Computationally Efficient Identification of Power Flow Alternative Solutions with Application to Geomagnetic Disturbance Analysis

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***Abstract*- It is common knowledge that the power flow can have multiple solutions; one is the normal (desired) solution while the others are alternative solutions, which are sometimes characterized by low voltages. Usually these alternative solutions represent erroneous or unstable operation conditions. The paper presents a method for quickly assessing whether the power flow has likely converged to an alternative solution by considering, on a bus by bus basis, the sensitivity of the bus voltage magnitude to a change in the bus's reactive power injection, coupled with whether the bus is closely connected with negative reactance branches. Sparse vector methods are used to calculate these sensitivities quickly even for large systems with 10,000+ buses. The paper shows that these solutions can be particularly problematic when assessing voltage stability impacts of geomagnetic disturbances on the grid, and uses the method to identify problematic areas of a system under a GMD event.**

Index Terms—alternative solutions, low voltage solutions, geomagnetically disturbance, voltage stability, sensitivity

I. INTRODUCTION

The power flow is one of the most important power system analysis applications. It used to determine the voltage angle and magnitude at every bus in an electric grid model such that the voltages satisfy the real and reactive power balance equations. It has been well known that the power flow equations can have multiple solutions [1]. These can be divided into one normal (or desired) solution, and a potentially large number of alternative solutions. Since they are not always characterized by low voltages, here they will be referred to as alternative power flow solutions. These alternative solutions have sometimes been known as low-voltage solutions due to the common presence of relatively low voltage magnitudes at some buses. Sometimes the alternative solutions are actually desired, such as in the use of energy methods to assess power system voltage stability [2]. However, for the vast majority of solutions they represent an erroneous condition. The focus of this paper is on a computationally efficient algorithm to detect these solutions.

Traditionally these alternative power flow solutions were identified by the presence of positive eigenvalues in the power flow Jacobian [3], [4], with those having a single positive eigenvalue denoted as type-one solutions. However, recently in [5] it was shown that positive eigenvalues appear in the normal power flow solution for systems with negative reactance branches. Nowadays such branches are quite common in realistic power system models, with the statistics and examples provided in Section III.

This paper presents a method for quickly assessing whether the power flow has likely converged to an alternative solution by considering, on a bus-by-bus basis, the sensitivity of the bus voltage magnitude to a change in the bus's reactive power injection coupled with whether the bus is closely connected with negative reactance branches. Since this sensitivity is a diagonal element of the power flow Jacobian, we show how it can be calculated quickly using sparse vector methods [6].

While alternative solutions can occur with essentially any power flow solution, the paper shows that they are particularly problematic when doing power flow studies to assess the voltage stability impacts of geomagnetic disturbances (GMDs) on the electric grid. GMDs impact power system operation by causing quasi-dc geomagnetically induced currents (GICs) to flow in transformers and transmission lines; these lead to increased reactive power consumption in high voltage transformers. Techniques for including GICs in the power flow are discussed in [7], [8]. In considering GMDs in the power flow, the standard from [9] requires the consideration of a local GMD enhancement, which can result in substantially varying reactive power injections from one power flow solutions to the next. The paper shows how such studies can result in convergence to alternative solution, and presents examples demonstrating the proposed algorithm.

The main contribution and novelty of this work is essentially two fold, 1) a fast method to detect "candidate" alternative solution buses, taking advantage of sparsity techniques to calculate sensitivities, and 2) automatically detecting the actual alternative solution buses from these candidate buses by calculating proximity to negative reactance branches. Such a method can be a useful tool in the analysis of large-scale systems to quickly detect voltage issues such as a local voltage collapse, or undesirable solution due to bad model data. Further action can then be taken to "correct" this solution by adjusting model parameters, adding var support, etc.

