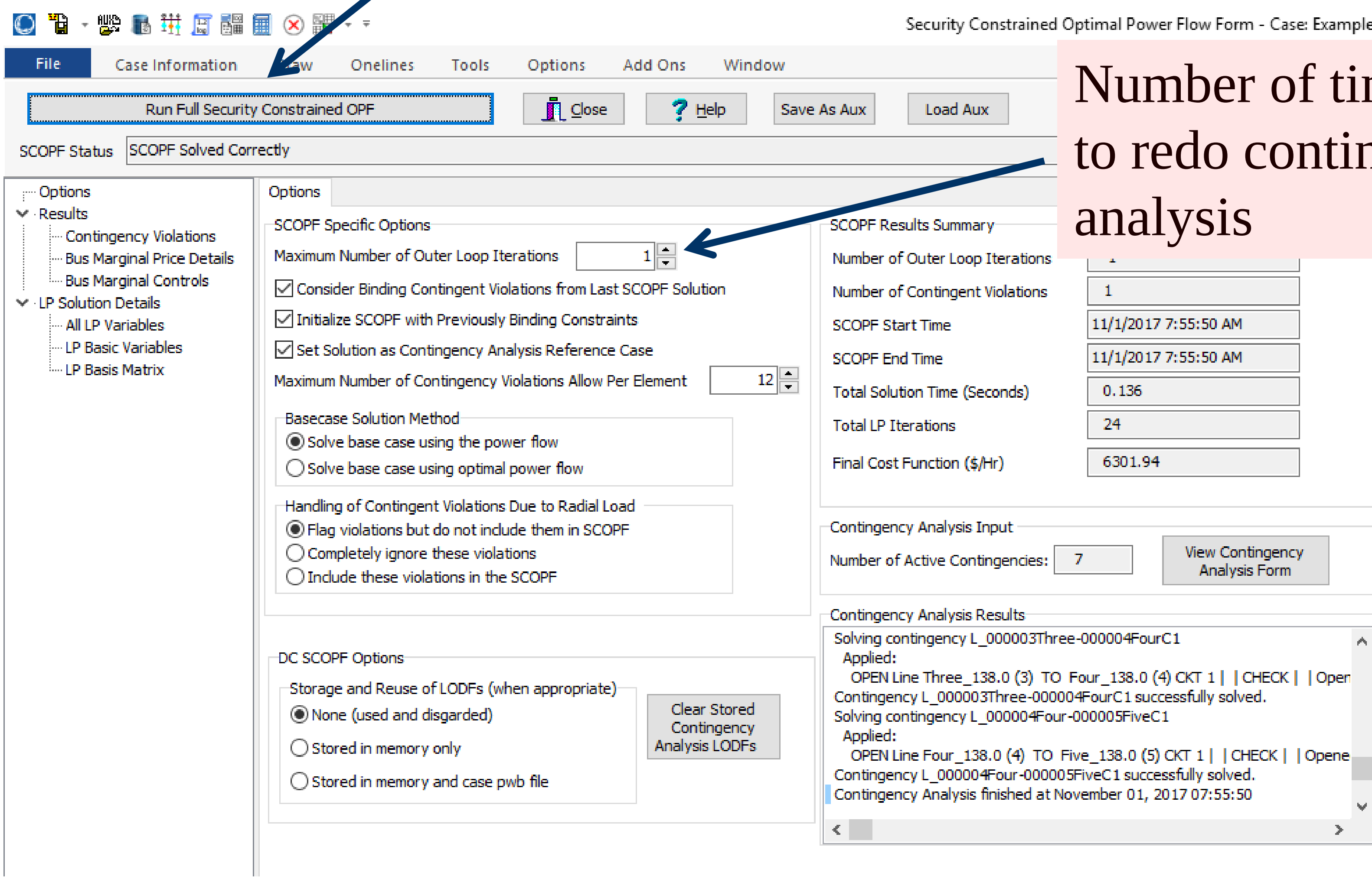


Modified with line 4-5 limit of 200 MVA with 2-5 out

PowerWorld SCOPF Application

Just click the button to solve

Number of times to redo contingency analysis



LP OPF and SCOPF Issues

- The LP approach is widely used for the OPF and SCOPF, particularly when implementing a dc power flow approach
- A key issue is determining the number of binding constraints to enforce in the LP tableau
  - Enforcing too many is time-consuming, enforcing too few results in excessive iterations
- The LP approach is limited by the degree of linearity in the power system
  - Real power constraints are fairly linear, reactive power constraints much less so

OPF Solution by Newton's Method

- An alternative to using the LP approach is to use Newton's method, in which all the equations are solved simultaneously
- Key paper in area is
  - D.I. Sun, B. Ashley, B. Brewer, B.A. Hughes, and W.F. Tinney, "Optimal Power Flow by Newton Approach", *IEEE Trans. Power App and Syst.*, October 1984
- Problem is

$$\text{Minimize } f(\mathbf{x})$$
$$\text{s.t.} \quad \mathbf{g}(\mathbf{x})=\mathbf{0}$$
$$\mathbf{h}(\mathbf{x}) \leq \mathbf{0}$$

For simplicity **x** represents all the variables and we can use **h** to impose limits on individual variables

OPF Solution by Newton's Method

- During the solution the inequality constraints are either binding (=0) or nonbinding (<0)
  - The nonbinding constraints do not impact the final solution
- We'll modify the problem to split the **h** vector into the binding constraints, $\mathbf{h}_1$ and the nonbinding constraints, $\mathbf{h}_2$

$$\text{Minimize } f(\mathbf{x})$$

$$\begin{aligned} \text{s.t.} \quad & \mathbf{g}(\mathbf{x}) = \mathbf{0} \\ & \mathbf{h}_1(\mathbf{x}) = \mathbf{0} \\ & \mathbf{h}_2(\mathbf{x}) < \mathbf{0} \end{aligned}$$

OPF Solution by Newton's Method

- To solve first define the Lagrangian

$$L(\mathbf{x},\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2) = f(\mathbf{x}) + \boldsymbol{\mu}^T \mathbf{g}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{h}_1(\mathbf{x})$$

