Iterative Matrix Pencil Method of Electric Grid Modal and Oscillation Analysis and Visualization

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Signal-Based Modal Analysis

- With the advent of large numbers of PMUs, measurement based modal analysis is increasingly used to understand power system behavior
  – The goal is to determine the damping associated with the dominant oscillatory modes in the system
  – Approaches seek to approximate a sampled signal by a series of exponential functions (usually damped sinusoidals)
- Techniques can also be used in dynamic simulation applications
- Several techniques are available
  – Prony is the oldest, dating to 1795, with power system applications from about 1980's

Presentation Overview

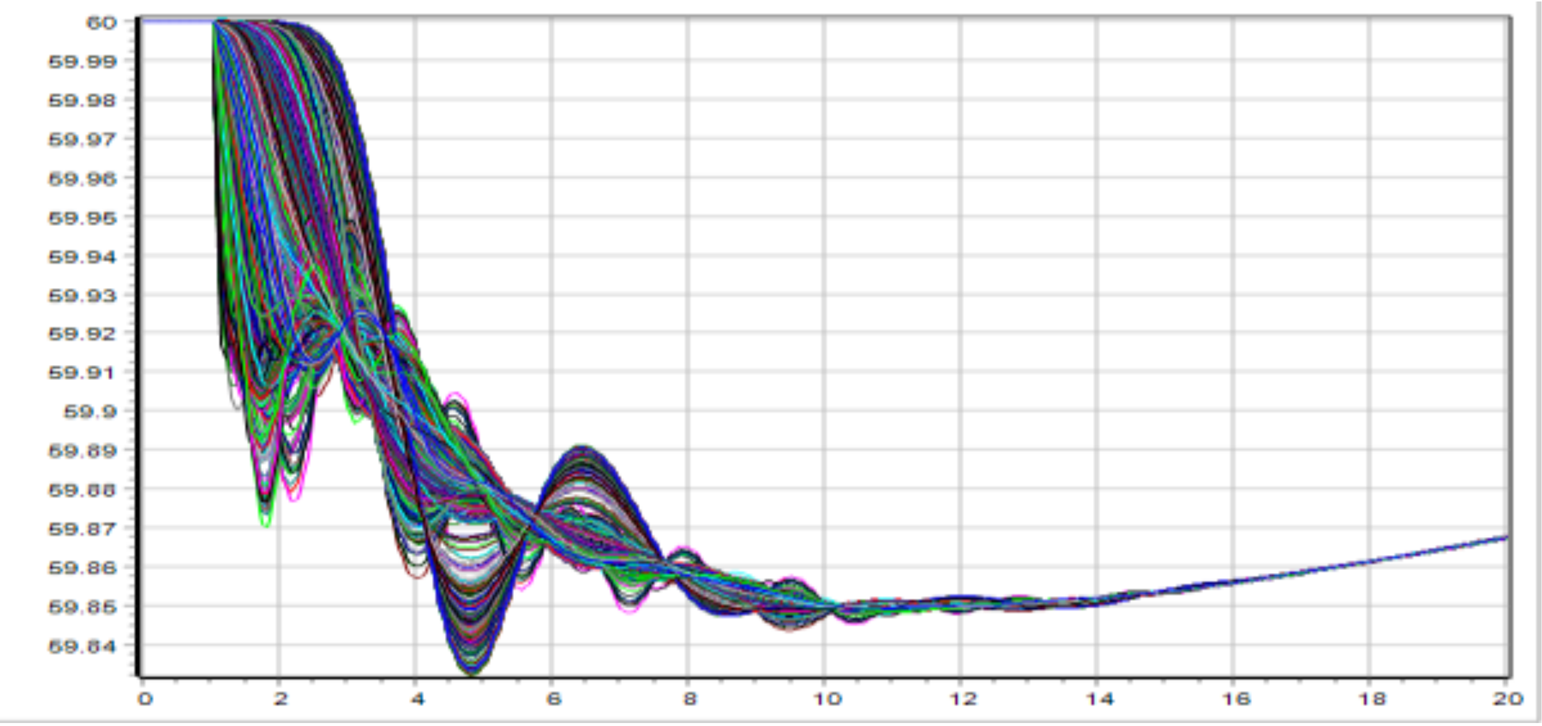
- Presentation discusses an approach to quickly handle a large number of signals
  - Goal is to make modal analysis and visualization an easy part of standard engineering studies; fully integrated into PowerWorld Simulator
  - Results can be extended to determine and visualize the source of oscillations
- Many of results are from
  - W. Trinh, K. Shetye, I. Idehen, T.J. Overbye, "Iterative Matrix Pencil Method for Power System Modal Analysis", Proc. HICSS Jan 2019
  - I. Idehen, B. Wang, K.S. Shetye, T.J. Overbye, J.D. Weber, "Visualization of Large-Scale Electric Grid Oscillation Modes," Proc. 2018 North American Power Symposium, Fargo, ND, September 2018

Ring-down Modal Analysis

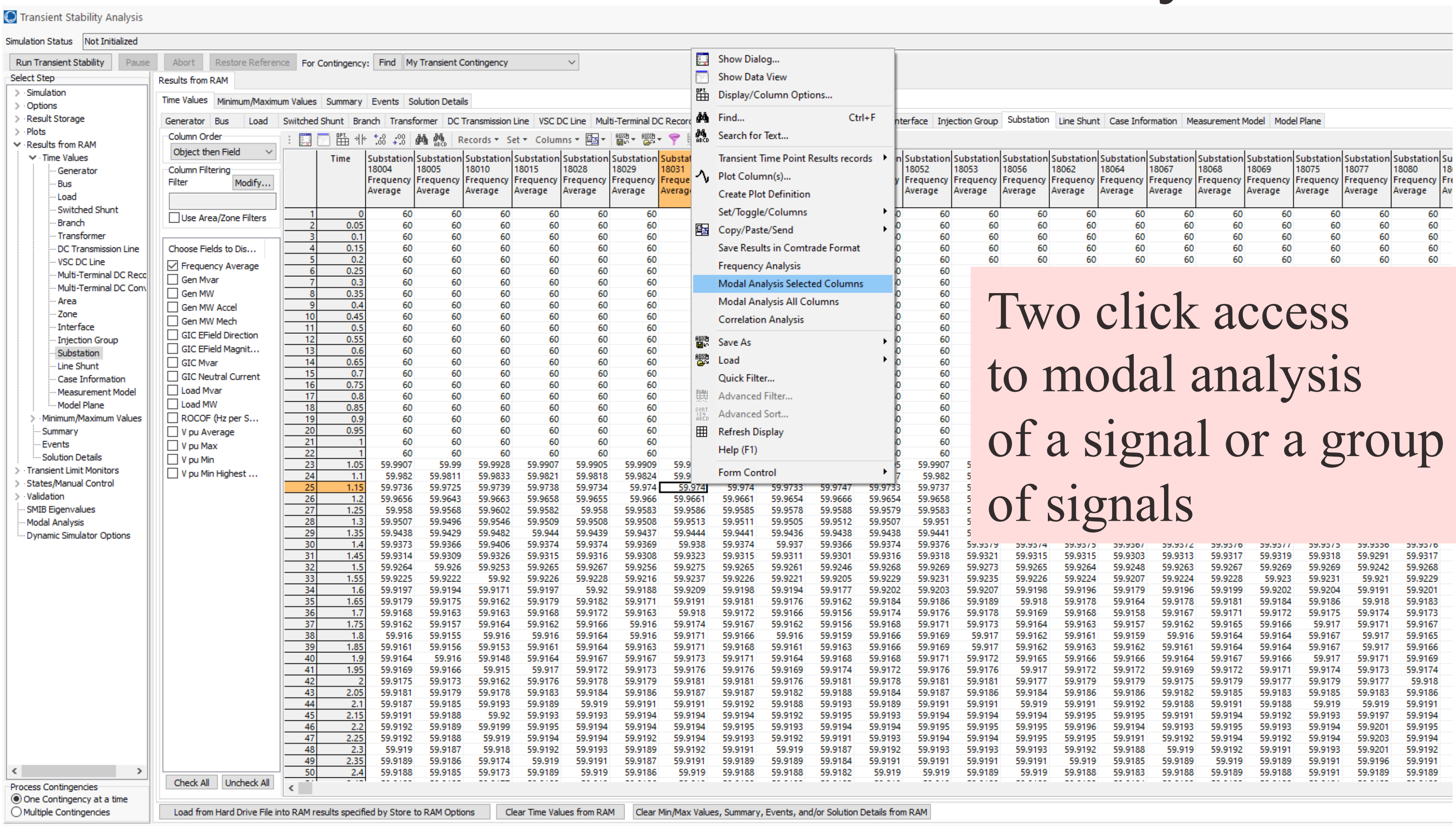
- A variety of different techniques can be used to approximate a signal, $y_{org}(t)$, by the sum of other, simpler signals (basis functions)
  - Basis functions are usually exponentials, with linear and quadratic functions used to detrend the signal
  - Properties of the original signal can be quantified from basis function properties
    - Examples are frequency and damping
  - Signal is considered over some time window, with t=0 defined as the beginning of the window
- Starting point is time-varying signal, $y_{org}(t)$, that is then assumed to be uniformly sampled

Example Application

- An example application is to make sense of the frequency response following a contingency
  - Below example at the shows the frequency variation at 8400 substations

