ECEN 667
Power System Stability

Lecture 8: Synchronous Machine Modeling

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Announcements

- Read Chapter 5 and Appendix A
- Homework 2 is due today
- Homework 3 is due on Tuesday October 1
- Exam 1 is Thursday October 10 during class

Subtransient Models

- The two-axis model is a transient model
- Essentially all commercial studies now use subtransient models
- First models considered are GENSAL and GENROU, which require $X''_d = X''_q$
- This allows the internal, subtransient voltage to be represented as

$$\bar{E}'' = \bar{V} + (R_s + jX'')\bar{I}$$

$$E''_d + jE''_q = \left(-\psi''_q + j\psi''_d\right)\omega$$

Subtransient Models

- Usually represented by a Norton Injection with

$$I_d + jI_q = \frac{E_d'' + jE_q''}{R_s + jX''} = \frac{\left(-\psi_q'' + j\psi_d''\right)\omega}{R_s + jX''}$$

- May also be shown as

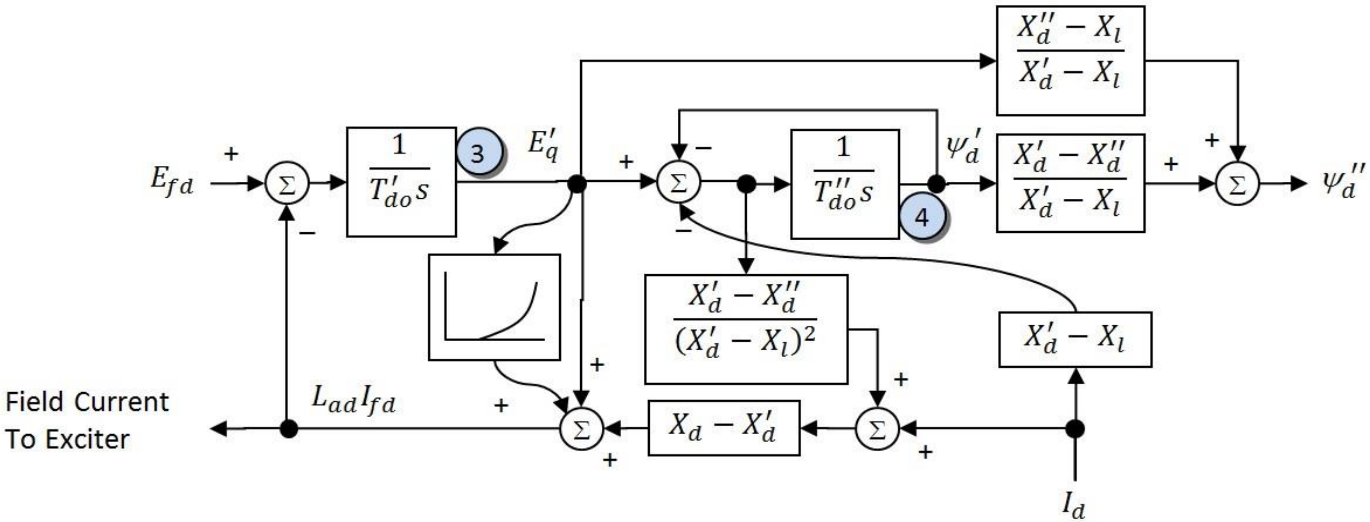
$$-j\left(I_d + jI_q\right) = I_q - jI_d = \frac{-j\left(-\psi_q'' + j\psi_d''\right)\omega}{R_s + jX''} = \frac{\left(\psi_d'' + j\psi_q''\right)\omega}{R_s + jX''}$$

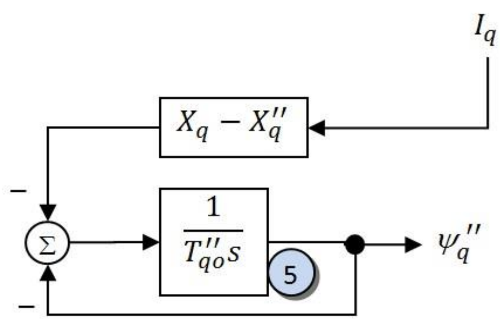
In steady-state ω = 1.0

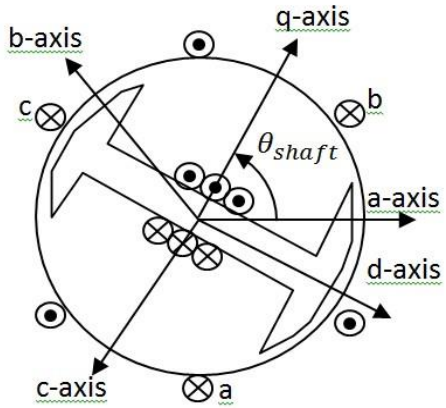
GENSAL

- The GENSAL model had been widely used to model salient pole synchronous generators
  - In salient pole models saturation is only assumed to affect the d-axis
  - In the 2010 WECC cases about 1/3 of machine models were GENSAL; in 2013 essentially none are, being replaced by GENTPF or GENTPJ
  - A 2014 series EI model had about 1/3 of its machines models set as GENSAL
  - In November 2016 NERC issued a recommendation to use GENTPJ rather than GENSAL for new models. See www.nerc.com/comm/PC/NERCModelingNotifications/Use%20of%20GENTPJ%20Generator%20Model.pdf

GENSAL Block Diagram







A quadratic saturation function is used; for initialization it only impacts the E_{fd} value

GENSAL Example

- Assume same system as before with same common generator parameters: H=3.0, D=0, R_a = 0, X_d = 2.1, X_q = 2.0, X'_d = 0.3, X''_d=X''_q=0.2, X_l = 0.13, T'_{do} = 7.0, T''_{do} = 0.07, T''_{qo} =0.07, S(1.0) =0, and S(1.2) = 0.
- Same terminal conditions as before
  - Current of 1.0-j0.3286 and generator terminal voltage of 1.072+j0.22 = 1.0946 $\angle 11.59°$
- Use same equation to get initial δ

Same delta as with the other models

$$|E|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$
$$= 1.072 + j0.22 + (0.0 + j2)(1.0 - j0.3286)$$
$$= 1.729 + j2.22 = 2.81\angle 52.1°$$

Saved as case **B4_GENSAL**

GENSAL Example

- Then as before

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

and $\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$

$$\bar{V} + (R_s + jX'')\bar{I}$$

$$= 1.072 + j0.22 + (0 + j0.2)(1.0 - j0.3286)$$

$$= 1.138 + j0.42$$

GENSAL Example

- Giving the initial fluxes (with ω = 1.0) of

$$\begin{bmatrix} -\psi''_q \\ \psi''_d \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.138 \\ 0.420 \end{bmatrix} = \begin{bmatrix} 0.6396 \\ 1.031 \end{bmatrix}$$