Let $\mathbf{z} = \begin{bmatrix} \mathbf{x} & \boldsymbol{\mu} & \boldsymbol{\lambda} \end{bmatrix}$

- A necessary condition for a minimum is that the gradient is zero

Both $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are Lagrange Multipliers

$$\nabla L(\mathbf{z}) = \mathbf{0} = \begin{bmatrix} \dfrac{\partial L(\mathbf{z})}{\partial z_1} \\ \dfrac{\partial L(\mathbf{z})}{\partial z_2} \\ \vdots \\ \end{bmatrix}$$

OPF Solution by Newton's Method

- Solve using Newton's method. To do this we need to define the Hessian matrix

$$\nabla^2 L(\mathbf{z}) = \mathbf{H}(\mathbf{z}) = \left[\frac{\partial^2 L(\mathbf{z})}{\partial z_i \partial z} \right] = \begin{bmatrix} \frac{\partial^2 L(\mathbf{z})}{\partial x_i \partial x_j} & \frac{\partial^2 L(\mathbf{z})}{\partial x_i \partial \mu_j} & \frac{\partial^2 L(\mathbf{z})}{\partial x_i \partial \lambda_j} \\ \frac{\partial^2 L(\mathbf{z})}{\partial \mu_i \partial x_j} & \mathbf{0} & \mathbf{0} \\ \frac{\partial^2 L(\mathbf{z})}{\partial \lambda \partial x_{ji}} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

- Because this is a second order method, as opposed to a first order linearization, it can better handle system nonlinearities

OPF Solution by Newton's Method

- Solution is then via the standard Newton's method. That is

Set iteration counter k=0, set k_{max}

Set convergence tolerance ε

Guess $\mathbf{z}^{(k)}$

While $\left(\left\|\nabla L(\mathbf{z})\right\| \geq \varepsilon\right)$ and $\left(k < k_{max}\right)$

$$\mathbf{z}^{(k+1)} = \mathbf{z}^{(k)} - \left[\mathbf{H}(\mathbf{z})\right]^{-1} \nabla L(\mathbf{z})$$

k = k + 1

End While

No iteration is needed for a quadratic function with linear constraints

Example

- Solve

Minimize $x_1^2 + x_2^2$ such that $x_1 + x_2 - 2 \geq 0$

Solve initially assuming the constraint is binding

$$L(\mathbf{x},\lambda) = x_1^2 + x_2^2 + \lambda(3x_1 + x_2 - 2)$$

$$\nabla L(\mathbf{x},\lambda) = \begin{bmatrix} \frac{\partial L}{\partial x_1} \\ \frac{\partial L}{\partial x_2} \\ \frac{\partial L}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} 2x_1 + 3\lambda \\ 2x_2 + \lambda \\ 3x_1 + x_2 - 2 \end{bmatrix}$$

No iteration is needed so any "guess" is fine. Pick (1,1,0)

$$\nabla^2 L(\mathbf{x},\lambda) = \mathbf{H}(\mathbf{x},\lambda) = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.2 \\ 0.4 \end{bmatrix}$$

Because λ is positive the constraint is binding

Newton OPF Comments

- The Newton OPF has the advantage of being better able to handle system nonlinearities
- There is still the issue of having to deal with determining which constraints are binding
- The Newton OPF needs to implement second order derivatives plus all the complexities of the power flow solution
  - The power flow starts off simple, but can rapidly get complex when dealing with actual systems
- There is still the issue of handling integer variables

Mixed-Integer Programming

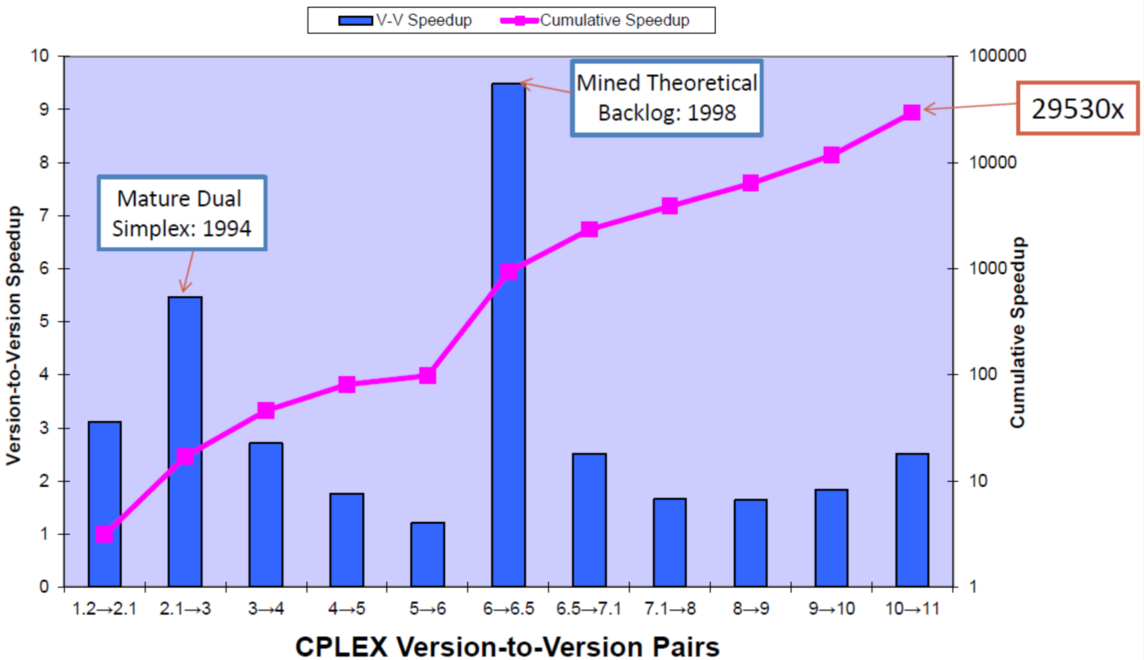
- A mixed-integer program (MIP) is an optimization problem of the form

$$
\begin{aligned}
&\text{Minimize} && \mathbf{c}\,\mathbf{x} \\
&\text{s.t.} && \mathbf{A}\mathbf{x} = \mathbf{b} \\
& && \mathbf{x} \geq \mathbf{0} \\
&\text{where} && \mathbf{x} = \text{n-dimensional column vector} \\
& && \mathbf{c} = \text{n-dimensional row vector} \\
& && \mathbf{b} = \text{m-dimensional column vector} \\
& && \mathbf{A} = \text{m} \times \text{n matrix} \\
& && \text{some or all } x_j \text{ integer}
\end{aligned}
$$

Mixed-Integer Programming

- The advances in the algorithms have been substantial

Speedups 1991-2008



Speedups from 2009 to 2015 were about a factor of 30!

