ECEN 667
Power System Stability

Lecture 23: Small Signal Stability, Oscillations

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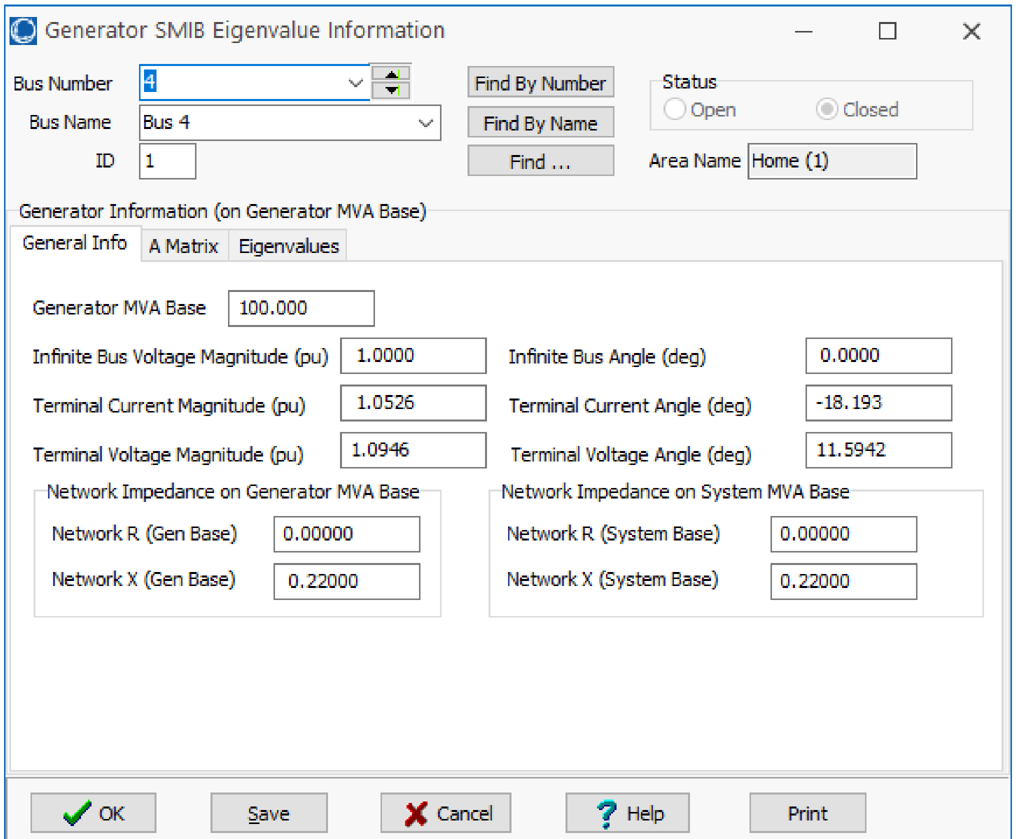
Announcements

- Read Chapter 8
- Homework 5 is due Today
- Homework 6 is due on Tuesday December 3
- Final is at scheduled time here (December 9 from1pm to 3pm)

Example: Bus 4 SMIB Dialog

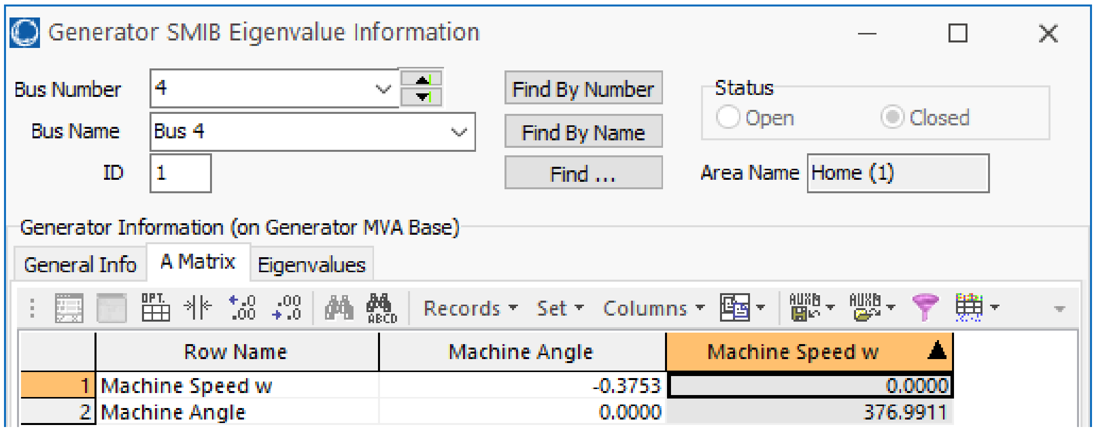
- On the SMIB dialog, the General Information tab shows information about the two bus equivalent



PowerWorld case **B4_SMIB**

Example: Bus 4 SMIB Dialog

- On the SMIB dialog, the **A** Matrix tab shows the $\mathbf{A}_{sys}$ matrix for the SMIB generator



- In this example A_{21} is showing

$$\frac{\partial \Delta\omega_{4,pu}}{\partial \delta_4} = \frac{1}{2H_4}\left(\frac{-\partial P_{E,4}}{\partial \delta_4}\right) = -\left(\frac{1}{6}\right)\left(\left(\frac{-1}{0.3+0.22}\right)\left(-1.2812\cos\left(23.94^\circ\right)\right)\right)$$

$$= -0.3753$$

Example: Bus 4 with GENROU Model

- The eigenvalues can be calculated for any set of generator models
- The example can be extended by replacing the bus 4 generator classical machine with a GENROU model
  - There are now six eigenvalues, with the dominate response coming from the electro-mechanical mode with a frequency of 1.84 Hz, and damping of 6.9%

General Info | A Matrix | Eigenvalues

Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Sp w
1	-0.4248	0.0000	0.4248	1.0000	0.0000		0.0676	0.0027	0.
2	-0.8040	-11.5563	11.5842	0.0694	-1.8392	-0.5437	1.8437	0.7055	0.
3	-0.8040	11.5563	11.5842	0.0694	1.8392	0.5437	1.8437	0.7055	0.
4	-3.7087	0.0000	3.7087	1.0000	0.0000		0.5903	0.0155	0.
5	-14.2256	0.0000	14.2256	1.0000	0.0000		2.2641	0.0044	0.
6	-21.2472	0.0000	21.2472	1.0000	0.0000		3.3816	0.0159	0.