The paper is organized as follows. Section II gives some background on the concept of multiple power flow solutions and their connection to bus dV/dQ sensitivities. Section III delves further into these sensitivities namely 1) what their negative values mean, 2) their statistics from power system models, and 3) the use of sparsity techniques to quickly

This work was supported by the National Science Foundation under Award Number ECCS-1916142. This paper was presented at the Texas Power and Energy Conference, College Station, TX, February 2020

978-1-7281-4436-8/20/$31.00 ©2020 IEEE

calculate them on a bus-by-bus basis. Section IV describes the overall method of determining convergence to a low voltage solution based on the proximity of the negative sensitivity bus to a negative reactance branch. This is shown with some examples of large-scale systems. Section V summarizes the paper with directions for future work.

II. Multiple Solutions and Sensitivities

A simple way to visualize the existence of multiple solutions can be a PV curve, shown in Figure 1. For a given load demand, the solution can converge to either a high voltage solution V_H or a low voltage solution V_L, depending on the initial values. V_H is generally the desrired solution as it is usually in the power grid operating range, i.e. closer to 1 pu. When we refer to alternative solutions which are not necessary very low in voltage, we can see that they appear closer to the point of maximum loadability point, P_{max} (also called the stability boundary). This point is known to lie at the midpoint of high and low voltage solutions [10].

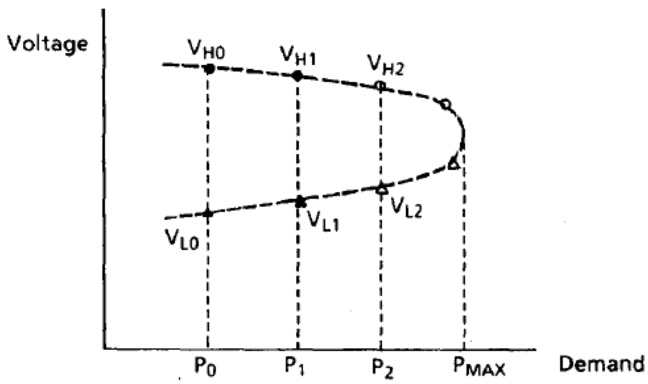

Figure 1. Relationship between real power demand versus voltage magnitude at a bus, from [12]

For a purely reactive load, as in the case of GMD induced losses, which is discussed further, consider the following example depicted in Figure 2.

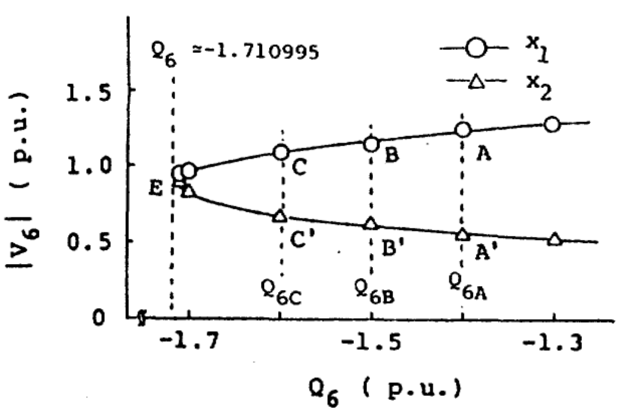

Figure 2. Relationship between Var injection and voltage magnitude at a bus, from [13]

There are several well-established methods for finding multiple power flow solutions [11], one of them being the exhaustive method from [12]. This was implemented in [13] to obtain the three low-voltage solutions A', B', C' corresponding to the "desired" solutions A, B, and C respectively. This paper was one of the earliest ones to make a reference to voltage sensitivities in this context, and noted that the dV/dQ sensitivities for the high and low voltage solutions were opposite in sign.

At the maximum reactive loadability point, Q_{max}, the sensitivity i.e. the slope tends to infinity. Beyond this load value there is no solution, and the slope becomes negative when the load is decreased. This is the key concept used in this paper to detect alternative power flow solutions based on negative dV/dQ sensitivities. However, some special cases need to be considered where system parameters can cause negative sensitivity values for a "regular" solution. The proposed alternative solution detection method accounts for these exceptions to isolate the "actual" alternative solutions.