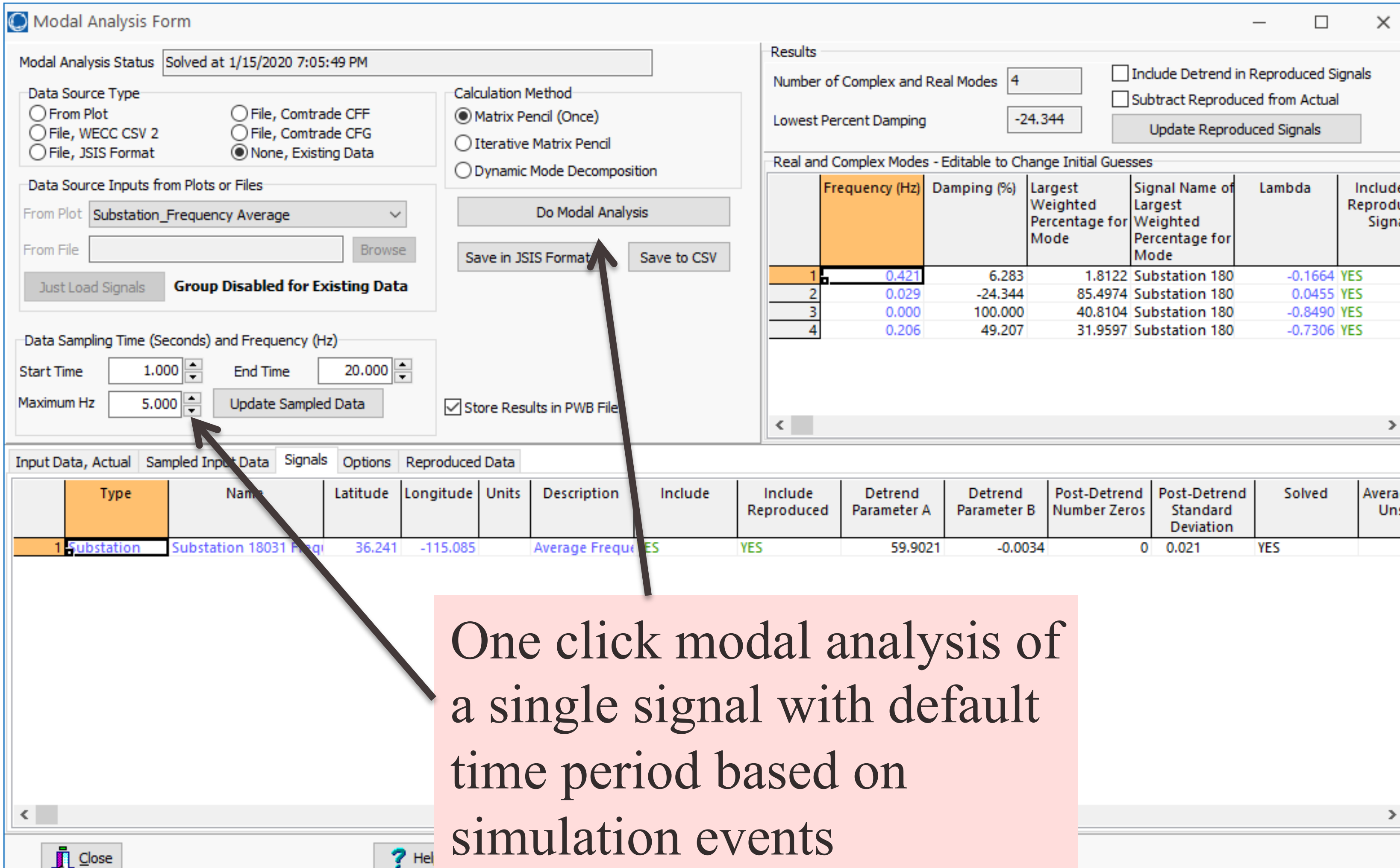


Quick Access to Modal Analysis



Two click access to modal analysis of a signal or a group of signals

Quick Access to Modal Analysis



One click modal analysis of a single signal with default time period based on simulation events

And to Accuracy Verification

Modal Analysis Signal Dialog

Name: Substation 18031 Frequ
Type: Substation
Units:
Description: Average Frequency, Hz
☑ Include in Modal Analysis

Data Detrend Parameters

Detrend Model = A + B*(t-t0) + C*(t-t0)^2
☑ Use Case Default Detrend Model
Signal Specific Detrend Model: ◉ None ○ Linear ○ Constant ○ Quadratic

Used Detrend Model: Linear
Parameter A: 59.9021
Parameter B: -0.0034
Parameter C: 0.0000
Standard Deviation (SD): 0.0207

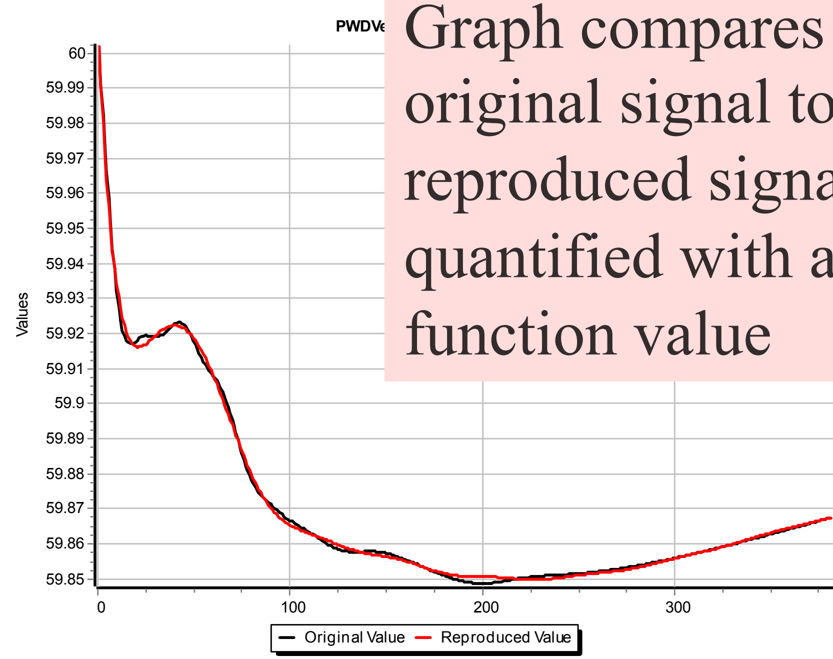
Output Summary

Average Error, Scaled by SD: 0.0008
Average Error, Unscaled: 0.0000
Cost Function Value, Scaled: 0.0036
☐ Include Detrend in Reproduced Signal
Update Reproduced

Actual Input | Sampled Input | Fast Fourier Transform Results | Modal Results | Original and Reproduced Signal

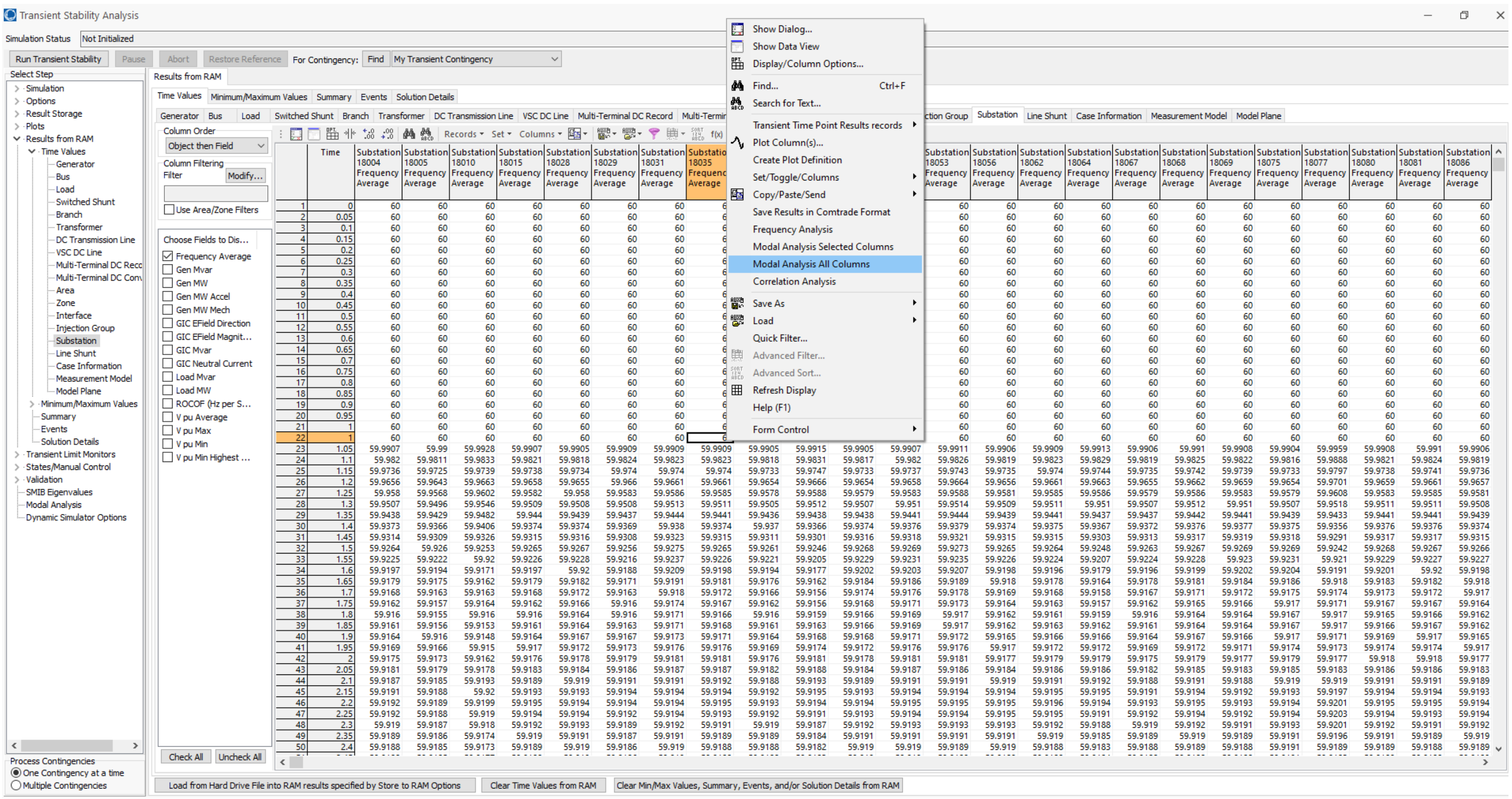
	Time (Seconds)	Original Value	Reproduced Value	Difference
1	1.000	60.000	60.002	0.002
2	1.050	59.991	59.991	-0.000
3	1.100	59.982	59.981	-0.002
4	1.150	59.974	59.971	-0.003
5	1.200	59.966	59.963	-0.003
6	1.250	59.959	59.956	-0.003
7	1.300	59.951	59.949	-0.002
8	1.350	59.944	59.944	-0.001
9	1.400	59.938	59.939	0.001
10	1.450	59.932	59.934	0.002
11	1.500	59.927	59.931	0.003
12	1.550	59.924	59.927	0.004
13	1.600	59.921	59.925	0.004
14	1.650	59.919	59.922	0.003
15	1.700	59.918	59.921	0.003
16	1.750	59.917	59.919	0.002
17	1.800	59.917	59.918	0.001
18	1.850	59.917	59.917	0.000
19	1.900	59.917	59.917	-0.001
20	1.950	59.918	59.916	-0.001
21	2.000	59.918	59.916	-0.002

