Assessment of Multirate Method for Power System Dynamics Analysis

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***Abstract*—As many power electronic converter interfaced technologies have recently been integrated into power systems, it has resulted in substantial computation demands and potential numerical instability issues in power system dynamics studies. Using a proper integration time step is key to solve these issues and multirate methods, which utilize a smaller time step to capture the system behavior associated with fast states and a larger time step for slow states, are an efficient way to represent current power system dynamics. This paper proposes a methodology to evaluate the system dynamic behaviors depending on different integration methods: a single rate method and a multirate method. These methods are compared with the results of rotor angle dynamic simulations using two different sizes of systems. Computational time and performance accuracy of the two methods are also assessed for the comparison.**

***Index Terms*—Multirate integration methods, rotor angle dynamic simulation, eigenvalue analysis, subinterval size**

I. Introduction

VARIOUS dynamic models are utilized in power system simulations to see if the system can appropriately respond within the first few seconds to possible contingencies. Conducting dynamic studies can help system operators to prevent unpredictable accidents as well as maintain stability. Different classes of dynamic phenomena in power systems are defined through the time scales of each dynamic model, ranging from microseconds to minutes [1].

In recent years, new power electronic devices and renewable generation have been increasingly integrated into power systems. Due to their fast-response and inertia-less characteristics, these new dynamic models can lead to high computational burdens, complexities, and stability limits in dynamics analysis [2]. Therefore, it has become important to develop methods to efficiently conduct dynamics studies, as well as realistically represent current power systems in dynamic simulations.

Different numerical integration methods are used in simulations to determine dynamic states at the next time step, and utilize iterative approaches for finding the solution of nonlinear algebraic equations. The two main categories of numerical methods are implicit and explicit integration methods. Multirate approaches, one of the explicit integration methods, offer an efficient integration technique for systems involving a wide range of diverse time responses. Integrating different states with different step sizes is the fundamental basis of this method. Only fast devices selectively use a small time step, while the remainder of the system uses a larger time step. The multirate method, thus, helps to avoid numerical instability issues and shorten computational time [2]. A substantial amount of power dynamics studies have been conducted over the years using multirate methods.

The multirate method was first proposed in [3], [4]. The multirate method is applied to a linear system of n distinct time scales, and then extended to a nonlinear power system that demonstrates a time scale separation into two and three distinct time scales in [5]. A multirate time stepping approach for time integration of differential and algebraic equation (DAE) systems contributes to automatically adjusting the partitioning of the variables via the local time variation of the system solution, and decides the optimal size of the global time step to minimize global computational costs [6], [7]. A methodology to divide the algebraic variables into two groups, fast and slow algebraic variables, using power mismatch equations is proposed in [8].

A methodology for the design of a Kalman filter for a multirate control system with a fast input is presented in [9]. Noise variance and the oversampling ratio of the multirate controller is confirmed using Monte Carlo method. A multirate extended Kalman filter (EKF) algorithm, involving both input and output algorithms, is proposed for load torque estimation in the induction motor, which can be performed in real-time on a PC-cluster node. A multirate model reference adaptive system (MRAS) is also presented to compute the rotor time constant for assurance of high-performance control of an induction motor [10].

Multirate stabilizers are designed satisfying three properties; periodicity, causality, and finite dimensionality. Implementing multirate control systems provides a cost advantage in achieving the infinite optimal design [11]. The synchronous generators and controllers in the holomorphic embedding (HE) formulation are modeled by an adjustable generator interface. The switch between generator and network coordinates is designed and solved using HE [12].

While many applications of multirate methods in dynamics studies have been conducted as listed, there is little written in the way of assessment. Thus, this paper proposes a method to

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compute and evaluate the power system dynamics by comparing the simulation results both with the single rate and multirate methods. Rotor angle dynamics are mostly considered for the analysis, where its time scales are ranging from about ten milliseconds to minutes [1]. For the approach with the single rate method, a standard time step and an incredibly reduced time step are considered.

The rest of the paper is structured as follows. Section II presents a brief description and an example of the multirate integration method and a methodology to determine the time step for the fast components. Section III applies these techniques to two different case studies. Section IV demonstrates our assessment approach of the multirate method for dynamics analysis with numerical results such as computational time/accuracy comparisons. Finally, the conclusion is included in Section V.

