Iterative Matrix Pencil Method of Power System Modal Analysis

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Measurement Based Modal Analysis

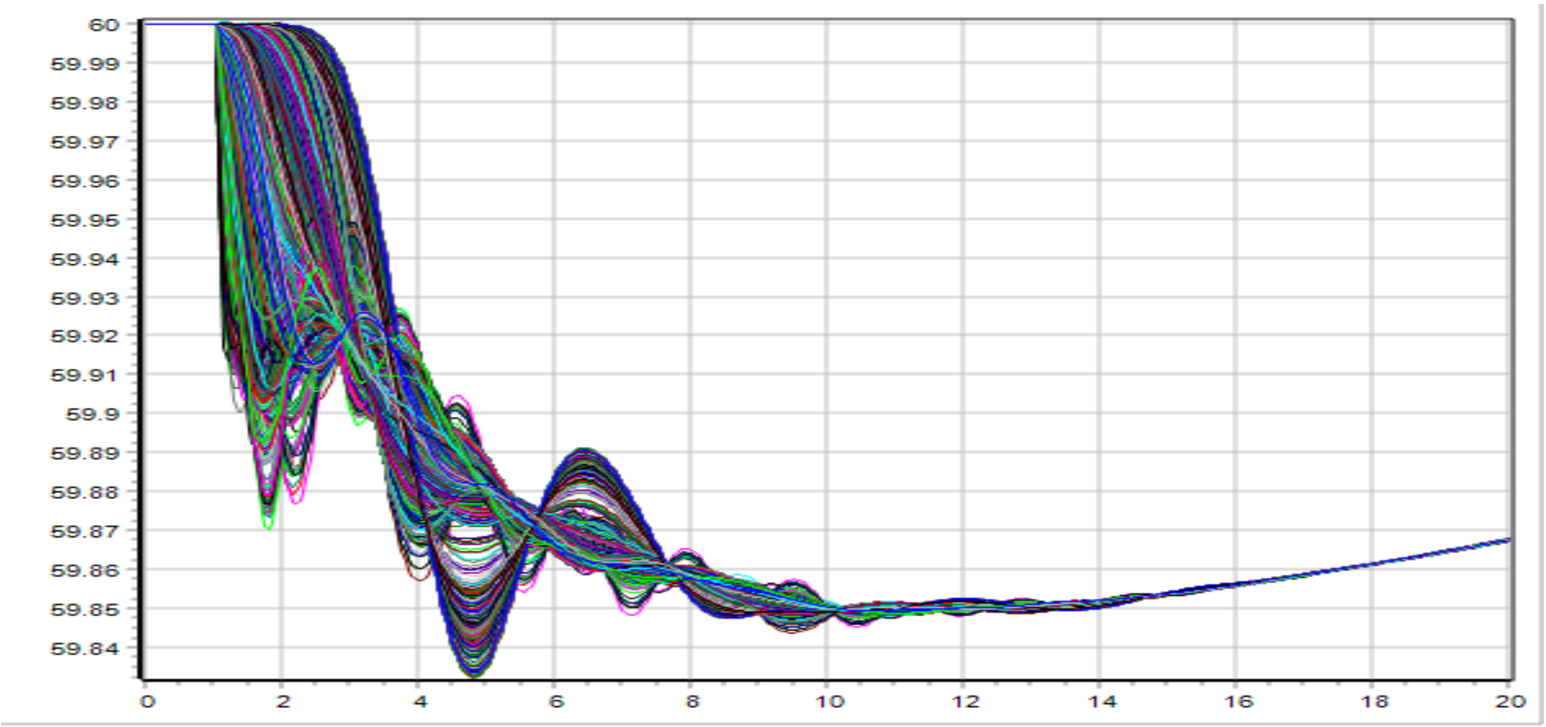
- With the advent of large numbers of PMUs, measurement based modal analysis is increasingly used to understand power system behavior
  - The goal is to determine the damping associated with the dominant oscillatory modes in the system
  - Approaches seek to approximate a sampled signal by a series of exponential functions (usually damped sinusoidals)
- Several techniques are available
  - Prony is the oldest, dating to 1795, with power system applications from about 1980's
- Paper discusses an approach to quickly handle a large number of signals

