#### ECEN 667 Power System Stability

#### Lecture 20: Voltage Stability and Modal Analysis

Prof. Tom Overbye Dept. of Electrical and Computer Engineering Texas A&M University <u>overbye@tamu.edu</u>

Special Guest Lecture by TA Hanyue Li



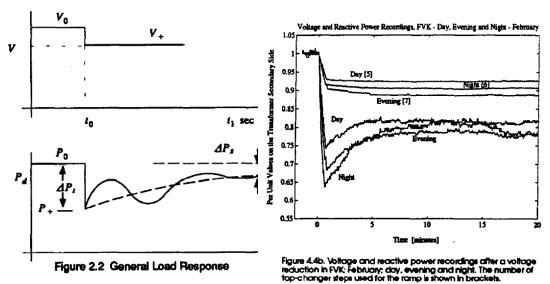
#### Announcements

- Read Chapter 8
- Homework 5 is due on Thursday Nov 14



# **Dynamic Load Response**

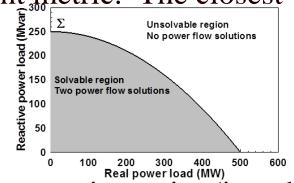
- As first reported in the below paper, following a change in voltage there will be a dynamic load response
  - Residential supply voltage should be between 114 and 126 V  $\,$
- If there is a heating load the response might be on the order of ten minutes
- Longer term issues can also come into play



Useful paper and figure reference: D. Karlsson, D.J. Hill, "Modeling and Identification of Nonlinear Dynamic Loads in Power Systems," IEEE. Trans. on Power Systems, Feb 1994, pp. 157-166

#### **Determining a Metric to Voltage** Collapse

- The goal of much of the voltage stability work was to determine an easy to calculate metric (or metrics) of the current operating point to voltage collapse
  - PV and QV curves (or some combination) can determine such a metric along a particular path
  - Goal was to have a path independent metric. The closest boundary point was considered, but this could be quite misleading if the system was not going to move in that direction



- Any linearization about the current operating point (i.e., the Jacobian) does not consider important nonlinearities like generators hitting their reactive power limits

#### Assessing Voltage Margin Using PV and QV Curve Analysis



- A common method for assessing the distance in parameter space to voltage instability (or an undesirable voltage profile) is to trace how the voltage magnitudes vary as the system parameters (such as the loads) are changed in a specified direction
  - If the direction involves changing the real power (P) this is known as a PV curve; if the change is with the reactive power (Q) then this is a QV curve
- PV/QV curve analysis can be generalized to any parameter change, and can include the consideration of contingencies

# **PV and QV Analysis in PowerWorld**



- Requires setting up what is known in PowerWorld as an injection group
  - An injection group specifies a set of objects, such as generators and loads, that can inject or absorb power
  - Injection groups can be defined by selecting Case Information, Aggregation, Injection Groups
- The PV and/or QV analysis then varies the injections in the injection group, tracing out the PV curve
- This allows optional consideration of contingencies
- The PV tool can be displayed by selecting Add-Ons, PV

This has already been done in the **Bus2\_PV** case

#### PV and QV Analysis in PowerWorld: Two Bus Example



• Setup page defines the source and sink and step size

PV CURVES      Common Options     Injection Group Ramp     Interface Ramping Oc     Advanced Options      Quantities to track     Unit violations     PV output     QV setup     PV Results     Plots     Plot Designer     Plot Definition Grids	Ramping Method       Source         Injection Group Source/Sink       Interface         Interface MW Flow       Sink         Common Options       Injection Group Ramping Options       Interface         Critical Scenarios       Stop after finding at least       1 Critical scenarios         Base Case and Contingencies       Source       Source			
	Kip contingencies     Manage contingency list     Run base case to completion     Base Case Solution Option     Vary the transfer as follows:     Initial Step Size (MW):     10.00     Minimum Step Size (MW):     2.00		<ul> <li>Optionally concerning</li> <li>can be considered</li> </ul>	•
< >> Save Auxiliary Load Au	When convergence fails, reduce step by a factor of       2.00 ÷         Stop when transfer exceeds       0.00 ÷         xiliary       Launch OV curve tool	<b>?</b> Help	Cos	