Notes are partially based on a presentation at Feb 2015 US National Academies Analytic Foundations of the Next Generation Grid by Robert Bixby from Gurobi Optimization titled "Advances in Mixed-Integer Programming and the Impact on Managing Electrical Power Grids"

Mixed-Integer Programming

- Suppose you were given the following choices?
  - Solve a MIP with today's solution technology on a 1991 machine
  - Solve a MIP with a 1991 solution on a machine from today?
- The answer is to choose option 1, by a factor of approximately 300
- This leads to the current debate of whether the OPF (and SCOPF) should be solved using generic solvers or more customized code (which could also have quite good solvers!)

Notes are partially based on a presentation at Feb 2015 US National Academies Analytic Foundations of the Next Generation Grid by Robert Bixby from Gurobi Optimization titled "Advances in Mixed-Integer Programming and the Impact on Managing Electrical Power Grids"

More General Solvers Overview

- OPF is currently an area of active research
- Many formulations and solution methods exist…
  - As do many ***tools*** for highly complex, large-scale computing!

- While many options exist, some may work better for certain problems or with certain programs you already use

- Consider experimenting with a new language/solver!

Gurobi and CPLEX

- Gurobi and CPLEX are two well-known commercial optimization solvers/packages for **linear programming (LP),** quadratic programming (QP), quadratically constrained programming (QCP), and the mixed integer (MI) counterparts of LP/QP/QCP
- Gurobi and CPLEX are accessible through object-oriented interfaces (C++, Java, Python, C), matrix-oriented interfaces (MATLAB) and other modeling languages (AMPL, GAMS)

Solver Comparison

Algorithm Type ----------------- *Solver*	**LP/MILP** **linear/mixed integer linear program**	**QP/MIQP** quadratic/mixed integer quadratic program	**SOCP** second order cone program	**SDP** semidefinite program
*CPLEX**	**x**	**x**	**x**	
GLPK	**x**			
*Gurobi**	**x**	**x**	**x**	
IPOPT		**x**		
*Mosek**	**x**	**x**	**x**	**x**
SDPT3/SeDuMi			**x**	**x**

Linear programming can be solved by quadratic programming, which can be solved by second-order cone programming, which can be solved by semidefinite programming.