Ring-down Modal Analysis

- A variety of different techniques can be used to approximate a signal, $y_{org}(t)$, by the sum of other, simpler signals (basis functions)
  - Basis functions are usually exponentials, with linear and quadratic functions used to detrend the signal
  - Properties of the original signal can be quantified from basis function properties
    - Examples are frequency and damping
  - Signal is considered over some time window, with t=0 defined as the beginning of the window
- Starting point is time-varying signal, $y_{org}(t)$, that is then assumed to be uniformly sampled

Example Application

- An example application is to make sense of the frequency response following a contingency
  - Below example at the shows the frequency variation at 8400 substations



Measurement-Based Modal Analysis

- Vector **y** consists of m uniformly sampled points from $y_{org}(t)$ at a sampling value of DT, starting with t=0, with values y_j for j=1…m
  - Times are then $t_j = (j-1)DT$
  - At each time point j, the approximation of y_j is

$$\hat{y}_j(\boldsymbol{\alpha}, \mathbf{b}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}$ is a vector with the real and imaginary eigenvalue components,

with $\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j}$ for α_{i} corresponding to a real eigenvalue, and

$$\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j} \cos(\alpha_{\mathrm{i+1}} \mathrm{t}_j) \text{ and } \phi_{i+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \sin(\alpha_{\mathrm{i+1}} \mathrm{t}_j)$$

for a complex eigenvector value

Measurement-Based Modal Analysis

- Error (residual) value at each point j is

$$r_j(t_j, \boldsymbol{\alpha}, \mathbf{b}) = y_j - \hat{y}_j(t_j, \boldsymbol{\alpha}, \mathbf{b})$$

- Closeness of fit can be quantified using the Euclidean norm of the residuals as a cost function

$$\frac{1}{2}\sum_{j=1}^{m}(y_j - \hat{y}_j(t_j, \boldsymbol{\alpha}, \mathbf{b}))^2 = \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\alpha}, \mathbf{b})\right\|_2^2$$

- Hence we need to determine **a** and **b**
- Approaches can be used with multiple signals, with **a** common to all signals, and **b** signal specific

Matrix Pencil Method

- The **a** vector can be calculated using the Matrix Pencil Method (MPM)
- First, with m samples, let L=m/2
- Then form a Hankel matrix, **Y** such that

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{L+1} \\ y_2 & y_3 & \cdots & y_{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L} & y_{m-L+1} & \cdots & y_m \end{bmatrix}$$

The computational complexity increases with the cube of the number of measurements!

- Calculate its singular values with an economy SVD

Matrix Pencil Method (MPM)

- The ratio of each singular value is then compared to the largest singular value; retain the M ones with a ratio greater than a threshold
  - This determines the modal order
  - Assuming **V** is ordered by singular values (highest to lowest), let $\mathbf{V}_p$ be then matrix with the first M columns of **V**
- Then form the matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ such that
  - $\mathbf{V}_1$ is the matrix consisting of all but the last row of $\mathbf{V}_p$
  - $\mathbf{V}_2$ is the matrix consisting of all but the first row of $\mathbf{V}_p$
  - Discrete-time poles are found as the generalized eigenvalues of the pair $\{\mathbf{V}_2^T\mathbf{V}_1, \mathbf{V}_1^T\mathbf{V}_1\}$
- Then calculate the eigenvalues

Computational Considerations

- MPM can be applied to multiple signals, with computational order scaling according to the cube of the number of samples and linearly with the number of signals
- The MPM can become computationally difficult with large numbers of signals
- A key insight is just a small number of signals are needed to calculate **a**; **b** can then be quickly calculated for each signal

Quick Determination of **b**

- A key insight from a technique know as the variable projection method (VPM) is

$$\hat{\mathbf{y}}(\boldsymbol{\alpha},\mathbf{b}) = \boldsymbol{\Phi}(\boldsymbol{\alpha})\mathbf{b}$$