#### **PV and QV Analysis in PowerWorld: Two Bus Example**

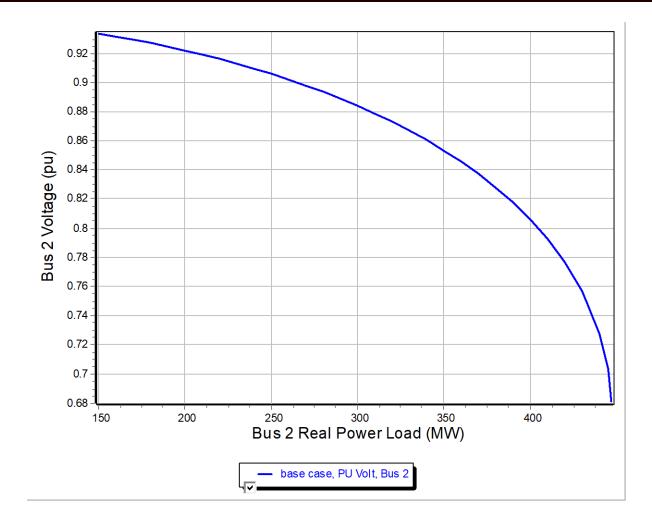


- The PV Results Page does the actual solution
  - Plots can be defined to show the results
    - This should be done beforehand •
  - **Other Actions**, **Restore initial state** restores the pre-study state

Setup     PV Results     Ountities to track	Click the Run button
Limit violations     PV output     QV setup     PV Results     PV Results     Construct and Data	to run the PV analysis;
Present step size  Source  150.00  0.00  0.00  0.00  View detailed results	Check the <b>Restore</b>
Image: Contract of the state of the st	Initial State on
	Completion of Run to
	restore the pre-PV
	state (by default it is
	not restored)
Save Auxiliary Load Auxiliary Launch QV curve tool 7 Help	

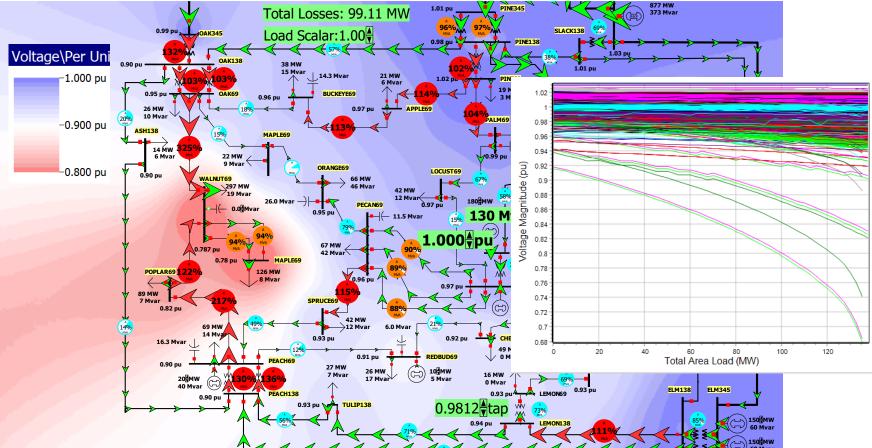
#### PV and QV Analysis in PowerWorld: Two Bus Example





#### PV and QV Analysis in PowerWorld: 37 Bus Example





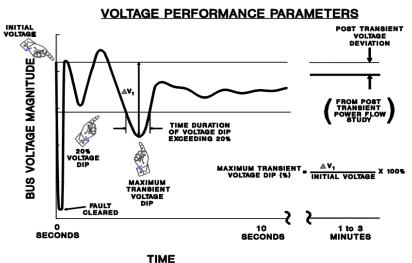
Usually other limits also need to be considered in doing a realistic PV analysis; example case is **Bus37\_PV** 

# **Shorter Term Dynamics**



- On a shorter time-scale (minutes down to seconds) voltage stability is impacted by controls hitting limits (such as the action of generator over excitation limiters), the movement of voltage control devices (such as LTC transformers) and load dynamics
  - Motor stalling can have a major impact
- The potential for voltage instability can be quantified by looking at the amount and duration of voltage dips following an event