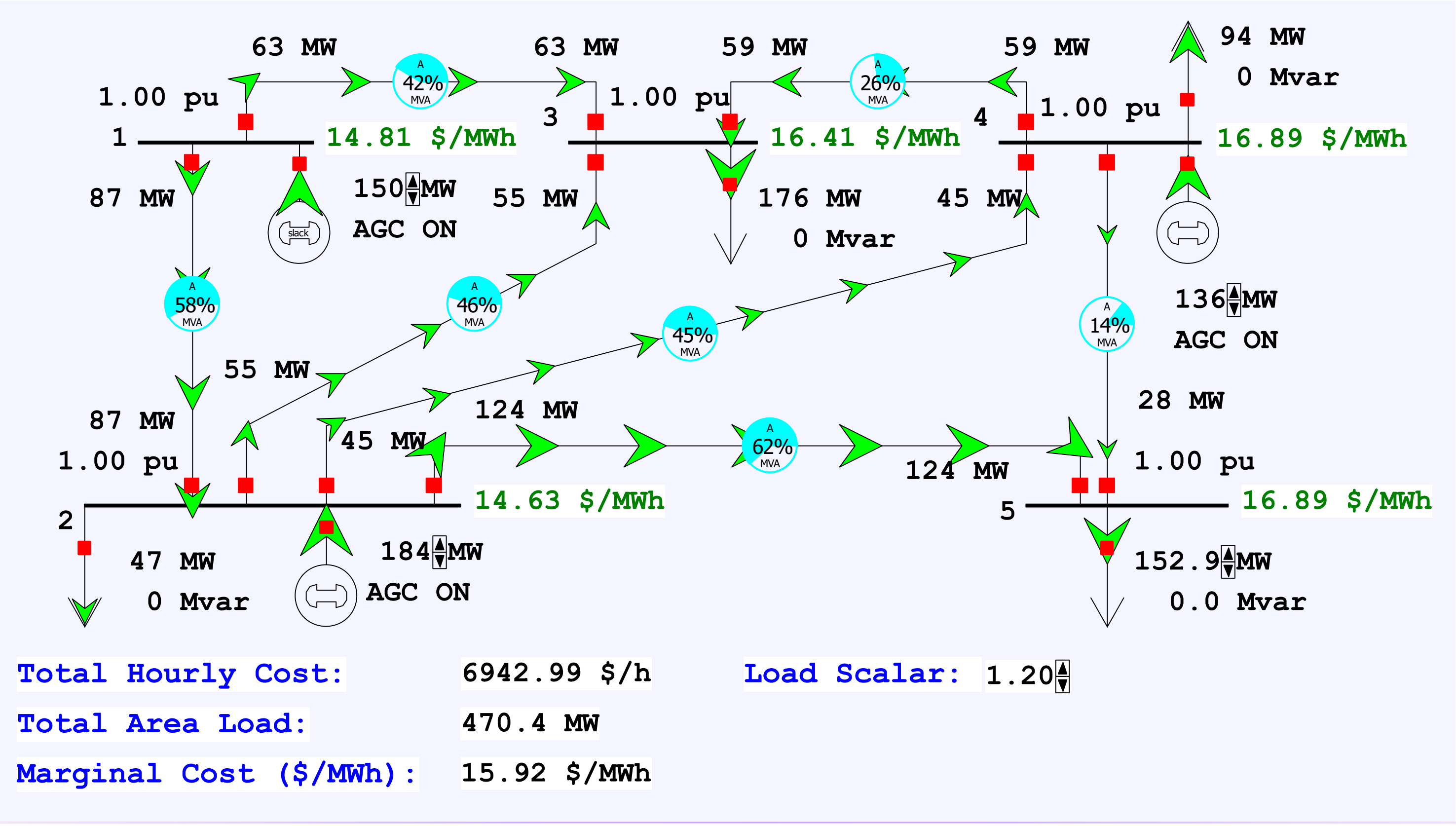
DC OPF and SCOPF

- Solving a full ac OPF or SCOPF on a large system is difficult, so most electricity markets actually use the more approximate, but much simpler DCOPF, in which a dc power flow is used
  - Interested students can learn more about what is actually done by reading the below paper (this is completely optional)
- PowerWorld includes this option in the **Options, Power Flow Solution, DC Options**

https://www.ferc.gov/legal/staff-reports/2017/marginallosscalculations.pdf

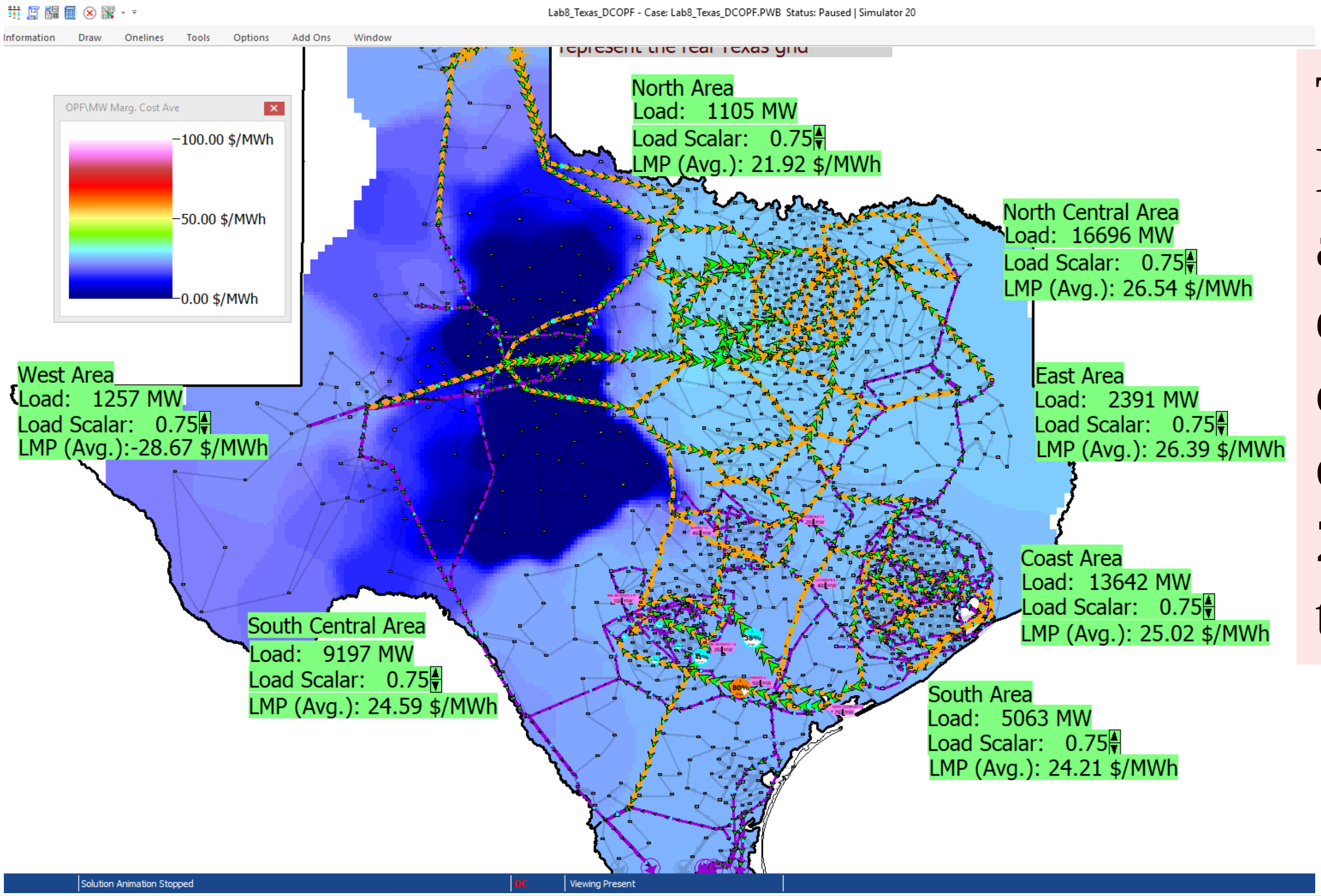
Example 6_13 DC SCOPF Results: Load Scalar at 1.20

- Now there is not an unenforceable constraint on the line between 4-5 (for the line 2-5 contingency) because the reactive losses are ignored