II. Multirate Integration Method

A. Methodology

The multirate approach, which uses a unique integration step depending on dynamic characteristics of each variable, is one of the main explicit numerical integration methods. This scheme is commonly utilized in some commercial dynamic simulation software [13]. By adopting the multirate methods into dynamic simulations, maintaining stability with very fast dynamics can be successfully achieved, since the methods allow these dynamic models to be integrated with a much smaller time step while slow variables constantly use a larger time step. Therefore, this method aids in increasing the stability as well as reducing the computational burdens. Single rate integration methods, however, can cause numerical instability issues because their time step can sometimes be too large for fast devices. If the time step is reduced to not miss what's happening with the fast components, then slow components will also be unnecessarily updated every time step, which results in long simulations.

An example of a multirate integration process in a linear system is demonstrated in Figure 1.

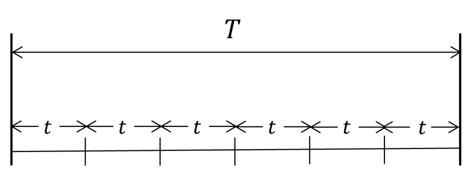

Fig. 1. The Multirate Integration Method by Slow and Fast Variables

In this example, the ratio of the smaller steps (t) to the larger step (T) is equal to six. Linear interpolation is used from slow components in order to solve the equations of each fast component. The number of subintervals (n) in the linear system can simply be obtained by computing the ratio of the main time step (T) to time step of the fast components (t), where T = nt. This linear relationship, however, cannot be directly applied to non-linear systems and thus they require more computational burden to decide the subinterval size.

To determine the subinterval size for non-linear power systems, a single machine infinite bus (SMIB) system is used based on modal analysis [14]. This analysis helps to determine eigenvalues, which can identify the fast dynamic states. Then, the subinterval is normally applied to all the differential equations for the specific model. Lastly, the minimum subinterval size for the fast dynamic variables is decided by Equation 1 [15].

$$n_{min} = \frac{main\ time\ step}{required\ time\ step\ for\ fast\ states} \tag{1}$$

Note that the user determines the particular value of the main time step, which is the slow time step. The required time step for fast variables can be obtained via the range of eigenvalues based on time step using the 2nd order Runge-Kutta (RK2) method [16]. This range has been presented in Table I.

TABLE I
The Required Time Step for Fast Variables by Range of Eigenvalues

Range of Eigenvalue	Time Step (Cycle)
$-120 < \lambda < 0$	1
$-240 < \lambda < 0$	0.5
$-480 < \lambda < 0$	0.25
$-1200 < \lambda < 0$	0.1

B. Example: two-bus system

A two-bus test case is used as an example and its one-line diagram is presented in Figure 2. This system involves one synchronous machine and one exciter per each bus. GENROU and EXST1 dynamic models [2] are used for the synchronous machine and the exctier, respectively. The system consists of a 138-kV network.

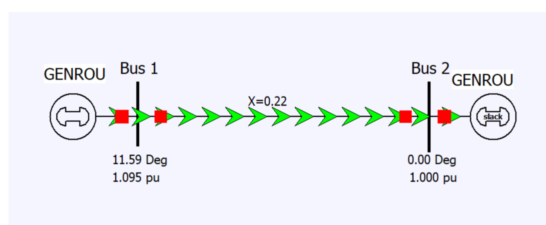

Fig. 2. One-line Diagram of the Two Bus System

From the SMIB analysis, the most negative eigenvalues are -1014.41 and -32.63 at bus 1 and bus 2, respectively. Since a large negative eigenvalue represents a very fast mode, it implies that there would be very fast dynamics at bus 1. It can be explained by the exciter dynamic state V_A of participation factor at bus 1, which has a high magnitude. It contributes to the exciter having a fast mode, and thus gaining a huge negative value at the same bus. As a result, the exciter at bus 1 uses 16 subintervals to avoid numerical instability. The block diagram of this exciter model, EXST1, is shown in Figure 3.

The required time step for the fast state V_A can be decided using the eigenvalue corresponding to its bus. Given that the time step is 0.5 cycles, only the exciter at bus 1 will need to use the required step size 0.1 cycle based on its eigenvalue. Therefore, its subinterval size should be greater than 5.