Image from WECC Planning and Operating Criteria



## Fault Induced Delayed Voltage Recovery (FIDVR)



- FIDVR is a situation in which the system voltage remains significantly reduced for at least several seconds following a fault (at either the transmission or distribution level)
  - It is most concerning in the high voltage grid, but found to be unexpectedly prevalent in the distribution system
- Stalled residential air conditioning units are a key cause of FIDVR – they can stall within the three cycles needed to clear a fault

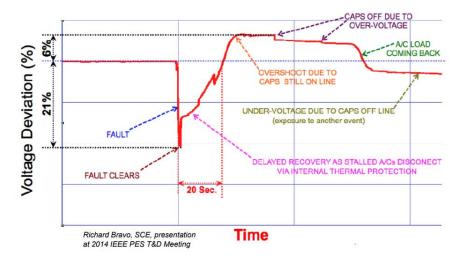


Image Source: NERC, Fault Induced Delayed Voltage Recovery (FIDVR) Advisory, July 2015

## **Measurement-Based Modal Analysis**



- Measurement-based modal analysis determines the observed dynamic properties of a system
  - Input can either be measurements from PMUs or transient stability results
  - This is now more common, and has been implemented in PowerWorld Simulator

# **Ring-down Modal Analysis**

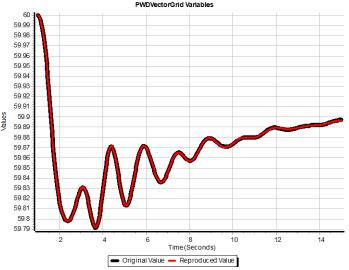


- Ring-down analysis seeks to determine the frequency and damping of key power system modes following some disturbance
- There are several different techniques, with the Prony approach the oldest (from 1795)
- Regardless of technique, the goal is to represent the response of a sampled signal as a set of exponentially damped sinusoidals (modes)

$$y(t) = \sum_{i=1}^{q} A_i e^{\sigma_i t} \cos\left(\omega_i t + \phi_i\right) \qquad \text{Damping (\%)} = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \omega_i^2}} \times 100$$

# **Ring-down Modal Analysis Example**

- The image on this slide shows an example in which the frequency response at a bus for 15 seconds after a contingency (thick black line) is approximated by a set of exponentially decaying sinusoids (thinner red line)
  - In this example six sinusoids and a linear detrend are used
- This approach assumes the 5000 system behavior is linear during the time period of interest



- It isn't fully linear, but it can be a quite useful approximation

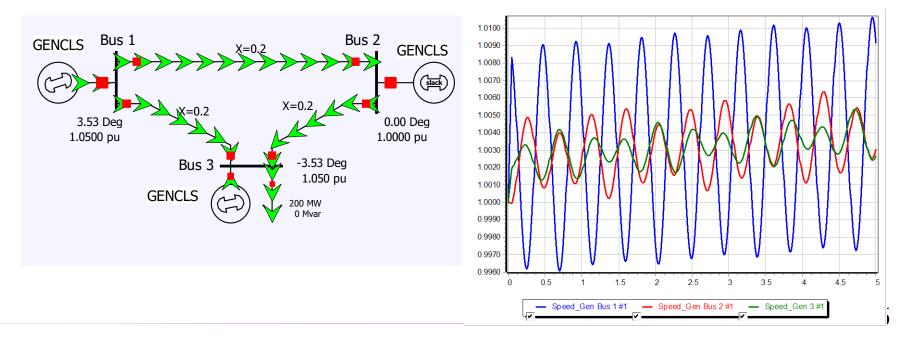
# Modal Analysis in PowerWorld



- Goal is to make modal analysis easy to use, and easy to visualize the results
- Provided tool can be used with either transient stability results or actual system signals (e.g., from PMUs)
- Two ways to access in PowerWorld (with a quicker third way coming soon!)
  - On the Transient Stability Analysis form left menu, Modal Analysis (right below SMIB Eigenvalues)
  - By right-clicking on a transient stability or plot case information display, and selecting Modal Analysis Selected Columns or Modal Analysis All Columns

#### Modal Analysis: Three Generator Example

- A M
- A short fault at t=0 gets the below three generator case oscillating with multiple modes (mostly clearly visible for the red and the green curve)