And then the residual is minimized by selecting

$$\mathbf{b} = \boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\mathbf{y}$$

where $\boldsymbol{\Phi}(\boldsymbol{\alpha})$ is the m by M matrix with values

$\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ if α_{i} corresponds to a real eigenvalue,

and $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{i+1} t_j)$ and $\Phi_{ji+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{i+1} t_j)$

for a complex eigenvalue; $t_j = \left(j-1\right)\Delta T$

Finally, $\boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}$ is the pseudoinverse of $\boldsymbol{\Phi}(\boldsymbol{\alpha})$

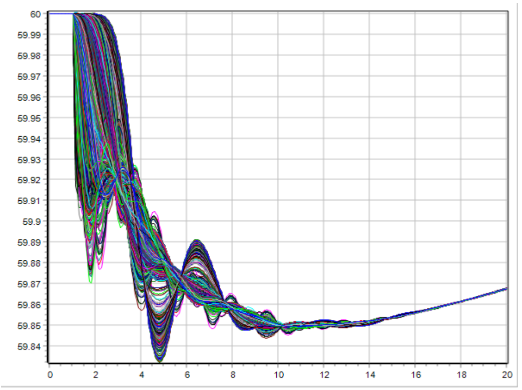
M is the number of retained modes, and is usually very small

Iterative Matrix Pencil

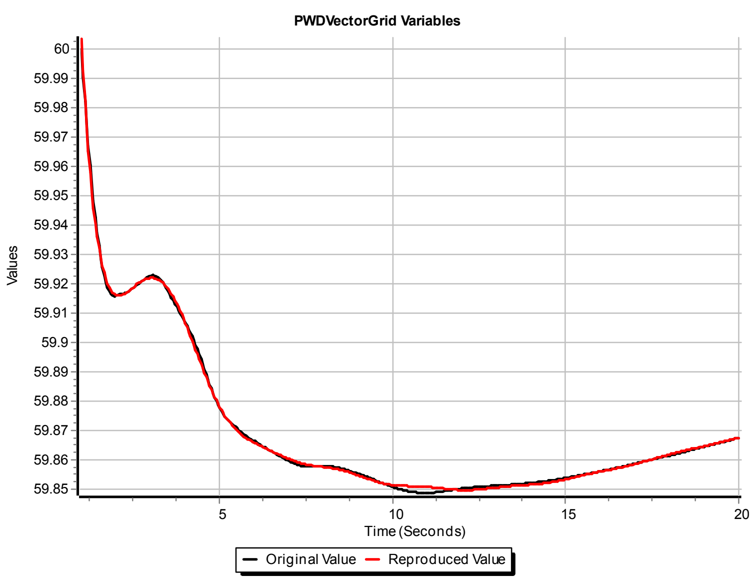
- The Iterative Matrix Pencil (IMP) is used to iteratively improve **a** to better match a large number of signals by sequentially adding signals to be included in the calculation of **a**
  - The **b** for each signal, and its associated costs function, can be quickly calculated
- The paper algorithm arbitrarily selects one signal, and then sequentially adds the signal with the highest cost function (i.e., the worst fit); usually only a small number of signals needs to be considered (approximately 10)

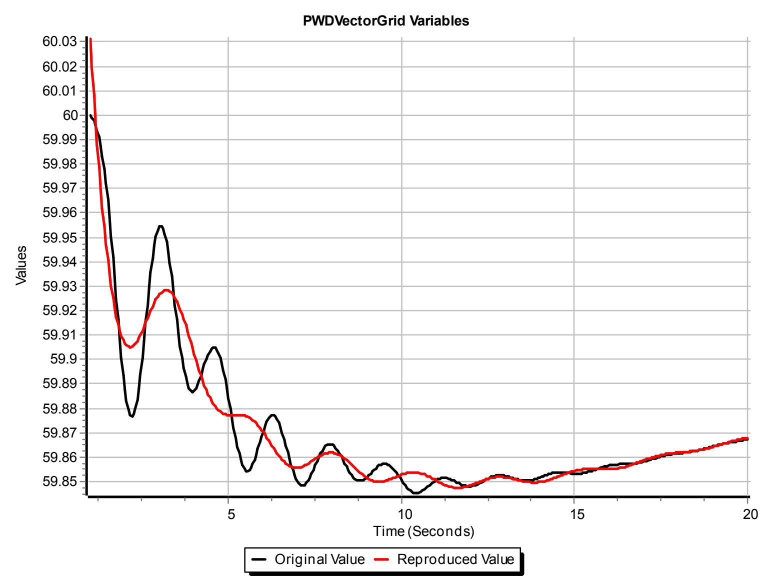
8400 Signal Example

- In previous 8400 signal example if just one signal is included in the calculation of **a** then just four modes are found: (0, 0.029, 0.21, 0.42Hz)