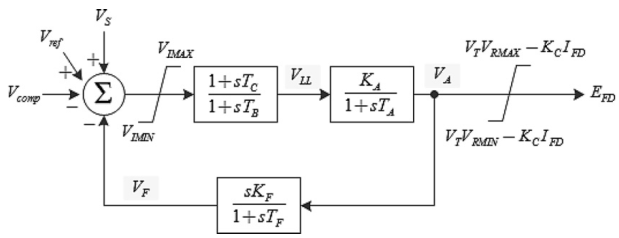

Fig. 3. Block Diagram of EXST1 Exciter Model [2]

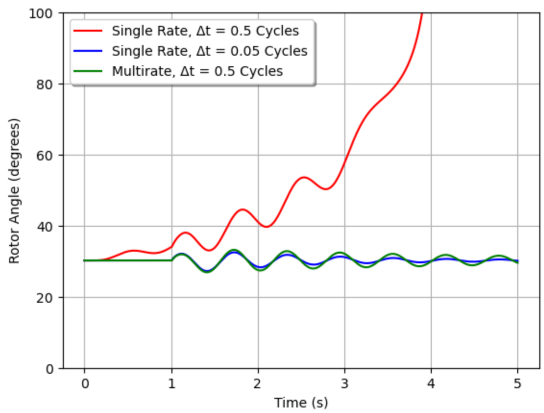

Fig. 4. Generator Rotor Angle for 2 Bus System

To demonstrate the differences between single rate and multirate integration techniques, Figure 4 shows the rotor angle for the generator at bus 1. Here, a solid 3 phase fault is applied at 1 second, and cleared it at 1.01 seconds. It can be seen that if dynamics analysis with a time step of 0.5 cycles is attempted, the rotor angle diverges quickly. However, if the time step is greatly reduced, the behavior of the rotor angle stabilizes, and converges to a new value. If multirate integration at the larger time step is applied, the rotor angle behavior closely matches that of the reduced time step.

III. Case Studies

Different integration methods are applied to two different study cases. Dynamics studies and eigenvalue analysis have been conducted to compare the simulation results depending on the integration methods and see if the multirate technique has been implemented into the systems properly.

Dynamics studies look at the behavior of a system after a contingency occurs. When comparing single rate and multirate integration, the ability of dynamic studies to accurately model the behavior of the system, including variables like bus voltage and generator rotor angle, is observed. Also, eigenvalues provide a way to determine if a system is nearing instability and thus are indicators to its stability. They can identify the existing frequencies and modes, and show interactions between a system and its modes. Therefore, the eigenvalues can show how the system uses appropriate values of the subintervals to escape numerical instability issues and curtail the computational requirement.

To compare the impact of the different integration methods on power systems, three different approaches have been conducted: (1) with the single rate method and a standard time step, (2) with the single rate method and a greatly reduced time step, and (3) with the multirate method and a standard time step.

These three methods are applied to two synthetic grids: a 42-bus system and a 2000-bus system. These synthetic grids are publicly available test cases built on the footprint of Illinois and Texas, respectively. They are fictitious but possess characteristics of an actual grid statistically and functionally [17], [18].

A. Case Study 1: 42-bus Synthetic Grid

The first test case is the 42-bus synthetic system modeling a 345/138 kv network. The case has 14 generators and 55 loads, and includes 14 synchronous machines, exciters, and governors for each generator, but no stabilizers. The one-line diagram of the 42-bus system is illustrated in Figure 5.

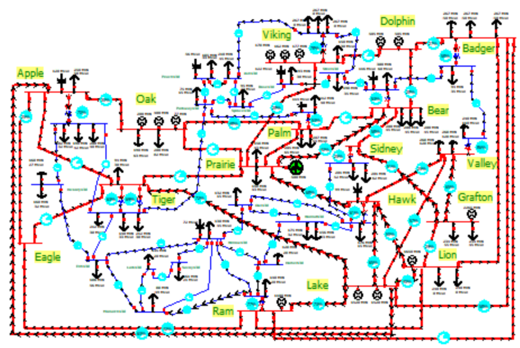

Fig. 5. One-line Diagram of the 42 Bus System

The dynamics analysis results for both single rate and multirate integration can be seen in Figure 6.