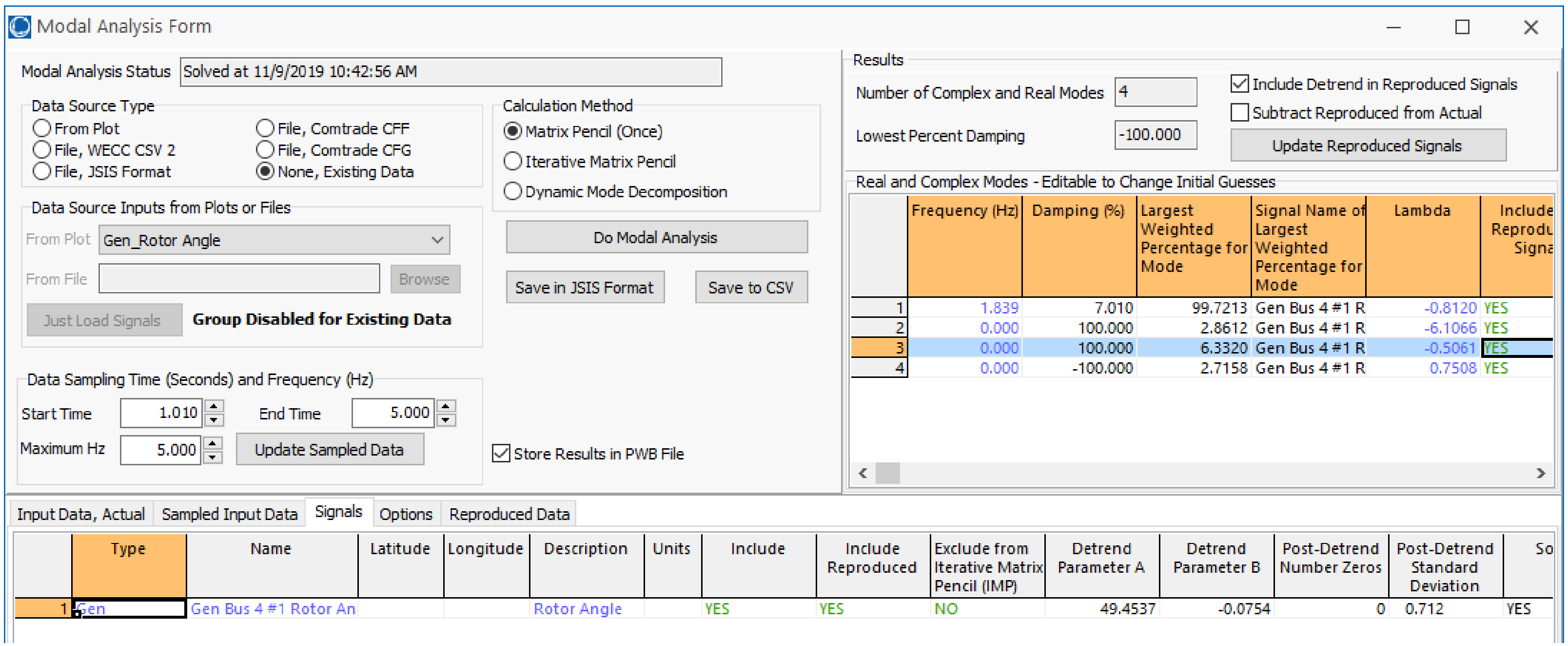
PowerWorld case **B4_SMIB_GENROU**

Relation to the Signal-Based Approaches

- The results from the SMIB analysis can be compared with the signal-based approach by applying a short self-clearing fault to disturb the system
- This can be done easily in PowerWorld by running the transient stability simulation, looking at the **Results from RAM** page, right-clicking on the desired signal(s) and selecting **Modal Analysis Selected Column**.
- The next slide shows the results for the Bus 4 Generator rotor angle (which would not be directly observable)

Bus 4 Generator Rotor Angle

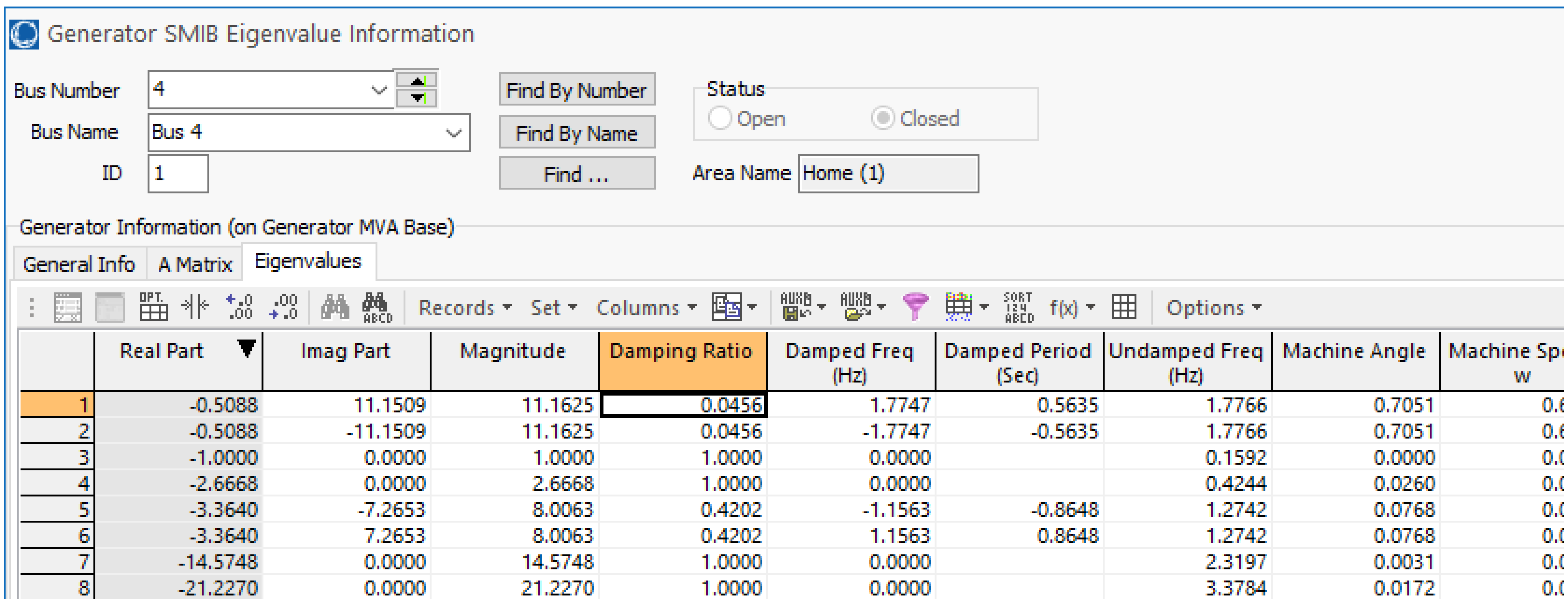
- Notice that the main mode, at 1.84 Hz with 7% damping closely matches