OK | Help | Print

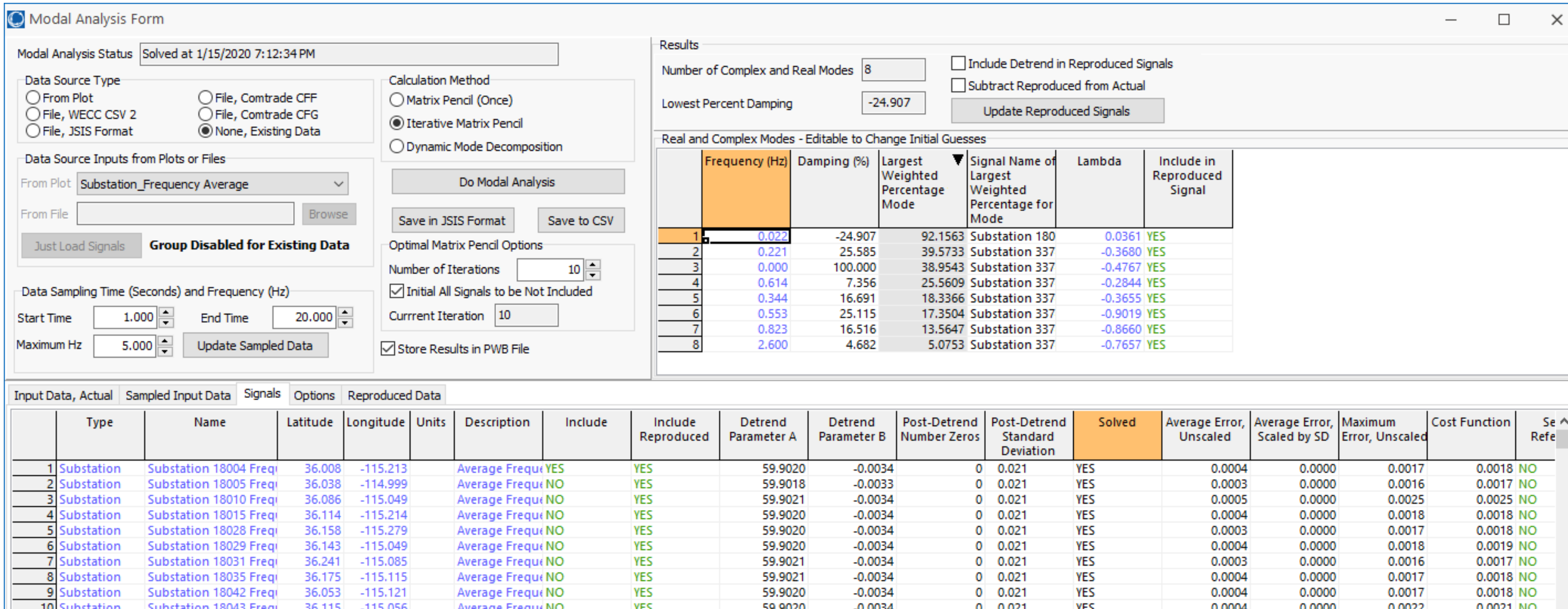


Graph compares the original signal to the reproduced signal, quantified with a cost function value

Issue is How to Handle a Large Number of Signals



There are Now 8400 Signals to Consider



Next few slides show how this can be done with good computational efficiency, and the results both quantified and visualized.

Measurement-Based Modal Analysis

- Vector **y** consists of m uniformly sampled points from $y_{org}(t)$ at a sampling value of DT, starting with t=0, with values y_j for j=1…m
  - Times are then t_j= (j-1)DT
  - At each time point j, the approximation of y_j is

$$\hat{y}_j(\boldsymbol{\alpha},\mathbf{b}) = \sum_{i=1}^{n} b_i \phi_i(t_j,\boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}$ is a vector with the real and imaginary eigenvalue components,

with $\phi_i(t_j,\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ for α_{i} corresponding to a real eigenvalue, and

$\phi_i(t_j,\boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{\mathrm{i+1}}\mathrm{t}_j)$ and $\phi_{i+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{\mathrm{i+1}}\mathrm{t}_j)$

for a complex eigenvector value

Measurement-Based Modal Analysis

- Error (residual) value at each point j is

$$r_j(t_j, \boldsymbol{\alpha}, \mathbf{b}) = y_j - \hat{y}_j(t_j, \boldsymbol{\alpha}, \mathbf{b})$$

- Closeness of fit can be quantified using the Euclidean norm of the residuals

$$\frac{1}{2}\sum_{j=1}^{m}(y_j - \hat{y}_j(t_j, \boldsymbol{\alpha}, \mathbf{b}))^2 = \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\alpha}, \mathbf{b})\right\|_2^2$$

- Hence we need to determine **a** and **b**; PowerWorld has three techniques for determining **a**, and then one for **b**
- Approaches can be quickly used with multiple signals

Matrix Pencil Method

- The **a** vector can be calculated using the Matrix Pencil Method (MPM)
- First, with m samples, let L=m/2
- Then form a Hankel matrix, **Y** such that

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{L+1} \\ y_2 & y_3 & \cdots & y_{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L} & y_{m-L+1} & \cdots & y_m \end{bmatrix}$$

The computational complexity increases with the cube of the number of measurements!

- Calculate its singular values with an economy SVD

Matrix Pencil Method (MPM)

- The ratio of each singular value is then compared to the largest singular value; retain the M ones with a ratio greater than a threshold
  - This determines the modal order
  - Assuming **V** is ordered by singular values (highest to lowest), let $\mathbf{V}_p$ be then matrix with the first M columns of **V**
- Then form the matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ such that
  - $\mathbf{V}_1$ is the matrix consisting of all but the last row of $\mathbf{V}_p$
  - $\mathbf{V}_2$ is the matrix consisting of all but the first row of $\mathbf{V}_p$
  - Discrete-time poles are found as the generalized eigenvalues of the pair $\{\mathbf{V}_2^T\mathbf{V}_1, \mathbf{V}_1^T\mathbf{V}_1\}$
- Then calculate the eigenvalues

Computational Considerations

- MPM can be applied to multiple signals, with computational order scaling according to the cube of the number of samples and linearly with the number of signals
- The MPM can become computationally difficult with large numbers of signals
- A key insight is just a small number of signals are needed to calculate **a**; **b** can then be quickly calculated for each signal

Quick Determination of b

- A key insight from a technique know as the variable projection method (VPM) is

$\hat{\mathbf{y}}(\boldsymbol{\alpha},\mathbf{b}) = \boldsymbol{\Phi}(\boldsymbol{\alpha})\mathbf{b}$

And then the residual is minimized by selecting

$\mathbf{b} = \boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\mathbf{y}$

where $\boldsymbol{\Phi}(\boldsymbol{\alpha})$ is the m by M matrix with values

$\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ if α_{i} corresponds to a real eigenvalue,

and $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{i+1}t_j)$ and $\Phi_{ji+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{i+1}t_j)$

for a complex eigenvalue; $t_j = \left(j-1\right)\Delta T$

Finally, $\boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}$ is the pseudoinverse of $\boldsymbol{\Phi}(\boldsymbol{\alpha})$