### Modal Analysis: Three Generator Example



- Open the case **B3\_CLS\_UnDamped** 
  - This system has three classical generators without damping; the default event is a self clearing fault at bus 1
- Run the transient stability for 5 seconds
- To do modal analysis, on the Transient Stability page select Results from RAM, view just the generator speed fields, right-click and select **Modal Analysis All Columns** 
  - This display the Modal Analysis Form

# **Modal Analysis Form**

#### First click on **Do Modal Analysis** to run the modal analysis

💭 Modal Analysis Fo	orm											_		×
Modal Analysis Status	Solved						Results		_			- Proved		
Data Source Type     File, Comtrade CFF       From Plot     File, Comtrade CFF       File, WECC CSV 2     File, Comtrade CFG       File, JSIS Format     None, Existing Data       Data Source Inputs from Plots or Files     From Plot       Gen_Speed     ✓				ulation Method /ariable Projectio /atrix Pencil (On terative Matrix R Dynamic Mode Do Do Moo		Number of Complex and Real Modes     2       Lowest Percent Damping     -0.013       Real and Complex Modes - Editable to Change Initial Complex Modes - Editable to Change Initial Complex (Hz)     Damping (%)       Frequency (Hz)     Damping (%)     Largest Weighted			0.013	Signal Name of Lambda Largest			-	
From File		Brows	e Sa	ve in JSIS Form	at	Save to CSV				Mode	Percentage for Mode			
Start Time 0.05	Group Disabled for Est econds) and Frequency (F 50 T End Time	iz)					1	2.233	-0.009 -0.013				.0012 YI .0013 YI	
Data Sampling Time (Se Start Time 0.09 Maximum Hz 5.00	econds) and Frequency (F 50 - End Time 10 - Update Sample	iz) 5.000 d Data	•				1 2 <							
Data Sampling Time (Se Start Time 0.09 Maximum Hz 5.00	conds) and Frequency (F	iz) 5.000 d Data S Options			Units	Include	Include Reproduced		-0.013	Post-Detre		0.		
Data Sampling Time (Se Start Time 0.09 Maximum Hz 5.00 Input Data, Actual San	conds) and Frequency (F 50  F End Time Update Sample npled Input Data Signal Name Gen Bus 1 #1 Speed	iz) 5.000 d Data S Options	Reproduced	Data		Include YES	Include Reproduced YES	1.511 Detrend	-0.013 Detrend Parameter 24 0.00	B Post-Detrei Number Zei	o 0.005	d So	.0013 Y	ES >
Data Sampling Time (Se Start Time 0.03 Maximum Hz 5.00 Input Data, Actual San Type	econds) and Frequency (F 50 - End Time Update Sample Update Sample Name	iz) 5.000 d Data S Options	Reproduced	Data Description	Units	Include	Include Reproduced	1.511 Detrend Parameter A	-0.013 Detrend Parameter 24 0.00 24 0.00	B Post-Detree Number Zer 2004	ros Standard Deviation	d So	.0013 Y	ES Average

Right-click on signal to view its dialog

7 Help

<u>C</u>lose

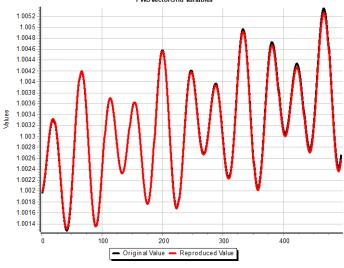
Key results are shown in the upper-right of the form. There are two main modes, one at 2.23Hz and one at 1.51; both have very little damping.

#### Three Generator Example: Signal Dialog

• The Signal Dialog provides details about each signal, including its modal components and a comparison between the original and reproduced signals (example for gen 3)