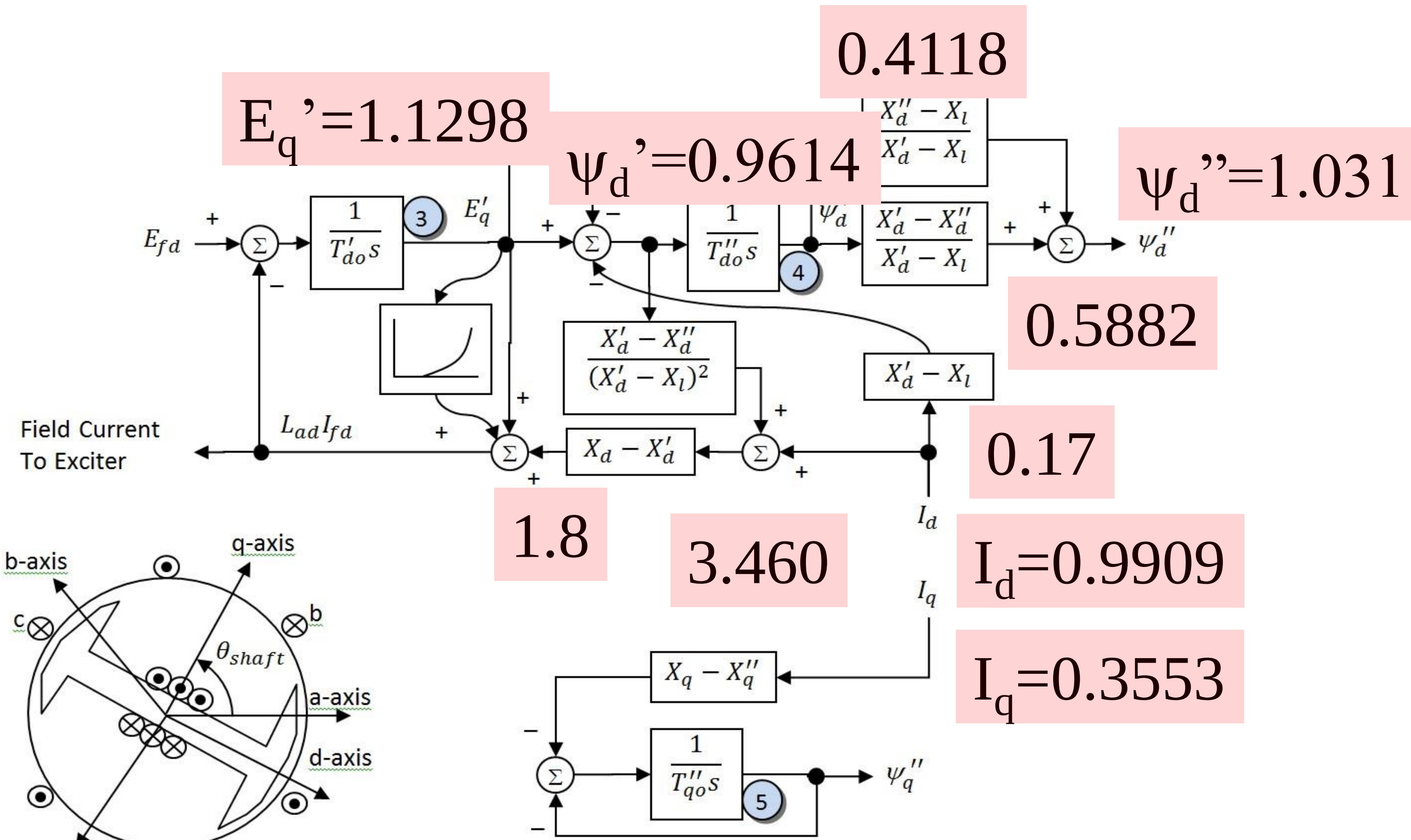
- To get the remaining variables set the differential equations equal to zero, e.g.,

$$\psi''_q = -\left(X_q - X''_q\right) I_q = -\left(2 - 0.2\right)\left(0.3553\right) = -0.6396$$

$$E'_q = 1.1298, \quad \psi'_d = 0.9614$$

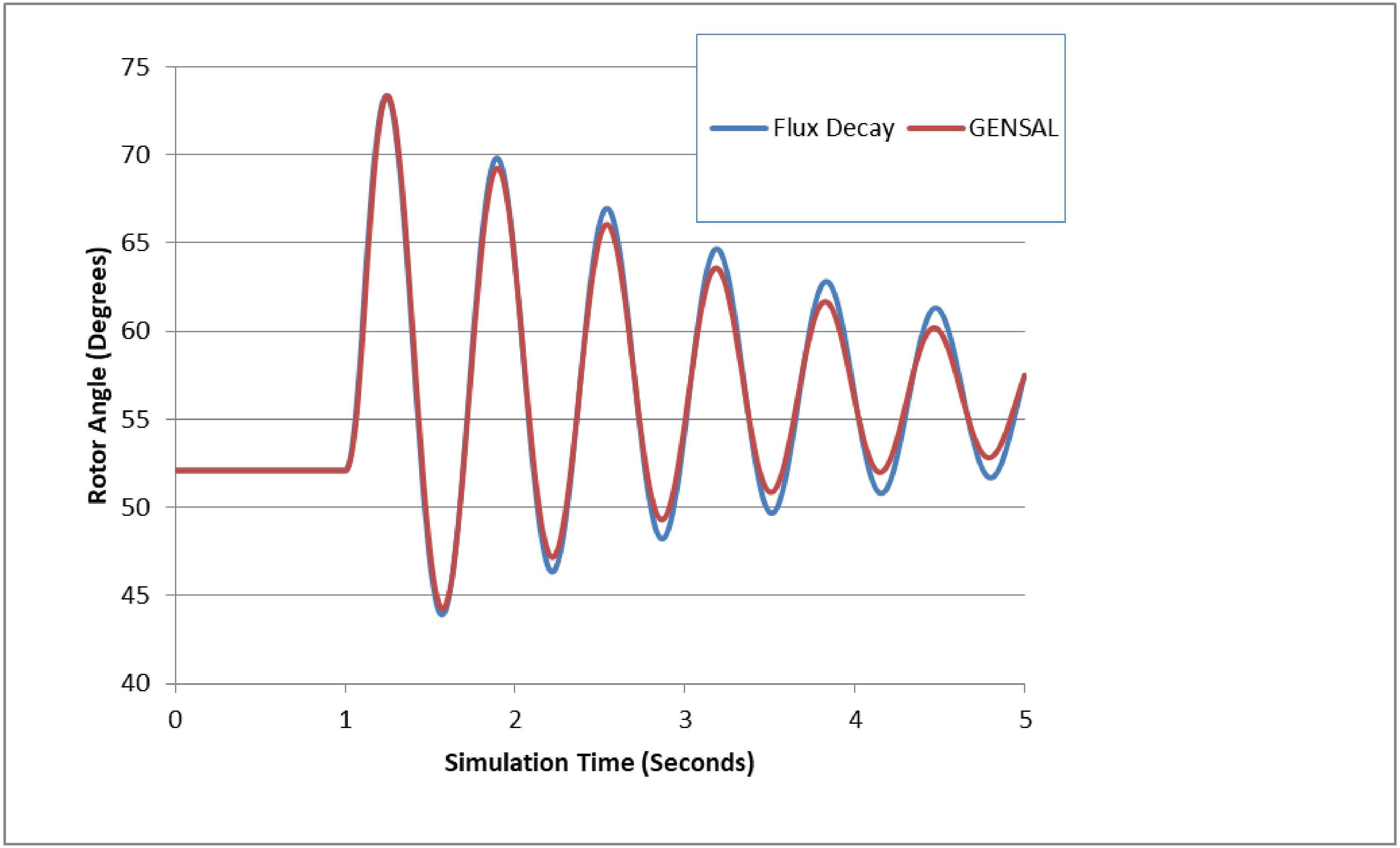
Solving the d-axis requires solving two linear equations for two unknowns

GENSAL Example



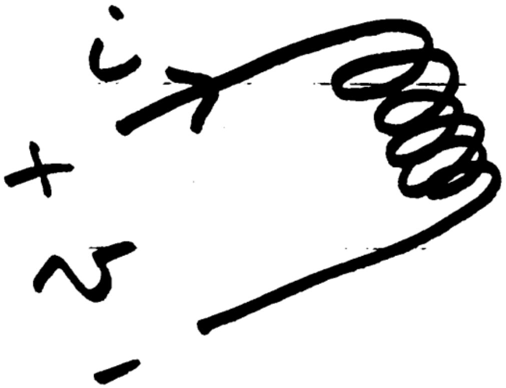
E_{fd} = 1.1298+1.8*0.991=2.912

Comparison Between Gensal and Flux Decay



Nonlinear Magnetic Circuits

- Nonlinear magnetic models are needed because magnetic materials tend to saturate; that is, increasingly large amounts of current are needed to increase the flux density