- The best and worst signal matches are shown

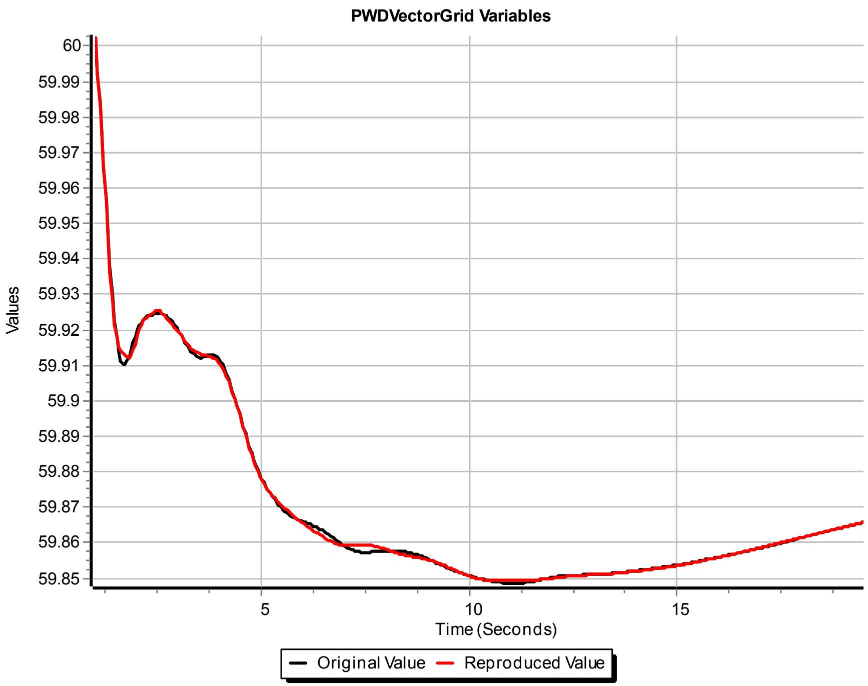




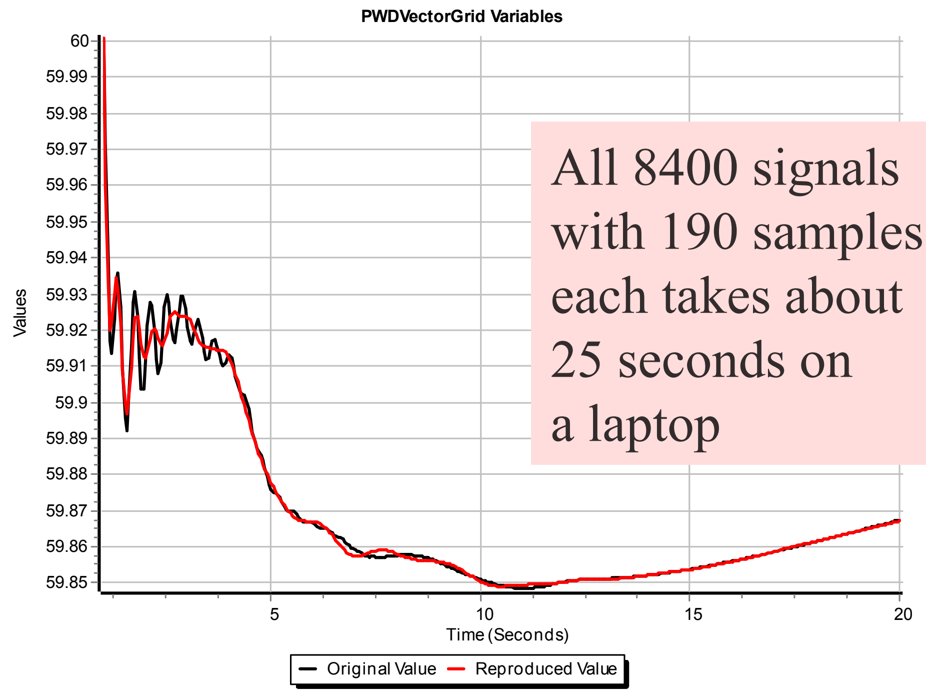
8400 Signal Example

- When ten signals are included there are eight modes, and the overall match for all the 8400 signals is much improved

