ECEN 615
Methods of Electric Power Systems Analysis

Lecture 10: Advanced Power Flow, Sensitivity

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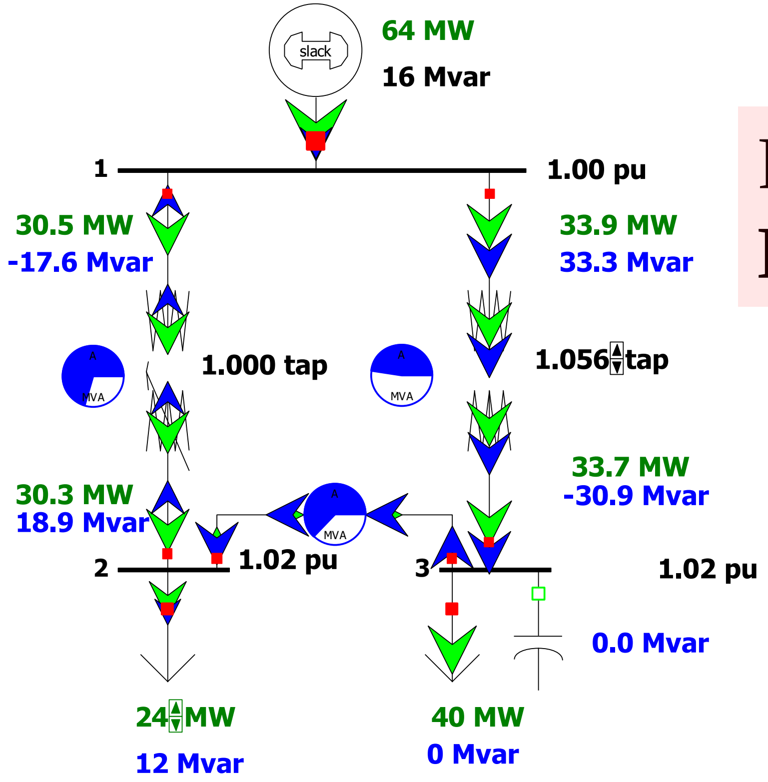
Announcements

- Homework 3 should be done before the first exam but need not be turned in
- Start reading Chapter 7 (the term reliability is now often used instead of security)
- First exam is in class on Thursday Oct 1
  - Distance learning students do not need to take the exam during the class period
  - Closed book, notes. One 8.5 by 11 inch notesheet and calculators allowed
  - Last’s years exam is available in Canvas

Circulating Reactive Power

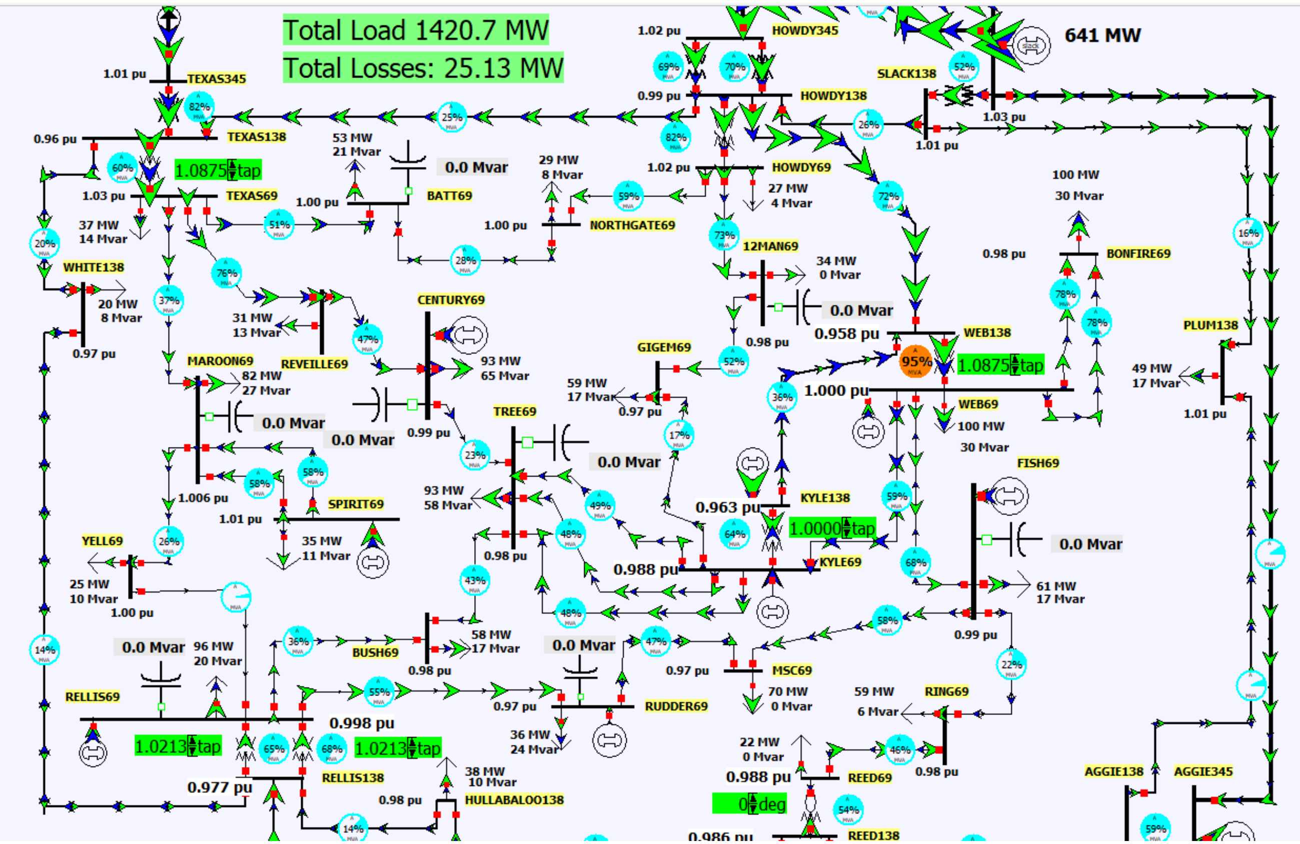
- Unbalanced transformer taps can cause large amounts of reactive power to circulating, increasing power system losses and overloading transformers



PowerWorld Case: **Bus3CirculatingVars**

LTC Tap Coordination

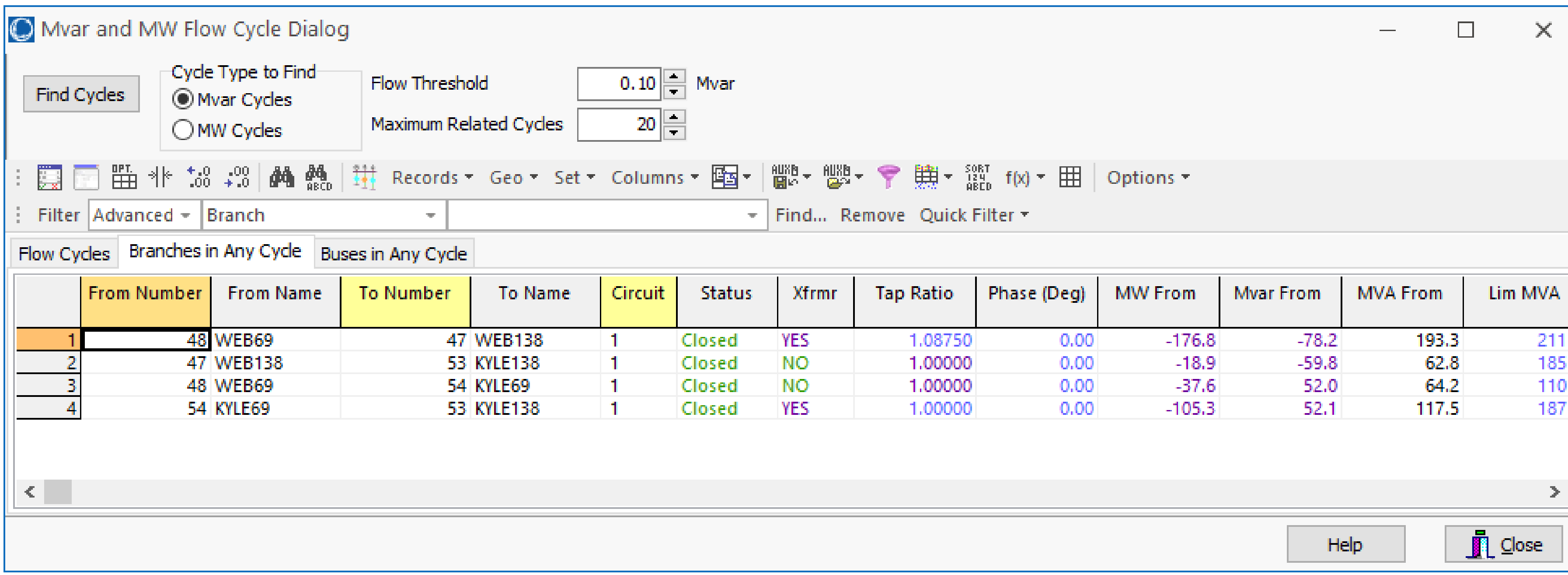
- Changing tap ratios can affect the voltages and var flow at nearby buses; hence coordinated control is needed



PowerWorld Case: **Aggieland37_LTC**

Auto Detection of Circulating Reactive or Real Power

- Select **Tools, Connections, Find Circulating MW or Mvar Flows** to do an automatic determination of the circulating power in a case



	From Number	From Name	To Number	To Name	Circuit	Status	Xfrmr	Tap Ratio	Phase (Deg)	MW From	Mvar From	MVA From	Lim MVA
1	48	WEB69	47	WEB138	1	Closed	YES	1.08750	0.00	-176.8	-78.2	193.3	211
2	47	WEB138	53	KYLE138	1	Closed	NO	1.00000	0.00	-18.9	-59.8	62.8	185
3	48	WEB69	54	KYLE69	1	Closed	NO	1.00000	0.00	-37.6	52.0	64.2	110
4	54	KYLE69	53	KYLE138	1	Closed	YES	1.00000	0.00	-105.3	52.1	117.5	187

Coordinated Reactive Control

- A number of different devices may be doing automatic reactive power control. They must be considered in some control priority
  - One example would be 1) generator reactive power, 2) switched shunts, 3) LTCs
- You can see the active controls in PowerWorld with **Case Information, Solution Details, Remotely Regulated Buses**