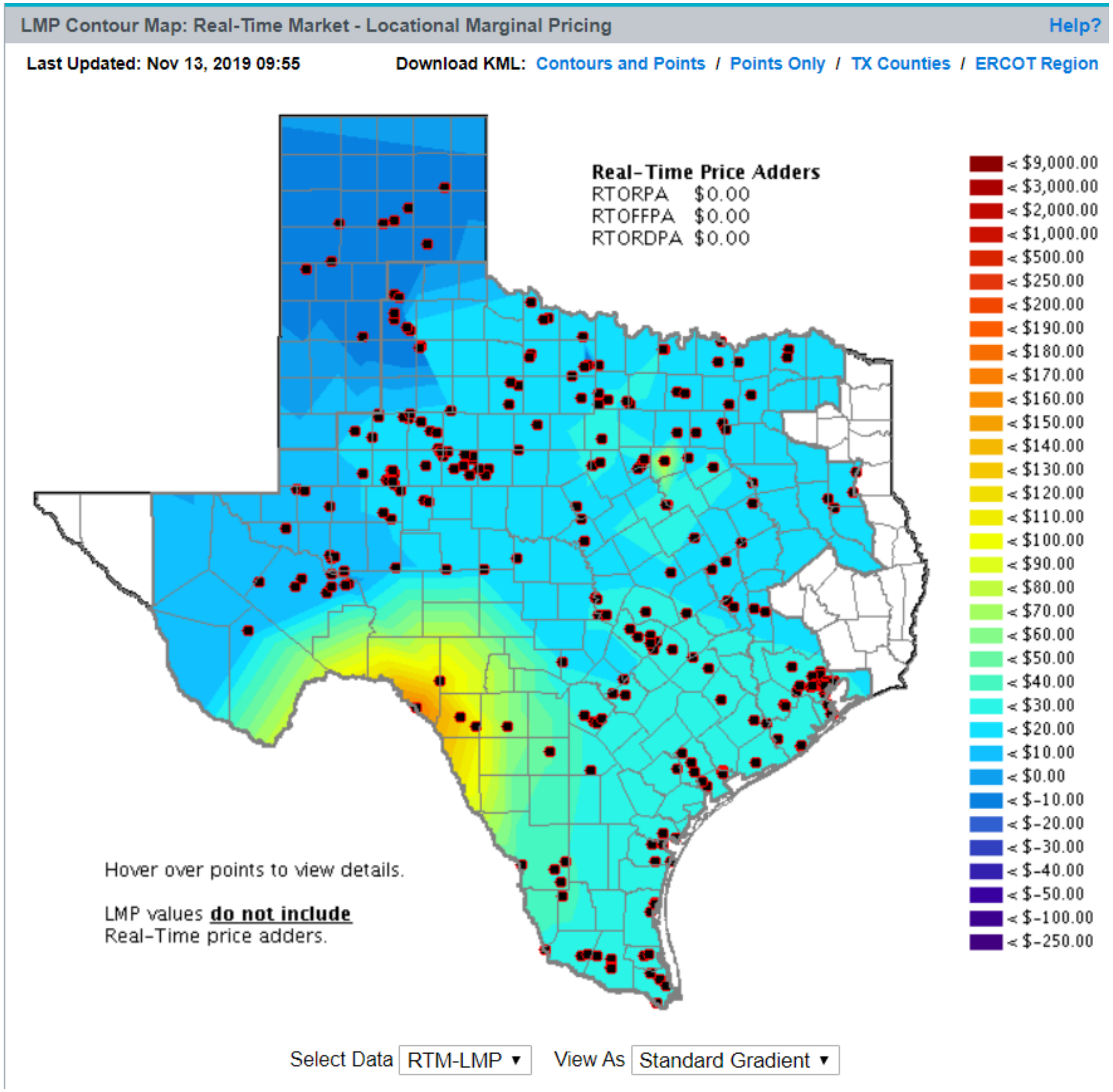
2000 Bus Texas Synthetic DC OPF Example

- This system does a DC OPF solution, with the ability to change the load in the areas



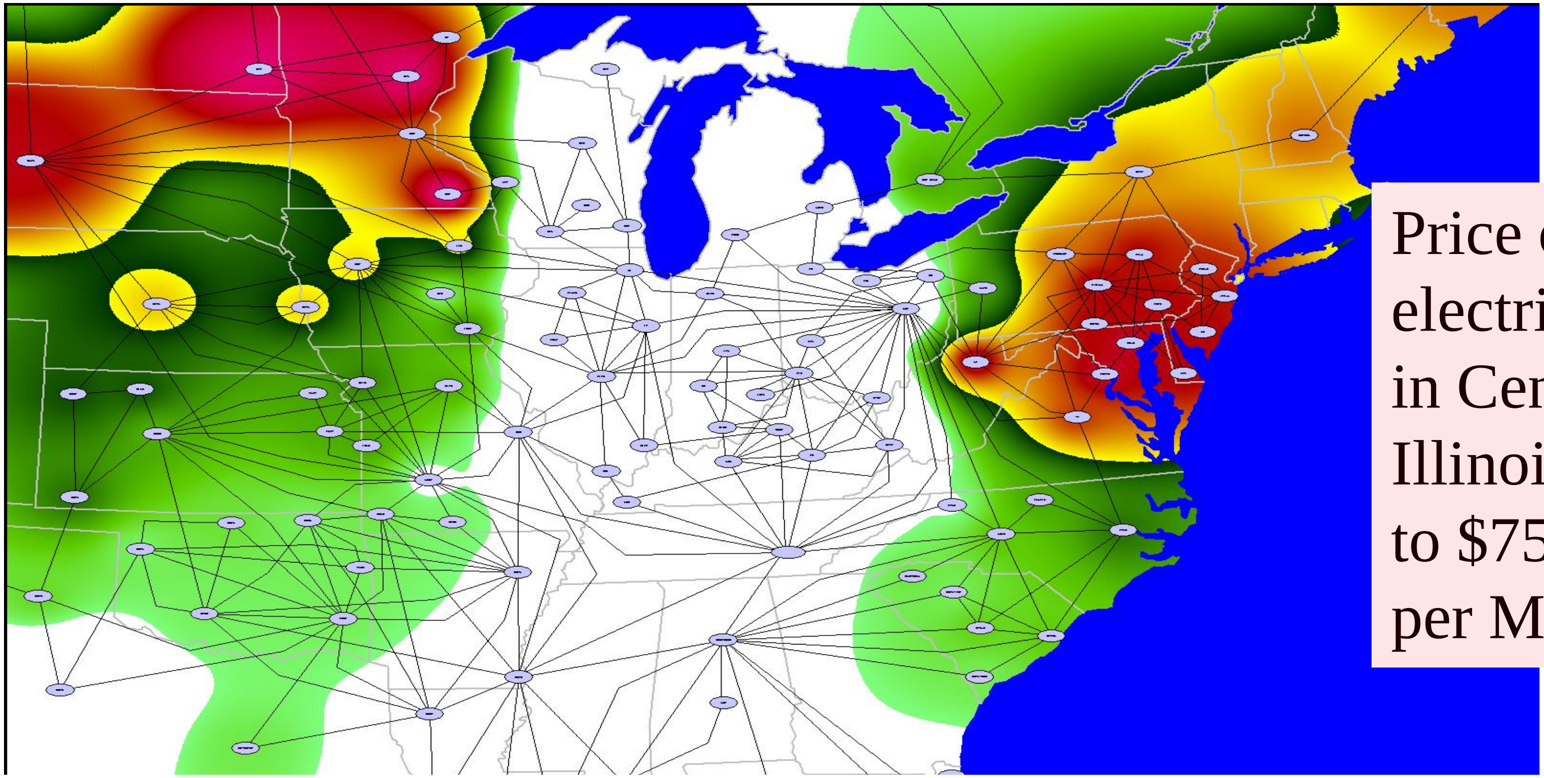
The quite low LMPs are actually due to a constraint on a single 230/115 kV transformer