Average Match

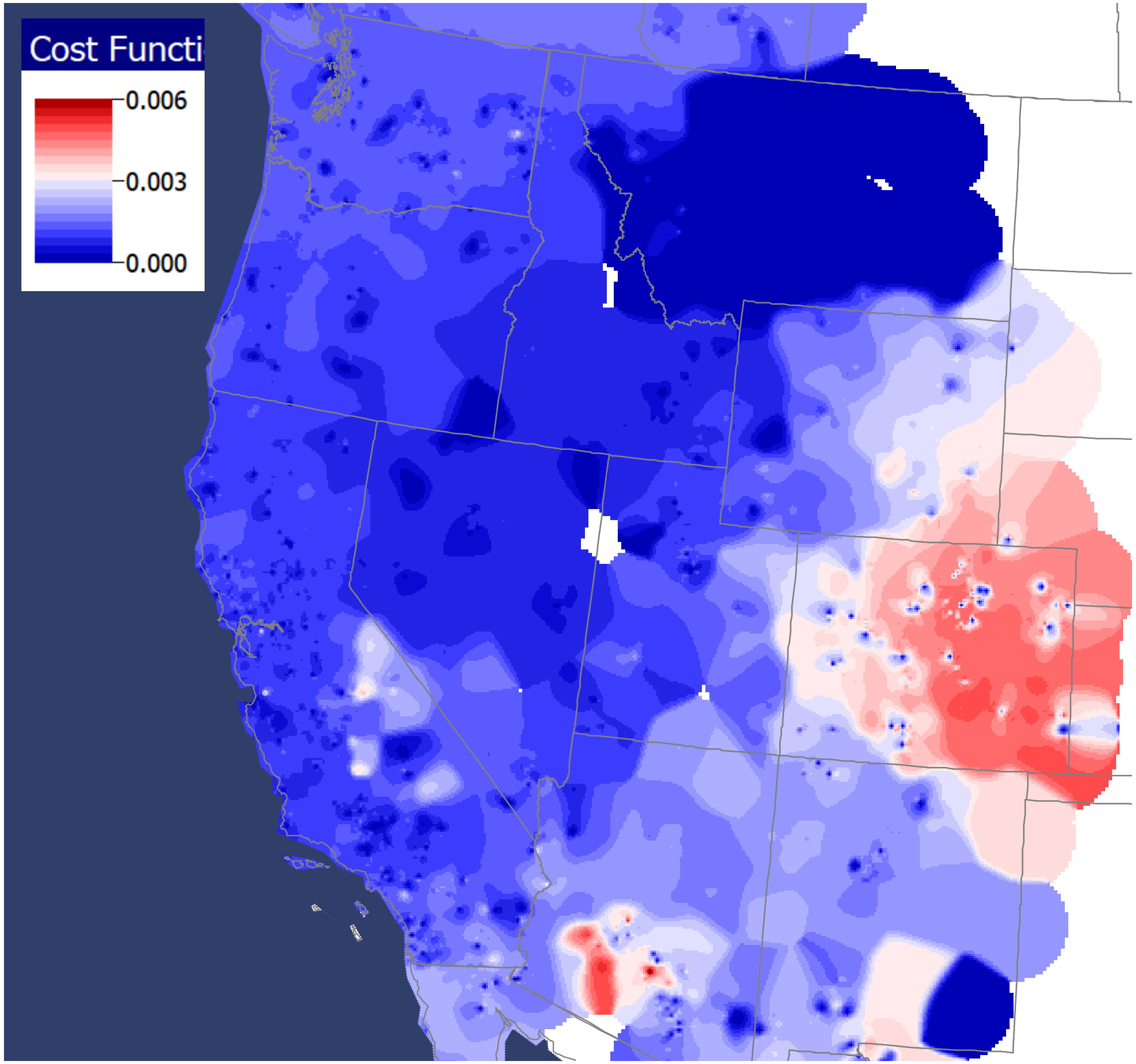


Worst Match



All 8400 signals with 190 samples each takes about 25 seconds on a laptop

8400 Signal Example: Contouring the Cost Functions