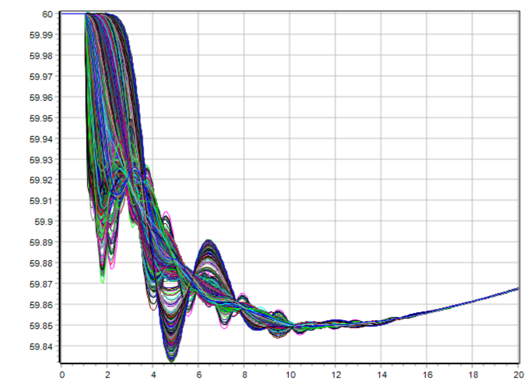
M is the number of retained modes, and is usually very small

Iterative Matrix Pencil

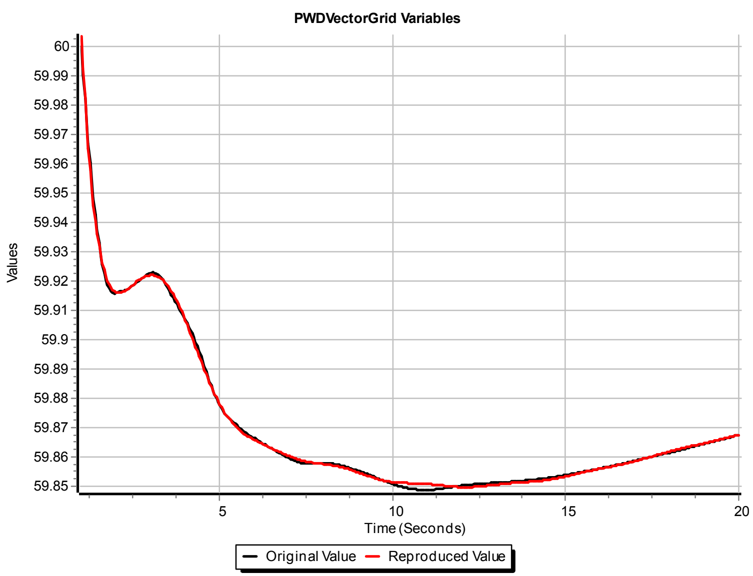
- The Iterative Matrix Pencil (IMP) is used to iteratively improve **a** to better match a large number of signals by sequentially adding signals to be included in the calculation of **a**
  - The **b** for each signal, and its associated costs function, can be quickly calculated
- The algorithm arbitrarily selects one signal, and then sequentially adds another signal, often the one with the highest cost function (i.e., the worst fit); usually only a small number of signals needs to be considered (approximately 10)

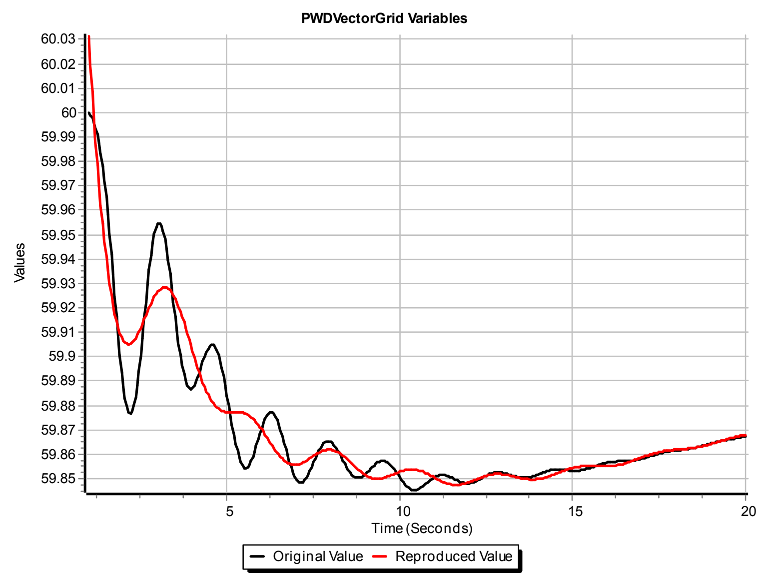
8400 Signal Example

- In previous 8400 signal example if just one signal is included in the calculation of **a** then just four modes are found: (0, 0.029, 0.21, 0.42Hz)



- The best and worst signal matches are shown

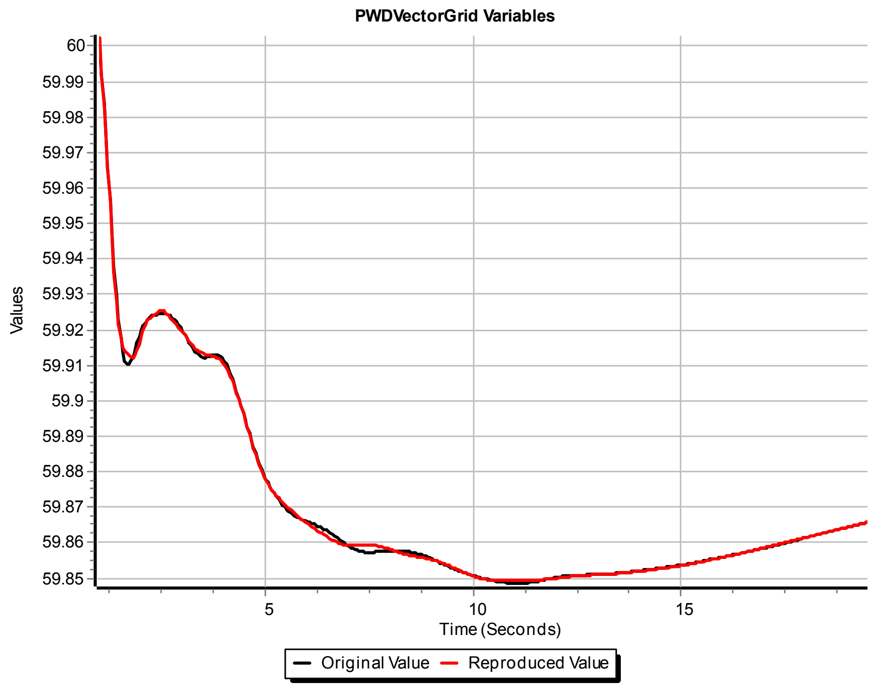




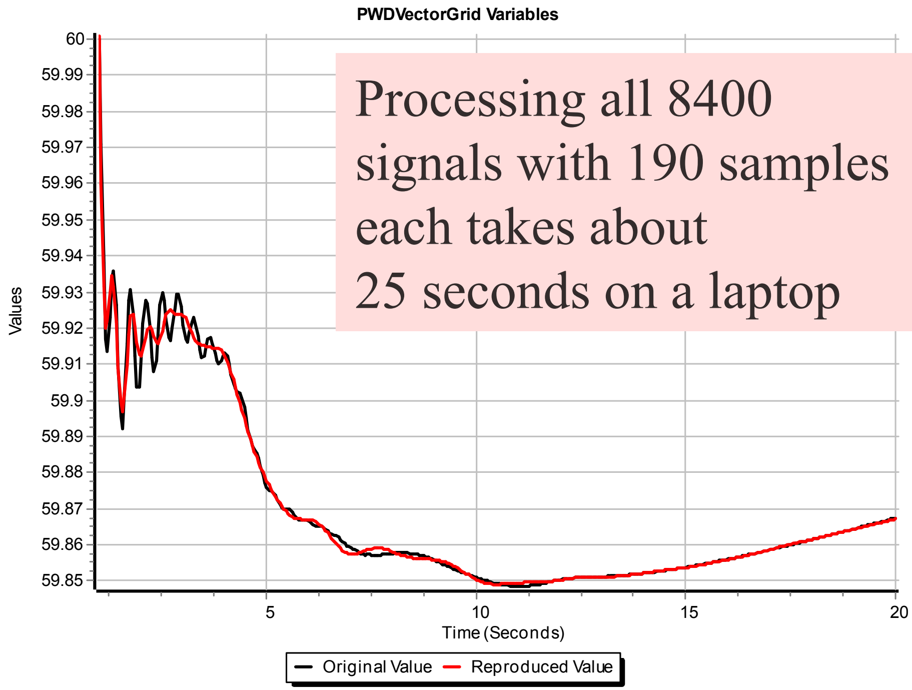
8400 Signal Example

- When ten signals are included there are eight modes, and the overall match for all the 8400 signals is much improved