Filter | Advanced | Bus | Find... | Remove | Quick Filter

	Number	Name	Area Name	PU Volt	Set Volt	Volt Diff	AVR	Total Mvar	Mvar Min	Mvar Max	Rem Regs (gen)	Rem Regs (XFMR)	Rem Regs (SS)	Rem Regs (VSC DC)	HDR::Bus_Bus	Rem Reg Gen Bus 1
1	1	TEXAS345	1	1.01689				0.0	0.0	0.0						
2	10	HOWDY69	1	1.02108				0.0	0.0	0.0		LTC: 10 TO 39				
3	12	TEXAS69	1	1.01865				0.0	0.0	0.0		LTC: 12 TO 40				
4	13	12MAN69	1	0.99348				0.0	0.0	0.0			Switched Shu			
5	14	RUDDER69	1	1.02000				0.0	0.0	0.0	Generator: 14		Switched Shu			14
6	15	TREE69	1	1.00058				0.0	0.0	0.0			Switched Shu			
7	16	CENTURY69	1	1.00286				0.0	0.0	0.0	Generator: 16		Switched Shu			16
8	17	BATT69	1	1.00165				0.0	0.0	0.0			Switched Shu			
9	18	MAROON69	1	1.00636				0.0	0.0	0.0						
10	20	FISH69	1	1.03000				0.0	0.0	0.0	Generator: 20		Switched Shu			20
11	28	AGGIE345	1	1.03000				0.0	0.0	0.0	Generator: 28					28
12	31	SLACK345	1	1.03000				0.0	0.0	0.0	Generator: 31					31
13	33	REED69	1	1.00564				0.0	0.0	0.0						
14	35	SLACK138	1	1.01706				0.0	0.0	0.0						
15	37	SPIRIT69	1	1.01000				0.0	0.0	0.0	Generator: 37					37
16	39	HOWDY138	1	0.99759				0.0	0.0	0.0		LTCs: 39 TO 3				
17	44	RELLIS69	1	1.02000				0.0	0.0	0.0	Generator: 44	LTCs: 44 TO 4				44
18	48	WEB69	1	1.02134				0.0	0.0	0.0		LTC: 48 TO 47				
19	53	KYLE138	1	0.98467				0.0	0.0	0.0	Generator: 53					53

Coordinated Reactive Control

- The challenge with implementing tap control in the power flow is it is quite common for at least some of the taps to reach their limits
  - Keeping in mind a large case may have thousands of LTCs!
- If this control was directly included in the power flow equations then every time a limit was encountered the Jacobian would change
  - Also taps are discrete variables, so voltages must be a range
- Doing an outer loop control can more directly include the limit impacts; often time sensitivity values are used
- We'll return to this once we discuss sparse matrices and sensitivity calculations

Phase-Shifting Transformers

- Phase shifters are transformers in which the phase angle across the transformer can be varied in order to control real power flow
  - Sometimes they are called phase angle regulars (PAR)
  - Quadrature booster (evidently British though I've never heard this term)
- They are constructed by include a delta-connected winding that introduces a 90º phase shift that is added to the output voltage

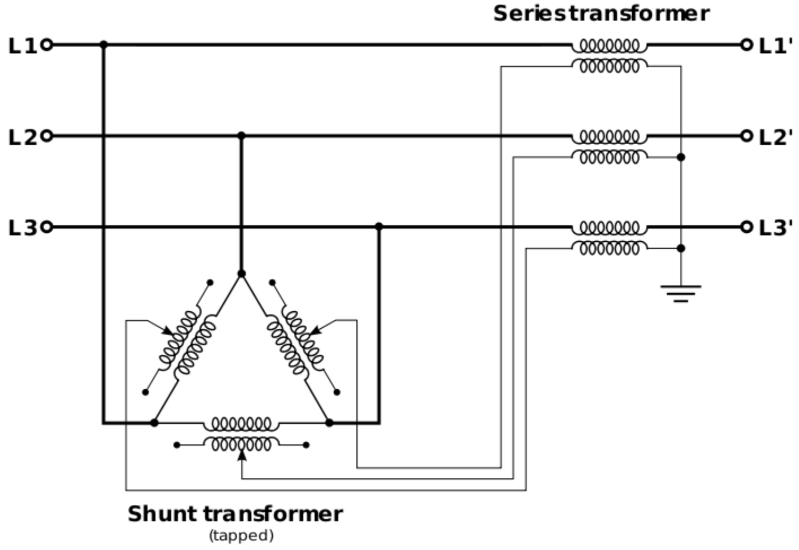


Image: http://en.wikipedia.org/wiki/Quadrature_booster

Phase-Shifter Model

- We develop the mathematical model of a phase shifting transformer as a first step toward our study of its simulation
- Let buses *k* and *m* be the terminals of the phase–shifting transformer, then define the phase shift angle as Φ_{km}
- The latter differs from an off–nominal turns ratio LTC transformer in that its tap ratio is a complex quantity, i.e., a complex number, $t_{km}\angle\Phi_{km}$
- The phase shift angle is a discrete value, with one degree a typical increment

Phase-Shifter Model

- For a phase shifter located on the branch (k, m), the admittance matrix representation is obtained analogously to that for the LTC

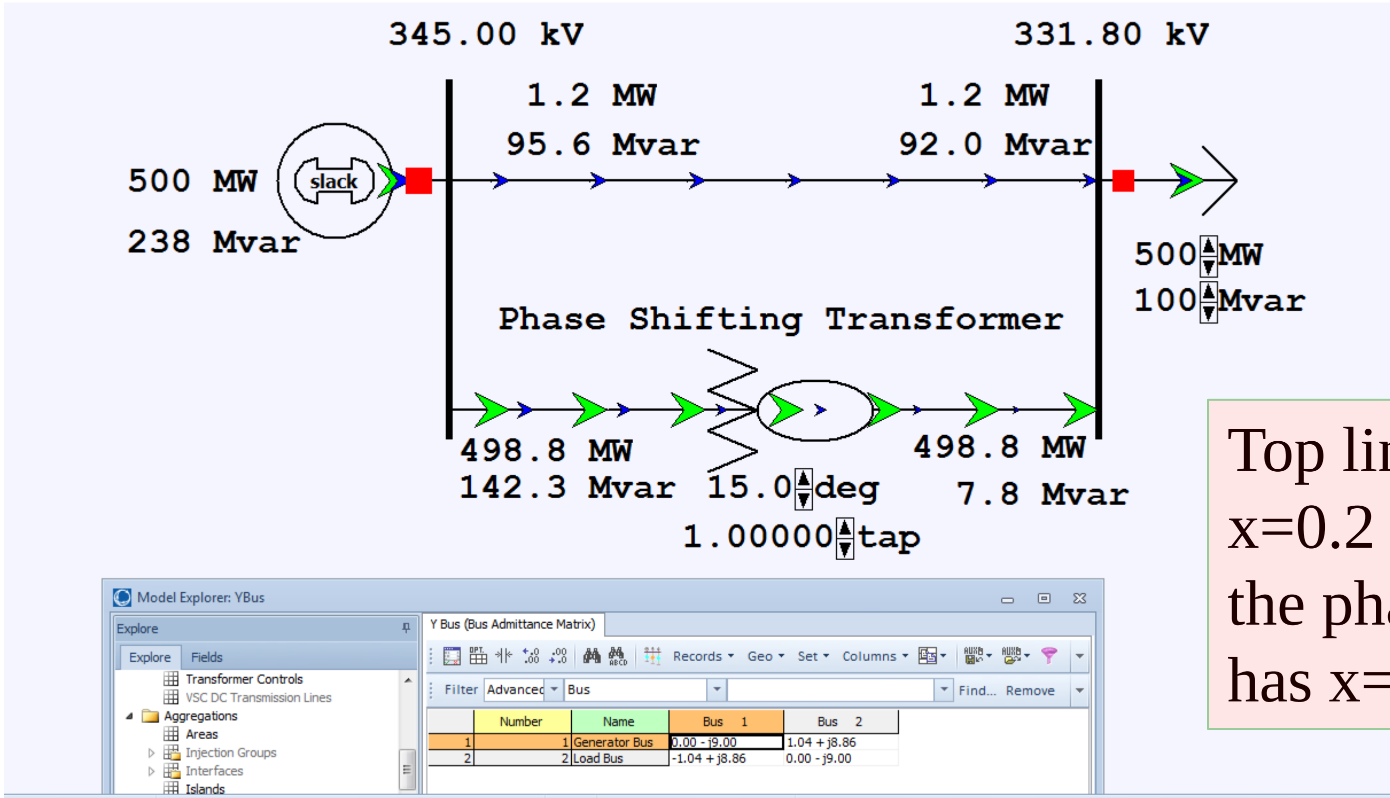
$$\begin{bmatrix} \bar{I}_k \\ \bar{I}_m \end{bmatrix} = \begin{bmatrix} \dfrac{\bar{y}_{km}}{t^2} & -\dfrac{\bar{y}_{km}}{te^{j\phi_{km}}} \\ -\dfrac{\bar{y}_{km}}{te^{-j\phi_{km}}} & \bar{y}_{km} \end{bmatrix} \begin{bmatrix} \bar{E}_k \\ \bar{E}_m \end{bmatrix}$$

- Note, if there is a phase shift then $\mathbf{Y}_{\text{bus}}$ is no longer symmetric!! In a large case there are almost always some phase shifters. Y-Δ transformers also introduce a phase shift that is often not modeled

Integrated Phase-Shifter Control

- Phase shifters are usually used to control the real power flow on a device
- Similar to LTCs, phase-shifter control can either be directly integrated into the power flow equations (adding an equation for the real power flow equality constraint, and a variable for the phase shifter value), or they can be handled in with an outer loop approach
- As was the case with LTCs, limit enforcement often makes the outer loop approach preferred
- Coordinated control is needed when there are multiple, close by phase shifters

Two Bus Phase Shifter Example



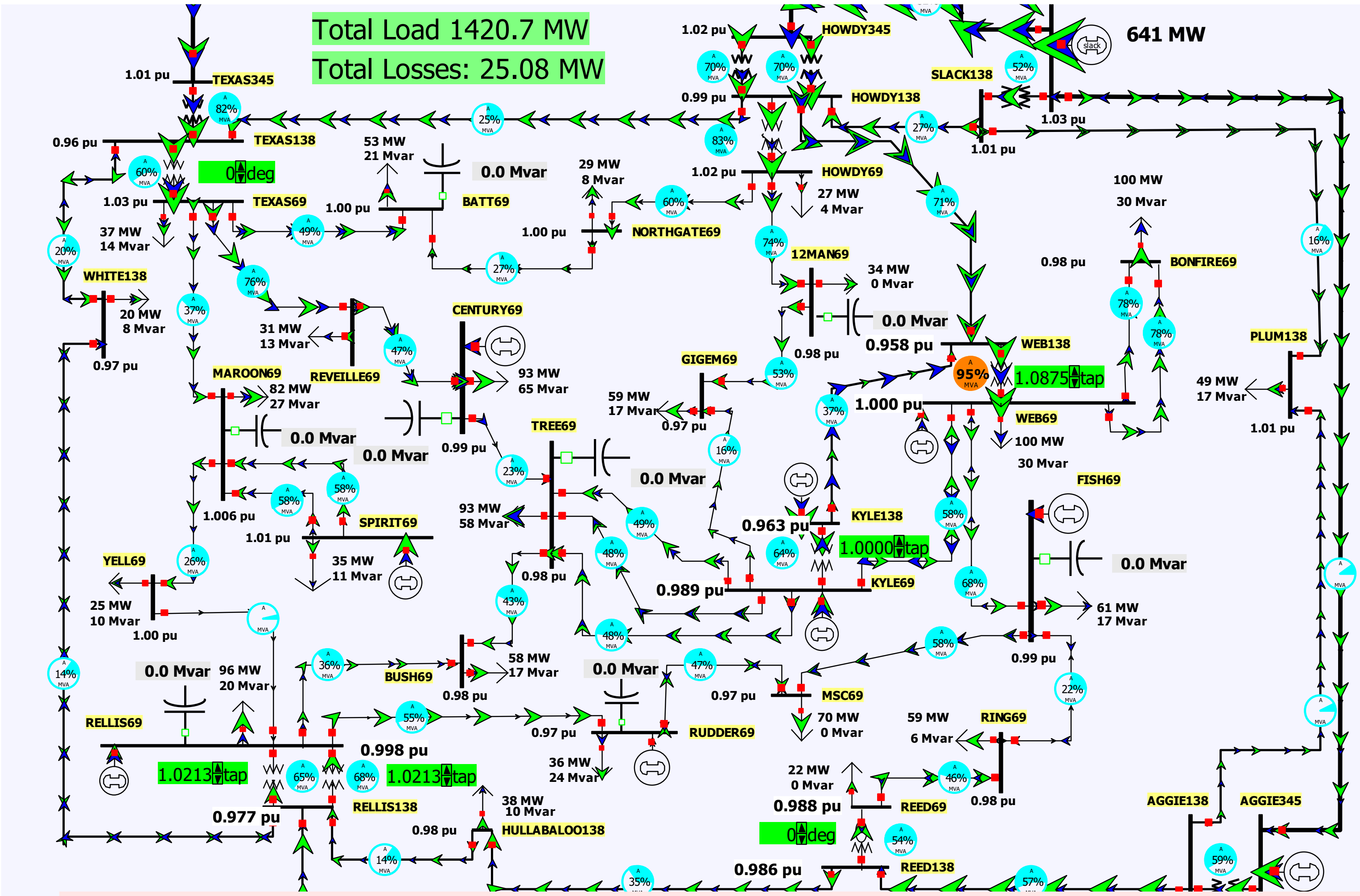
Top line has x=0.2 pu, while the phase shifter has x=0.25 pu.