III. Negative Sensitivities and Reactances

A. *Interpretation and Statistics*

The main reason behind negative dV/dQ sensitivities despite the convergence to V_H is the presence of a negative series reactance at or near the bus(es) being analyzed. Hence in order to detect a true alternative solution convergence, our method needs to calculate and rule out proximity to these branches when a negative dV/dQ sensitivity is encountered at a bus. As seen later, these systems can have up to 5% of their total number of branches with negative reactance values, accounting for hundreds or thousands of branches, hence including them in the methodology is of utmost importance.

The main sources of negative reactance branches are 1) series capacitors, and 2) fictitious "star" buses of three-winding transformers, and 3) equivalent (EQ) lines. These parameters are appearing more commonly in models of real, large-scale power systems. The details of these systems cannot be shared due to confidentiality concerns; however we have summarized some of the statistics in this section to demonstrate the prevalence of this issue and why our method is important. Such statistics have been used to build synthetic but realistic test systems that can be made publicly available for education and research, including reproducibility of results.

The star buses mentioned above arise from star equivalents of three-winding transformers, which are actually delta connected out in the real grid. The reactances of these mathematically equivalent branches can, hence, be negative at times. The star equivalent is able represent a 3-winding transformer as three 2-winding transformers and simplify calculations. Similarly, equivalent lines also do not represent an actual, physical reactance value but are important to track in proximity to negative sensitivities.

Table 1 shows some relevant statistics of negative reactance (NR) branches from three actual power grid models, and two synthetic but realistic test systems of similar scale as the actual grid (10,000 buses and 70,000 buses) derived from

[14], [15]. Due to confidentiality requirements, the authors are unable to reveal the source(s) of the actual grids' data. The purpose of this subsection is just to demonstrate some properties compared to synthetic grids. A key difference between the real and the synthetic systems is that the latter do not have series capacitors modeled in them. Another 2000-bus synthetic system (not mentioned in this table since it has no NR branches) is considered later in the paper as a simple example to illustrate the method.

TABLE I. Negative reactance (NR) branch statistics

	Actual Grid 1	**Actual Grid 2**	**Actual Grid 3**	**10K**	**70K**
Number of branches	97344	26397	8972	12707	88207
Number of branches with NR	2114 (2.17%)	698 (2.64%)	187 (2.08%)	193 (1.52%)	1365 (1.55%)
Number of transformer branches with NR	2004 (2.06%)	529 (2%)	166 (1.85%)	193 (1.52%)	1365 (1.55%)
Number of line branches with NR (EQ lines)	50 (0.05%)	9 (0.03%)	1 (0.01%)	0	0
Number of series capacitor branches	60 (0.06%)	160 (0.61%)	20 (0.22%)	0	0

EQ = Equivalent

B. Sensitivity Calculations

Here we discuss how these sensitivities are calculated, and how quickly it is possible to do so even for systems with tens of thousands of buses. Sensitivities are linearized relationships, which are used to determine the impact of small changes in a variable on the system [16]. The negative inverse of the power flow Jacobian, $\mathbf{J}$, describes the way the power flow solution state variables $\boldsymbol{\theta}, \boldsymbol{V}$ i.e. the voltage angle and magnitude, change due to bus power injection mismatch.

$$\boldsymbol{\Delta s}_{(\theta,V)} = [-\mathbf{J}]^{-1} \cdot \boldsymbol{f}_{(p,q)} \tag{1}$$

The sensitivity of the voltage magnitude $\boldsymbol{V}$ to the injected reactive power $\boldsymbol{Q}$ at all buses is given by the block matrix $\boldsymbol{\Lambda_{VQ}}$ of $\mathbf{J}^{-1}$ as,

$$\mathbf{J}^{-1} = \begin{bmatrix} \frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{P}} & \frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{Q}} \\ \frac{\partial \boldsymbol{V}}{\partial \boldsymbol{P}} & \frac{\partial \boldsymbol{V}}{\partial \boldsymbol{Q}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Lambda_{\theta P}} & \boldsymbol{\Lambda_{\theta Q}} \\ \boldsymbol{\Lambda_{VP}} & \boldsymbol{\Lambda_{VQ}} \end{bmatrix} \tag{2}$$