Average Match

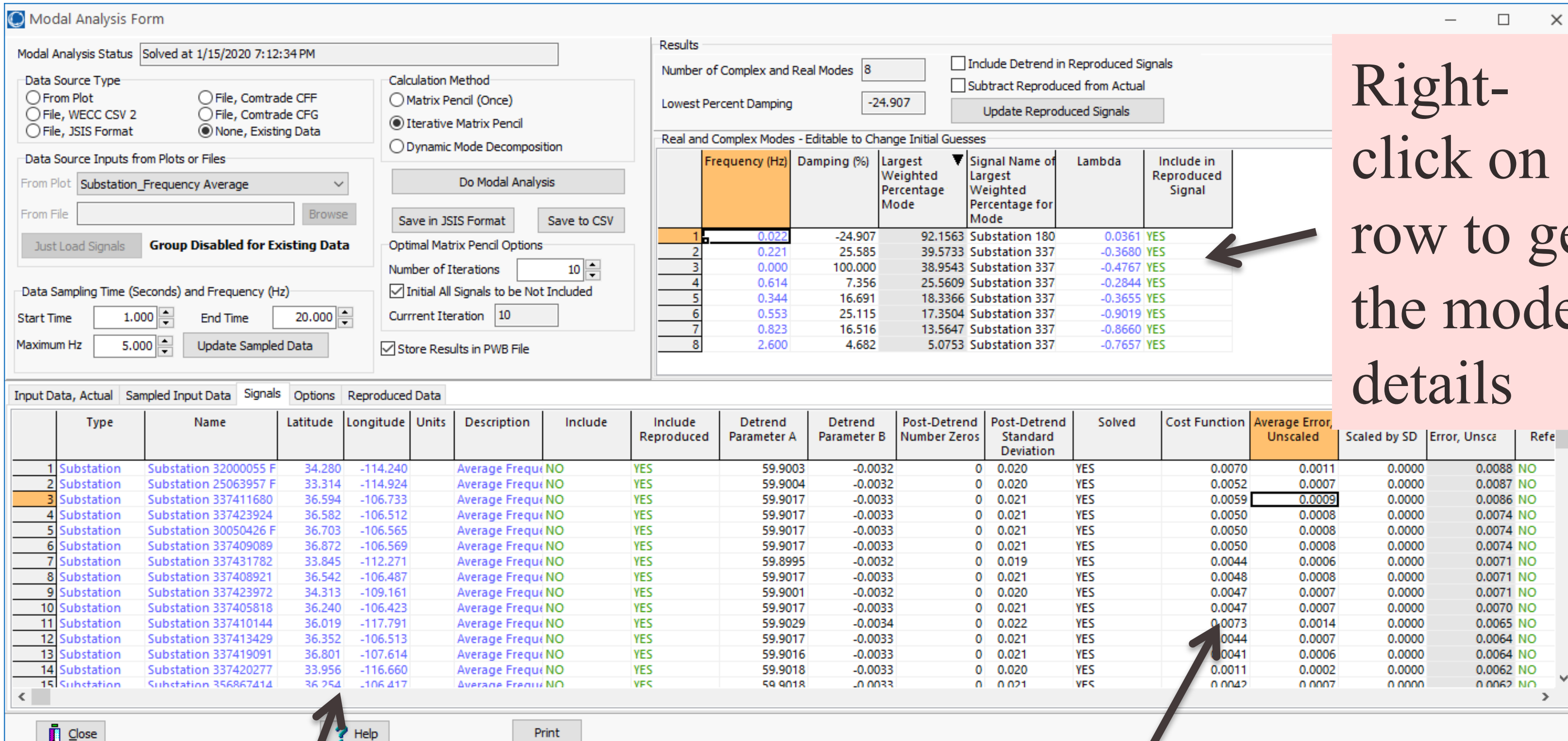


Worst Match



Processing all 8400 signals with 190 samples each takes about 25 seconds on a laptop

Results Screen for the 8400 Signals

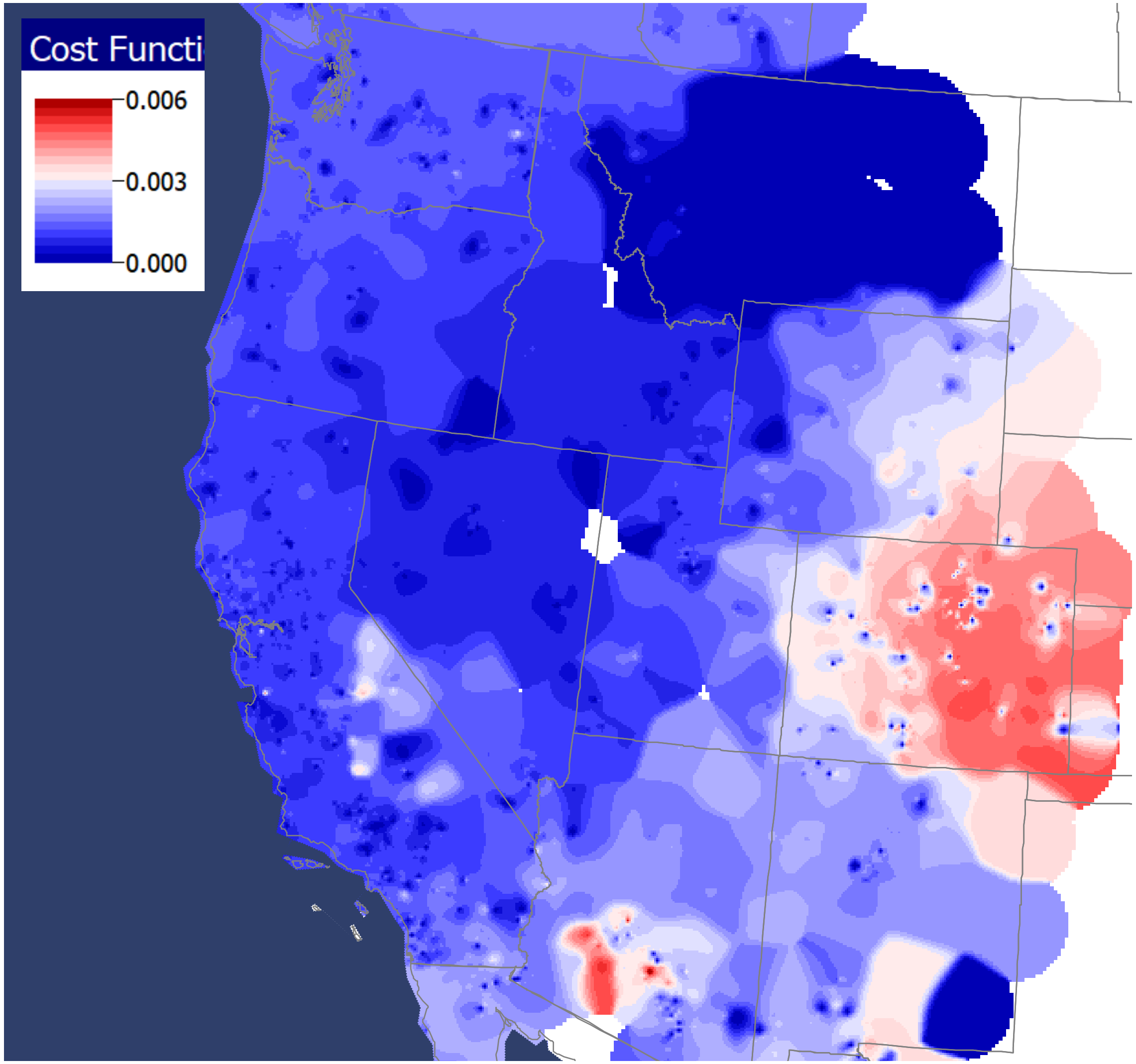


Right-click on row to get the mode details

Each signal is geo-mapped

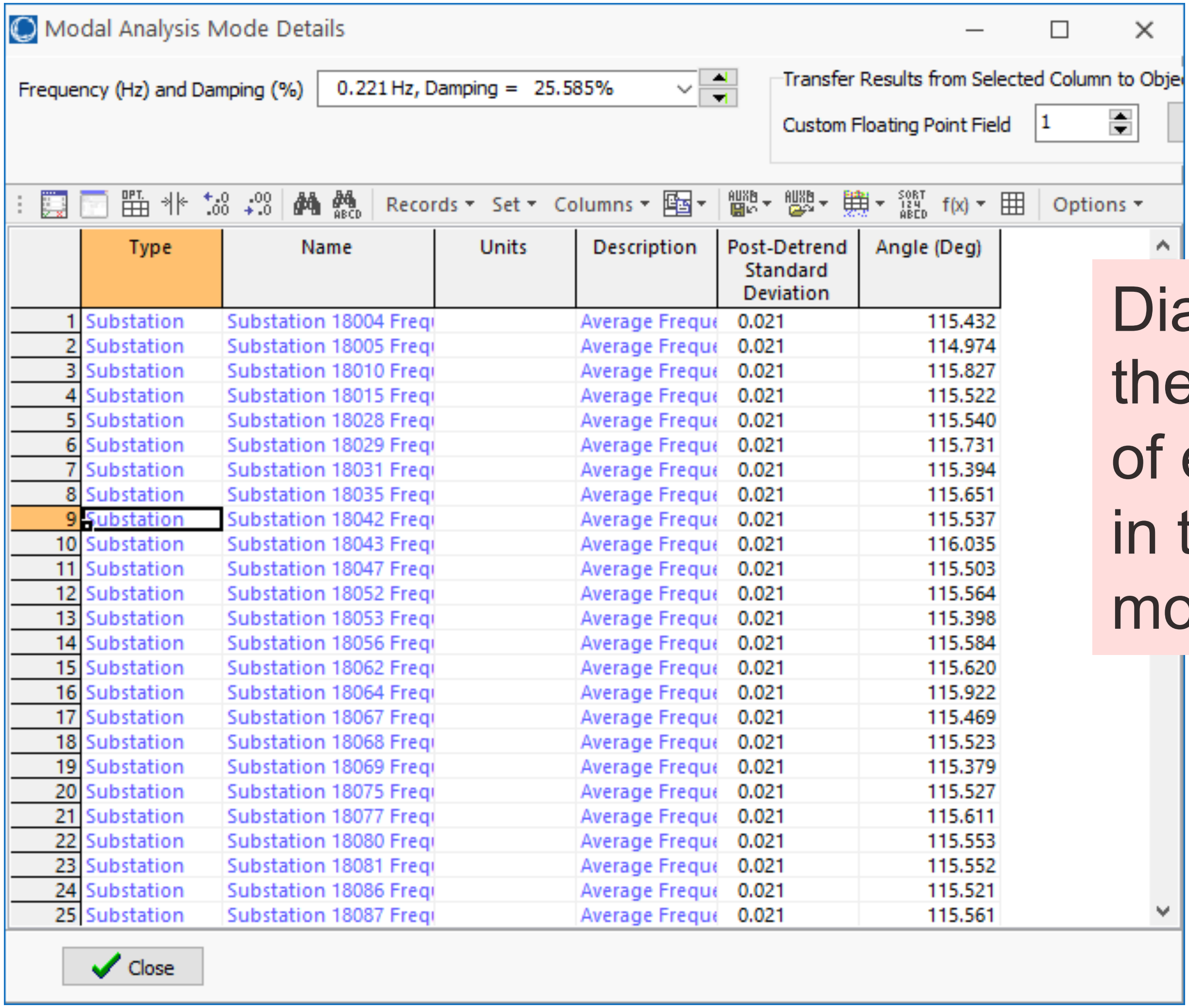
Results here are sorted by the cost function