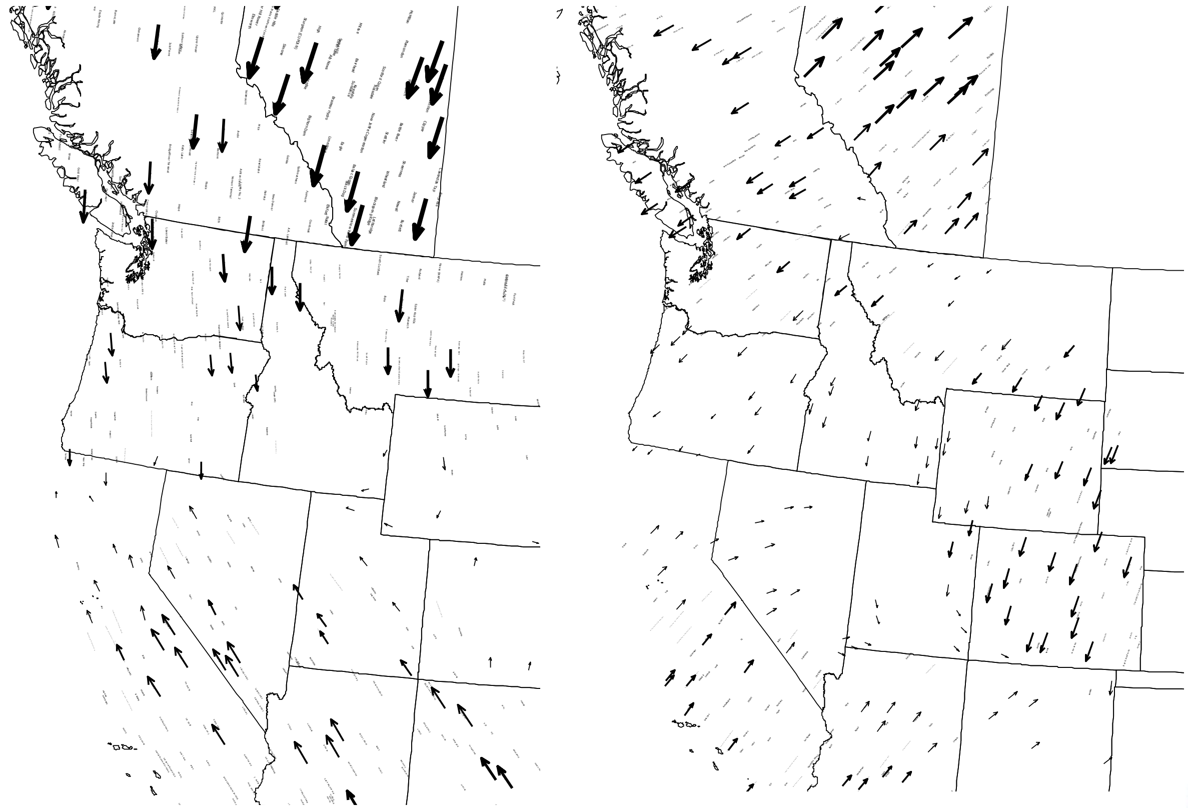


The contour shows the locations in which the signals are well matched and where they are less well matched (perhaps indicating bad measurements or unusual system behavior)