Actual ERCOT LMPs on Nov 13, 2019 at 9:55 am



Source: www.ercot.com/content/cdr/contours/rtmLmp.html

June 1998 Heat Storm: Two Constraints Caused a Price Spike

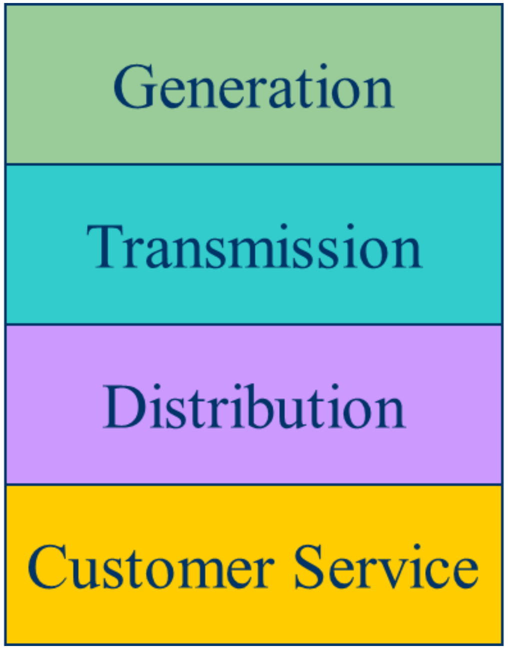


Price of electricity in Central Illinois went to $7500 per MWh!

Colored areas could NOT sell into Midwest because of constraints on a line in Northern Wisconsin and on a Transformer in Ohio

Electricity Markets History

- For decades electric utilities operated as vertical monopolies, with their rates set by state regulators
- Utilities had an obligation to serve and customers had no choice
  - There was little third party generation
- Major change in US occurred in 1992 with the National Energy Policy Act that mandated utilities provide "nondiscriminatory" access to the high voltage grid
- Goal was to setup true competition in generation



Markets Versus Centralized Planning

- With the vertically integrated utility, a small number of entities (typically utilities) did most of the planning
  - For example, which new generators and/or lines to build
  - Planning was coordinated and governed by regulators
  - Regulators needed to know the utilities actual costs so they could provide them with a fixed rate of return
- With markets the larger number of participants often make individual decisions in reaction to prices
  - For example, whether to build new generation
  - Generator owners in general to not need to reveal their true costs; rather they make offers into the market

Overall Goal

- Goal is to maximize the economic surplus (or total welfare), which is the sum of the consumer surplus and the producer surplus (i.e., their profit)
- Generation owners have to decide their offer prices
- If their price is too high, they are not selected to generate
- At the wholesale level, the consumers often just see a price, though there can be price responsive load bids

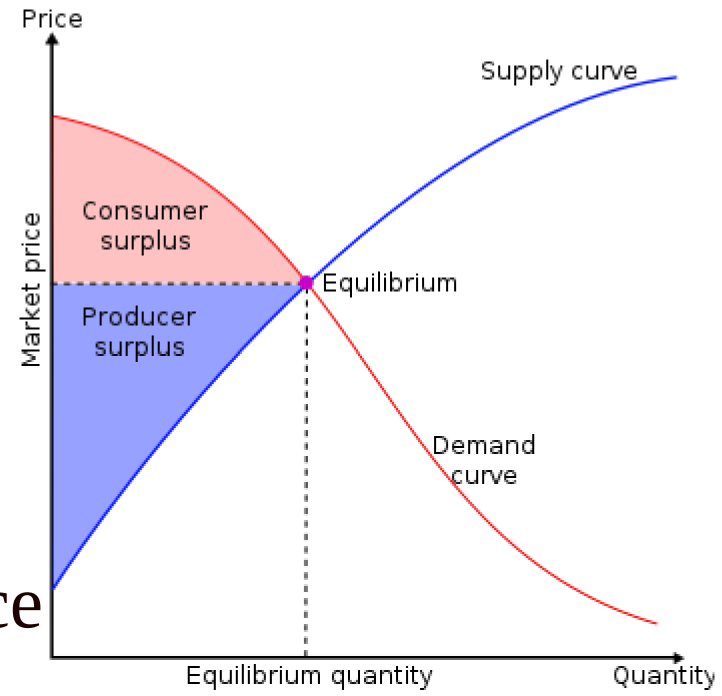


Image Source: en.wikipedia.org/wiki/Economic_surplus#/media/File:Economic-surpluses.svg

Electricity Markets Today

- Starting in about 1995 electricity markets gradually started to develop, both in the US and elsewhere
- In North America more than 60% of the load is supplied via wholesale electricity markets
- Markets differ but they all have certain common features