Not all modes will be easily observed in all signals

Example: Bus 4 with GENROU Model and Exciter

- Adding an relatively slow EXST1 exciter adds additional states (with K_A=200, T_A=0.2)
  - As the initial reactive power output of the generator is decreased, the system becomes unstable



	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Sp w
1	-0.5088	11.1509	11.1625	0.0456	1.7747	0.5635	1.7766	0.7051	0.6
2	-0.5088	-11.1509	11.1625	0.0456	-1.7747	-0.5635	1.7766	0.7051	0.6
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	0.0000	0.0
4	-2.6668	0.0000	2.6668	1.0000	0.0000		0.4244	0.0260	0.0
5	-3.3640	-7.2653	8.0063	0.4202	-1.1563	-0.8648	1.2742	0.0768	0.0
6	-3.3640	7.2653	8.0063	0.4202	1.1563	0.8648	1.2742	0.0768	0.0
7	-14.5748	0.0000	14.5748	1.0000	0.0000		2.3197	0.0031	0.0
8	-21.2270	0.0000	21.2270	1.0000	0.0000		3.3784	0.0172	0.0

Case is saved as **B4_GENROU_EXST1**

Example: Bus 4 with GENROU Model and Exciter

- With Q_4 = 25 Mvar the eigenvalues are

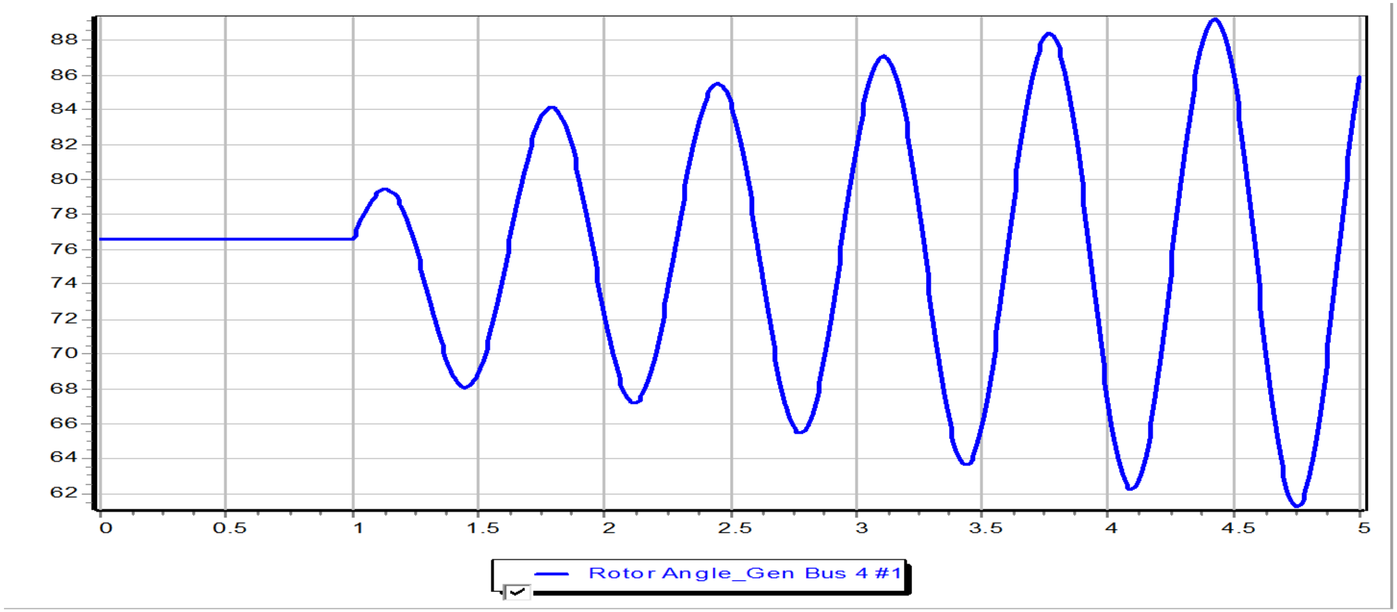
	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machi
1	-0.1239	-10.3955	10.3962	0.0119	-1.6545	-0.6044	1.6546	0.7021	
2	-0.1239	10.3955	10.3962	0.0119	1.6545	0.6044	1.6546	0.7021	
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	0.0000	
4	-2.6586	0.0000	2.6586	1.0000	0.0000		0.4231	0.0209	
5	-3.5938	-6.8580	7.7426	0.4642	-1.0915	-0.9162	1.2323	0.1110	
6	-3.5938	6.8580	7.7426	0.4642	1.0915	0.9162	1.2323	0.1110	
7	-14.5078	0.0000	14.5078	1.0000	0.0000		2.3090	0.0045	
8	-21.4739	0.0000	21.4739	1.0000	0.0000		3.4177	0.0097	

- And with Q_4=0 Mvar the eigenvalues are

	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machi
1	0.2704	-9.5336	9.5374	-0.0283	-1.5173	-0.6591	1.5179	0.6920	
2	0.2704	9.5336	9.5374	-0.0283	1.5173	0.6591	1.5179	0.6920	
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	0.0000	
4	-3.0137	0.0000	3.0137	1.0000	0.0000		0.4796	0.0071	
5	-3.6849	-6.4281	7.4094	0.4973	-1.0231	-0.9775	1.1792	0.1643	
6	-3.6849	6.4281	7.4094	0.4973	1.0231	0.9775	1.1792	0.1643	
7	-14.4234	0.0000	14.4234	1.0000	0.0000		2.2956	0.0054	
8	-21.6978	0.0000	21.6978	1.0000	0.0000		3.4533	0.0030	

Example: Bus 4 with GENROU Model and Exciter

- Graph shows response following a short fault when Q4 is 0 Mvar



- This motivates trying to get additional insight into how to increase system damping, which is the goal of modal analysis

Modal Analysis - Comments

- Modal analysis (analysis of small signal stability through eigenvalue analysis) is at the core of SSA software
- In Modal Analysis one looks at:
  - Eigenvalues
  - Eigenvectors (left or right)
  - Participation factors
  - Mode shape

Goal is to determine how the various parameters affect the response of the system