Thus, we only need to calculate a portion of the diagonal of $\mathbf{J}^{-1}$. Inverting this matrix, however is a computationally challenging task. Factorizing a full matrix is an O(n^3) operation. However, recognizing that $\mathbf{J}$ is also a sparse matrix similar to the network admittance matrix or $\mathbf{Y}_{\boldsymbol{bus}}$, we can leverage sparse vector methods [6], which can bring down the computational complexity to about O($n^{1.5}$). The main steps involved in applying sparse vector methods here are performing, 1) **LU** factorization of $\mathbf{J}$, 2) fast forward substitution with the lower triangular matrix **L**, and 3) a fast backward substitution using the upper triangular matrix **U** [17], [18]. In this case, the factorization is O($n^{1.4}$) whereas the forward/backward substitution is O($n^{1.2}$) [19]. This especially makes a huge difference while dealing with large-scale systems of tens of thousands of buses. Both these substitutions take advantage of the fact that we need only certain elements of the sensitivity vector $\boldsymbol{\Delta s}$ and the diagonal elements of the sparse Jacobian matrix $\mathbf{J}$.

As an example, consider the systems discussed in Table I. For the Actual Grid 1 (AG1), calculating the dV/dQ sensitivities at all the 77,000 buses took around 28 seconds on an Intel® Core™ i7-7820HQ CPU @ 2.90GHz with 32.0 GB RAM, 64-bit OS, x-64 based processor. For the synthetic 2000 bus system, this takes less a second. This clearly indicates how quick and computationally efficient this method is. In the next section, we consider certain systems and intentionally "stress" them from a reactive power perspective to induce unusual operating points, more specifically push them towards converging to alternative solutions, so that we can detect them using our proposed method.

IV. Method and Examples

A. Overall Method

As mentioned earlier, the key to detecting an alternative solution at a bus is to rule out the possibility that the negative dV/dQ sensitivity (if it is negative that is) at the bus is being caused by a nearby negative reactance branch. In other words, the negative sensitivity should be a property of the system or operating point, and not due to the negative reactance. Naturally, this process can be sped up if it is already known that there is no negative reactance branch in the power system model as we do not need to check for proximity to these branches. Following this approach, the overall process of automatically determining convergence to an alternative power flow solution can be summarized as follows-

```
Calculate dV/dQ sensitivities at all buses
    If negative sensitivities exist
        If the case has NR branches
            For each bus in the set of negative dV/dQ
                -If it is connected directly or through one
                 neighbor bus to a NR branch, reject that bus
                 (as likely alternative solution)
                -If not, flag this bus as converged to a likely
                 alternative solution
            End
        If there are no NR branches
            Flag all buses with negative dV/dQ as likely
            alternative solution
    If no buses have negative sensitivities
        There are likely no alternative solutions
End
```

Here, we chose number of bus hops as the metric to decide whether a bus is close to a NR branch. Other metrics such as electrical distance could be used as well. The common element among both these metrics is that if the value is large enough, the reason for negative sensitivity can be considered to be arising from a genuine alternative solution at that bus. In our experimental testing, the first neighbor metric has worked well to detect the proximity to NR branches.

B. Example 1 – 2000 bus system with a GMD

To illustrate the concept, we begin with a simple example i.e. the 2000 bus synthetic test system from [14]. This is based geographically on the footprint of Texas. As mentioned earlier, this simple model does not have a NR branch i.e. no three-winding transformer, series capacitor, or equivalent lines. Hence a negative dV/dQ in this case can be solely attributed to an alternative solution, eliminating the intermediate step of calculating the distance of the bus from the NR source. Calculating the sensitivities of the system as is, yields no negative values, indicating a "normal" operating point.

Now consider a stressed version of this system, where it is facing a major GMD event. GMDs are caused by solar winds and coronal mass ejections, which alter the earth's magnetic field, inducing electric fields at the earth's surface. These in turn induce quasi-dc GICs in grounded conducting paths of the power grid. The major effects of GICs are increased reactive power absorption by transformers, as well as harmonics that can lead to relay misoperation and tripping of var support devices when they are most needed. The goal of this subsection is to illustrate, 1) how a steady-state voltage stability study of a system under a GMD can cause multiple power flow solutions and, 2) whether our method is able to detect them.