Application: Mode Shape Visualization

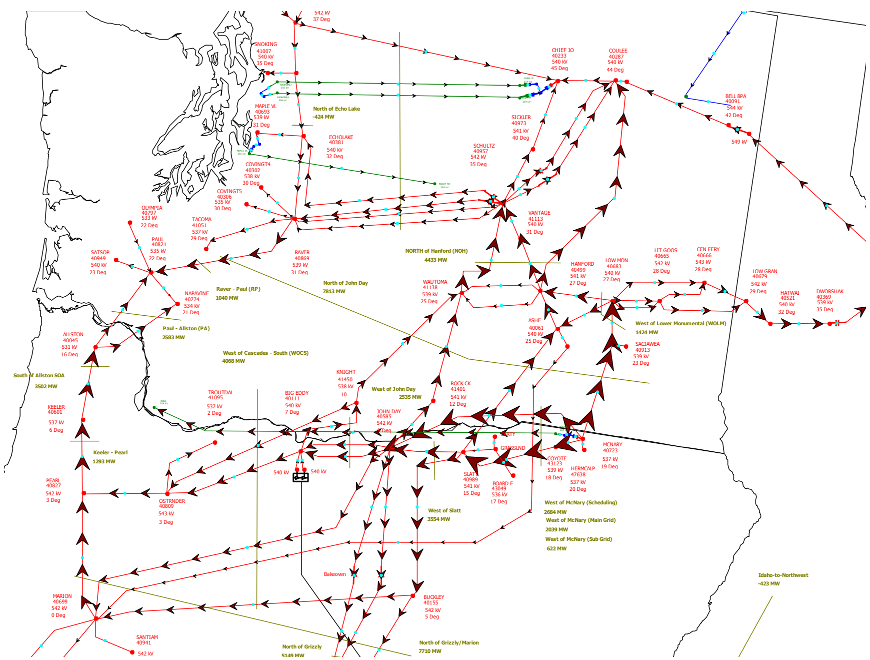
- The participation of each signal in each mode can be readily calculated and visualized

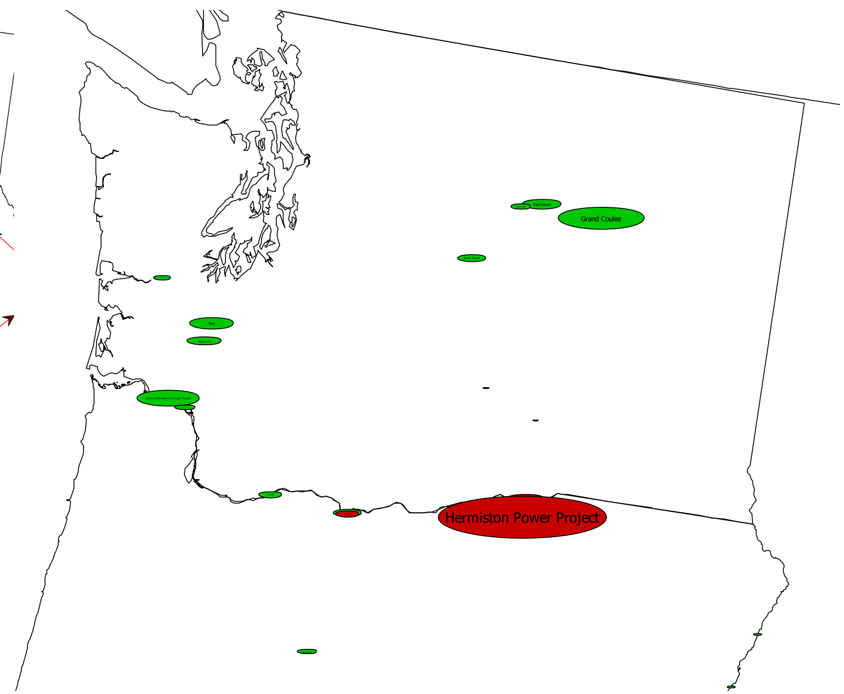
The displays show the 0.218 and the 0.348 Hz modes; pruning is used to reduce the number of vectors