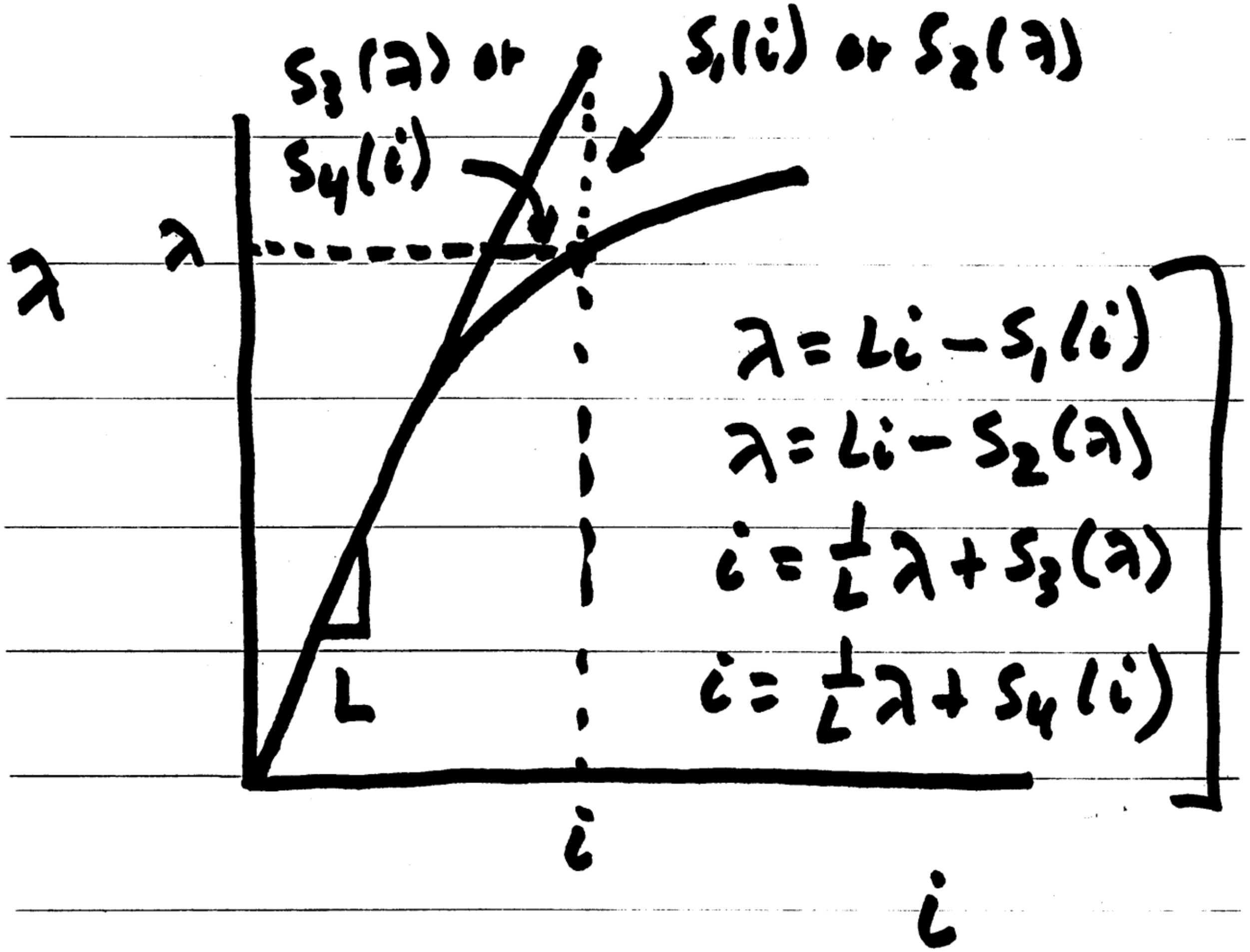


$$R = 0$$

$$v = \frac{d\lambda}{dt} = N\frac{d\phi}{dt}$$

When linear λ = Li

Saturation



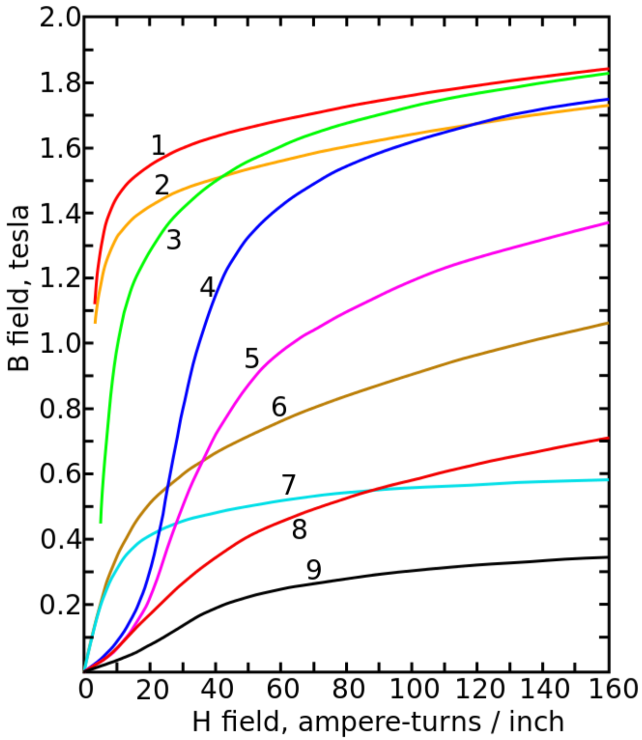
$S_3(\lambda)$ or
$S_4(i)$
$S_1(i)$ or $S_2(\lambda)$
λ
λ
L
i
i
$\lambda = Li - S_1(i)$
$\lambda = Li - S_2(\lambda)$
$i = \frac{1}{L}\lambda + S_3(\lambda)$
$i = \frac{1}{L}\lambda + S_4(i)$

Relative Magnetic Strength Levels

- Earth's magnetic field is between 30 and 70 μT (0.3 to 0.7 gauss)
- A refrigerator magnet might have 0.005 T
- A commercial neodymium magnet might be 1 T
- A magnetic resonance imaging (MRI) machine would be between 1 and 3 T
- Strong lab magnets can be 10 T
- Frogs can be levitated at 16 T (see www.ru.nl/hfml/research/levitation/diamagnetic
- A neutron star can have 100 MT!

Magnetic Saturation and Hysteresis

- The below image shows the saturation curves for various materials



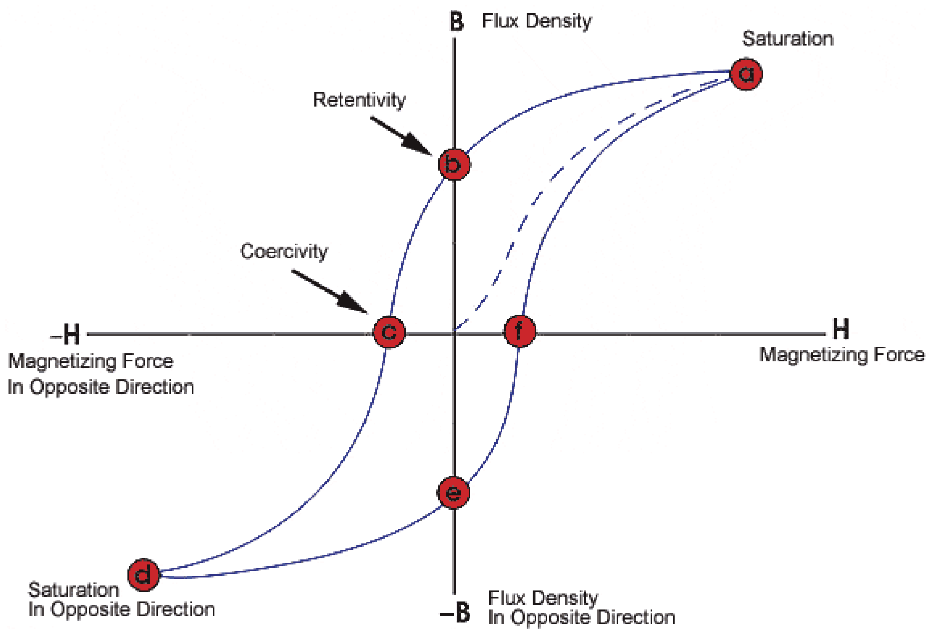
Magnetization curves of 9 ferromagnetic materials, showing saturation. 1.Sheet steel, 2.Silicon steel, 3.Cast steel, 4.Tungsten steel, 5.Magnet steel, 6.Cast iron, 7.Nickel, 8.Cobalt, 9.Magnetite; highest saturation materials can get to around 2.2 or 2.3T

H is proportional to current

Image Source: en.wikipedia.org/wiki/Saturation_(magnetic)

Magnetic Saturation and Hysteresis

- Magnetic materials also exhibit hysteresis, so there is some residual magnetism when the current goes to zero; design goal is to reduce the area enclosed by the hysteresis loop



To minimize the amount of magnetic material, and hence cost and weight, electric machines are designed to operate close to saturation

Image source: www.nde-ed.org/EducationResources/CommunityCollege/MagParticle/Graphics/BHCurve.gif