We perform a voltage stability study of this system applying a GMD scenario, using the methodologies described in [8], [20]. Similar to a QV analysis, the GIC-induced reactive load at transformer buses is increased slowly by increasing the input electric field for the simulation from 0 V/km in steps of 0.5 V/km. At each step, the dc GICs are calculated, followed by the GIC-induced transformer reactive power losses, and then the power flow is solved including these losses. In this example, a uniform electric field was assumed over the entire footprint. At an input of 4 V/km, a switched shunt in San Antonio was opened as a possible GIC harmonic-induced relay misoperation contingency. Then gradually increasing the field by 0.5 V/km, the solution failed to converge at 7.5 V/km. Hence, 7 V/km was noted as the maximum GIC reactive loadability point. From 7.5 V/km, reducing the electric field back to 7 V/km caused the system to converge to another solution, with slightly lower voltages. Sensitivities at this "load" value i.e. electric field = 7 V/km, before and after the loss of convergence were calculated.

Figure 3 describes this process and the solutions in more detail, considering one bus (Dallas #5252) as an example. Its voltage is close to 1 pu before the GMD i.e. before the electric field is applied. The pu voltage magnitude at this bus from each power flow solution, following the increase in electric field is shown by the markers on the "Forward" curve. Point V_m, i.e. when the input is 7 V/km, is where the last convergence occurs. At this stage, the dV/dQ sensitivities are calculated and they are all positive. The lowest voltage in the system is 0.91 pu at this point. The solution does not converge at the next step of 7.5 V/km, following which the electric field is reduced to 7 V/km and the solution V_m' is obtained at this bus. Continuing in this way, gradually reducing the electric field in steps of 0.5 V/km traces out the "Reverse" curve shown in the figure.

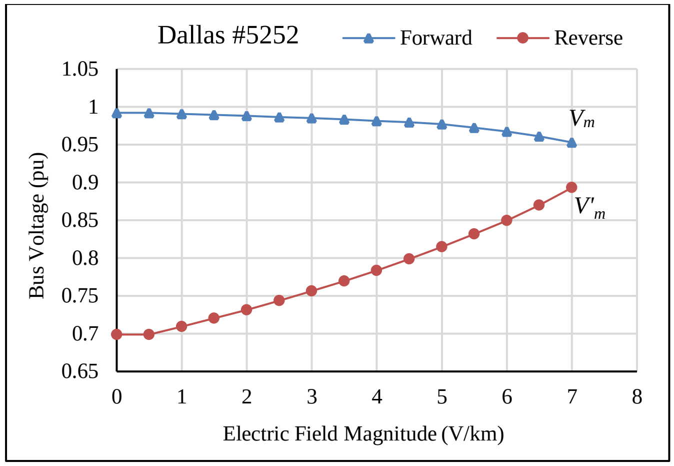

Figure 3. Steady state voltage stability analysis curve for a bus (Dallas, Bus number 5252) from the 2000 bus synthetic system

TABLE II. Ten lowest dV/dQ buses at solution V_m' for the 2000 bus system

No.	Name	Nom kV	Area Name	dV/dQ	Negative reactance lines	Likely Alternative Solution	PU Volt
5322	DALLAS 2 0	500	North Central	-4.46E-05	NO	YES	0.924
5384	DALLAS 3 0	500	North Central	-4.32E-05	NO	YES	0.924
5464	FRISCO 2 0	500	North Central	-4.32E-05	NO	YES	0.919
5063	ARLINGTON 1 0	500	North Central	-4.20E-05	NO	YES	0.927
5102	RICHARDSON 2 0	500	North Central	-4.18E-05	NO	YES	0.917
5304	ALLEN 1 0	500	North Central	-4.11E-05	NO	YES	0.923
5083	MCKINNEY 3 0	500	North Central	-4.10E-05	NO	YES	0.923
5448	GRAND PRAIRIE 3 0	500	North Central	-4.04E-05	NO	YES	0.921
5295	MCKINNEY 1 0	500	North Central	-3.97E-05	NO	YES	0.926
5033	DALLAS 1 0	500	North Central	-3.96E-05	NO	YES	0.929