8400 Signal Example: Contouring the Cost Functions



The contour shows the locations in which the signals are well matched and where they are less well matched (perhaps indicating bad measurements or unusual system behavior)

Mode Details



Modal Analysis Mode Details

Frequency (Hz) and Damping (%) 0.221 Hz, Damping = 25.585%

Transfer Results from Selected Column to Obje

Custom Floating Point Field 1

Records Set Columns Options

	Type	Name	Units	Description	Post-Detrend Standard Deviation	Angle (Deg)
1	Substation	Substation 18004 Freq		Average Freque	0.021	115.432
2	Substation	Substation 18005 Freq		Average Freque	0.021	114.974
3	Substation	Substation 18010 Freq		Average Freque	0.021	115.827
4	Substation	Substation 18015 Freq		Average Freque	0.021	115.522
5	Substation	Substation 18028 Freq		Average Freque	0.021	115.540
6	Substation	Substation 18029 Freq		Average Freque	0.021	115.731
7	Substation	Substation 18031 Freq		Average Freque	0.021	115.394
8	Substation	Substation 18035 Freq		Average Freque	0.021	115.651
9	Substation	Substation 18042 Freq		Average Freque	0.021	115.537
10	Substation	Substation 18043 Freq		Average Freque	0.021	116.035
11	Substation	Substation 18047 Freq		Average Freque	0.021	115.503
12	Substation	Substation 18052 Freq		Average Freque	0.021	115.564
13	Substation	Substation 18053 Freq		Average Freque	0.021	115.398
14	Substation	Substation 18056 Freq		Average Freque	0.021	115.584
15	Substation	Substation 18062 Freq		Average Freque	0.021	115.620
16	Substation	Substation 18064 Freq		Average Freque	0.021	115.922
17	Substation	Substation 18067 Freq		Average Freque	0.021	115.469
18	Substation	Substation 18068 Freq		Average Freque	0.021	115.523
19	Substation	Substation 18069 Freq		Average Freque	0.021	115.379
20	Substation	Substation 18075 Freq		Average Freque	0.021	115.527
21	Substation	Substation 18077 Freq		Average Freque	0.021	115.611
22	Substation	Substation 18080 Freq		Average Freque	0.021	115.553
23	Substation	Substation 18081 Freq		Average Freque	0.021	115.552
24	Substation	Substation 18086 Freq		Average Freque	0.021	115.521
25	Substation	Substation 18087 Freq		Average Freque	0.021	115.561

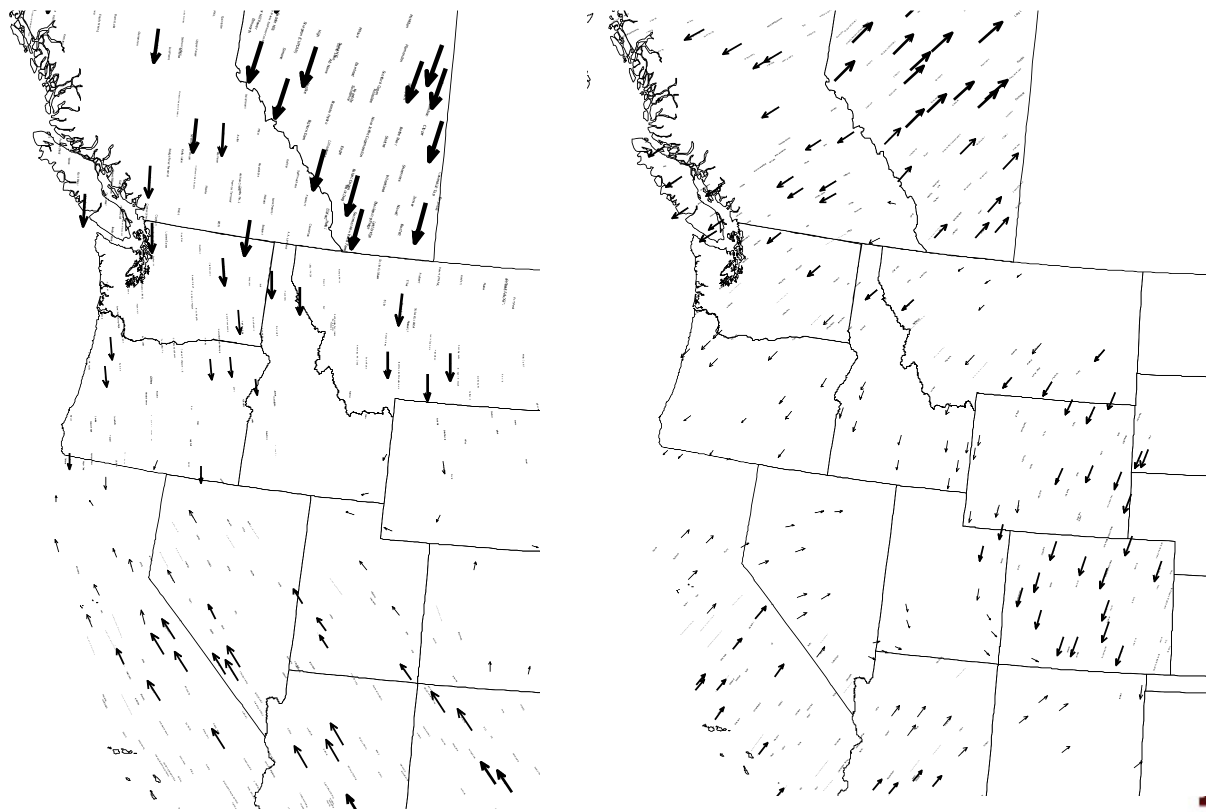
Close

Dialog shows the participation of each signal in the selected mode

Application: Mode Shape Visualization

- Participation of each signal in each mode can then be readily visualized using geographic data views (GDVs)

The displays show the 0.22 and the 0.34 Hz modes; pruning is used to reduce the number of vectors



Oscillation Source Visualization

- When the signals are bus voltage angles the mode j component of the voltage angle at bus k is

$$\theta_{k,j}(t) = A_{k,\mathrm{j}} e^{\sigma_j t} \cos\left(\omega_j t + \phi_{k,j}\right)$$

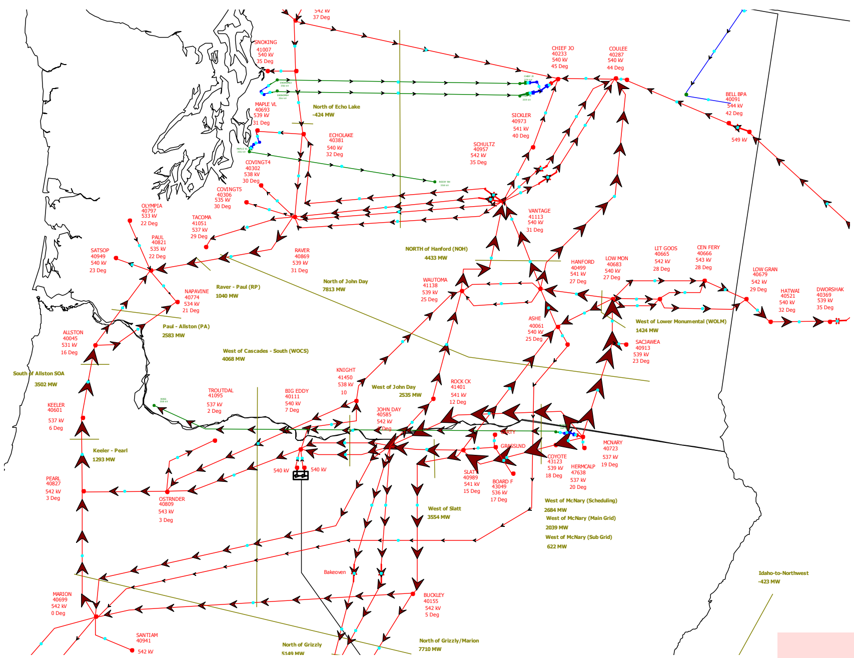
- For an undamped oscillation $s_j = 0$
- Using the dc power flow approximation, the flow of mode j power can be approximated as