$$\mathbf{Y}_{12} = -\frac{1}{j0.2} - \frac{1}{j0.25}\left(\cos\left(-15^\circ\right) + j\sin\left(-15^\circ\right)\right) = j5 + (j4)(0.966 - j0.259)$$

$$\mathbf{Y}_{12} = 1.036 + j8.864$$

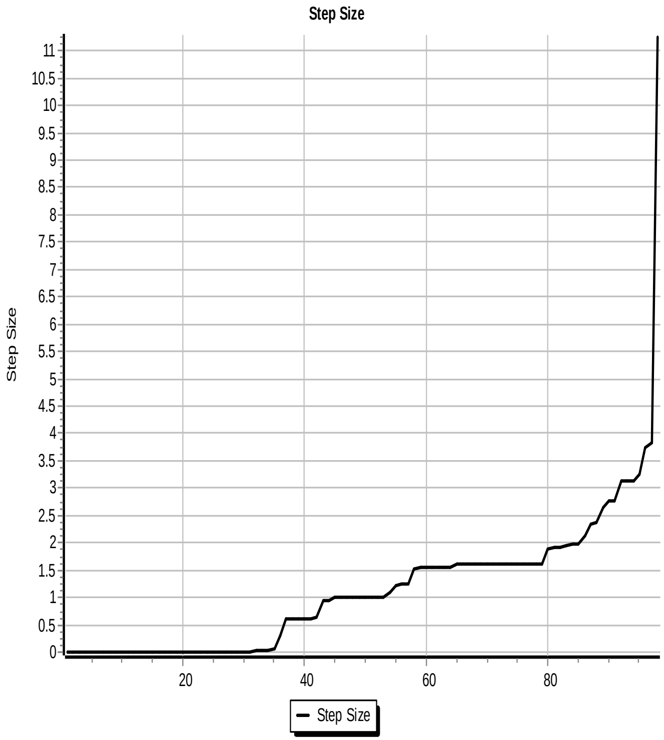
PowerWorld Case: **B2PhaseShifter**

Aggieland37 With Phase Shifters



PowerWorld Case: **Aggieland37_PhaseShifter**

Large Case Phase Shifter Limits and Step Size



Example of Phase Shifters in Practice

- The below report mentions issues associated with the Ontario-Michigan PARs

Ontario-Michigan Interface

LEC flow is affected by several factors including PARs in multiple locations around Lake Erie (*see Figure 1*). This report considered data only for PARs on the Ontario-Michigan interface.

Figure 1 – PAR Locations Which Impact Lake Erie Circulation Flow

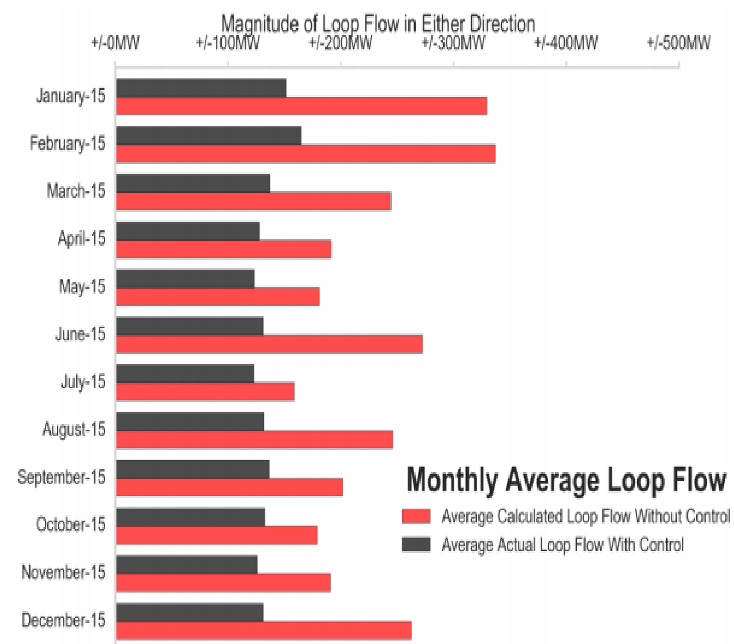


Figure 6 - Monthly Average Loop Flow

https://www.nyiso.com/public/webdocs/markets_operations/committees/bic_miwg/meeting_materials/2017-02-28/2016%20Ontario-Michigan%20Interface%20PAR%20Evaluation%20Final%20Report.pdf

Impedance Correction Tables

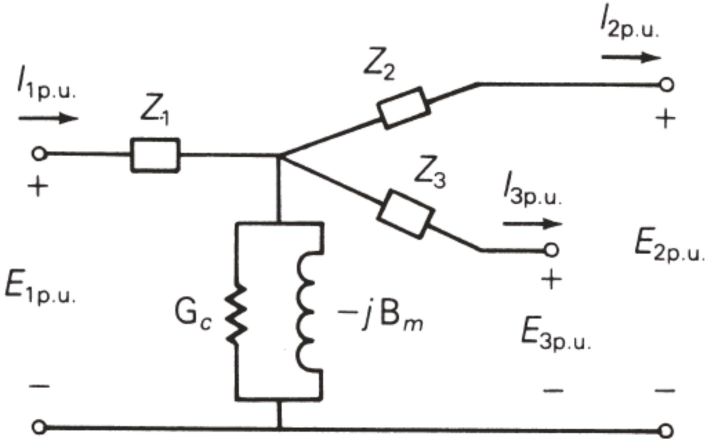
- With taps the impedance of the transformer changes; sometimes the changes are relatively minor and sometimes they are dramatic
  - A unity turns ratio phase shifter is a good example with essentially no impedance when the phase shift is zero
  - Often modeled with piecewise linear function with impedance correction varying with tap ratio or phase shift
  - Next lines give several examples, with format being (phase shift or tap ratio, impedance correction)
    - (-60,1), (0,0.01), (60,1)
    - (-25,2.43),(0,1),(25,2.43)
    - (0.941,0.5), (1.04,1), (1.15,2.45)
    - (0.937,1.64), (1,1), (1.1, 1.427)

Three-Winding Transformers

- Three-winding transformers are very common, with the third winding called the tertiary
  - The tertiary is often a delta winding
- Three-winding transformers have various benefits
  - Providing station service
  - Place for a capacitor connection
  - Reduces third-harmonics
  - Allows for three different transmission level voltages
  - Better handling of fault current

Three-Winding Transformers

- Usually modeled in the power flow with a star equivalent; the internal "star" bus does not really exist
- Star bus is often given a voltage of 1.0 or 999 kV



Per-unit equivalent circuit—practical transformer

Impedances calculated using the wye-delta transform can result in negative resistance (about 900 out of 97,000 in EI model)

The winding impedances are measured between the windings with one winding shorted and the other open; for example Z_{12} is measured from 1 with winding 2 shorted, 3 open

Image from *Power System Analysis and Design*, by Glover, Overbye, and Sarma 6th Edition

Three-Winding Transformer Example

EXAMPLE 3.9

Three-winding single-phase transformer: per-unit impedances

The ratings of a single-phase three-winding transformer are

winding 1: 300 MVA, 13.8 kV

winding 2: 300 MVA, 199.2 kV

winding 3: 50 MVA, 19.92 kV

The leakage reactances, from short-circuit tests, are

$X_{12} = 0.10$ per unit on a 300-MVA, 13.8-kV base

$X_{13} = 0.16$ per unit on a 50-MVA, 13.8-kV base

$X_{23} = 0.14$ per unit on a 50-MVA, 199.2-kV base

Winding resistances and exciting current are neglected. Calculate the impedances of the per-unit equivalent circuit using a base of 300 MVA and 13.8 kV for terminal 1.

SOLUTION

$S_{base} = 300$ MVA is the same for all three terminals. Also, the specified voltage base for terminal 1 is $V_{base1} = 13.8$ kV. The base voltages for terminals 2 and 3 are then $V_{base2} = 199.2$ kV and $V_{base3} = 19.92$ kV, which are the rated voltages of these windings. From the data given, $X_{12} = 0.10$ per unit was measured from terminal 1 using the same base values as those specified for the circuit. However, $X_{13} = 0.16$ and $X_{23} = 0.14$ per unit on a 50-MVA base are first converted to the 300-MVA circuit base.

(Continued)

$$Z_{12} = Z_1 + Z_2$$
$$Z_{13} = Z_1 + Z_3$$
$$Z_{23} = Z_2 + Z_3$$

Hence

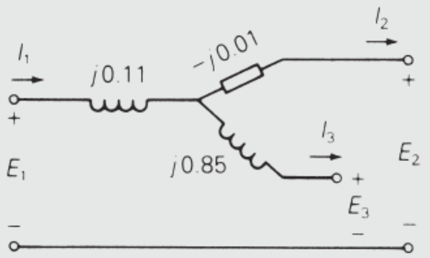
$$Z_1 = 0.5(Z_{12} + Z_{13} - Z_{23})$$
$$Z_2 = 0.5(Z_{12} + Z_{23} - Z_{13})$$
$$Z_3 = 0.5(Z_{13} + Z_{23} - Z_{12})$$

Image from *Power System Analysis and Design*, by Glover, Overbye, and Sarma 6th Edition

Three-Winding Transformer Example, cont.