- Power System Stabilizer (PSS) design in a multi-machine context can be done using modal analysis method (we'll see another method later)

Eigenvalues, Right Eigenvectors

- For an n by n matrix **A** the eigenvalues of **A** are the roots of the characteristic equation:

$$\det[\mathbf{A}-\lambda\mathbf{I}] = \left|\mathbf{A}-\lambda\mathbf{I}\right| = 0$$

- Assume $\lambda_1 \ldots \lambda_n$ as distinct (no repeated eigenvalues).
- For each eigenvalue λ_i there exists an eigenvector such that:

$$\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$$

- $\mathbf{v}_i$ is called a right eigenvector
- If λ_i is complex, then $\mathbf{v}_i$ has complex entries

Left Eigenvectors

- For each eigenvalue λ_i there exists a left eigenvector $\mathbf{w}_i$ such that:

$$\mathbf{w}_i^t \mathbf{A} = \mathbf{w}_i^t \lambda_i$$

- Equivalently, the left eigenvector is the right eigenvector of $\mathbf{A}^T$; that is,

$$\mathbf{A}^t \mathbf{w}_i = \lambda_i \mathbf{w}_i$$

Eigenvector Properties

- The right and left eigenvectors are orthogonal i.e.

$$\mathbf{w}_i^t \mathbf{v}_i \neq 0 \ , \ \mathbf{w}_i^t \mathbf{v}_j = 0 \quad (i \neq j)$$

- We can normalize the eigenvectors so that:

$$\mathbf{w}_i^t \mathbf{v}_i = 1 \ , \ \mathbf{w}_i^t \mathbf{v}_j = 0 \quad (i \neq j)$$

Eigenvector Example

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \ |\mathbf{A} - \lambda \mathbf{I}| = 0 \ \Rightarrow \begin{vmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 3\lambda - 10 = 0 \Rightarrow \ \lambda_{1,2} = \frac{3 \pm \sqrt{(3)^2 + 4(10)}}{2} = \frac{3 \pm \sqrt{49}}{2} = 5, -2$$

Right Eigenvectors $\lambda_1 = 5$

$$\mathbf{A}\mathbf{v}_1 = 5\mathbf{v}_1 \Rightarrow \ \mathbf{v}_1 = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \Rightarrow \begin{matrix} v_{11} + 4v_{21} = 5v_{11} \\ 3v_{11} + 2v_{21} = 5v_{21} \end{matrix}$$

$$\textit{choose } \mathbf{v}_{21} = 1 \Rightarrow \mathbf{v}_{11} = 1$$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Similarly,

$$\lambda_2 = -2 \Rightarrow \ \mathbf{v}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Eigenvector Example

- Left eigenvectors

$$\lambda_1 = 5 \;\; \mathbf{w}_1^t \mathbf{A} = \mathbf{w}_1^t 5 \Rightarrow [w_{11} \;\; w_{21}] \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} = 5[w_{11} \;\; w_{21}]$$

$$\begin{array}{r} w_{11} + 3w_{21} = 5w_{11} \\ 4w_{11} + 2w_{21} = 5w_{21} \end{array} \Rightarrow \textit{Let } \; w_{21} = 4, \; \textit{then} \;\; w_{11} = 3$$

$$\mathbf{w}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad \lambda_2 = -2 \Rightarrow \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \;\; \mathbf{v}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix} \;\; \mathbf{w}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \;\; \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\textit{Verify} \;\; \mathbf{w}_1^t \mathbf{v}_1 = 7 \;, \;\; \mathbf{w}_2^t \mathbf{v}_2 = 7 \;, \;\; \mathbf{w}_2^t \mathbf{v}_1 = 0 \;, \;\; \mathbf{w}_1^t \mathbf{v}_2 = 0$$

We would like to make $w_i^t v_i = 1.$

This can be done in many ways.

Eigenvector Example

$$\textit{Let} \quad \mathbf{W} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$\textit{Then} \quad \mathbf{W}^T \mathbf{V} = \mathbf{I}$$

$$\textit{Verify} \quad \frac{1}{7}\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- It can be verified that $\mathbf{W}^T=\mathbf{V}^{-1}$.
- The left and right eigenvectors are used in computing the participation factor matrix.

Modal Matrices

- The deviation away from an equilibrium point can be defined as

$$\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x}$$

- If the initial deviation corresponds to a right eigenvector, then the subsequent response is along this eigenvector since $\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$

Modal Matrices

- From this equation ($\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x}$) it is difficult to determine how parameters in **A** affect a particular **x** because of the variable coupling
- To decouple the problem first define the matrices of the right and left eigenvectors (the modal matrices)

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2 ..... \mathbf{v}_n] \;\; \& \;\; \mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, ... \mathbf{w}_n]$$

$$\mathbf{AV} = \mathbf{V\Lambda} \qquad \text{when} \;\; \mathbf{\Lambda} = Diag(\lambda_i)$$

Modal Matrices

- It follows that

$$\mathbf{V}^{-1}\mathbf{A}\mathbf{V} = \mathbf{\Lambda}$$

- To decouple the variables define $\mathbf{z}$ so

$$\Delta\mathbf{x} = \mathbf{V}\mathbf{z} \rightarrow \Delta\dot{\mathbf{x}} = \mathbf{V}\dot{\mathbf{z}} = \mathbf{A}\Delta\mathbf{x} = \mathbf{A}\mathbf{V}\mathbf{z}$$

- Then

$$\dot{\mathbf{z}} = \mathbf{V}^{-1}\mathbf{A}\mathbf{V}\mathbf{z} = \mathbf{W}\mathbf{A}\mathbf{V}\mathbf{z} = \mathbf{\Lambda}\mathbf{z}$$

- Since $\mathbf{\Lambda}$ is diagonal, the equations are now uncoupled with $\dot{z}_i = \lambda_i z_i$

- So $\Delta\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t)$

Example

- Assume the previous system with

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$$

$$\mathbf{V}^{-1}\mathbf{A}\mathbf{V} = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$$

Modal Matrices

- Thus the response can be written in terms of the individual eigenvalues and right eigenvectors as

$$\Delta \mathbf{x}(t) = \sum_{i=1}^{n} \mathbf{v}_i z_i(0) e^{\lambda_i t}$$

Note, we are requiring that the eigenvalues be distinct!