Saturation Models

- Many different models exist to represent saturation
  - There is a tradeoff between accuracy and complexity
- One simple approach is to replace

$$\frac{dE_q^{'}}{dt} = \frac{1}{T_{do}^{'}}\left(-E_q^{'} - (X_d - X_d^{'})I_d + E_{fd}\right)$$

- with

$$\frac{dE_q^{'}}{dt} = \frac{1}{T_{do}^{'}}\left(-E_q^{'} - (X_d - X_d^{'})I_d - Se(E_q^{'}) + E_{fd}\right)$$

Saturation Models

- In steady-state this becomes

$$E_{fd} = E_q' + (X_d - X_d')I_d + Se(E_q')$$

- Hence saturation increases the required E_{fd} to get a desired flux
- Saturation is usually modeled using a quadratic function, with the value of Se specified at two points (often at 1.0 flux and 1.2 flux)

$$Se = B(E_q' - A)^2$$

An alternative model is $Se = \dfrac{B(E_q' - A)^2}{E_q'}$

A and B are determined from two provided data points

Saturation Example

- If Se = 0.1 when the flux is 1.0 and 0.5 when the flux is 1.2, what are the values of A and B using the

$$Se = B(E_q^{'} - A)^2$$

To solve use the Se(1.2) value to eliminate B

$$B = \frac{Se(1.2)}{(1.2-A)^2} \rightarrow Se(1.0) = \frac{Se(1.2)}{(1.2-A)^2}(1.0-A)^2$$

$$(1.2-A)^2 Se(1.0) = Se(1.2)(1.0-A)^2$$

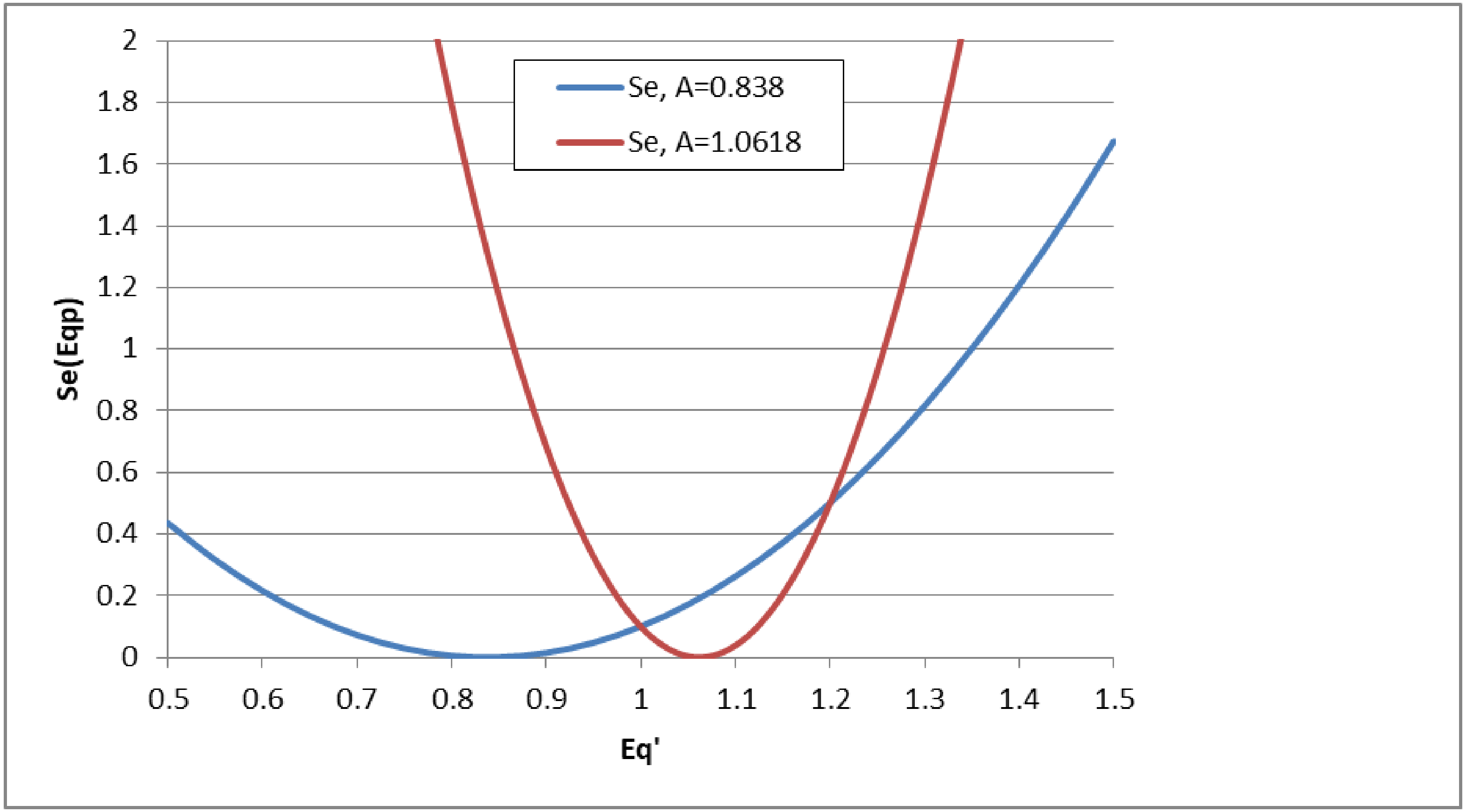
With the values we get

$$4A^2 - 7.6A + 3.56 = 0 \rightarrow A = 0.838 \text{ or } 1.0618$$

Use A=0.838, which gives B=3.820

Saturation Example: Selection of A

When selecting which of the two values of A to use, we do not want the minimum to be between the two specified values. That is Se(1.0) and Se(1.2).



Implementing Saturation Models

- When implementing saturation models in code, it is important to recognize that the function is meant to be positive, so negative values are not allowed
- In large cases one is almost guaranteed to have special cases, sometimes caused by user typos
  - What to do if Se(1.2) < Se(1.0)?
  - What to do if Se(1.0) = 0 and Se(1.2) <> 0
  - What to do if Se(1.0) = Se(1.2) <> 0
- Exponential saturation models have also been used

GENSAL Example with Saturation

- Once E'_q has been determined, the initial field current (and hence field voltage) are easily determined by recognizing in steady-state the E'_q is zero