e	Gen 3 #1 Speed	Data Detrend Parar	meters			Output Summary	
	Gen	Detrend Model = A	+ B*(t-t0) + C*(t-t0)^2	Used Detrend Model	Linear	Average Error. Scaled by SD	0.0000
s		Use Case Def	ault Detrend Model	Parameter A	1.0025	Average Error. Unscaled	0.0000
ription	Speed	Signal Specific (		Parameter B	0.0003	Cost Function Value, Scaled	0.000
Tachuda	in Modal Analysis	None	O Linear	Parameter C	0.0000	Include Detrend in Reprodu	iced Signa
Include	In Modal Analysis	O Constant	○ Quadratic	Standard Deviation (SD)	0.0008	Update Reproduced	
1	Time (Seconds) 0.050	Original Value 1.002	Reproduced Value	Difference 0.000			
2	0.060	1.002	1.002	0.000			
3	0.070	1.002	1.002	0.000			
4	0.080	1.002	1.002	0.000			
5	0.090	1.002	1.002	0.000			
6	0.100	1.002	1.002	0.000			
7	0.110	1.002	1.002	0.000			
8	0.120	1.003 1.003	1.003	0.000			
10	0.140	1.003	1.003	0.000			
	0.150	1.003	1.003	0.000			
11	0.160	1.003	1.003	0.000			
	0.170	1.003	1.003	0.000			
12 13		1.003	1.003	0.000			
12 13 14	0.180		1.003	0.000			
12 13 14 15	0.190	1.003	4 8 8 8				
12 13 14 15 16	0.190 0.200	1.003	1.003	0.000			
12 13 14 15 16 17	0.190 0.200 0.210	1.003 1.003	1.003	0.000			
12 13 14 15 16 17 18	0.190 0.200 0.210 0.220	1.003 1.003 1.003	1.003 1.003	0.000			
11 12 13 14 15 16 17 18 19 20	0.190 0.200 0.210	1.003 1.003	1.003	0.000			

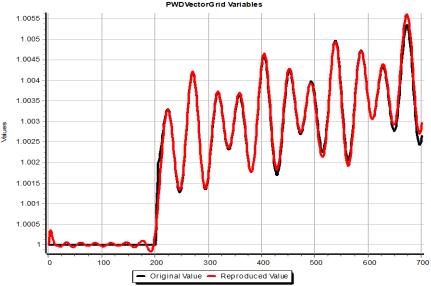
Plotting the original and reproduced signals shows a near exact match





#### Caution: Setting Time Range Incorrectly Can Result in Unexpected Results!

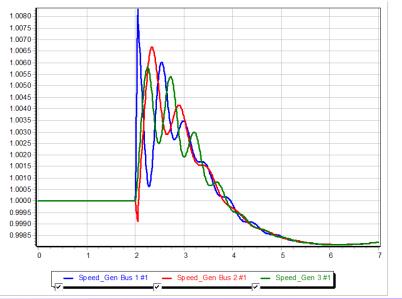
- Assume the system is run with no disturbance for two seconds, and then the fault is applied and the system is run for a total of seven seconds (five seconds post-fault)
  - The incorrect approach would be to try to match the entire signal; rather just match from after the fault
  - Trying to match the full signal between 0 and
    7 seconds required eleven modes!
  - By default the Modal
     Analysis Form sets the default start time to immediately after the last event



# **GENROU Example with Damping**



- Open the case **B3\_GENROU**, which changes the GENCLS to GENROU models, adding damping
  - Also each has an EXST1 exciter and a TGOV1 governor
  - The simulation runs for seven seconds, with the fault occurring at two seconds; modal analysis is done from the time the fault is cleared until the end of the simulation.



The image shows the generator speeds. The initial rise in the speed is caused by the load dropping during the fault, causing a power mismatch; this is corrected by the governors. Note the system now has damping; modal analysis tells us how much.