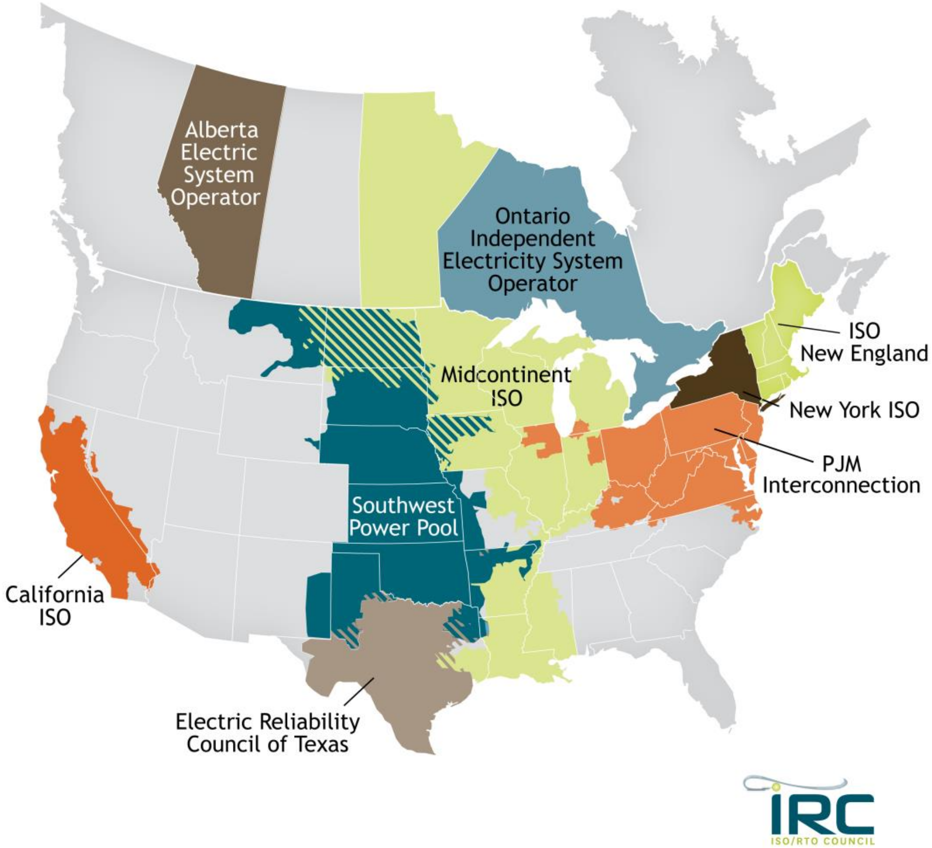


Image: www.isorto.org/about/default

Electricity Markets Common Features

- Day ahead market – this is needed because time is required to make decisions about committing generators
  - Generation owners submit offers for how much generation they can supply and at what price; accepted offers are binding
- Real-time energy market – needed because day ahead forecasts are never perfect, and unexpected events can occur
- Co-optimization with other "ancillary services" such as reserves

The source for much of this material "Analytic Research Foundations for the Next-Generation Electric Grid" (Chapter 2), The National Academies Press, 2016 (free download available)

Electricity Markets Common Features

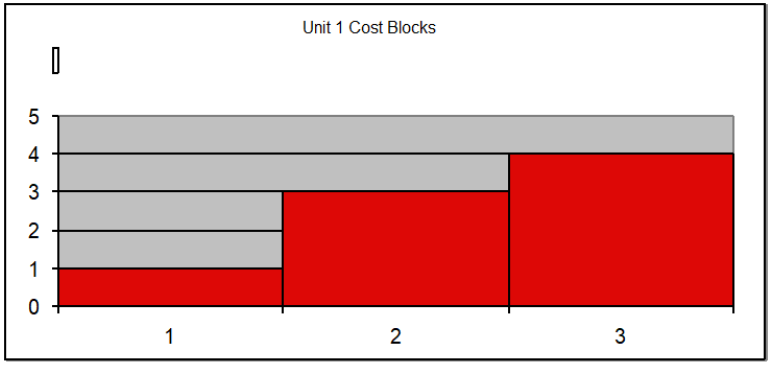
- Pricing is done using locational marginal prices, determined by an SCOPF
  - Most markets include a marginal losses component
- LMP markets are designed to send transparent price signal so people can make short and long-term decisions
  - Generators are free to offer their electricity at whatever price they desire; they do not have to reveal their "true" costs
  - Most of the times markets work as planned, and prices are competitive
  - During times of shortages (scarcity) there are limits on LMPs; ERCOT's is $9000/MWh
  - Markets are run by independent system operators (ISOs)

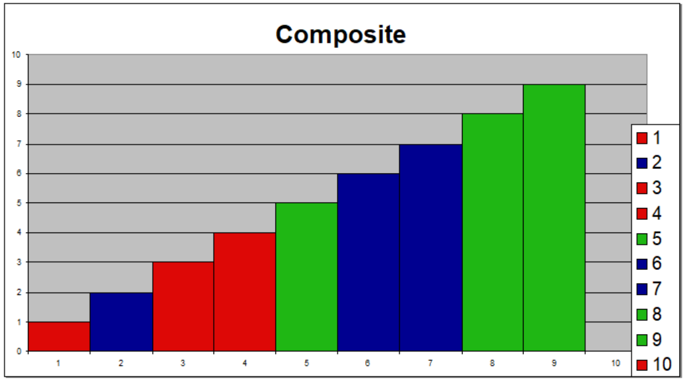
LMP Energy Markets

- In an LMP energy market the generation is paid the LMP at the bus, and the loads pay the LMP at the bus
  - This is done in both the day ahead market and in the real-time market (which makes up the differences between actual and the day ahead)
- The generator surplus (profit) is the difference between the LMP and the actual cost of generation
- Generators that offer too high are not selected to run, and hence make no profit
- A key decision for the generation owners is what values to offer

Generator Offers

- Generator offers are given in piecewise linear curves; that is, a fixed $/MWh for so much power for a time period
- In the absence of constraints (congestion) the ISO would just select the lowest offers to meet the anticipated load
- Actual dispatch is determined using an SCOPF