$$E_{fd} = E'_q\left(1 + Sat(E'_q)\right) + \left(X_d - X'_d\right) I_D$$

$$= 1.1298\left(1 + B(1.1298 - A)^2\right) + \left(2.1 - 0.3\right)(0.9909)$$

$$= 1.1298\left(1 + 3.82(1.1298 - 0.838)^2\right) + 1.784 = 3.28$$

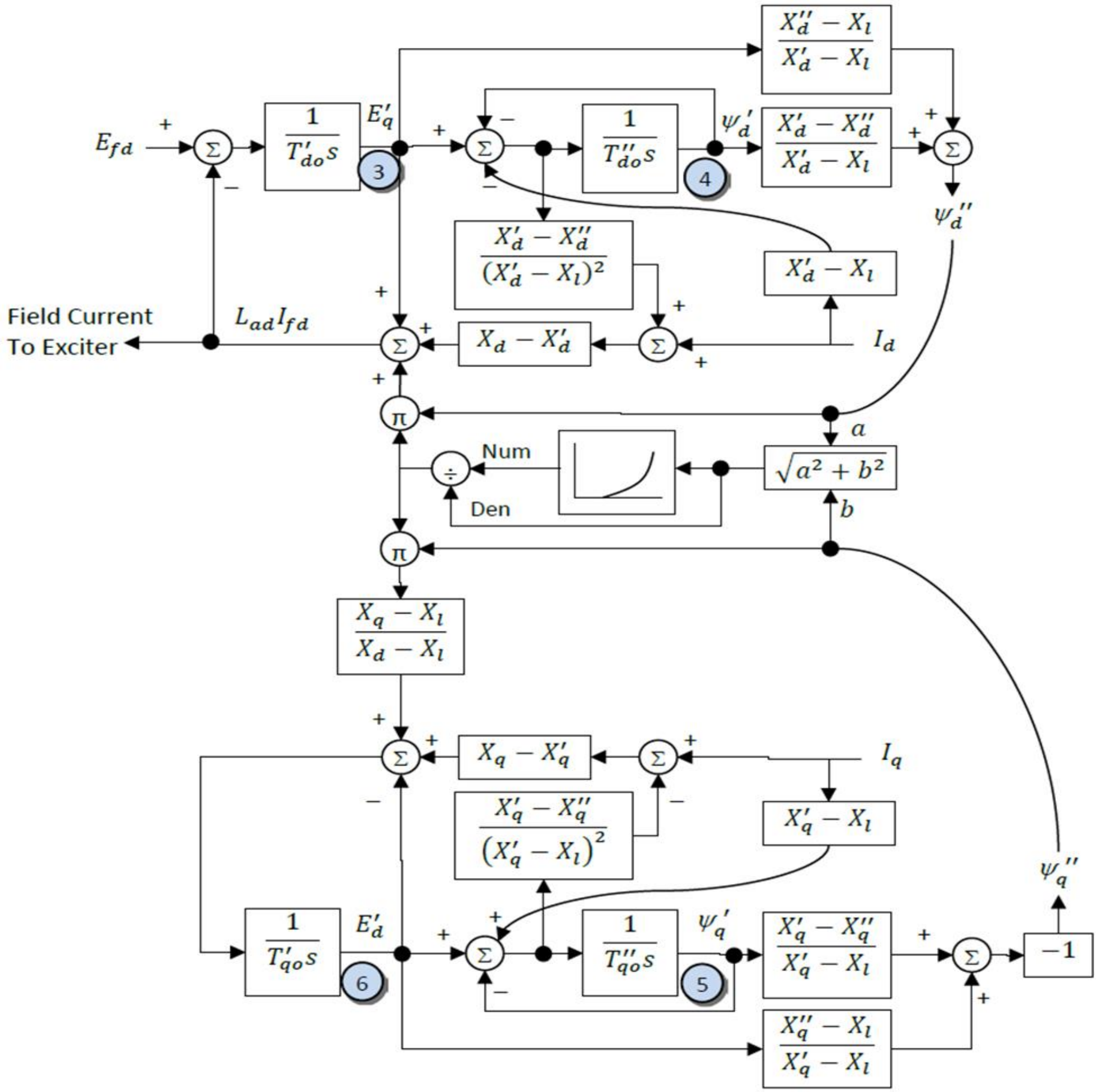
Saturation coefficients were determined from the two initial values

Saved as case **B4_GENSAL_SAT**

GENROU

- The GENROU model has been widely used to model round rotor machines
- Saturation is assumed to occur on both the d-axis and the q-axis, making initialization slightly more difficult

GENROU



The d-axis is similar to that of the GENSAL; the q-axis is now similar to the d-axis. Note that saturation now affects both axes.

GENROU Initialization

- Because saturation impacts both axes, the simple approach will no longer work
- Key insight for determining initial δ is that the magnitude of the saturation depends upon the magnitude of ψ'', which is independent of δ

$$\left|\psi''\right| = \left|\bar{V} + (R_s + jX'')\bar{I}\right|$$

This point is crucial!

- Solving for δ requires an iterative approach; first get a guess of δ using the unsaturated approach

$$\left|E\right|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$

GENROU Initialization

- Then solve five nonlinear equations for five unknowns
  - The five unknowns are δ, E'_q, E'_d, ψ'_q, and ψ'_d
- Five equations come from the terminal power flow constraints (which allow us to define ψ_d'' and ψ_q'' as a function of the power flow voltage, current and δ) and from the differential equations initially set to zero
  - The ψ_d'' and ψ_q'' block diagram constraints
  - Two differential equations for the q-axis, one for the d-axis (the other equation is used to set the field voltage
- Values can be determined using Newton's method, which is needed for the nonlinear case with saturation

GENROU Initialization

- Use dq transform to express terminal current as

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$

These values will change during the iteration as δ changes

- Get expressions for ψ''_q and ψ''_d in terms of the initial terminal voltage and δ
  - Use dq transform to express terminal voltage as

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$

Recall $X''_d = X''_q = X''$ and ω=1 (in steady-state)

  - Then from $-\psi''_q + j\psi''_d = \left(V_d + jV_q\right) + (R_s + jX'')\left(I_d + jI_q\right)$

$$-\psi''_q = V_d + R_s I_d - X'' I_q$$

$$\psi''_d = V_q + R_s I_a + X'' I_d$$

Expressing complex equation as two real equations

GENROU Initialization Example

- Extend the two-axis example
  - For two-axis assume H = 3.0 per unit-seconds, R_s=0, X_d = 2.1, X_q = 2.0, X'_d= 0.3, X'_q = 0.5, T'_{do} = 7.0, T'_{qo} = 0.75 per unit using the 100 MVA base.
  - For subtransient fields assume X''_d=X''_q=0.28, X_l = 0.13, T''_{do} = 0.073, T''_{qo} =0.07
  - for comparison we'll initially assume no saturation
- From two-axis get a guess of δ

$$\bar{E} = 1.0946\angle 11.59^\circ + (j2.0)(1.052\angle -18.2^\circ) = 2.814\angle 52.1^\circ$$

$$\rightarrow \delta = 52.1^\circ$$

Saved as case **B4_GENROU_NoSat**

GENROU Initialization Example

- And the network current and voltage in dq reference

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

- Which gives initial subtransient fluxes (with R_s=0),

$$\left(-\psi''_q + j\psi''_d\right)\omega = \left(V_d + jV_q\right) + (R_s + jX'')\left(I_d + jI_q\right)$$

$$-\psi''_q\,\omega = V_d + R_s I_d - X'' I_q = 0.7107 - 0.28 \times 0.3553 = 0.611$$

$$\psi''_d\,\omega = V_q + R_s I_a + X'' I_d = 0.8326 + 0.28 \times 0.9909 = 1.110$$

GENROU Initialization Example

- Without saturation this is the exact solution

Initial values are:

$\delta = 52.1°$,

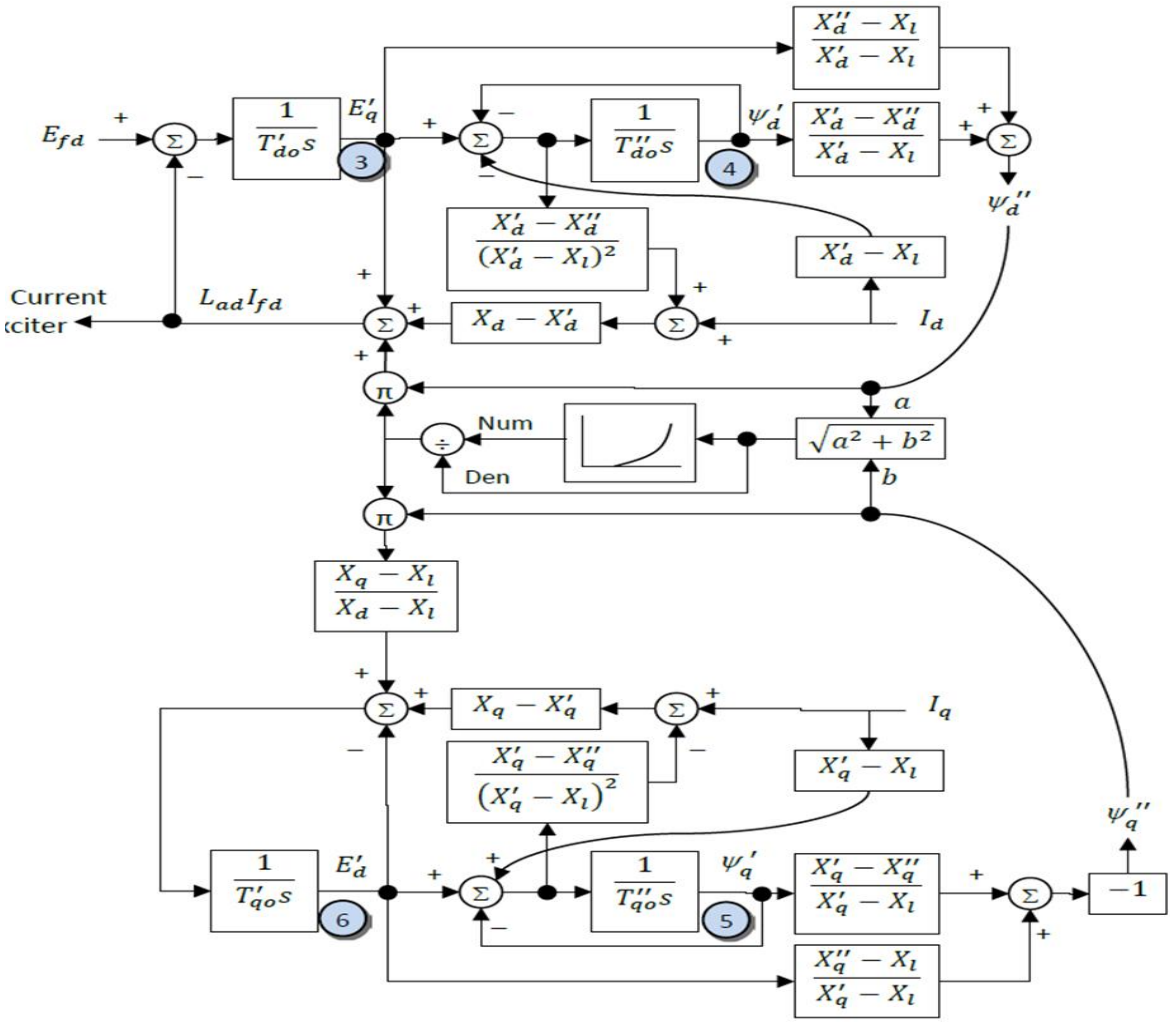
$E'_q = 1.1298$,

$E'_d = 0.533$,

$\psi'_q = 0.6645$,

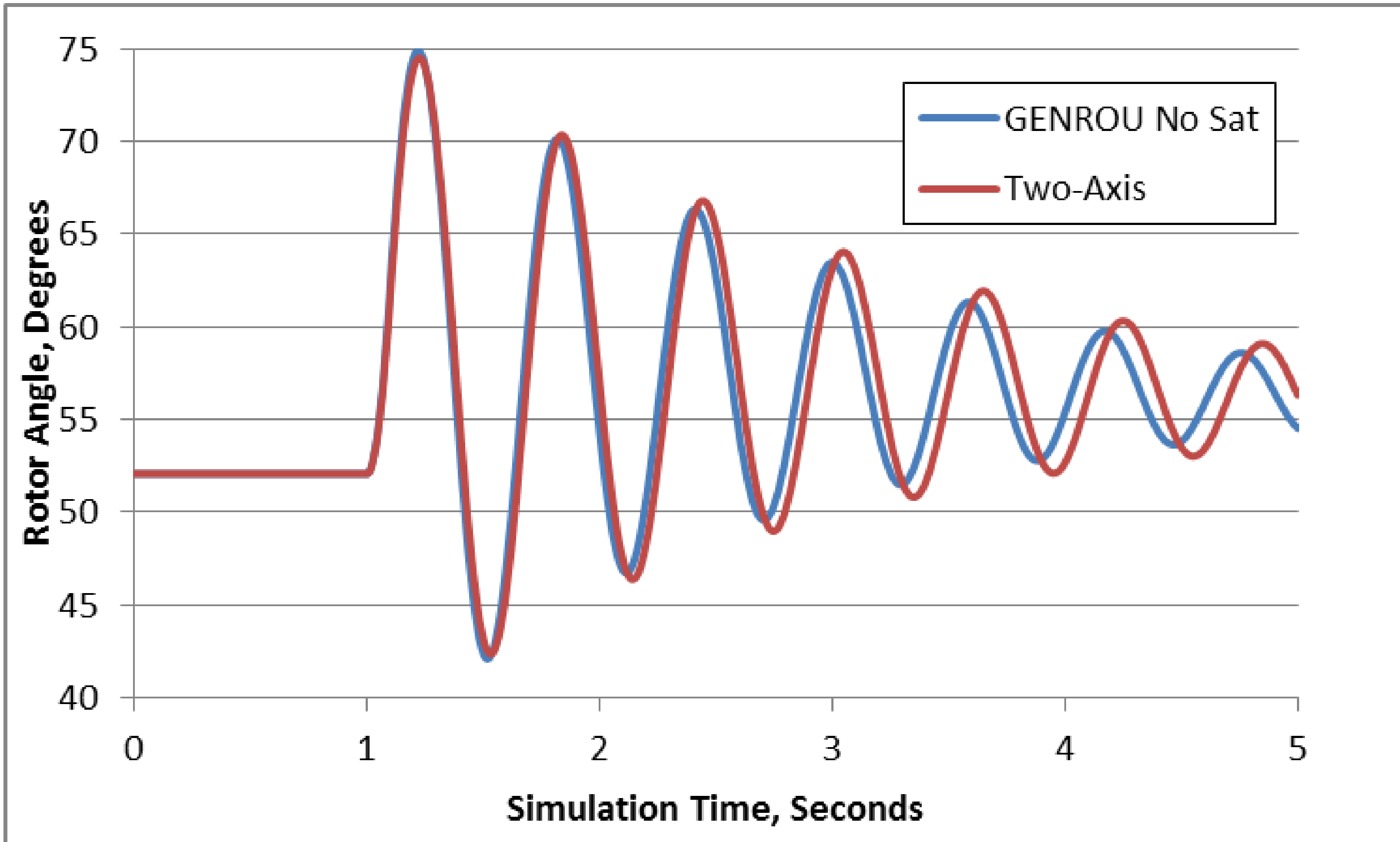
and $\psi'_d = 0.9614$

$E_{fd} = 2.9133$



Two-Axis versus GENROU Response

Figure compares rotor angle for bus 3 fault, cleared after 0.1 seconds

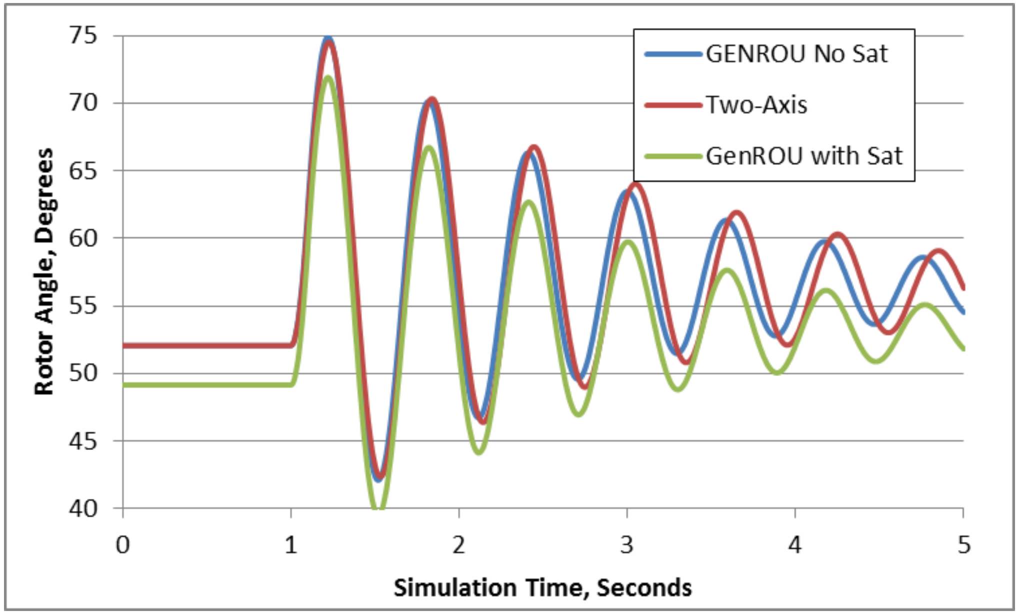


GENROU with Saturation

- Nonlinear approach is needed in common situation in which there is saturation
- Assume previous GENROU model with S(1.0) = 0.05, and S(1.2) = 0.2.
- Initial values are: $\delta = 49.2°$, E'_q=1.1591, E'_d=0.4646, ψ'_q=0.6146, and ψ'_d=0.9940
- E_{fd}=3.2186