- Furthermore with

$$\Delta \mathbf{x} = \mathbf{V}\mathbf{Z} \Rightarrow \quad \mathbf{z} = \mathbf{V}^{-1}\mathbf{x} = \mathbf{W}^T \mathbf{x}$$

- So z(t) can be written as using the left eigenvectors as

$$\mathbf{z}(t) = \mathbf{W}^t \mathbf{x}(t) = [\mathbf{w}_1 \;\; \mathbf{w}_2 \ldots . \mathbf{w}_n]^t \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

Modal Matrices

- We can then write the response x(t) in terms of the modes of the system

$$z_i(t) = w_i^t x(t)$$

$$z_i(0) = w_i^t x(0) \triangleq c_i$$

$$\text{so} \quad \mathbf{x}(t) = \sum_{i=1}^{n} \mathbf{v}_i c_i e^{\lambda_i t}$$

$$\text{Expanding} \quad \Delta x_i(t) = v_{i1} c_1 e^{\lambda_1 t} + v_{i2} c_2 e^{\lambda_2 t} + \dots v_{in} c_n e^{\lambda_n t}$$

- So c_i is a scalar that represents the magnitude of excitation of the i^{th} mode from the initial conditions

Numerical example

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}, \ \Delta\mathbf{x}(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

Eigenvalues are $\lambda_1 = -4, \ \lambda_2 = 2$

Eigenvectors are $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Modal matrix $\mathbf{V} = \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix}$

Normalize so $\mathbf{V} = \begin{bmatrix} 0.2425 & 0.4472 \\ -0.9701 & 0.8944 \end{bmatrix}$

Numerical example (contd)

Left eigenvector matrix is:

$$\mathbf{W}^{\mathrm{T}} = \mathbf{V}^{-1} = \begin{bmatrix} 1.3745 & -0.6872 \\ 1.4908 & 0.3727 \end{bmatrix}$$

$$\dot{\mathbf{z}} = \mathbf{W}^{\mathrm{T}}\mathbf{A}\mathbf{V}\mathbf{z}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Numerical example (contd)

$$\dot{z}_1 = -4z_1 \quad , \quad \mathbf{z}(0) = V^{-1}\mathbf{x}(0)$$

$$\dot{z}_2 = 2z_2 \quad , \quad \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix} = \begin{bmatrix} 4.123 \\ 0 \end{bmatrix}$$

$$z_1(t) = z_1(0)e^{-4t} \;\; ; \;\; z_2(t) = z_2(0)e^{2t}, \mathbf{C} = \mathbf{W}^T\mathbf{x}(0) = \begin{bmatrix} 4.123 \\ 0 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{Vz}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}$$

Because of the initial condition, the 2nd mode does not get excited

$$= c_1 \begin{bmatrix} 0.2425 \\ -0.9701 \end{bmatrix} z_1(t) + c_2 \begin{bmatrix} 0.4472 \\ 0.8944 \end{bmatrix} z_2(t) = \sum_{i=1}^{2} c_i \mathbf{v}_i z_i(0) e^{\lambda_i t}$$

Mode Shape, Sensitivity and Participation Factors

- So we have

$$\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t), \quad \mathbf{z}(t) = \mathbf{W}^t\mathbf{x}(t)$$

- $\mathbf{x}$(t) are the original state variables, $\mathbf{z}$(t) are the transformed variables so that each variable is associated with only one mode.
- From the first equation the right eigenvector gives the “mode shape” i.e. relative activity of state variables when a particular mode is excited.
- For example the degree of activity of state variable x_k in $\mathbf{v}_i$ mode is given by the element V_{ki} of the the right eigenvector matrix **V**

Mode Shape, Sensitivity and Participation Factors

- The magnitude of elements of $\mathbf{v}_i$ give the extent of activities of n state variables in the i^{th} mode and angles of elements (if complex) give phase displacements of the state variables with regard to the mode.
- The left eigenvector $\mathbf{w}_i$ identifies which combination of original state variables display only the i^{th} mode.

Eigenvalue Parameter Sensitivity

- To derive the sensitivity of the eigenvalues to the parameters recall $\mathbf{A}\mathbf{v}_i = \lambda_i\mathbf{v}_i$; take the partial derivative with respect to A_{kj} by using the chain rule

$$\frac{\partial \mathbf{A}}{\partial \mathbf{A}_{kj}}\mathbf{v}_i + \mathbf{A}\frac{\partial \mathbf{v_i}}{\partial A_{kj}} = \frac{\partial \lambda_i}{\partial A_{kj}}\mathbf{v}_i + \lambda_i \frac{\partial \mathbf{v}_i}{\partial A_{kj}}$$

Multiply by $\mathbf{w}_i^t$

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial \mathbf{A}_{kj}}\mathbf{v}_i + \mathbf{w}_i^t\mathbf{A}\frac{\partial \mathbf{v_i}}{\partial A_{kj}} = \mathbf{w}_i^t\frac{\partial \lambda_i}{\partial A_{kj}}\mathbf{v}_i + \mathbf{w}_i^t\lambda_i \frac{\partial \mathbf{v}_i}{\partial A_{kj}}$$

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial A_{kj}}\mathbf{v}_i + \mathbf{w}_i^t[\mathbf{A} - \lambda_i\mathbf{I}]\frac{\partial \mathbf{v}_i}{\partial A_{kj}} = \mathbf{w}_i^t\frac{\partial \lambda_i}{\partial A_{kj}}\mathbf{v}_i$$

Eigenvalue Parameter Sensitivity

- This is simplified by noting that $\mathbf{w}_i^t(\mathbf{A}-\lambda_i\mathbf{I})=0$ by the definition of $\mathbf{w}_i$ being a left eigenvector
- Therefore

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_i = \frac{\partial \lambda_i}{\partial A_{kj}}$$

- Since all elements of $\frac{\partial \mathbf{A}}{\partial A_{kj}}$ are zero, except the k^{th} row, j^{th} column is 1
- Thus $\frac{\partial \lambda_i}{\partial A_{kj}} = W_{ki} V_{ji}$

Sensitivity Example

- In the previous example we had

$$\mathbf{A}=\begin{bmatrix}1 & 4\\ 3 & 2\end{bmatrix},\quad \lambda_{1,2}=5,-2,\quad \mathbf{V}=\begin{bmatrix}1 & 4\\ 1 & -3\end{bmatrix},\quad \mathbf{W}=\frac{1}{7}\begin{bmatrix}3 & 1\\ 4 & -1\end{bmatrix}$$

- Then the sensitivity of λ_1 and λ_2 to changes in **A** are

$$\frac{\partial \lambda_i}{\partial A_{kj}}=W_{ki}V_{ji}\rightarrow\frac{\partial \lambda_1}{\partial \mathbf{A}}=\frac{1}{7}\begin{bmatrix}3 & 3\\ 4 & 4\end{bmatrix},\quad \frac{\partial \lambda_2}{\partial \mathbf{A}}=\frac{1}{7}\begin{bmatrix}4 & -3\\ -4 & 3\end{bmatrix}$$