General Guidelines

- Generators with high fixed costs and low operating costs (e.g., wind, solar, nuclear) benefit from running many hours
  - Usually they should submit offers close to their marginal costs
  - Wind (and some others) receive a production tax credit for their first ten years of operation
    - \$23/MWh for systems starting construction before 1/1/2017
    - \$18.4/MWh for systems starting construction in 2017 (a 20% reduction)
    - In 2018 the reduction is 40% and 60% in 2019; after that it is zero (unless, of course, changed by Congress)
- Generators with low fixed costs and high operating cost can do fine operating fewer hours (at higher prices)

Auctions

- In its simplest form, an auction is a mechanism of allocating scarce goods based upon competition
  - a seller wishes to obtain as much money as possible, and a buyer wants to pay as little as necessary.
- An auction is usually considered efficient if resources accrue to those who value them most highly
- Auctions can be either one-sided with a single monopolist seller/buyer or a double auction with multiple parties in each category
  - bid to buy, offer to sell
- Most people's experience is with one-side auctions with one seller and multiple buyers

Auctions, cont.

- Electricity markets can be one-sided, with the ISO functioning as a monopolist buyer, while multiple generating companies make offers to sell their generation, or two-sided with load participation
- Auction provides mechanism for participants to reveal their true costs while satisfying their desires to buy low and/or sell high.
- Auctions differ on the price participants receive and the information they see along the way

Types of Single-Sided Auctions with Multiple Buyers, One Seller

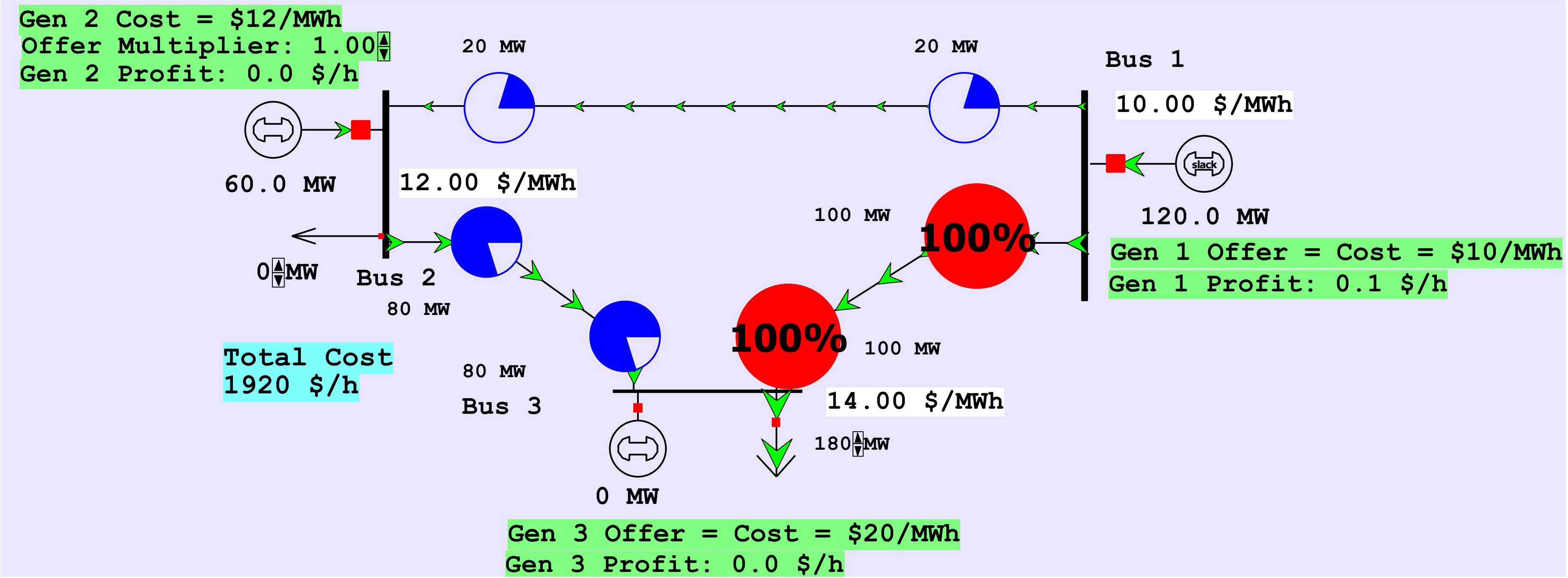
- Simultaneous auctions
  - English (ascending price to buy)
  - Dutch (descending price to buy)
- Sealed-bid auctions (all participants submit offers simultaneously)
  - First price sealed bid (pay highest price if one, discriminatory prices if multiple)
  - Vickrey (uniform second price) (pay the second highest price if one, all pay highest losing price if many); this approach gives people incentive to bid their true value

Uniform Price Auctions: Multiple Sellers, One Buyer

- Uniform price auctions are sealed offer auctions in which sellers make simultaneous decisions (done when submitting offers).
- Generators are paid the last accepted offer
- Provides incentive to offer at marginal cost since higher values cause offers to be rejected
  - reigning price should match marginal cost
- Price caps are needed to prevent prices from rising up to infinity during shortages
- Some generators offering above their marginal costs are needed to cover their fixed costs

What to Offer Example

- Below example shows 3 generator case, in which the bus 2 generator can vary its offer to maximize profit



Note, this example makes the unrealistic assumption that the other generators do not vary their offers in response

Horizontal Market Power

- One issue is whether a particular group of generators has market power
- Market power is the antithesis of competition
  - It is the ability of a particular group of sellers to maintain prices above competitive levels, usually by withholding supply
- The extreme case is a single supplier of a product (i.e., a monopoly)
- In the short run what a monopolistic producer can charge depends upon the price elasticity of the demand
- Sometimes market power can result in decreased prices in the long-term by quickening the entry of new players or new innovation

Market Power and Scarcity Rents

- A generator owner exercises market power when it is unwilling to make energy available at a price that is equal to that unit's variable cost of production, even thought there is currently unloaded generation capacity (i.e., there is no scarcity).
- Scarcity rents occur when the level of electric demand is such that there is little, if any, unused capacity
- Scarcity rents are used to recover fixed costs