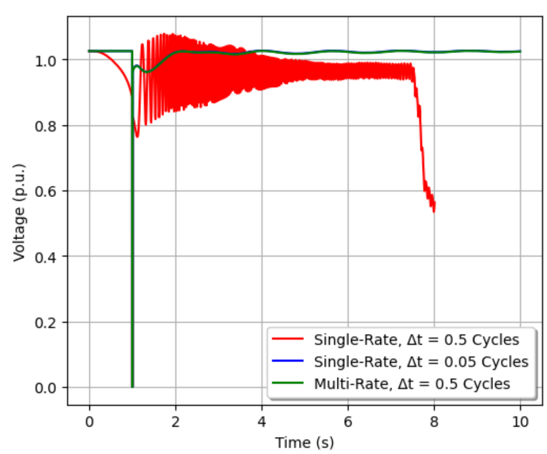

Fig. 6. Bus Voltage for 42 Bus System

In particular, the focus is on the voltage of a specific bus with an EXST1 exciter after a three-phase solid fault is applied at that bus. The fault is applied at 1 second, and cleared at 1.01 seconds. The time step here is 0.5 cycles, and without multirate integration, the solution diverges, and the dynamics of the system are not captured. In addition, the voltage does not stay flat indicating that there is an initial condition issue with the single rate integration method with a time step of 0.5 cycles. However, by implementing multirate integration at the same time step, there are dramatic improvements in the simulation's ability to capture the dynamics of the rotor angle and the issue with the initial condition also disappears. This is compared to a single rate approach with a greatly reduced time step of 0.05 cycles, and notice that the behavior of the voltage is very similar in both cases.

B. Case Study 2: 2000-bus Synthetic Grid

The one-line diagram for the 2000-bus synthetic system is shown in Figure 7. There are 435 synchronous machines, 444 exciters, 435 governors, and 434 stabilizers. The total number of dynamic loads is 1350. For the voltage levels, the orange, purple and green lines in the one-line diagram embody the 500-kV 230-kV and 115-kV network in the system respectively. The total load demand is 67 GW with 100 GW generation capacity.

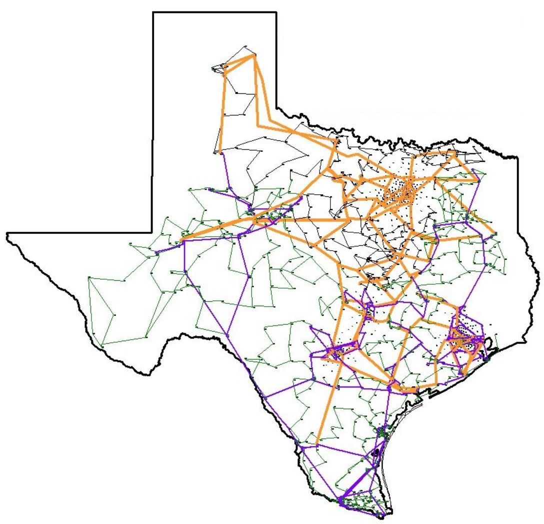

Fig. 7. One-line Diagram of 2000 Bus Synthetic Grid

The dynamics analysis results can be seen in Figure 8. The simulation lasts 30 seconds, with a three phase solid fault occurring at 1 second, and being cleared at 1.1 second. All measurements were taken at bus 7108, which has the largest negative eigenvalue from SMIB analysis. Looking at Figure 8, both the multirate approach with the larger time step of 0.5 cycles and the single rate approach with the reduced time step of 0.05 cycles converge to the same value, but the single rate approach with a larger time step does not converge.

While generator outages are generally found by looking at properties like rotor angle and frequency that are associated with generators being in sync, numerical stability of the single rate approach can also be evaluated by looking at bus voltage