FIGURE 3.21

Circuit for Example 3.9



$$X_{13} = (0.16)\left(\frac{300}{50}\right) = 0.96 \quad \text{per unit}$$

$$X_{23} = (0.14)\left(\frac{300}{50}\right) = 0.84 \quad \text{per unit}$$

Then, from (3.6.8) through (3.6.10),

$$X_1 = \tfrac{1}{2}(0.10 + 0.96 - 0.84) = 0.11 \quad \text{per unit}$$

$$X_2 = \tfrac{1}{2}(0.10 + 0.84 - 0.96) = -0.01 \quad \text{per unit}$$

$$X_3 = \tfrac{1}{2}(0.84 + 0.96 - 0.10) = 0.85 \quad \text{per unit}$$

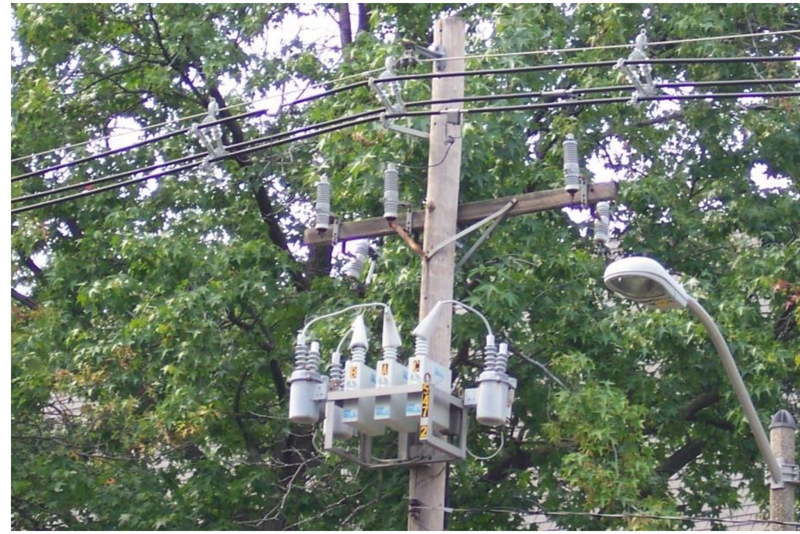
The per-unit equivalent circuit of this three-winding transformer is shown in Figure 3.21. Note that X_2 is negative. This illustrates the fact that X_1, X_2, and X_3 are *not* leakage reactances, but instead are equivalent reactances derived from the leakage reactances. Leakage reactances are always positive.

Note also that the node where the three equivalent circuit reactances are connected does not correspond to any physical location within the transformer. Rather, it is simply part of the equivalent circuit representation.

Image from *Power System Analysis and Design*, by Glover, Overbye, and Sarma 6th Edition

When Transformers go Bad

Switched Shunts and SVCs



- Switched capacitors and sometimes reactors are widely used at both the transmission and distribution levels to supply or (for reactors) absorb discrete amounts of reactive power
- Static var compensators (SVCs) are also used to supply continuously varying amounts of reactive power
- In the power flow SVCs are sometimes represented as PV buses with zero real power

Switched Shunt Control

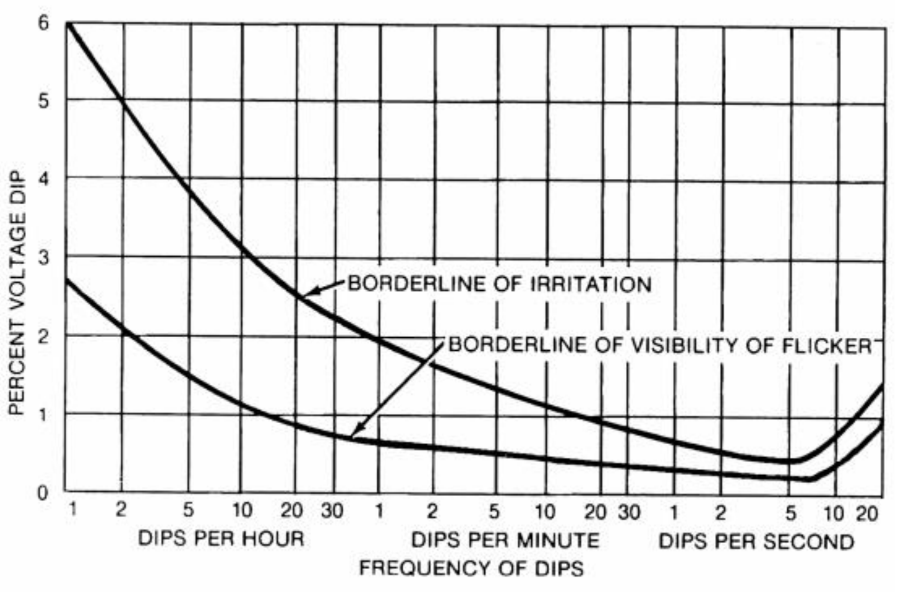
- The status of switched shunts can be handled in an outer loop algorithm, similar to what is done for LTCs and phase shifters
  – Because they are discrete they need to regulate a value to a voltage range
- Switches shunts often have multiple levels that need to be simulated
- Switched shunt control also interacts with the LTC and PV control
- The power flow modeling needs to take into account the control time delays associated with the various devices

Switched Shunt System Design

- Because switched shunts tend to have a local impact, there needs to be a coordinated design in their implementation at the transmission level
  - Shunt capacitors used to raise the voltage, shunt reactors used to lower the voltage; used with LTCs and gens
- Often in the transmission system they are switched manually by a system operator
- The size and number of banks depends
  - Change in the system voltages caused by bank switching
  - The availability of different sizes
  - Cost for the associated switchgear and protection system

Switched Shunt Sizing

- A goal with switched shunt sizing is to avoid human irritation caused by excessive changes in lighting
- IEEE Std 1453-2015 gives guidance on the percentage of voltage changes as a function of time; Table 3 of the standard suggests keeping the voltage changes below about 3%



- We determine analytic methods to calculate this percentage later in the semester

Image from IEEE Std. 1453-2015, "IEEE Recommended Practice for Measurement and Limits of Voltage Fluctuations and Associated Light Flicker on AC Power Systems"

Dynamic Reactive Capability

- Switched shunts are often used to maintain adequate dynamic reactive power from generators and SVCs
- FERC Order 827 (from June 2016, titled “Reactive Power Requirements for Non-Synchronous Generation”) states that the power factor of generators should be between 0.95 leading to 0.95 lagging
  - Hence the absolute value of the Mvar output of the machines should be no more than 31% of the MW output
  - Often a value substantially better for reactive reserves
- Switched shunts are used to keep the generator power factor within this range

Area Interchange Control

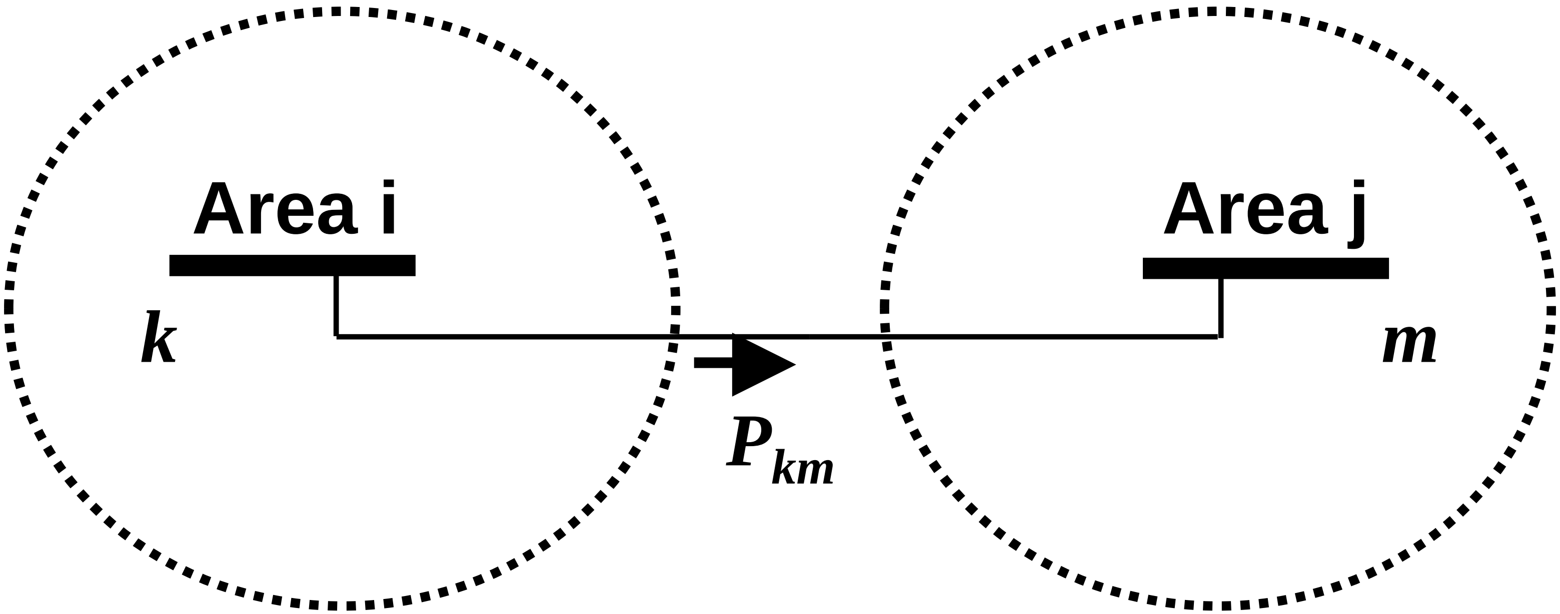
- The purpose of area interchange control is to regulate or control the interchange of real power between specified areas of the network
- Under area interchange control, the mutually exclusive subnetworks, the so-called areas, that make up a power system need to be explicitly represented
- These areas may be particular subnetworks of a power grid or may represent various interconnected systems
- The specified net power out of each area is controlled by the generators within the area
- A power flow may have many more areas than balancing authority areas

Area Interchange Control

- The net power interchange for an area is the algebraic sum of all its tie line real power flows
- We denote the real power flow across the tie line from bus k to bus m by P_{km}
- We use the convention that $P_{km} > 0$ if power leaves node k and $P_{km} \leq 0$ otherwise
- Thus the net area interchange S_i of area i is positive (negative) if area i exports (imports)
- Consider the two areas i and j that are directly connected by the single tie line (k, m) with the node k in area i and the node m in area j

Net Power Interchange

- Then, for the complex power interchange S_i, we have a sum in which P_{km} appears with a positive sign; for the area j power interchange it appears with a negative sign



Area i exports P_{km} and Area j imports P_{km}

Net Power Interchange

- Since each tie line flow appears twice in the net interchange equations, it follows that if the power system as *a* distinct areas, then

$$\sum_{i=1}^{a} S_i = 0$$

- Consequently, the specification of S_i for a collection of (*a*-1) areas determines the system interchange; we must leave the interchange for one area unspecified
  - This is usually (but not always) the area with the system slack bus