# **GENROU Example with Damping**

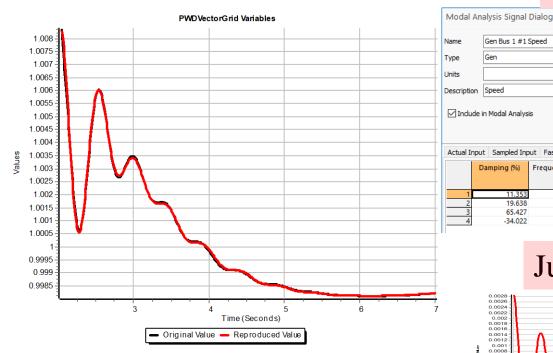
🔘 Modal Analysis Form									- 🗆	×
Modal Analysis Status Solved a	t 9/15/2019 7:28:01 AM			Results						
Data Source Type	IC 9/15/2019 7:28:01 AM	Calculation Method		Number of Complex and Real Modes 4 Indude Detrend in Reproduced Signals						
O From Plot File, WECC CSV 2	○ File, Comtrade CFF ○ File, Comtrade CFG	<ul> <li>Variable Projection</li> <li>Matrix Pencil (Once)</li> </ul>	Lowest Percent Damping -34 022					roduced Signals		
○ File, JSIS Format	None, Existing Data	O Iterative Matrix Pencil		Real and Co	mplex Modes	- Editable to Cha	ange Initial Guess	ses		
Data Source Inputs from Plots	or Files	O Dynamic Mode Decompo	sition	Free	quency (Hz)	Damping (%)	Largest	Signal Name of	Lambda	Include
From Plot Gen_Speed	✓ Browse	Do Modal Anal	ysis Save to CSV			F I I I I I I I I I I I I I I I I I I I	Weighted Percentage for Mode	Largest Weighted Percentage for Mode		Reprodu Signa
The second second	Disabled for fairting Date			1	2.053	11.353	25.5481	Gen 3 #1 Spee	-1.4737	YES
Just Load Signals Group	Disabled for Existing Data			2	1.649	19.638		Gen Bus 2 #1 S	-2.0754	
				3	0.236	65.427		Gen Bus 2 #1 S	-1.2833	
Data Sampling Time (Seconds) a	and Frequency (Hz)			4	0.098	-34.022	83.2537	Gen Bus 1 #1 S	0.2224	YES
Start Time 2.050 Maximum Hz 5.000	End Time 7.000			<	T					>
Input Data, Actuar Sempled Inp	put Data Signals Options Repr	oduced Data								
Nime	Latitude Longitude Descrip	tion Units Include	Include Reproduced	Detrend Parameter A	Detrend Birameter B	Post-Detreno Number Zero		d Solved	Average Erro Unscaled	or, Averaç Scalei
1 Gen Bus 1 #1 Speed	Speed	YES	YES	1.0037	-0.001		0 0.001	YES	0.00	
2 Gen Bus #1 Speed	Speed	YES	YES	1.0034	-0.001		0 0.001	YES	0.00	
3 Gen 3 #1 Speed	Speed	YES	YES	1.0035	-0.001	4	0 0.001	YES	0.00	00
<										>
	7 Hel	p	Print							
			M	ode fr	eque	ency,	dam	ping,		

Start time default value Mode frequency, damping, and largest contribution of each mode in the signals **A**M

# **GENROU Example with Damping**



#### • Left image show how well the speed for generator 1 is approximated by the modes



#### More signal details

me	Gen Bus 1 #1 Speed	Data Detrend Param	eters			Output Su
be	Gen	Detrend Model = A -	+ B*(t-t0) + C*(t-t0)^2	Used Detrend Model	Linear	Average E
its		Use Case Defa	ult Detrend Model	Parameter A	1.0037	Average E
scription	Speed	Signal Specific De	-	Parameter B	-0.0014	Cost Funct
_		None	Linear	Parameter C	0.0000	Include
Include i	in Modal Analysis	◯ Constant	○ Quadratic	Standard Deviation (SD)	0.0013	Upda

#### Actual Input Sampled Input Fast Fourier Transform Results Modal Results Original and Reproduced Signal Comparison

	Damping (%)	Frequency (Hz)	Magnitude Scaled by SD	Magnitude, Unscaled	Angle (Deg)	Lambda	Include in Reproduced Signal
1	11.353	2.053	2.300	0.003	13.82	-1.474	YES
2	19.638	1.649	2.038	0.003	10.46	-2.075	YES
3	65.427	0.236	4.757	0.006	-91.36	-1.283	YES
4	-34.022	0.098	0.689	0.001	135.64	0.222	YES

#### Just the 2.05 Hz mode



# Signal-Based Modal Analysis



- Idea of all techniques is to approximate a signal, y<sub>org</sub>(t), by the sum of other, simpler signals (basis functions)
  - Basis functions are usually exponentials, with linear and quadratic functions used to detrend the signal
  - Properties of the original signal can be quantified from basis function properties
    - Examples are frequency and damping
  - Signal is considered over time with t=0 as the start
- Approaches sample the original signal  $y_{org}(t)$