Sensitivities calculated at point V_m' indicate 70 buses with negative sensitivities, from a total of 2000 buses in the system. As mentioned earlier, this system has no NR branches. Hence there are 70 likely alternative solution buses. They all belong to the North Central area of the system, indicating a local voltage collapse. Some more interesting observations are:

1. Buses with the lowest voltages do not necessarily converge to alternative solutions. In fact, the 53 lowest voltage buses (0.88 pu – 0.89 pu) had positive dV/dQ

values. For the overall system, there are 162 low voltage (< 0.9 pu) buses in total. Only 16 out of 70 negative sensitivity buses had voltages < 0.9 pu. Hence, focusing on the lowest voltage bus(es) may not be a good heuristic measure to detect alternative solutions.

2. On the other hand, several seemingly "normal" voltage level buses seem to converge to alternative solutions, in this GMD scenario. E.g. for the 70 negative sensitivity buses, the average voltage was 0.913 pu, with a maximum value of 0.948 pu and minimum of 0.893 pu Table II shows 10 buses with the lowest sensitivity values.
3. There are 36 buses with voltage < 0.9 pu in the East area but all have $dV/dQ > 0$. Hence, not considering sensitivities could also lead to a wider search space for alternative solutions.

Note that it is not necessary that a system will converge to an alternative solution when the electric field is reduced after the maximum GIC reactive loadability point is reached. This was confirmed by keeping the shunt capacitor in San Antonio closed (that was opened in the previous simulation as a GIC contingency) and repeating the voltage stability study. In this case, there is only one solution at 7 V/km, which is the desired high voltage solution. Solution characteristics in GMD voltage assessments can thus vary depending on the reactive support in the system, for a given electric field profile.

C. *70,000 bus system*

Next, we consider a more complex example consisting of more than 30 times the number of buses in the previous case. This is the same 70K bus synthetic system mentioned earlier in Section III. Unlike the 2000 bus example, this case has NR branches due to the presence of three-winding transformers. Here we can show how buses close to NR branches are used in the process of determining buses with alternative solutions.

The 70K system covers a major portion of the Eastern part of the US. A procedure similar to the previous GMD study was applied to study the voltage impacts. Due to the complexity of the case in terms of interconnected areas, and differences in reactive power planning across generators and other devices, the last convergence point was obtained at a much lower electric field input of 0.5 V/km, compared to the 2000 bus system. The main cause for the non-convergence was generators reaching their reactive power limits. Note that this does not mean that the power grid of similar scale or geographic location would experience a blackout at this electric field magnitude. It most likely points to the need for additional reactive power planning in designing this synthetic system, especially under the influence of a specific event such as a GMD. Another point to consider is whether performing a voltage stability study across such a large footprint is practical, since reactive power is a localized phenomenon. We use this example purely for illustrative purposes.

The algorithm identified a total 167 likely alternative solution buses from the 508 buses with $dV/dQ < 0$, i.e. about a third of these, and 0.24% of the total number of buses in the system. In other words, these 167 buses were not found to be near any NR branch. The 15 lowest dV/dQ values were from buses near NR branches, with relatively high voltages between 1.02 pu and 1.06 pu. These branches are in fact connected to the fictitious star buses of three-winding transformers. These star buses are typically represented in real power grid cases by a nominal voltage level of 1 kV. Table III shows these 15 lowest-sensitivity-valued buses, plus five more for reference. We can see among the five that are included, four buses (i.e. Gulfport) are not near negative reactances and are hence flagged as alternative solutions. Note the difference in voltages between the first alternative solution bus encountered (#30334) in this table and the previous values (1.04 pu vs 0.836 pu).

Of the 167 alternative solution buses, 58 from Alabama, 3 from Louisiana, and 106 from Mississippi. In this system, the geographic states have been divided into different control areas. Of these 167 buses, the 60 buses with the lowest voltages are in Mississippi, with 0.82 pu as the lowest and 0.99 pu as the maximum with alternative solution. These 60 buses in Mississippi also happen to be the lowest voltage buses in the entire system, which was not the case in the 2000 bus study where alternative solutions were found at higher voltages.