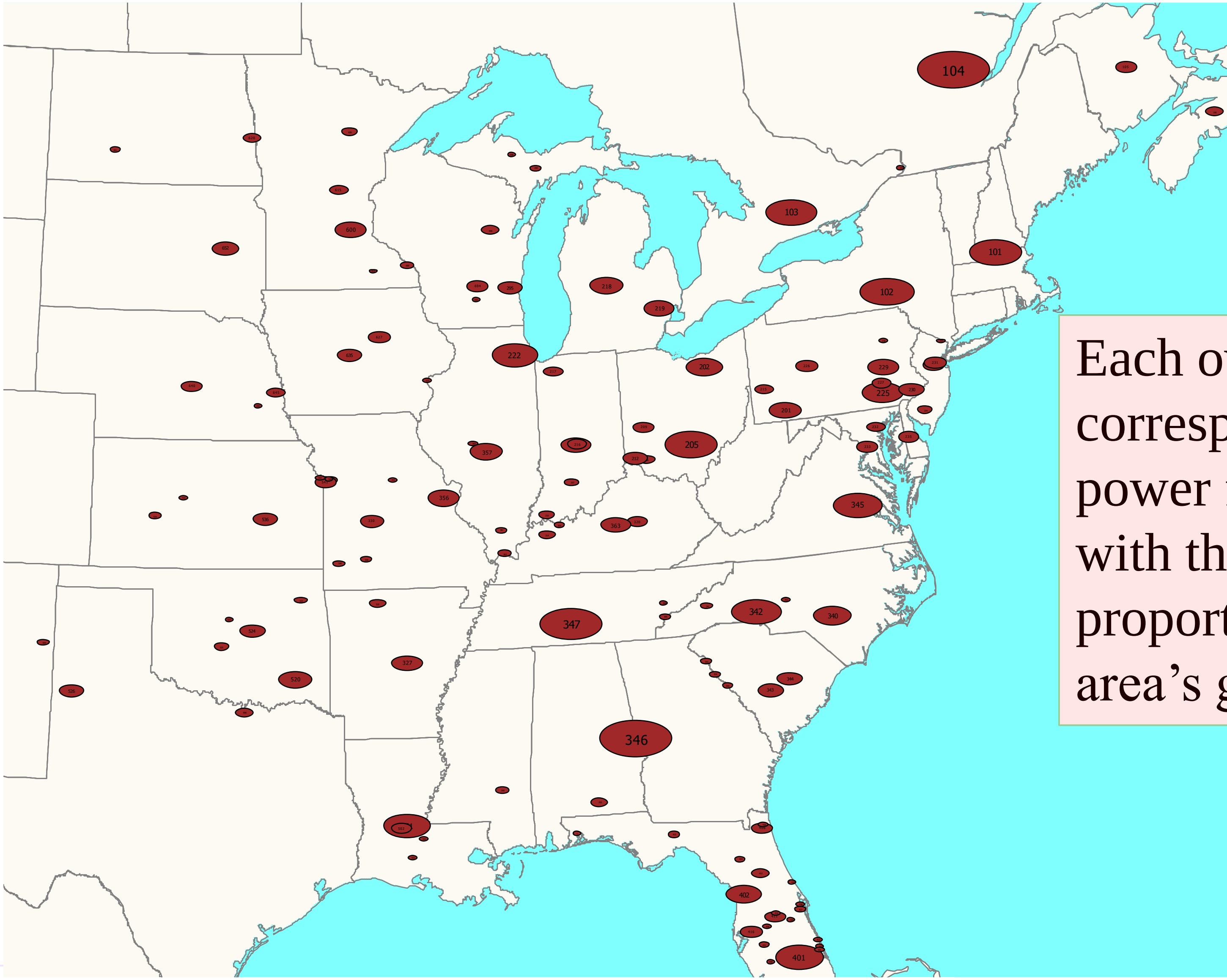
Modeling Area Interchange

- Area interchange is usually modeled using an outer loop control
- The net generation imbalance for an area can be handled using several different approach
  - Specify a single area slack bus, and the entire generation change is picked up by this bus; this may work if the interchange difference is small
  - Pick up the change at a set of generators in the area using constant participation factors; each generator gets a share
  - Use some sort of economic dispatch algorithm, so how generation is picked up depends on an assumed cost curve
  - Min/max limits need to be enforced

Including Impact on Losses

- A change in the generation dispatch can also change the system losses. These incremental impacts need to be included in an area interchange algorithm
- We'll discuss the details of these calculations later in the course when we consider sensitivity analysis

Example Large System Areas

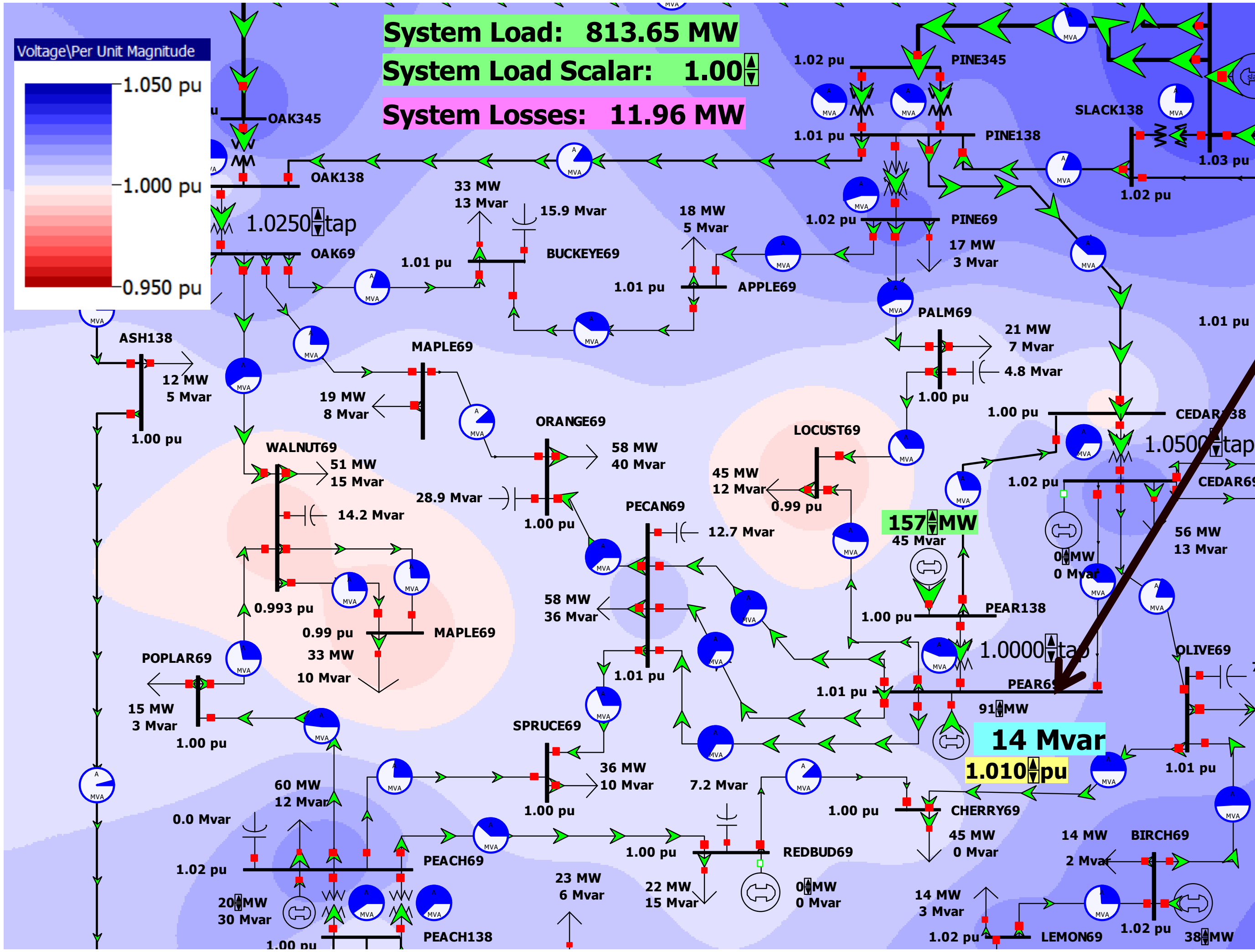


Each oval corresponds to a power flow area with the size proportional to the area's generation

Generator Volt/Reactive Control

- Simplest situation is a single generator at a bus regulating its own terminal
  - Either PV, modeled as a voltage magnitude constraint, or as a PQ with reactive power fixed at a limit value. If PQ the reactive power limits can vary with the generator MW output
- Next simplest is multiple generators at a bus. Obviously they need to be regulating the bus to the same voltage magnitude
  - From a power flow solution perspective, it is similar to a single generator, with limits being the total of the individual units
  - Options for allocation of vars among generators; this can affect the transient stability results

Generator Voltage Control



This example uses the case **PSC_37Bus** with a voltage contour. Try varying the voltage setpoint for the generator at PEAR69

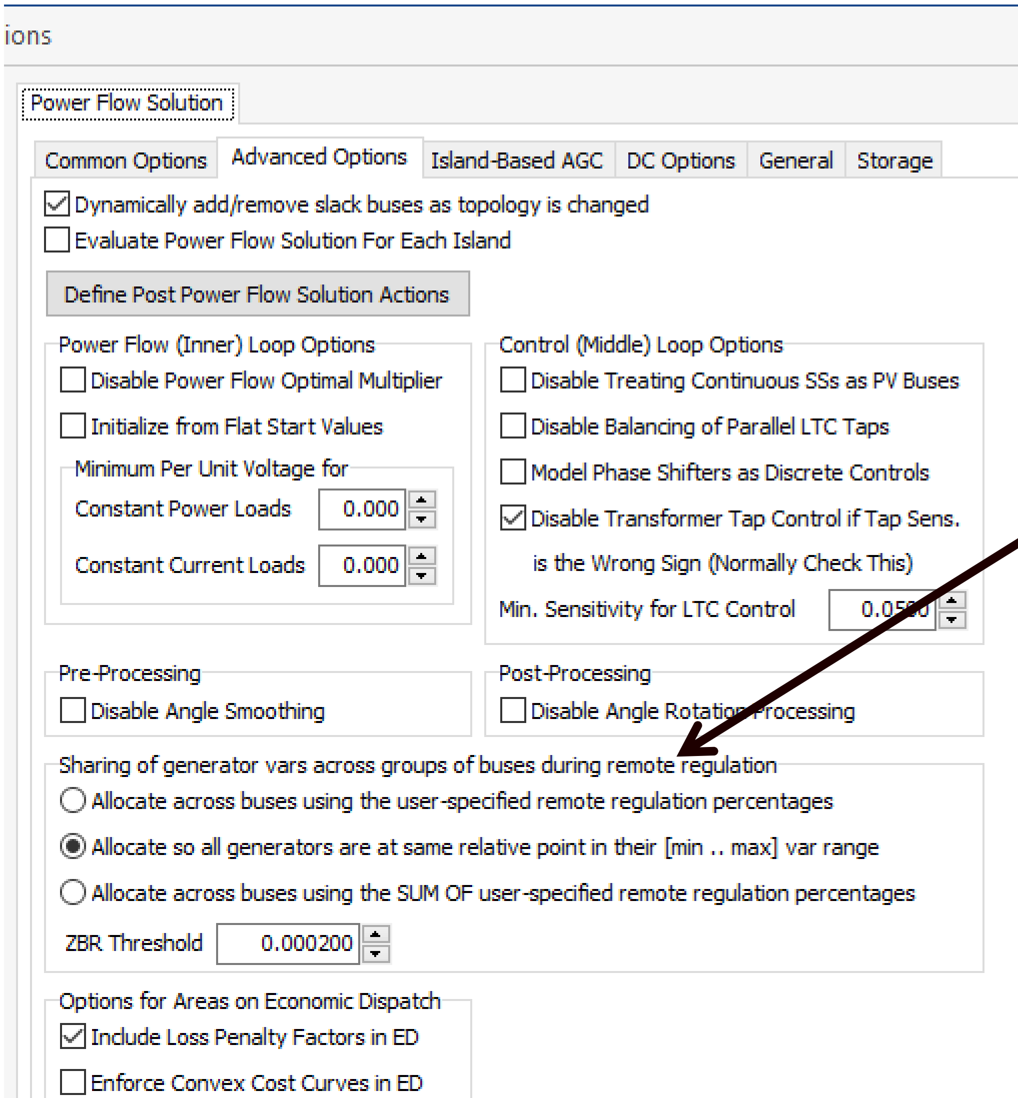
Generator Remote Bus Voltage Control

- Next complication is generators at a single bus regulating a remote bus; usually this is the high side of their generator step-up (GSU) transformer
  - When multiple generators regulate a single point their exciters need to have a dual input
  - This can be implemented in the power flow for the generators at bus j regulating the voltage at bus k by changing the bus j voltage constraint equation to be