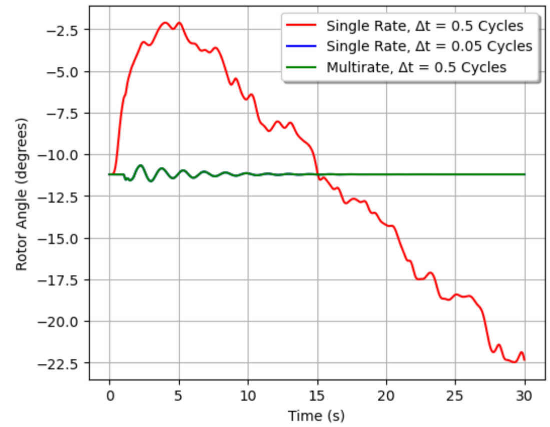

Fig. 8. Bus 7108 Generator Rotor Angle for 2000 Bus System

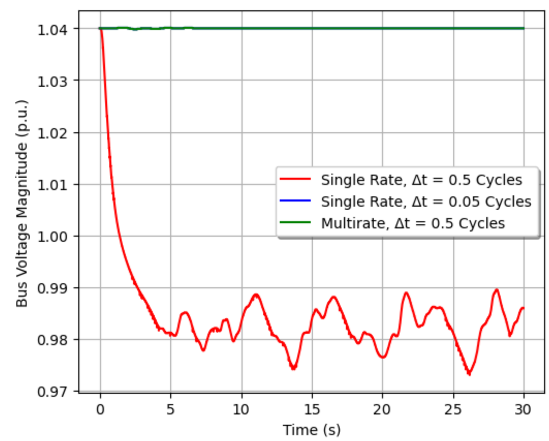

Fig. 9. Bus 7108 Voltage Magnitude for 2000 Bus System

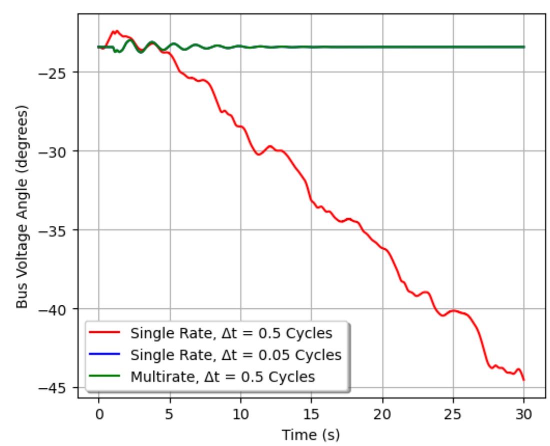

Fig. 10. Bus 7108 Voltage Angle for 2000 Bus System

magnitude and angle. Figures 9 and 10 show the magnitude and angle of the bus voltage, respectively.

Similar to rotor angle, it can be seen that both the bus voltage magnitude and angle do not converge to a specific value when considering a single rate approach with a larger time step, indicating numerical instability. However, when using a reduced time step with the single rate approach, both the voltage magnitude and angle converge, and similar behavior is seen when using the multirate approach with the larger time step, aligning almost exactly with the single rate approach with the reduced time step.

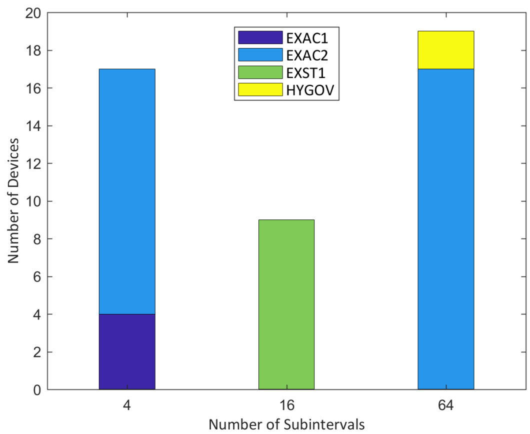

Fig. 11. The Number of Subintervals by Devices on 2000 Bus System

The subinterval results shown in Figure 11 reveal that very few fast devices, including exciters and governors, used the subintervals for their solution, although many other dynamic models were implemented into the system as well. The number of each device on the 2000-bus system is presented in Table II. Also, all devices using the subintervals have large negative eigenvalues ranging from -72.42 to -1014.93, which implies the system needed small integration time steps for them to avoid numerical instability since the large negative eigenvalues embody extremely fast modes.

TABLE II
THE INFORMATION OF DYNAMIC MODELS ON 2000 BUS SYSTEM

Exciters		**Governors**	
Device Name	**Number of Devices**	**Device Name**	**Number of Devices**
ESAC1A	4	GGOV1	367
ESAC6A	7	HYGOV	25
ESDC1A	12	IEEEG1	43
ESDC2A	1	**Machines**	
ESST4B	278	**Device Name**	**Number of Devices**
EXAC1	6	GENROU	410
EXAC2	38	GENSAL	25
EXPIC1	61	**Stabilizers**	
EXST1	9	**Device Name**	**Number of Devices**
IEEET1	23	IEEEST	434
SCRX	5	-	-

IV. Assessment

When evaluating multirate integration techniques, two primary considerations have been addressed; accuracy and computation time. Since multirate integration techniques strive to achieve equivalent results to single rate techniques, but at a larger time step for slow variables, it should be observed that multirate techniques should be faster and have a higher degree of accuracy when compared to single rate integration at the same time step.