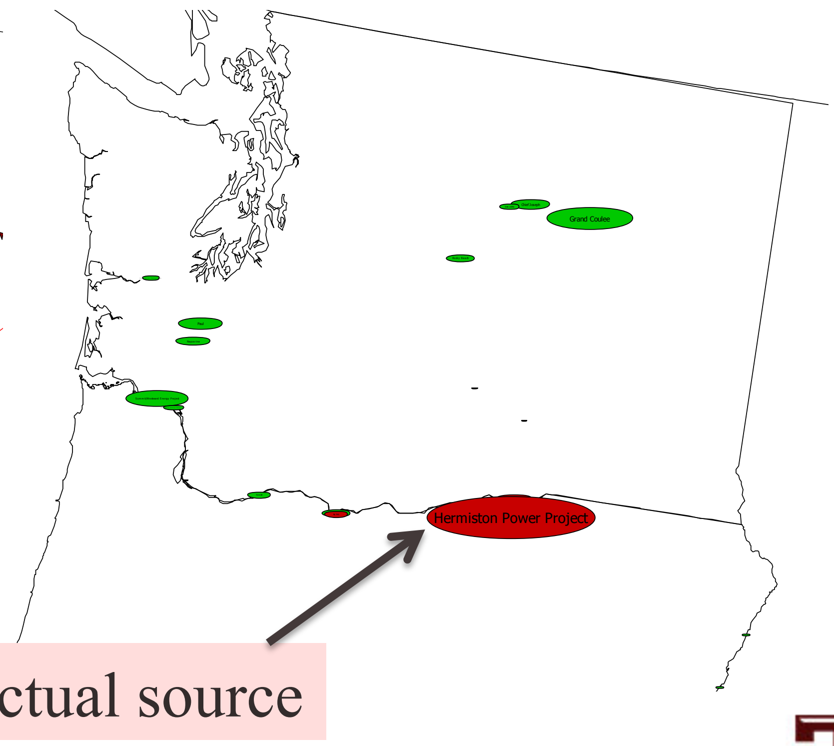
$$P_{mn} = \frac{1}{X_{mn}}\left(A_{\mathrm{m,j}} \cos\left(\phi_{\mathrm{m},j}\right) - A_{\mathrm{n,j}} \cos\left(\phi_{\mathrm{n},j}\right)\right)$$

- The source is found by tracing the flows

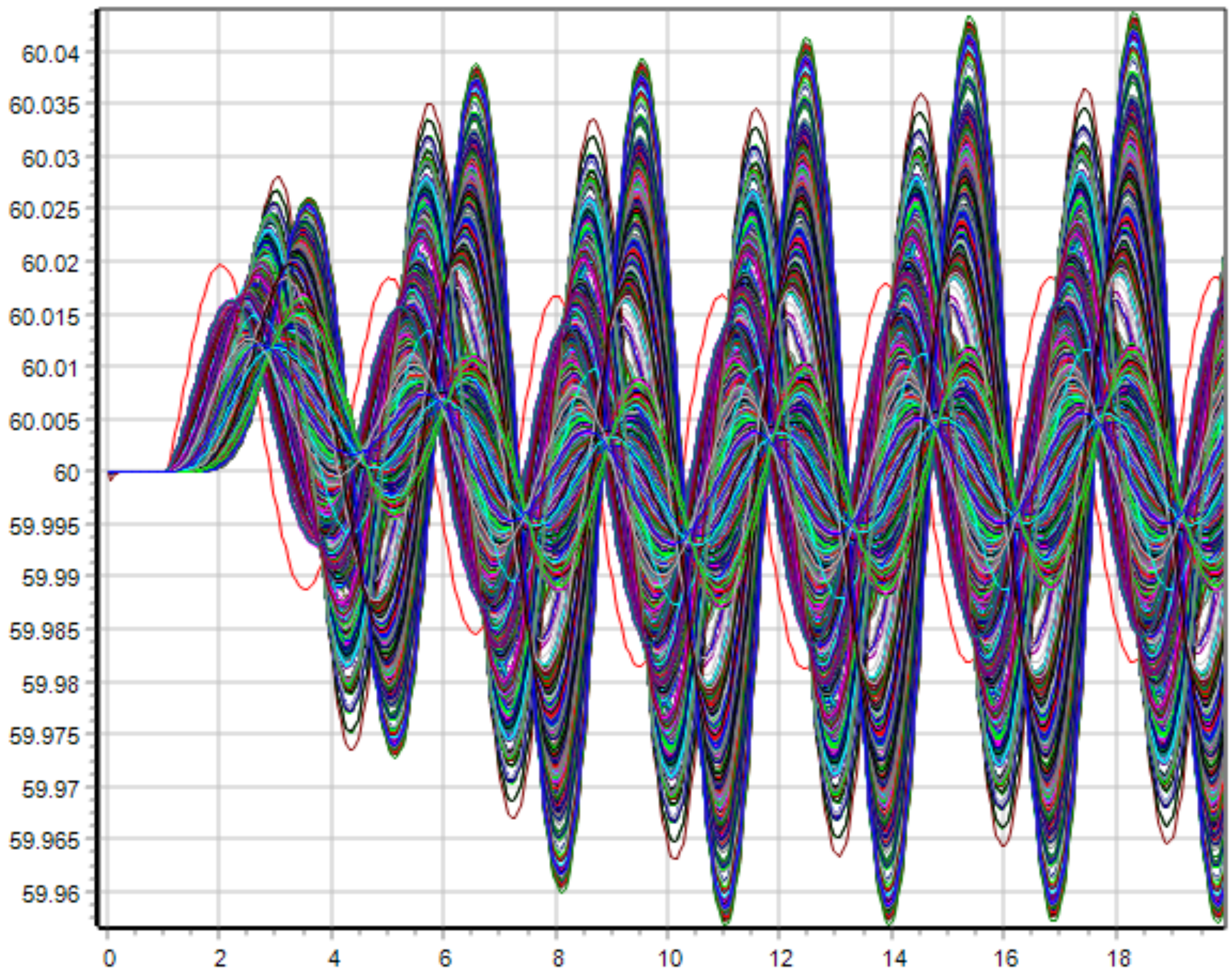
Application: Visualizing the Source of Oscillations

- The results can also be used to visualize the source of sustained oscillations; images show results for a 1 Hz forced oscillation