Application: Visualizing the Source of Oscillations

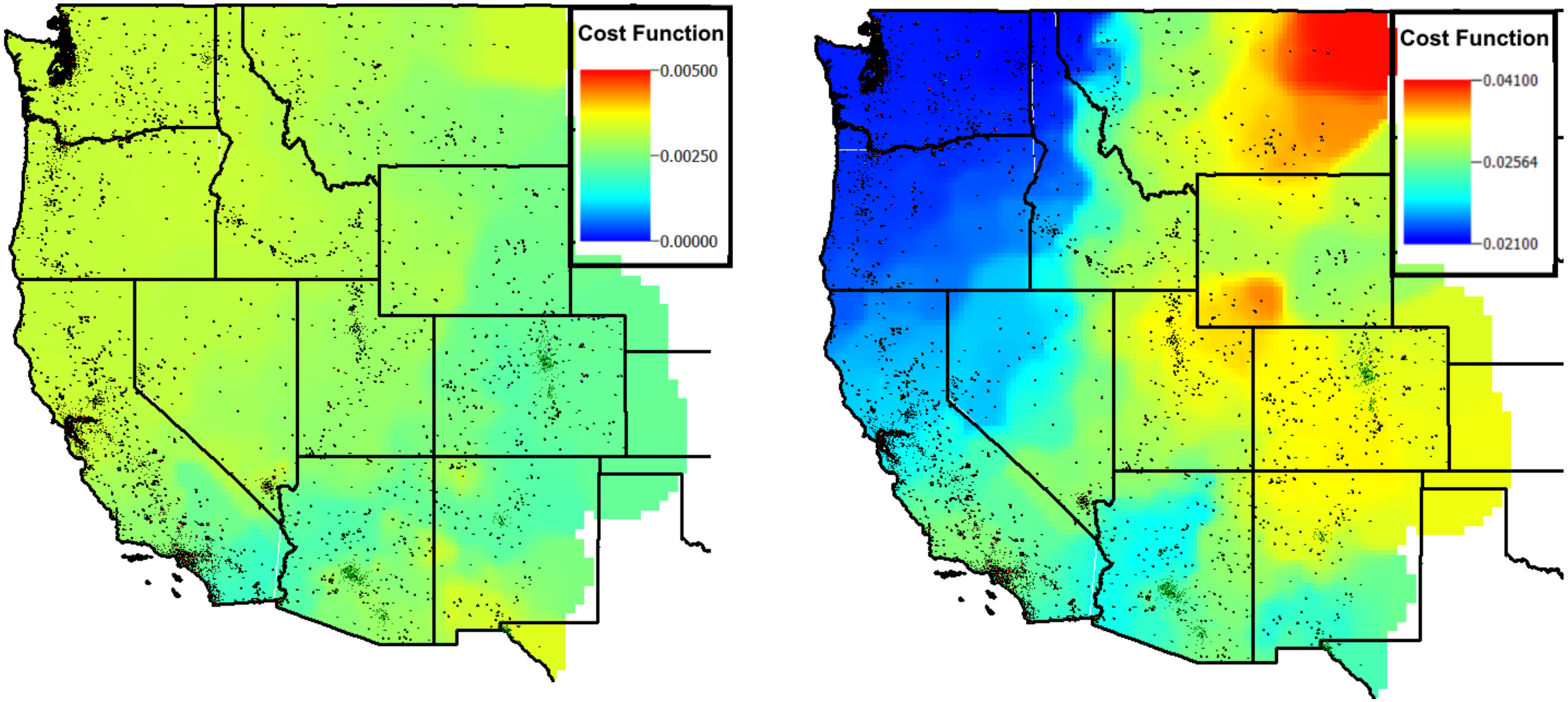
- The results can also be used to visualize the source of sustained oscillations





Comparison Between IMP and DMD

Ongoing work is looking comparisons between the different methods and developing public datasets (image on the left is cost contour for IMP and the right for dynamic mode decomposition)



Conclusion and Questions

- Measurement based modal analysis is becoming widely available for the analysis of power system data, both from actual measurements and simulation results
- Paper has presented an iterative matrix pencil approach that can be used to quickly calc
- However, much can and should be done to reduce to reduce this risk
- A broad, sustained effort is needed in this area including the entire electric grid sector
- Synthetic electric grids will play a crucial role in this effort