- For example with

$$\hat{\mathbf{A}}=\begin{bmatrix}1 & 4\\ 3 & 3\end{bmatrix},\quad \hat{\lambda}_{1,2}=5.61,-1.61,$$

Eigenvalue Parameter Sensitivity

- This is simplified by noting that $\mathbf{w}_i^t(\mathbf{A}-\lambda_i\mathbf{I})=0$ by the definition of $\mathbf{w}_i$ being a left eigenvector
- Therefore

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_i = \frac{\partial \lambda_i}{\partial A_{kj}}$$

- Since all elements of $\frac{\partial \mathbf{A}}{\partial A_{kj}}$ are zero, except the kth row, jth column is 1
- Thus $\frac{\partial \lambda_i}{\partial A_{kj}} = W_{ki} V_{ji}$

Participation Factors

- The participation factors, P_{ki}, are used to determine how much the k^{th} state variable participates in the i^{th} mode

$$P_{ki} = V_{ki} W_{ki}$$

- The sum of the participation factors for any mode or any variable sum to 1
- The participation factors are quite useful in relating the eigenvalues to portions of a model

Participation Factors

- For the previous example with $P_{ki} = V_{ki}W_{ik}$ and

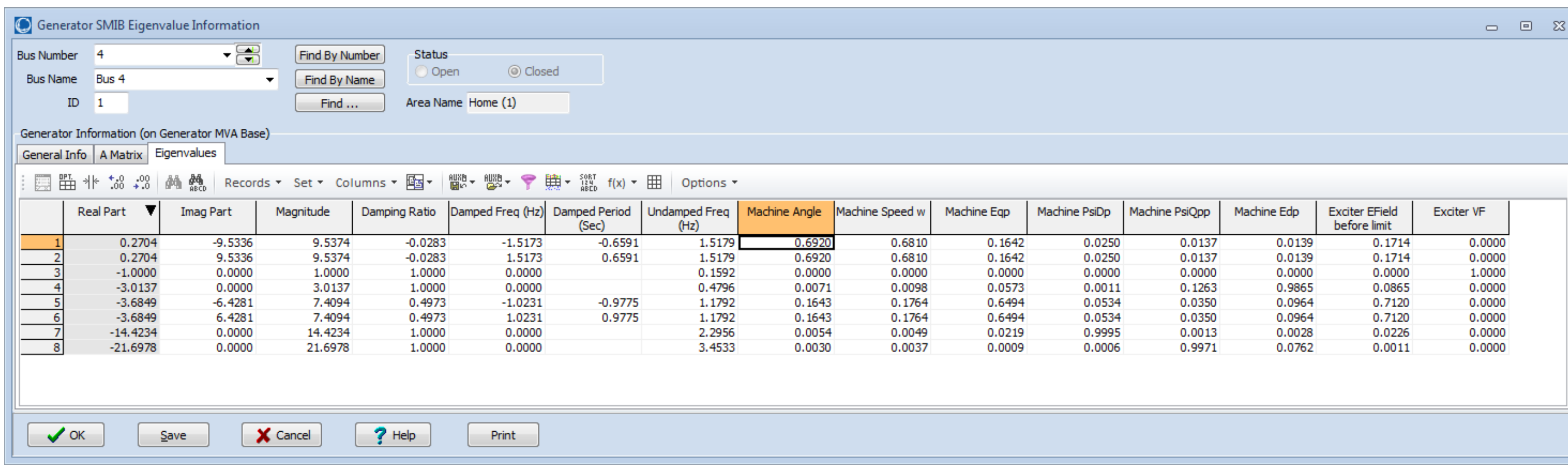
$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{W} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

- We get

$$\mathbf{P} = \frac{1}{7}\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

PowerWorld SMIB Participation Factors

- The magnitudes of the participation factors are shown on the PowerWorld SMIB dialog
- The below values are shown for the four bus example with $Q_4 = 0$



	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Speed w	Machine Eqp	Machine PsiDp	Machine PsiQpp	Machine Edp	Exciter EField before limit	Exciter VF
1	0.2704	-9.5336	9.5374	-0.0283	-1.5173	-0.6591	1.5179	0.6920	0.6810	0.1642	0.0250	0.0137	0.0139	0.1714	0.0000
2	0.2704	9.5336	9.5374	-0.0283	1.5173	0.6591	1.5179	0.6920	0.6810	0.1642	0.0250	0.0137	0.0139	0.1714	0.0000
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
4	-3.0137	0.0000	3.0137	1.0000	0.0000		0.4796	0.0071	0.0098	0.0573	0.0011	0.1263	0.9865	0.0865	0.0000
5	-3.6849	-6.4281	7.4094	0.4973	-1.0231	-0.9775	1.1792	0.1643	0.1764	0.6494	0.0534	0.0350	0.0964	0.7120	0.0000
6	-3.6849	6.4281	7.4094	0.4973	1.0231	0.9775	1.1792	0.1643	0.1764	0.6494	0.0534	0.0350	0.0964	0.7120	0.0000
7	-14.4234	0.0000	14.4234	1.0000	0.0000		2.2956	0.0054	0.0049	0.0219	0.9995	0.0013	0.0028	0.0226	0.0000
8	-21.6978	0.0000	21.6978	1.0000	0.0000		3.4533	0.0030	0.0037	0.0009	0.0006	0.9971	0.0762	0.0011	0.0000

Case is saved as **B4_GENROU_Sat_SMIB_QZero**

Oscillations

- An oscillation is just a repetitive motion that can be either undamped, positively damped (decaying with time) or negatively damped (growing with time)
- If the oscillation can be written as a sinusoid then

$$e^{\alpha t}\left(a\cos(\omega t)+b\sin(\omega t)\right)=e^{\alpha t}C\cos(\omega t+\theta)$$

$$\text{where } C=\sqrt{A^2+B^2} \text{ and } \theta=\tan\left(\frac{-b}{a}\right)$$