    $$\left|V_k\right| - V_{k,set} = 0$$

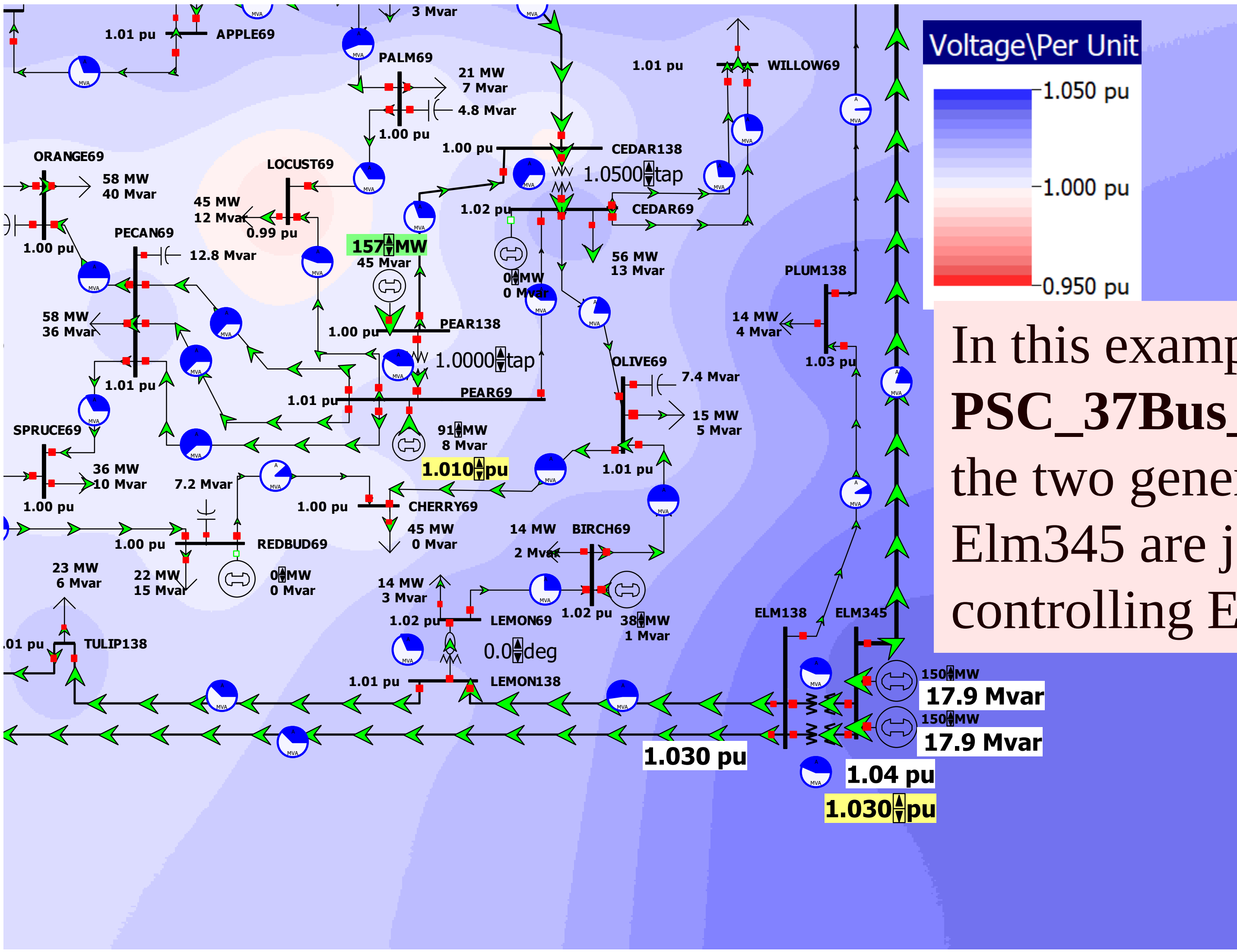
    (however, this does create a zero on the diagonal of the Jacobian)
  - Helps with power system voltage stability

Reactive Power Sharing Options



Different software packages use different approaches for allocating the reactive power; PowerWorld has several options.

Reactive Power Sharing

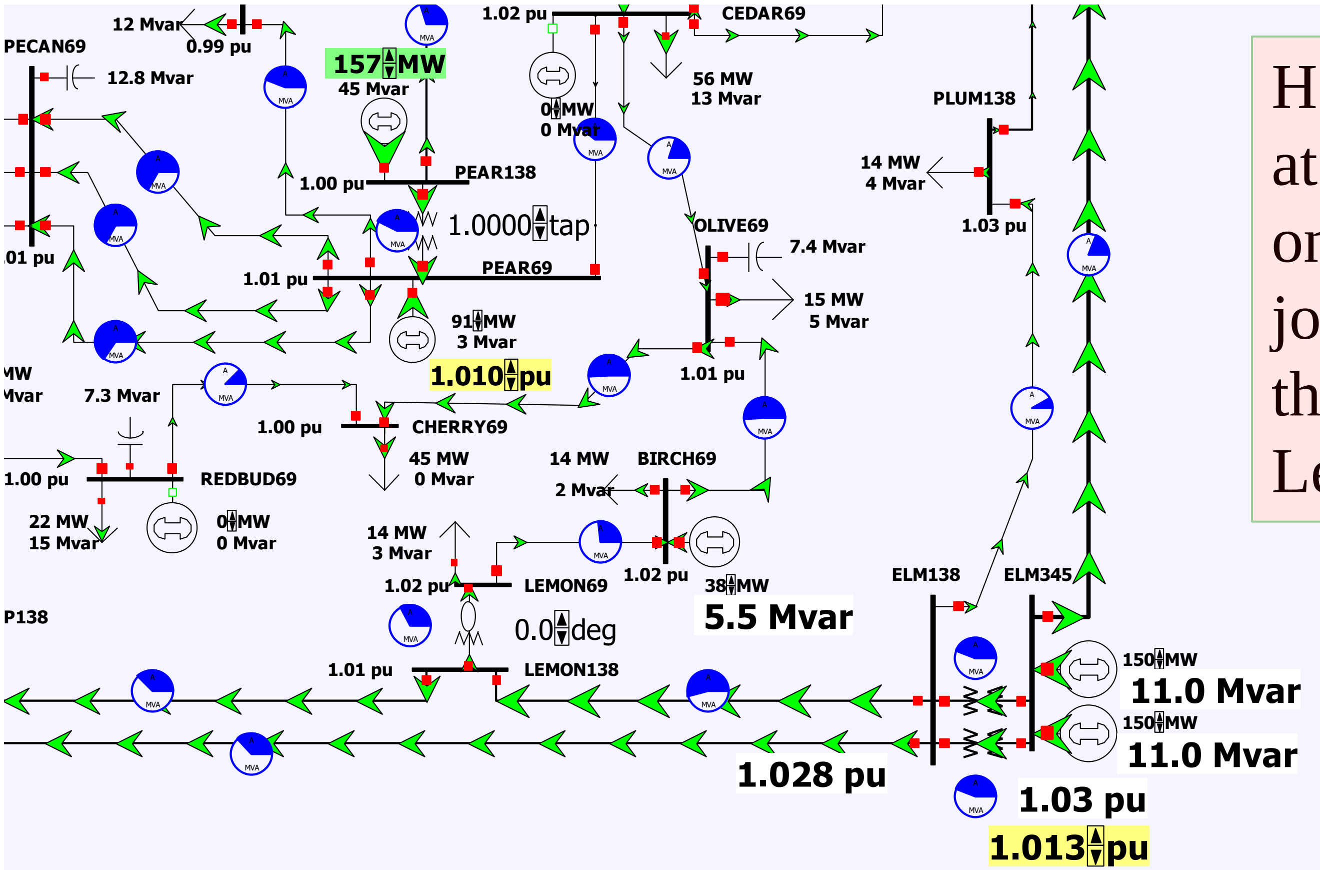


In this example, case **PSC_37Bus_Varsharing**, the two generators at Elm345 are jointly controlling Elms138

Generator Remote Bus Voltage Control

- The next complication is to have the generators at multiple buses doing coordinated voltage control
  - Controlled bus may or may not be one of the terminal buses
- There must be an a priori decision about how much reactive power is supplied by each bus; example allocations are a fixed percentage or placing all generators at the same place in their regulation range
- Implemented by designating one bus as the master; this bus models the voltage constraint
- All other buses are treated as PQ, with the equation including a percent of the total reactive power output of all the controlling bus generators

Remote and Coordinated Var Control Example



Here the generators at Elm345 and the one at Birch69 are jointly controlling the voltage at the Lemon138 bus.