# Sampling Rate and Aliasing



- The Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency
  - For example, to see a 5 Hz frequency we need to sample the signal at a rate of at least 10 Hz
- Sampling shifts the frequency spectrum by 1/T (where T is the sample time), which causes frequency overlap
- This overlapping of frequencies is known as aliasing, which can cause a high frequency signal to appear to be a lower frequency signal
  - Aliasing can be reduced by fast sampling and/or low pass filters

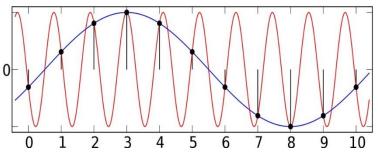


Image: upload.wikimedia.org/wikipedia/commons/thumb/2/28/AliasingSines.svg/2000px-AliasingSines.svg.png

# Signal-Based Modal Analysis



• Vector y consists of m uniformly sampled points from  $y_{org}(t)$  at a sampling value of  $\Delta T$ , starting with t=0, with values  $y_j$  for j=1...m

- Times are then 
$$t_j = (j-1)\Delta T$$

- At each time point j, the approximation of  $y_i$  is

$$\hat{y}_j(\boldsymbol{\alpha}) = \sum_{i=1}^n b_i \phi_i(t_j, \boldsymbol{\alpha})$$

where  $\boldsymbol{\alpha}$  is a vector with the real and imaginary eigenvalue components, with  $\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j}$  for  $\alpha_i$  corresponding to a real eigenvalue, and  $\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j} \cos(\alpha_{i+1} t_j)$  and  $\phi_{i+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \sin(\alpha_{i+1} t_j)$ for a complex eigenvector value

# Signal Based Modal Analysis



- Error (residual) value at each point j is  $r_j(t_j, \boldsymbol{\alpha}) = y_j - \hat{y}_j(t_j, \boldsymbol{\alpha})$
- Closeness of fit can be quantified using the Euclidean norm of the residuals

$$\frac{1}{2} \sum_{j=1}^{m} (y_j - \hat{y}_j(t_j, \boldsymbol{\alpha}))^2 = \frac{1}{2} \| \mathbf{r}(\boldsymbol{\alpha}) \|_2^2$$

- Hence we need to determine α and b; PowerWorld has three techniques for determining α, and then one for b
- Approaches can be used with multiple signals

# **Algorithm Details**

- The modes are found using the Matrix Pencil method
  - This is a newer technique for determining modes from noisy signals (from about 1990, introduced to power system problems in 2005); it is an alternative to the Prony Method (which dates back to 1795, introduced into power in 1990 by Hauer, Demeure and Scharf)
- Given m samples, with L=m/2, the first step is to form the Hankel Matrix, **Y** such that

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \dots & y_{L+1} \\ y_2 & y_3 & \dots & y_{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L} & y_{m-L+1} & \dots & y_m \end{bmatrix}$$

Reference: A. Singh and M. Crow, "The Matrix Pencil for Power System Modal Extraction," IEEE Transactions on Power Systems, vol. 20, no. 1, pp. 501-502, Institute of Electrical and Electronics Engineers (IEEE), Feb 2005.

# Algorithm Details, cont.

- Then calculate **Y**'s singular values using an economy singular value decomposition (SVD)  $\mathbf{Y} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^{\mathrm{T}}$  The computational
- The ratio of each singular value is then compared to the largest singular value  $\sigma_c$ ; retain the ones with a ratio > than a threshold (e.g., 0.16)
  - This determines the modal order, M
  - Assuming V is ordered by singular values (highest to lowest), let V<sub>p</sub> be then matrix with the first M columns of V

The computational complexity increases with the cube of the number of measurements!

> This threshold is a value that can be changed; decrease it to get more modes.