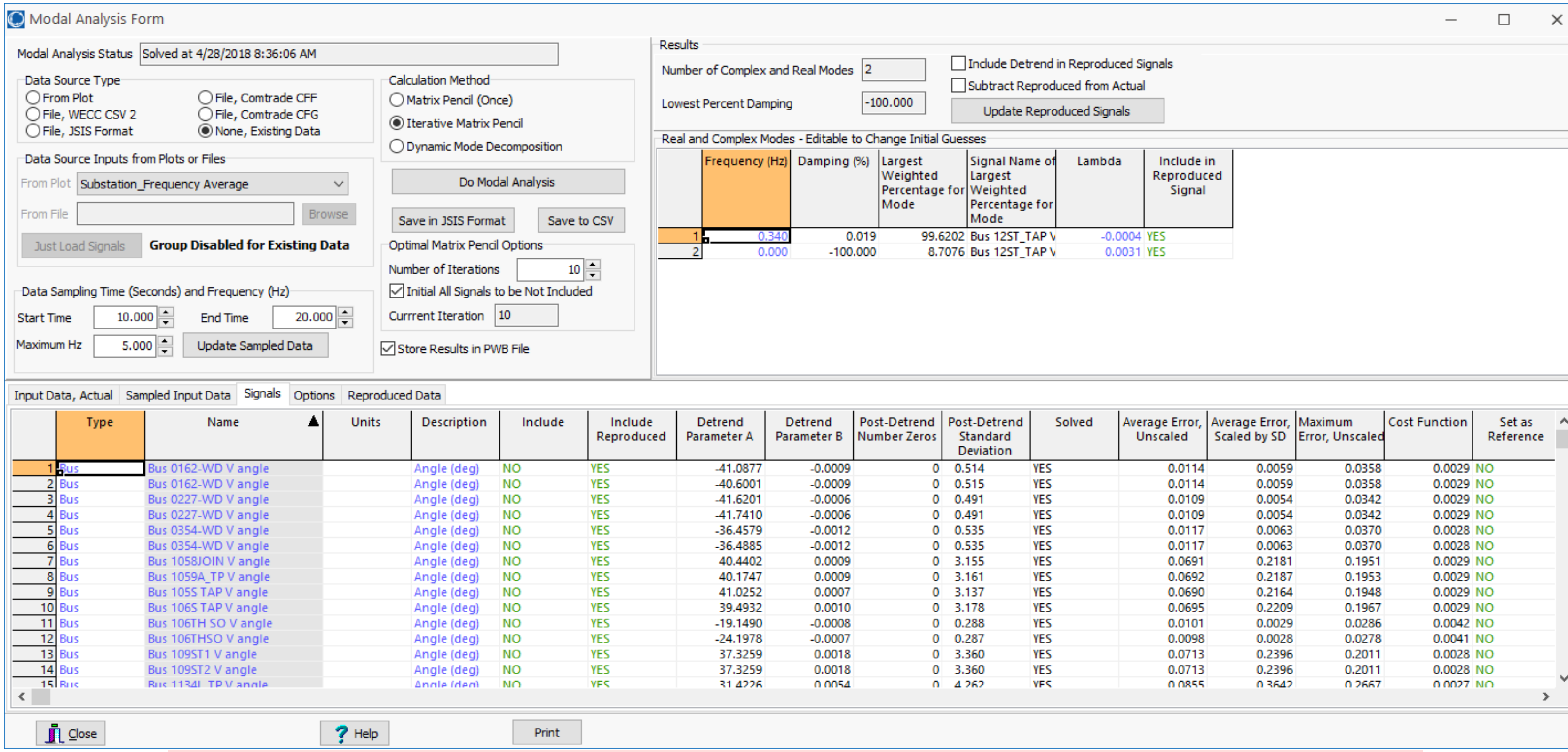


Application: Visualizing the Source of Oscillations at 0.34 Hz Forced



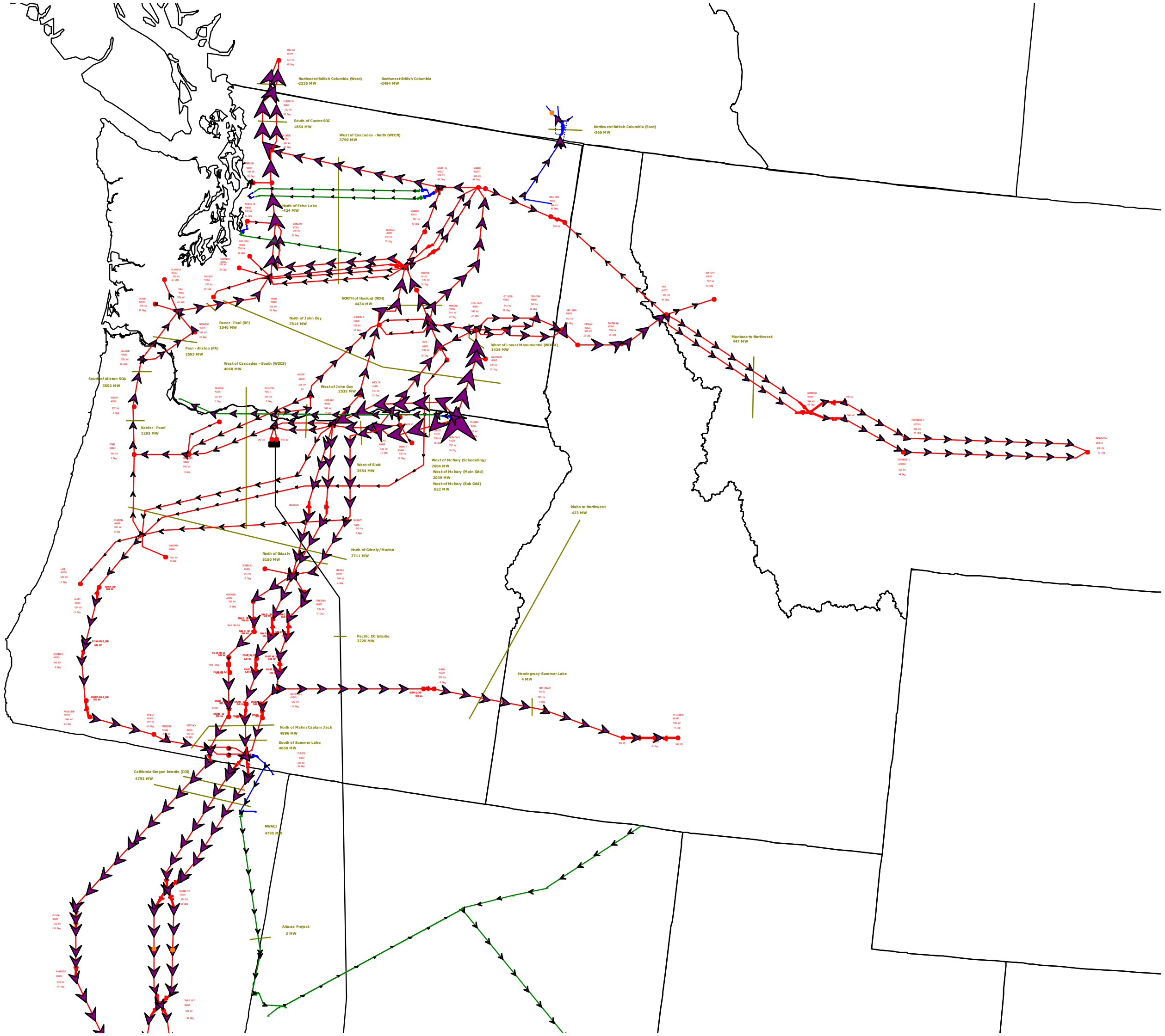
Graph shows the frequency response at all 6400 substations

Modal Results, Iterative Matrix Pencil

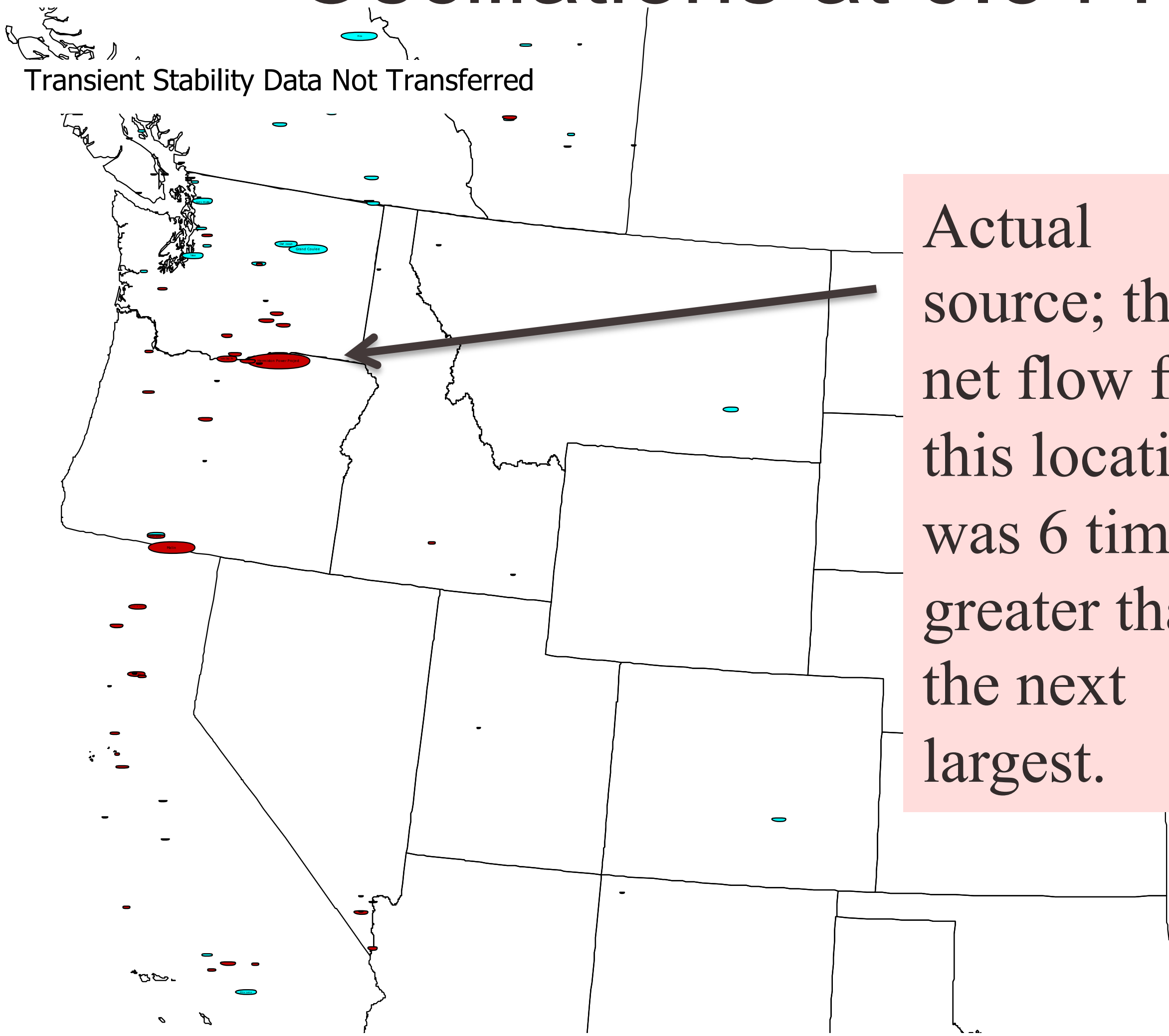


Analysis was done from 10 to 20 seconds

Visualization of Modal Line Flows



Application: Visualizing the Source of Oscillations at 0.34 Hz



Actual source; the net flow from this location was 6 times greater than the next largest.

Conclusion and Questions

- Measurement based modal analysis is becoming widely available for the analysis of power system data, both from actual measurements and simulation results
- Presentation has shown an iterative matrix pencil approach that can be used to quickly calculate values
- Results can also be easily visualized
- A simple but seemingly quite useful approach for locating the source of oscillations was presented