Same fault as before

Saved as case **B4_GENROU_Sat**

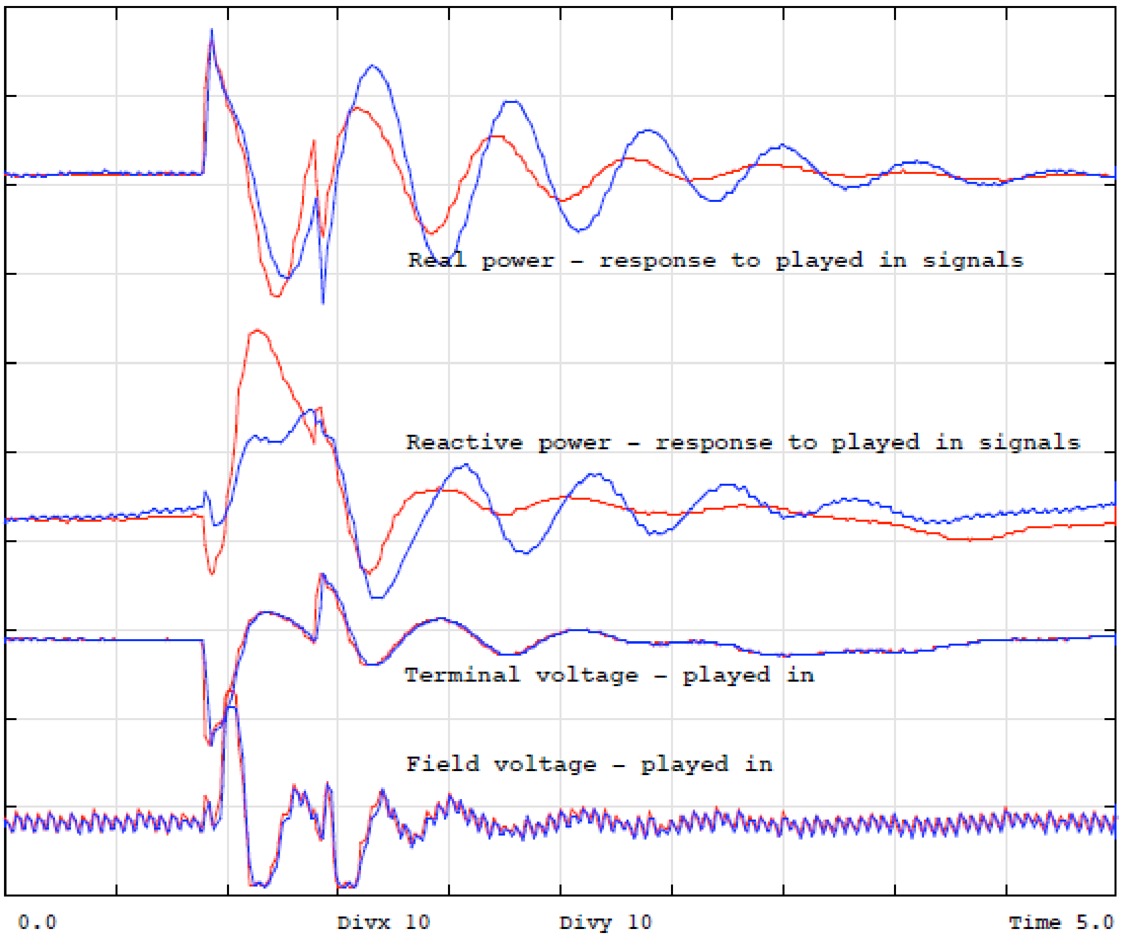


GENTPF and GENTPJ Models

- These models were introduced in 2009 to provide a better match between simulated and actual system results for salient pole machines
  - Desire was to duplicate functionality from old BPA TS code
  - Allows for subtransient saliency ($X''_d <> X''_q$)
  - Can also be used with round rotor, replacing GENSAL and GENROU
- Useful reference is available at below link; includes all the equations, and saturation details

https://www.wecc.biz/Reliability/gentpj-typej-definition.pdf

Motivation for the Change: GENSAL Actual Results



Chief Joseph disturbance playback GENSAL

BLUE = MODEL

RED = ACTUAL

(Chief Joseph is a 2620 MW hydro plant on the Columbia River in Washington)

Image source :https://www.wecc.biz/library/WECC%20Documents/Documents%20for %20Generators/Generator%20Testing%20Program/gentpj%20and%20gensal%20morel.pdf

GENTPJ Results

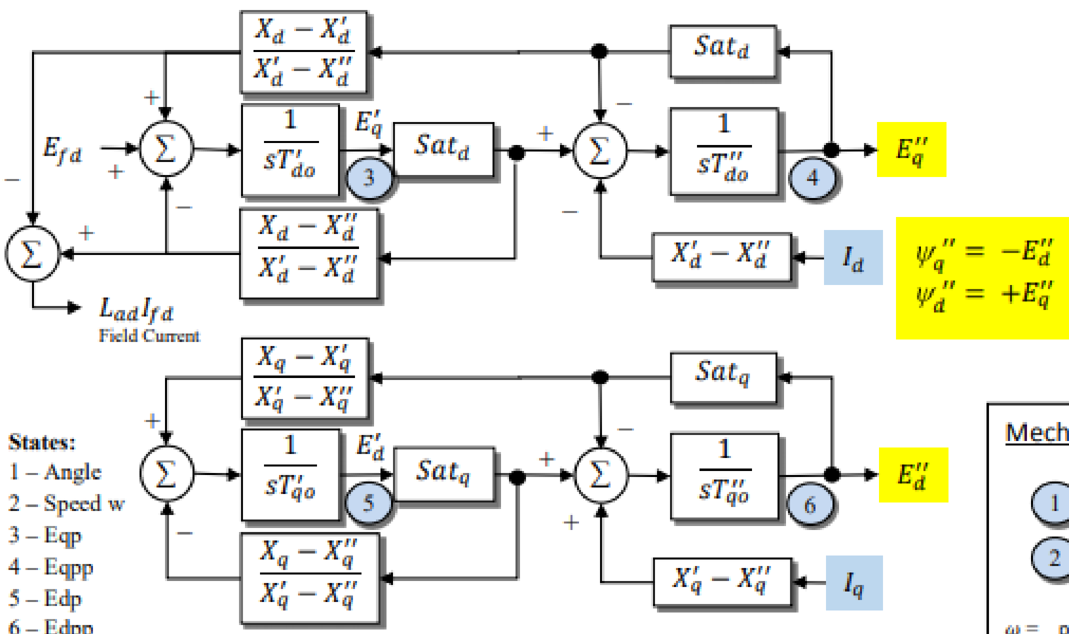
Chief Joseph disturbance playback

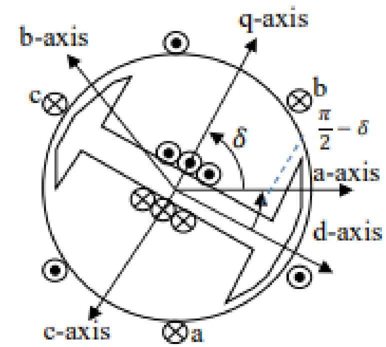
GENTPJ

BLUE = MODEL

RED = ACTUAL

GENTPF and GENTPJ Models





$$\psi_{ag} = \sqrt{\left(V_{qterm} + I_q R_a + I_d X_l\right)^2 + \left(V_{dterm} + I_d R_a - I_q X_l\right)^2}$$

D-Axis: $Sat_d = 1 + SaturationFunction\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

Q-Axis: $Sat_q = 1 + \frac{X_q}{X_d} SaturationFunction\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

Mechanical Swing Equations

1 $\dot{\delta} = \omega * \omega_0$

2 $\dot{\omega} = \frac{1}{2H}\left(\frac{P_{mech} - D\omega}{1 + \omega} - T_{elec}\right)$

$\omega =$ per unit speed deviation, so $\omega = 0$ means we are at synchronous speed and $\omega = 1$ would mean it's spinning at double synchronous speed

$\omega_0 =$ synchronous speed $2\pi f_0$ where f_0 is the nominal system frequency in Hz