## Aside: Matrix Singular Value Decomposition (SVD)



• The SVD is a factorization of a matrix that generalizes the eigendecomposition to any m by n matrix to produce  $\mathbf{Y} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^{\mathbf{T}}$ The original concept is more than 100 years old, but has founds lots

of recent applications where  $\Sigma$  is a diagonal matrix of the singular values

- The singular values are non-negative real numbers that can be used to indicate the major components of a matrix (the gist is they provide a way to decrease the rank of a matrix
- A key application is image compression

# **SVD Image Compression Example**



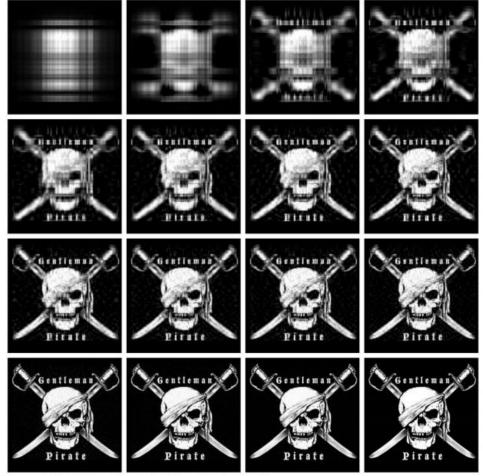


Figure 3.1: Image size 250x236 - modes used {{1,2,4,6},{8,10,12,14},{16,18,20,25},{50,75,100,original image}}

Images can be represented with matrices. When an SVD is applied and only the largest singular values are retained the image is compressed.

# Algorithm Details, cont.

- Then form the matrices  $\mathbf{V}_1$  and  $\mathbf{V}_2$  such that
  - $\mathbf{V}_1$  is the matrix consisting of all but the last row of  $\mathbf{V}_p$
  - $\mathbf{V}_2$  is the matrix consisting of all but the first row of  $\mathbf{V}_p$
- Discrete-time poles are found as the generalized eigenvalues of the pair  $(\mathbf{V}_2^T \mathbf{V}_1, \mathbf{V}_1^T \mathbf{V}_1) = (\mathbf{A}, \mathbf{B})$

The log of a complex

number  $z=r \angle \theta$  is

• These eigenvalues are the discrete-time poles, z<sub>i</sub> with the modal eigenvalues then

 $\ln(\mathbf{r}) + \mathbf{j}\theta$ 

 $\lambda_i = \frac{\ln(z_i)}{\Lambda T}$ 

If **B** is nonsingular (the situation here) then the generalized eigenvalues are the eigenvalues of  $\mathbf{B}^{-1}\mathbf{A}$ 

#### Matrix Pencil Method with Many Signals

- The Matrix Pencil approach can be used with one signal or with multiple signals
- Multiple signals are handled by forming a Y<sub>k</sub> matrix for each signal k using the measurements for that signal and then combining the matrices so that for N signals

$$\mathbf{Y}_{k} = \begin{bmatrix} y_{1,k} & y_{2,k} & \cdots & y_{L+1,k} \\ y_{2,k} & y_{3,k} & \cdots & y_{L+2,k} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L,k} & y_{m-L+1,k} & \cdots & y_{m,k} \end{bmatrix}$$
$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{1} \\ \vdots \\ \mathbf{Y}_{N} \end{bmatrix}$$

The required computation scales linearly with the number of signals

#### Matrix Pencil Method with Many Signals

- However, when dealing with many signals, usually the signals are somewhat correlated, so vary few of the signals actually need to be included to determine the desired modes
   The α is found using
- Recall we are ultimately finding

$$\hat{y}_{j,k}(\boldsymbol{\alpha}) = \sum_{i=1}^{n} b_{i,k} \phi_i(t_j, \boldsymbol{\alpha})$$
 signals; the **b** v  
signal specific.

where  $\boldsymbol{\alpha}$  is a vector with the real and imaginary eigenvalue components, with  $\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j}$  for  $\alpha_i$  corresponding to a real eigenvalue, and  $\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j} \cos(\alpha_{i+1} t_j)$  and  $\phi_{i+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \sin(\alpha_{i+1} t_j)$ for a complex eigenvector value



the matrix pencil method and is common to all the signals; the **b** vector is signal specific.

## **Iterative Matrix Pencil Method**



- When there are a large number of signals the iterative matrix pencil method works by
  - Selecting an initial signal to calculate the  $\alpha$  vector
  - Quickly calculating the **b** vectors for all the signals, and getting a cost function for how closely the reconstructed signals match their sampled values
  - Selecting a signal that has a high cost function, and repeating the above adding this signal to the algorithm to get an updated  $\alpha$