Case is **PSC_37Bus_Varsharing_MultipleBuses**

Power Flow Topology Processing

- Commercial power flow software must have algorithms to determine the number of asynchronous, interconnected systems in the model
  - These separate systems are known as Islands
  - In large system models such as the Eastern Interconnect it is common to have multiple islands in the base case (one recent EI model had nine islands)
  - Islands can also form unexpectedly as a result of contingencies
  - Power can be transferred between islands using dc lines
  - Each island must have a slack bus

Power Flow Topology Processing

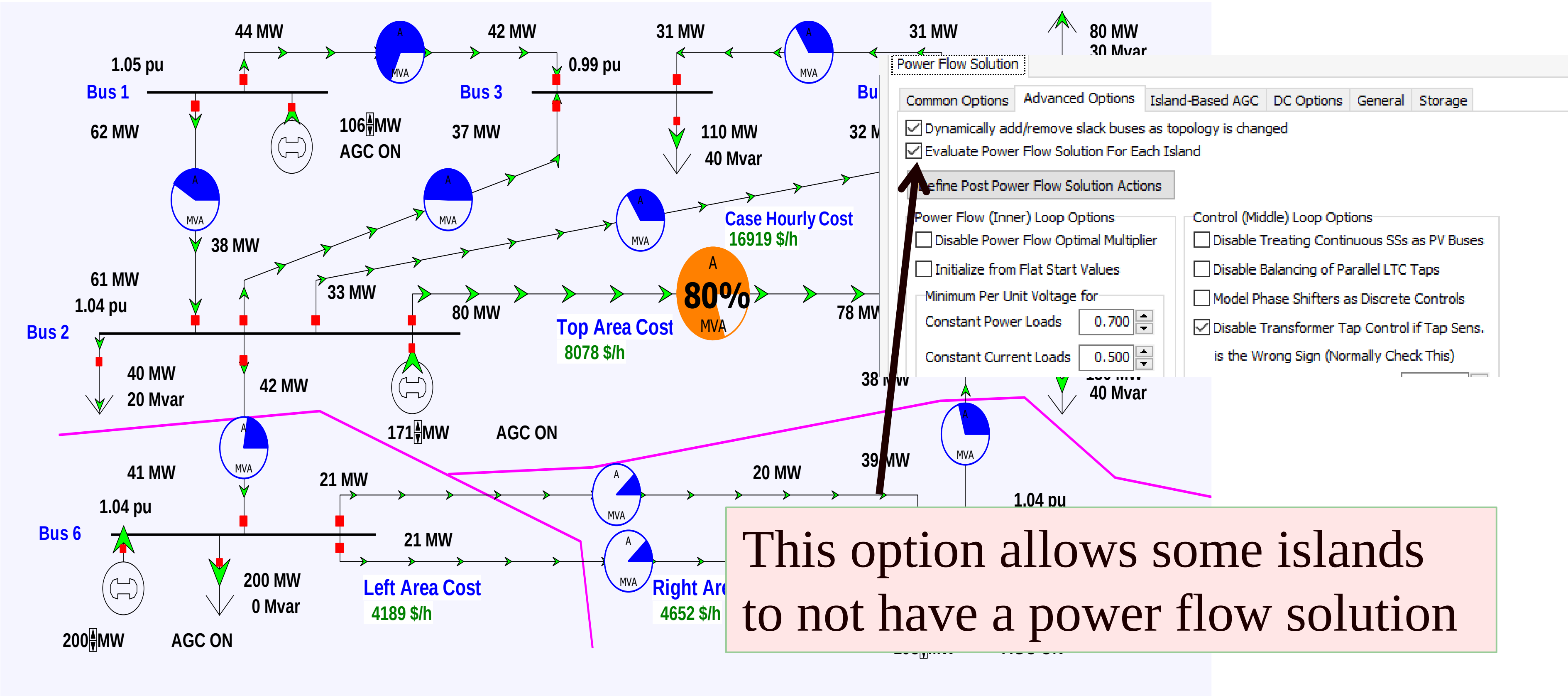
- Anytime a status change occurs the power flow must perform topology processing to determine whether there are either 1) new islands or 2) islands have merged
- Determination is needed to determine whether the island is "viable." That is, could it truly function as an independent system, or should the buses just be marked as dead
  - A quite common occurrence is when a single load or generator is isolated; in the case of a load it can be immediately killed; generators are more tricky

Topology Processing Algorithm

- Since topology processing is performed often, it must be quick (order n ln(n))!
- Simple, yet quick topology processing algoritm
  - Set all buses as being in their own island (equal to bus number)
  - Set ChangeInIslandStatus true
  - While ChangeInIslandStatus Do
    - Go through all the in-service lines, setting the islands for each of the buses to be the smaller island number; if the island numbers are different set ChangeInIslandStatus true
  - Determine which islands are viable, assigning a slack bus as necessary

This algorithm does depend on the depth of the system

Example of Island Formation



This option allows some islands to not have a power flow solution

Splitting large systems requires a careful consideration of the flow on the island tie-lines as they are opened

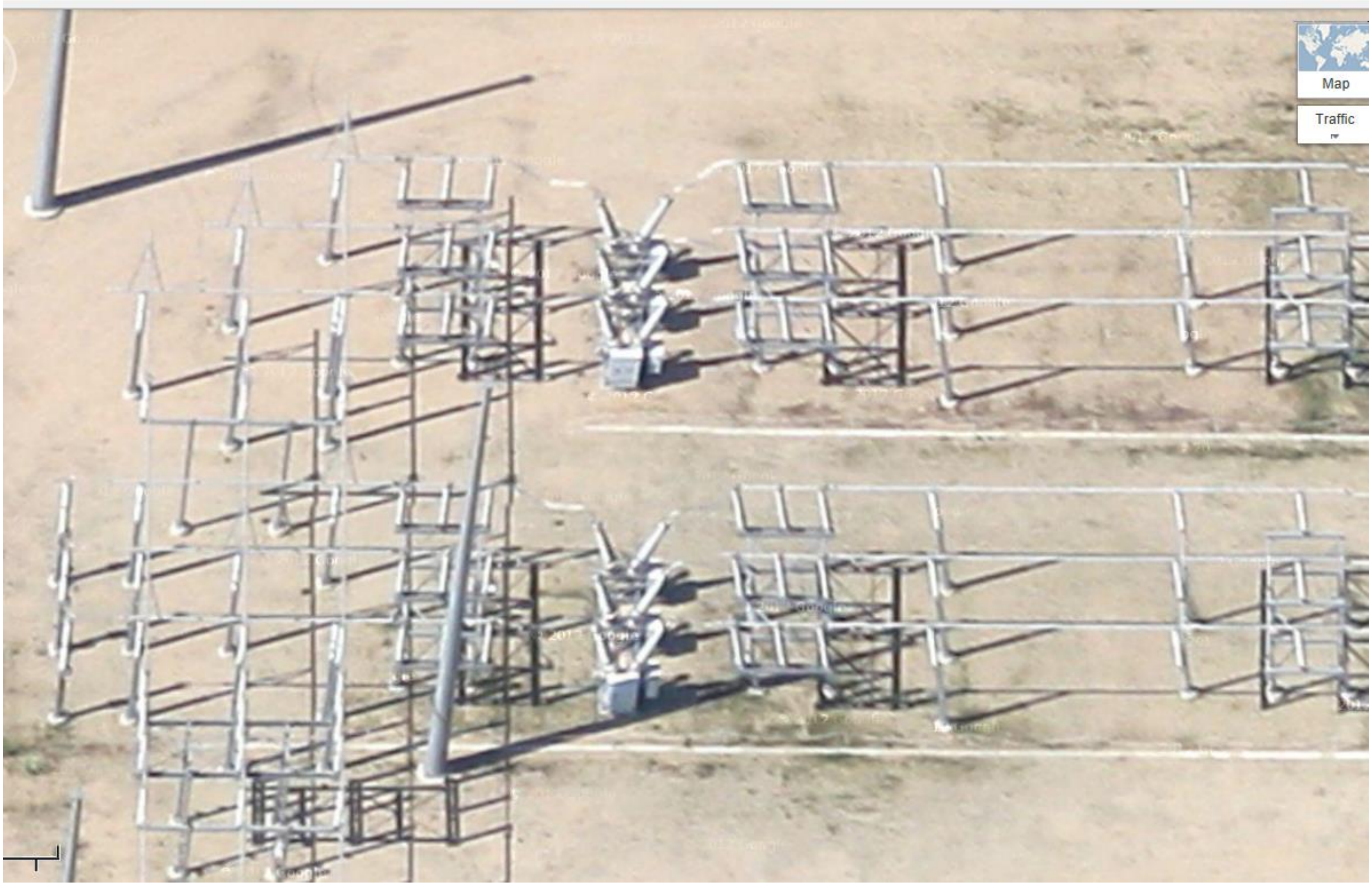
Bus Branch versus Node Breaker

- Due to a variety of issues during the 1970's and 1980's the real-time operations and planning stages of power systems adopted different modeling approaches

Real-Time Operations	Planning
Use detailed node/breaker model	Use simplified bus/branch model
EMS system as a set of integrated applications and processes	PC approach
Real-time operating system	Use of files
Real-time databases	Stand-alone applications

Entire data sets and software tools developed around these two distinct power system models

Google View of a 345 kV Substation



Example of Using a Disconnect to Break Load Current

Substation Configurations

- Several different substation breaker/disconnect configurations are common:
- Single bus: simple but a fault any where requires taking out the entire substation; also doing breaker or disconnect maintenance requires taking out the associated line

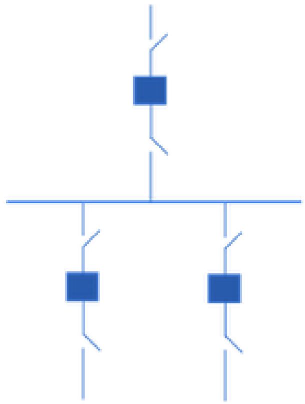


Fig B: Single Bus

Source: http://www.skm-eleksys.com/2011/09/substation-bus-schemes.html

Substation Configurations, cont.

- Main and Transfer Bus:
  Now the breakers can be taken out for maintenance without taking out a line, but protection is more difficult, and a fault on one line will take out at least two

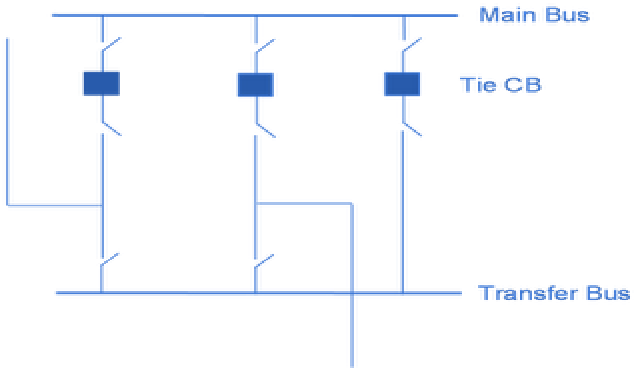


Fig C: Main and Transfer Bus

- Double Bus Breaker:
  Now each line is fully protected when a breaker is out, so high reliability, but more costly

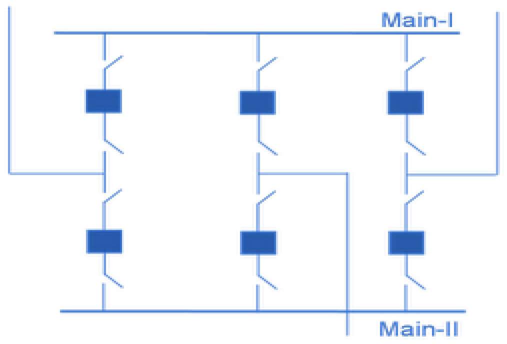


Fig D: Double Bus Double Breaker

Source: http://www.skm-eleksys.com/2011/09/substation-bus-schemes.html

Ring Bus, Breaker and Half

- As the name implies with a ring bus the breakers form a ring; number of breakers is same as number of devices; any breaker can be removed for maintenance
- The breaker and half has two buses and uses three breakers for two devices; both breakers and buses can be removed for maintenance