A. Computation Times

Table III shows how long each dynamics study took for the two different combinations of integration techniques and time steps. Looking at the computation time differences for the 42-bus system, which had a simulation time of 10 seconds, it is seen that using single rate integration with a reduced time step, while converging to the correct solution, takes 2.02 seconds, which is almost seven times as long as when multirate integration is used with the larger time step. When looking at the 2000-bus case, with a simulation time of 30 seconds, implementation of a single rate approach with a reduced time step takes over eight times as long at 147.95 seconds, when compared to multirate integration with a larger time step, which takes 17.52 seconds.

These results display one of the largest benefits of multirate integration techniques; a reduction in computation time. While it is entirely feasible in the context of dynamics studies to arrive at an accurate solution if you provide a small enough time step, especially for larger systems, this process can be highly inefficient in terms of time and computational load. Implementation of multirate integration techniques help to circumvent these problems, allowing for both fast and accurate solutions for large systems.

TABLE III
COMPUTATION TIMES BY INTEGRATION METHODS

System	**Single Rate Reduced Time Step (s)**	**Multirate Larger Time Step (s)**
42 Bus	2.02	0.24
2000 Bus	147.95	17.52

B. Accuracy of Multirate Integration

Table IV shows the accuracy of the different combinations of integration techniques. In this instance, the single rate integration approach with a greatly reduced time step of 0.05 cycles is used as the ”true” data, as multi-integration techniques are looking to achieve equivalent results in less time. Mean squared error (MSE) is used as the metric to determine accuracy, and is calculated using Equation 2, where $Y(t)$ is the ”true” data mentioned above, and $\hat{Y}(t)$ is the data we are comparing against it, generated from either a single rate or multirate integration technique at the time step of 0.5 cycles.

$$MSE = \frac{1}{n}\sqrt{\sum_{t=1}^{n}((Y(t) - \hat{Y}(t))^2} \tag{2}$$

For the 42 bus system, the single rate integration approach with a larger time step does not converge to a solution. Combining that knowledge with the computation times indicated in Table III, it can be concluded that multirate integration achieves relatively accurate results with significant time saved for smaller systems.

When looking at larger systems, the accuracy results are looked at through the different properties of the system discussed in Section III. Starting with rotor angle, it is noted that there is a significant difference between the single rate and multirate approaches at the same time step. This difference of over 8 orders of magnitude is attributed to the divergence of the single rate approach. However, due to the smaller discrepancies, in terms of the numerical difference in values, the single rate approach and the multirate approach differ by a smaller amount for voltage magnitude. But, MSE calculations indicate that the multirate approach perfectly matches the results from the single rate approach at a reduced time step for voltage magnitude. Finally, when examining the differences between the approaches for voltage angle, it is seen in Figure 10 that the voltage angle does not converge, whereas the multirate approach does. Similar to rotor angle, due to divergence, the MSE difference between the single rate and multirate approach is over 8 orders of magnitude.

In all instances, irrespective of system size, it is seen that the multirate approach is always significantly better than the single rate approach when it comes to accuracy. This, in conjunction with considerations for computation time, as seen in Table III, re-affirm that a multirate integration approach is ideal when considering fast dynamics.

TABLE IV
MEAN SQUARED ERROR ESTIMATES BY INTEGRATION METHODS

System	Single rate Larger Time Step	Multirate Larger Time Step
42 Bus	-	7.60E-7
2000 Bus Rotor Angle	39.27	2.98E-7
2000 Bus Voltage Magnitude	0.003	0
2000 Bus Voltage Angle	130.86	2.14E-7

V. CONCLUSIONS

This paper presents a methodology to assess the impact of different integration methods on dynamics solutions. The results of the proposed assessment show that the multirate approach has considerable potential as an integration method when power system involves very fast dynamic devices. The simulations with the single rate method and a larger time step become numerically unstable. Although this instability issue can be solved by greatly reducing the time step, it causes another problem in the form of an increase in computation time. On the other hand, the simulations with the multirate method achieve numerical stability with a high accuracy; the multirate methods also alleviate computational burden. This was able to be successfully achieved by applying the smaller subintervals only to a select few fast devices and utilizing the large step for the remainder of the system. This paper also shows that the subintervals are employed appropriately through the eigenvalues analysis. All dynamic models using the subintervals have large negative eigenvalues, which indicate that the subintervals were only used for devices that had very fast modes.

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