TABLE III. Twenty lowest dV/dQ buses at 0.5 V/km for the 70,000 bus system

No.	Name	Nom kV	Area Name	dV/dQ	Negative reactance lines	Likely Alternative Solution	PU Volt
56476	SAINT LOUIS 11 4	1	Missouri East	-8.37E-03	YES	NO	1.059
55173	KNOXVILLE 47 4	1	Iowa	-5.79E-03	YES	NO	1.053
56576	SAINT LOUIS 56 4	1	Missouri East	-5.78E-03	YES	NO	1.059
56425	FLORISSANT 4 4	1	Missouri East	-5.23E-03	YES	NO	1.057
68963	FARGO 6 4	1	North Dakota	-4.94E-03	YES	NO	1.020
20226	DURHAM 13 4	1	North Carolina	-4.83E-03	YES	NO	1.019
54183	AMES 8 4	1	Iowa	-4.78E-03	YES	NO	1.038
54513	WINTERSET 4 4	1	Iowa	-3.97E-03	YES	NO	1.031
22421	ANDERSON 1 4	1	South Carolina	-3.65E-03	YES	NO	1.038
23584	WILLIAMSTON 10 4	1	South Carolina	-3.57E-03	YES	NO	1.018
64373	OLATHE 7 4	1	Kansas	-3.54E-03	YES	NO	1.028
65192	KANSAS CITY 84 4	1	Kansas	-3.46E-03	YES	NO	1.047
58630	BELLA VISTA 3 4	1	Arkansas	-3.21E-03	YES	NO	1.033
58164	KIMBERLING CITY 3 4	1	Missouri West	-3.14E-03	YES	NO	1.046
65922	LAKIN 4 4	1	Kansas	-3.08E-03	YES	NO	1.040
30334	GULFPORT 4 1	100	Mississippi	-2.99E-03	NO	YES	0.836
30335	GULFPORT 5 1	100	Mississippi	-2.99E-03	NO	YES	0.844
67236	BELLEVUE 14 5	1	Nebraska	-2.98E-03	YES	NO	1.057
30337	GULFPORT 7 1	100	Mississippi	-2.98E-03	NO	YES	0.851
30338	GULFPORT 8 1	100	Mississippi	-2.96E-03	NO	YES	0.854

V. SUMMARY AND FUTURE WORK

This paper has provided a methodology to rapidly determine whether a system has converged to an alternative or low voltage solution by leveraging sparsity techniques towards calculating sensitivities of voltage magnitude to the reactive power injection at each of the buses in the system. Negative values indicate likely convergence of a bus to an alternative solution, unless it is close to negative reactance branch. This method is much faster compared to previous exhaustive and iterative methods of finding alternative solutions. It can help narrow down "problem areas" efficiently, such as areas in a system that are prone to voltage collapse. Such tools can be helpful in large system studies, especially steady-state voltage stability and GMD vulnerability assessments.

Ongoing work is looking into how to "correct" these undesired solutions. Several directions are being considered. First, convergence to a high or low voltage solution depends on the initial guess of the solution. This idea can be used to alter the starting point, with some additional reactive support perhaps in cases such as GMD studies. Pre-emptive dispatch of shunt capacitors and cancelling generator outages is already an established and recommended GMD operating procedure. This can be simulated in studies to prevent these low voltage conditions. On a more analytical front, techniques such as continuation power flow [11] can be explored to avoid the singularity of the Jacobian matrix encountered, for instance, between points V_m and V_m' in the 2000 bus study. This involves determining and using a continuation parameter to increase the load gradually to avoid the non-convergence near the voltage instability or maximum loadability point. This will be particularly useful in QV curve analysis or GMD voltage stability studies where the user has control of the sequential increase of the load. It will be interesting to determine how many more high or low voltage solutions, if any, are possible beyond the maximum loadability point found by manual methods in this paper (i.e. between 7 and 7.5 V/km for the 2000 bus system). This is also indicative of the stability boundary of the system, opening directions for application of this method in that area of studies and research.

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