Note: If option *Ignore Speed Effects in Generator Swing Equation* is true, then instead use

$$\dot{\omega} = \frac{1}{2H}\left(P_{mech} - D\omega - T_{elec}\right)$$

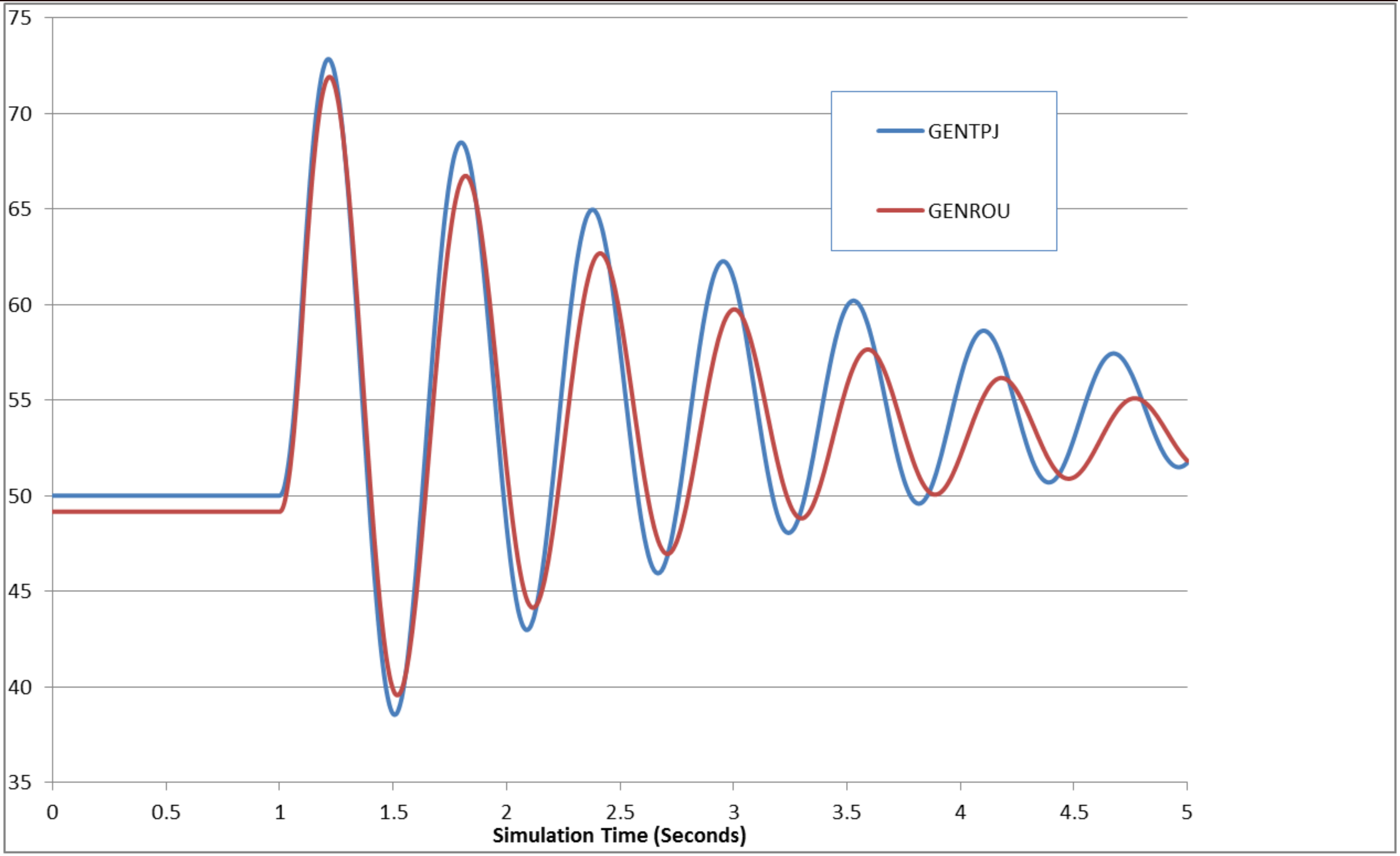
Most of WECC machine models are now GENTPF or GENTPJ

If nonzero, K_{is} typically ranges from 0.02 to 0.12

Theoretical Justification for GENTPF and GENTPJ

- In the GENROU and GENSAL models saturation shows up purely as an additive term of E'_q and E'_d
  - Saturation does not come into play in the network interface equations and thus with the assumption of $X''_q = X''_d$ a simple circuit model can be used
- The advantage of the GENTPF/J models is saturation really affects the entire model, and in this model it is applied to all the inductance terms simultaneously
  - This complicates the network boundary equations, but since these models are designed for $X''_q \neq X''_d$ there is no increase in complexity

GENROU/GENTPJ Comparison



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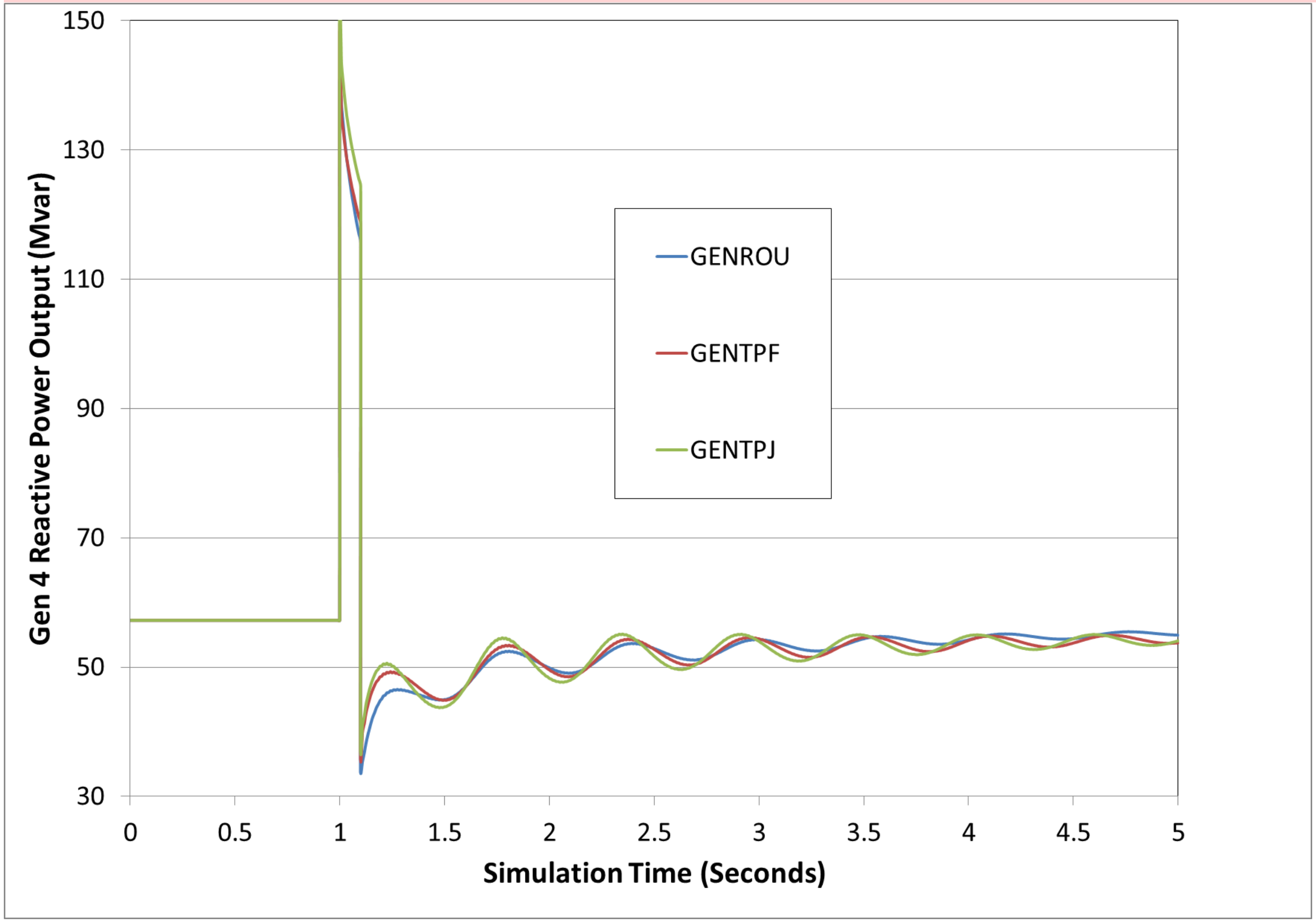
GENROU, GenTPF, GenTPJ

Figure compares gen 4 reactive power output for the 0.1 second fault



Why does this even matter?

- GENROU and GENSAL models date from 1970, and their purpose was to replicate the dynamic response the synchronous machine
  - They have done a great job doing that
- Weaknesses of the GENROU and GENSAL model has been found to be with matching the field current and field voltage measurements
  - Field Voltage/Current may have been off a little bit, but that didn't effect *dynamic* response
  - It just *shifted* the values and gave them an offset
- Shifted/Offset field voltage/current didn't matter too much in the past