- And the damping ratio is defined as (see Kundur 12.46)

$$\xi=\frac{-\alpha}{\sqrt{\alpha^2+\omega^2}}$$

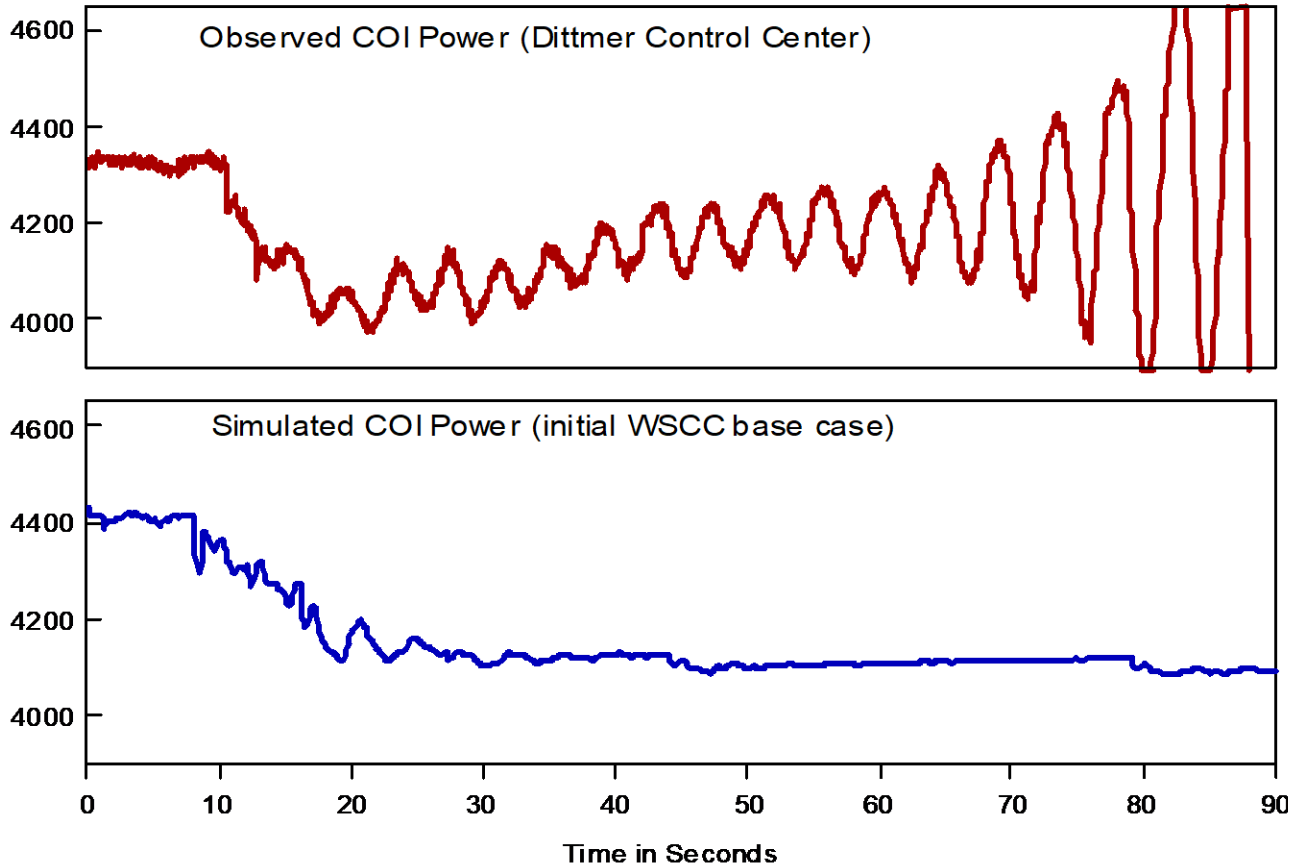
The percent damping is just the damping ratio multiplied by 100; goal is sufficiently positive damping

Power System Oscillations

- Power systems can experience a wide range of oscillations, ranging from highly damped and high frequency switching transients to sustained low frequency (< 2 Hz) inter-area oscillations affecting an entire interconnect
- Types of oscillations include
  - Transients: Usually high frequency and highly damped
  - Local plant: Usually from 1 to 5 Hz
  - Inter-area oscillations: From 0.15 to 1 Hz
  - Slower dynamics: Such as AGC, less than 0.15 Hz
  - Subsynchronous resonance: 10 to 50 Hz (less than synchronous)

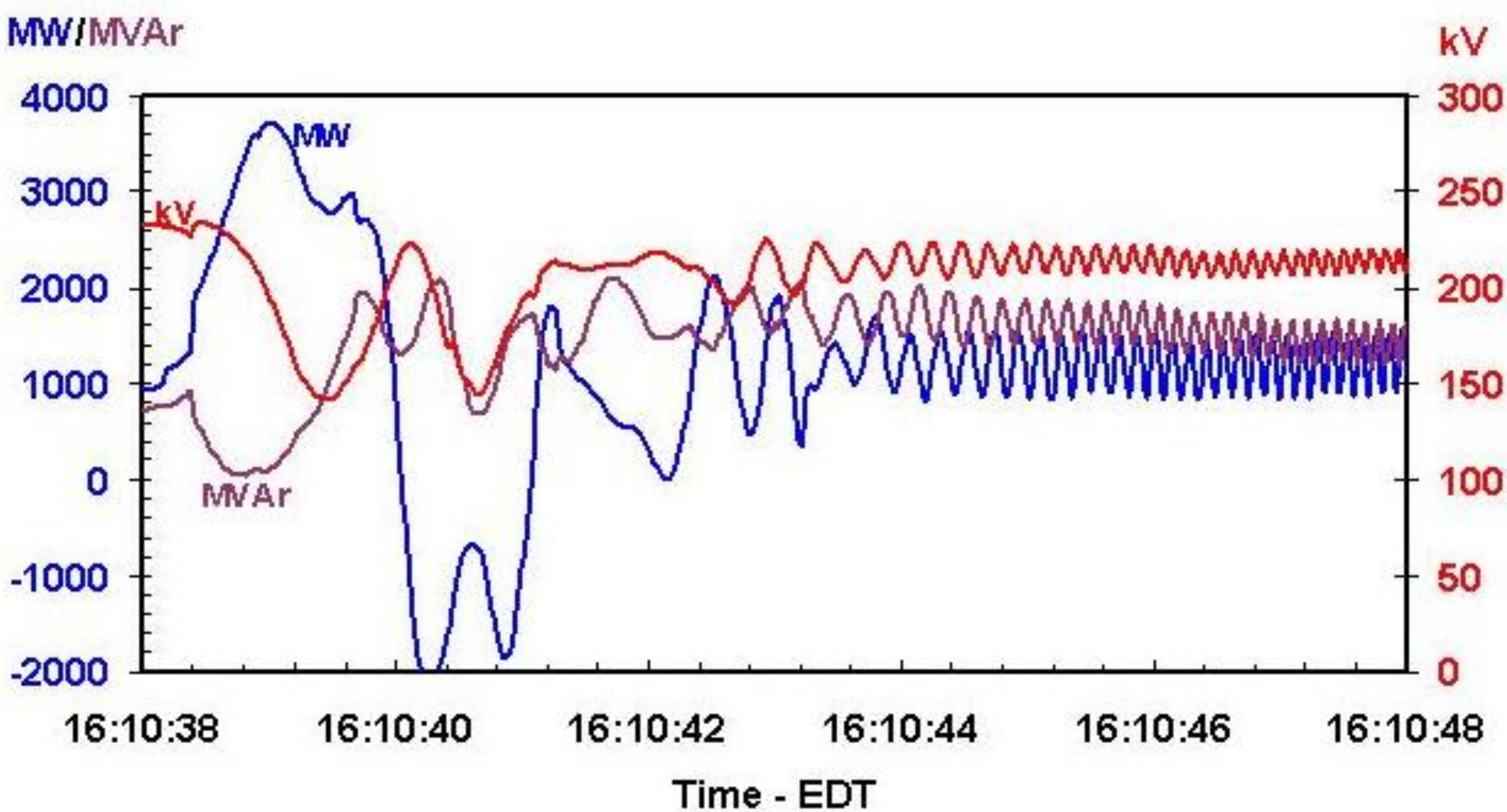
Example Oscillations

- The below graph shows an oscillation that was observed during a 1996 WECC Blackout

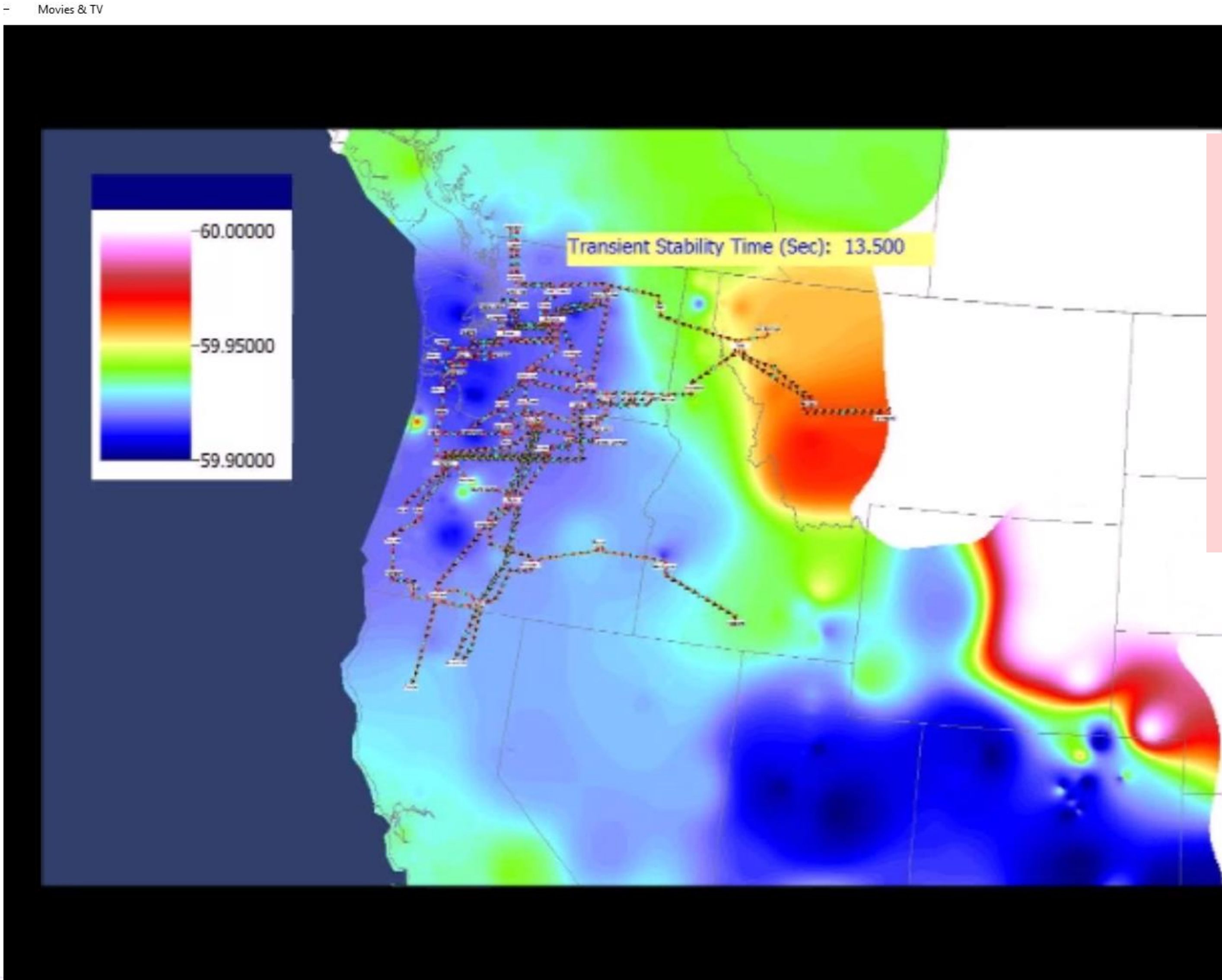


Example Oscillations

- The below graph shows oscillations on the Michigan/Ontario Interface on 8/14/03



Fictitious System Oscillation



Movie shows an example of sustained oscillations in an equivalent system

Forced Oscillations in WECC (from [1])

- Summer 2013 24 hour data: 0.37 Hz oscillations observed for several hours. Confirmed to be forced oscillations at a hydro plant from vortex effect.
- 2014 data: Another 0.5 Hz oscillation also observed. Source points to hydro unit as well. And 0.7 Hz. And 1.12 Hz. And 2 Hz.
- Resonance is possible when a system mode is poorly damped and close. Resonance can be observed in model simulations

1. M. Venkatasubramanian, "Oscillation Monitoring System", June 2015
http://www.energy.gov/sites/prod/files/2015/07/f24/3.%20Mani%20Oscillation%20Monitoring.pdf

Inter-Area Modes in the WECC

- The dominant inter-area modes in the WECC have been well studied
- A good reference paper is D. Trudnowski, "Properties of the Dominant Inter-Area Modes in the WECC Interconnect," 2012
  - Four well known modes are NS Mode A (0.25 Hz), NS Mode B (or Alberta Mode), (0.4 Hz), BC Mode (0.6 Hz), Montana Mode (0.8 Hz)

Below figure from paper shows NS Mode A On May 29, 2012