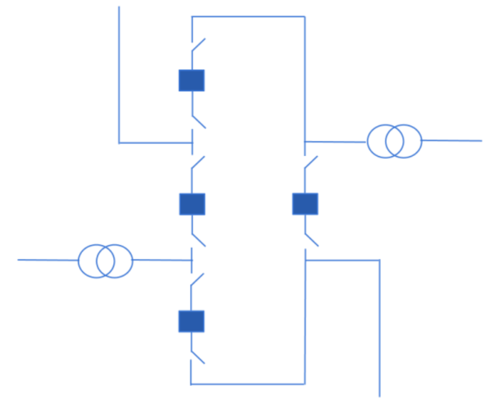


Fig F: Ring Bus

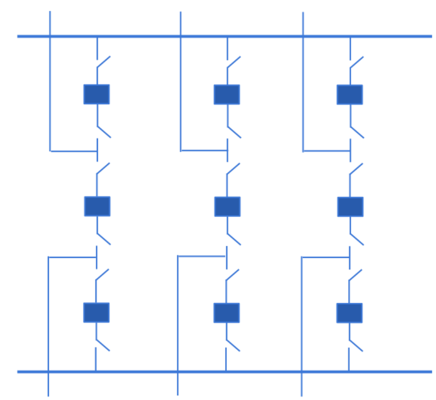
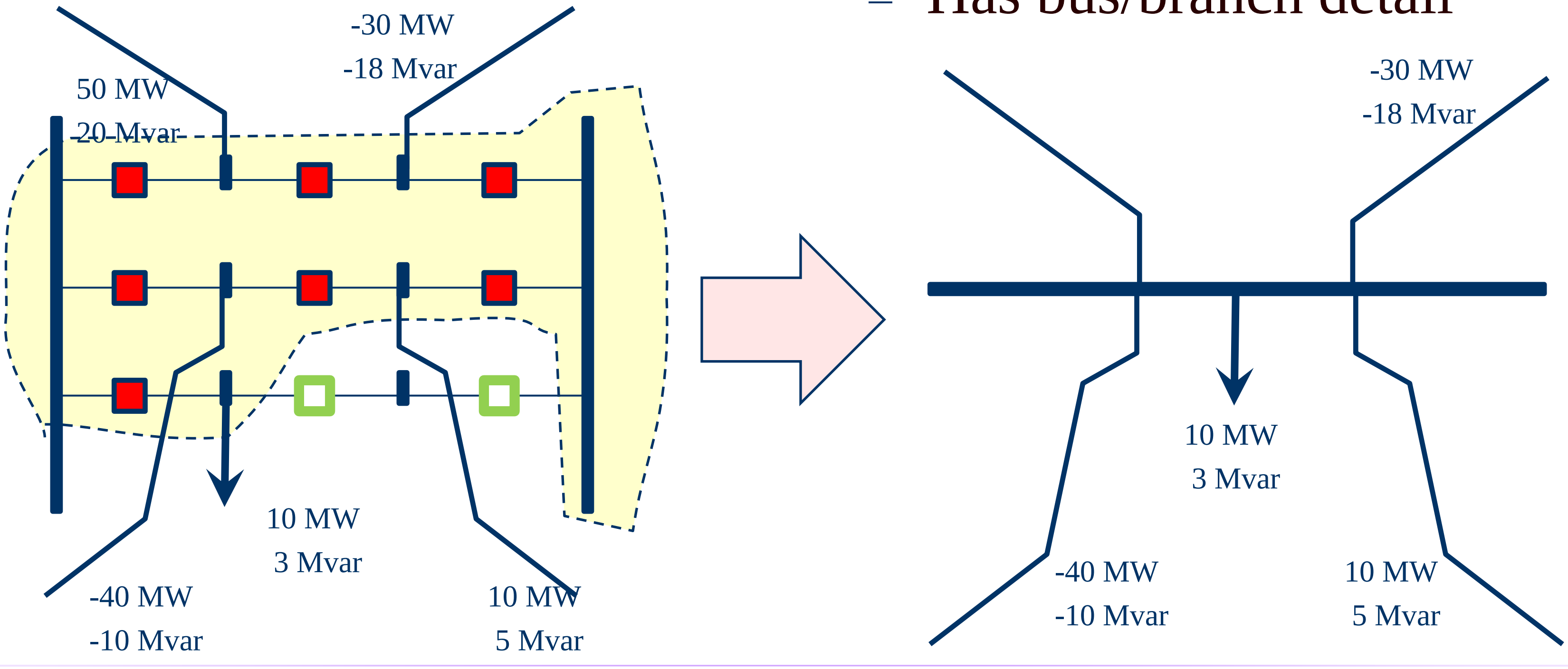


Fig G: Breaker and Half

Source: http://www.skm-eleksys.com/2011/09/substation-bus-schemes.html

EMS and Planning Models

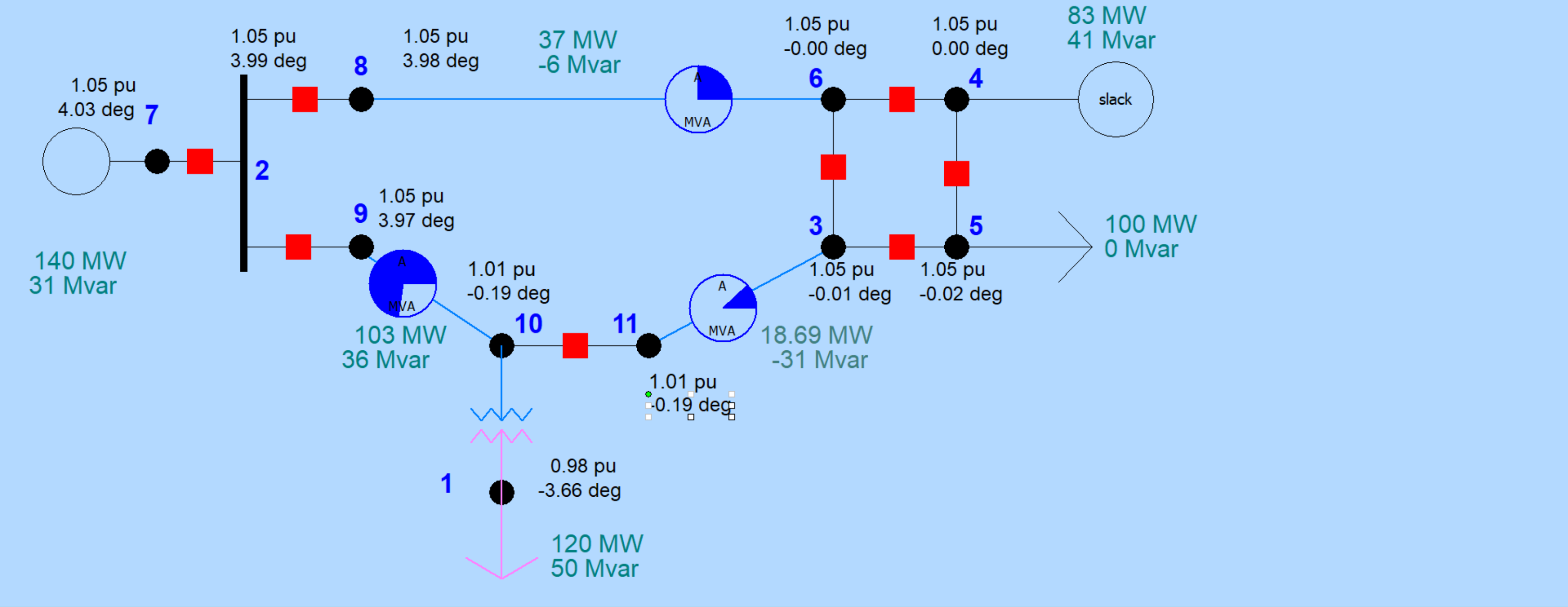
- EMS Model
  - Used for real-time operations
  - Called full topology model
  - Has node-breaker detail
- Planning Model
  - Used for off-line analysis
  - Called consolidated model by PowerWorld
  - Has bus/branch detail



Node-Breaker Consolidation

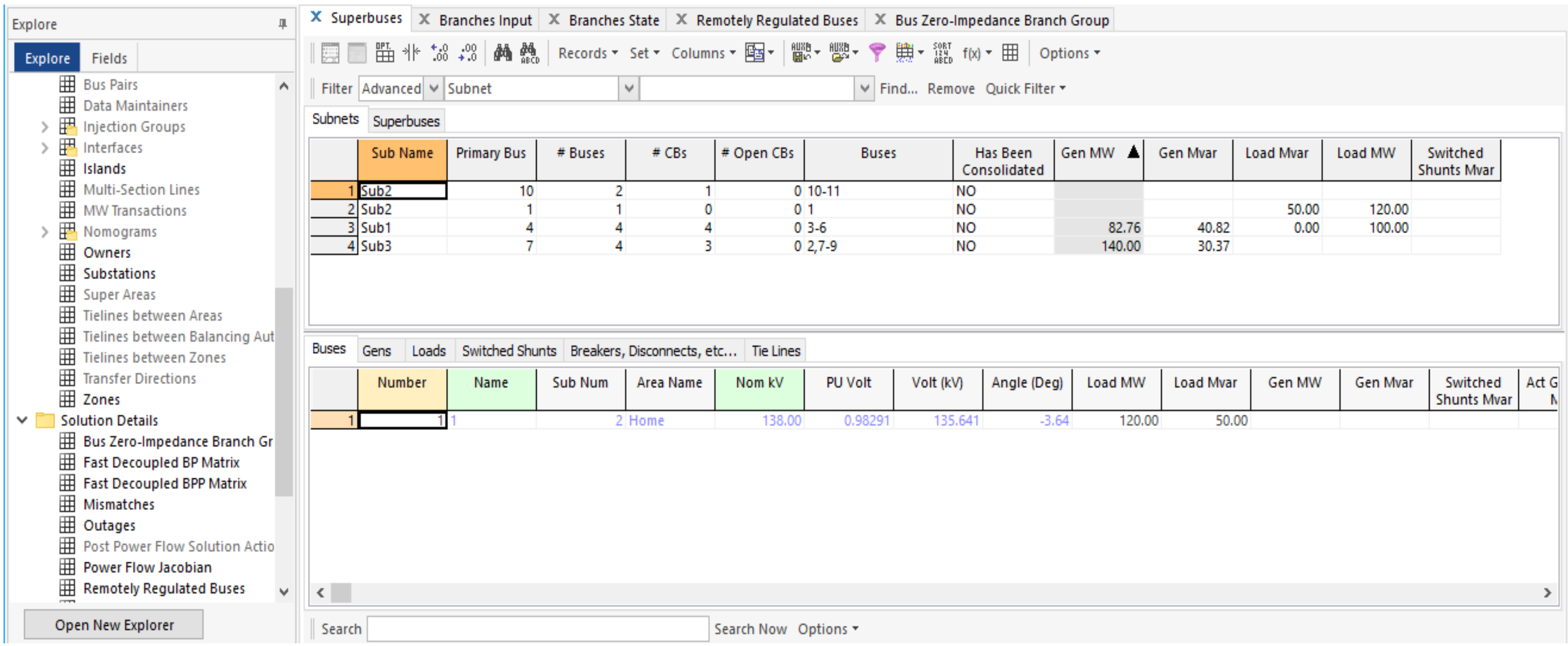
- One approach to modeling systems with large numbers of ZBRs (zero branch reactances, such as from circuit breakers) is to just assume a small reactance and solve
  - This results in lots of buses and branches, resulting in a much larger problem
  - This can cause numerical problems in the solution
- The alterative is to consolidate the nodes that are connected by ZBRs into a smaller number of buses
  - After solution all nodes have the same voltage; use logic to determine the device flows

Node-Breaker Example



Case name is **FT_11Node**. PowerWorld consolidates nodes (buses) into super buses; available in the Model Explorer: Solution, Details, Superbuses.

Node-Breaker Example



Note there is ambiguity on how much power is flowing in each device in the ring bus (assuming each device really has essentially no impedance)

Contingency Analysis

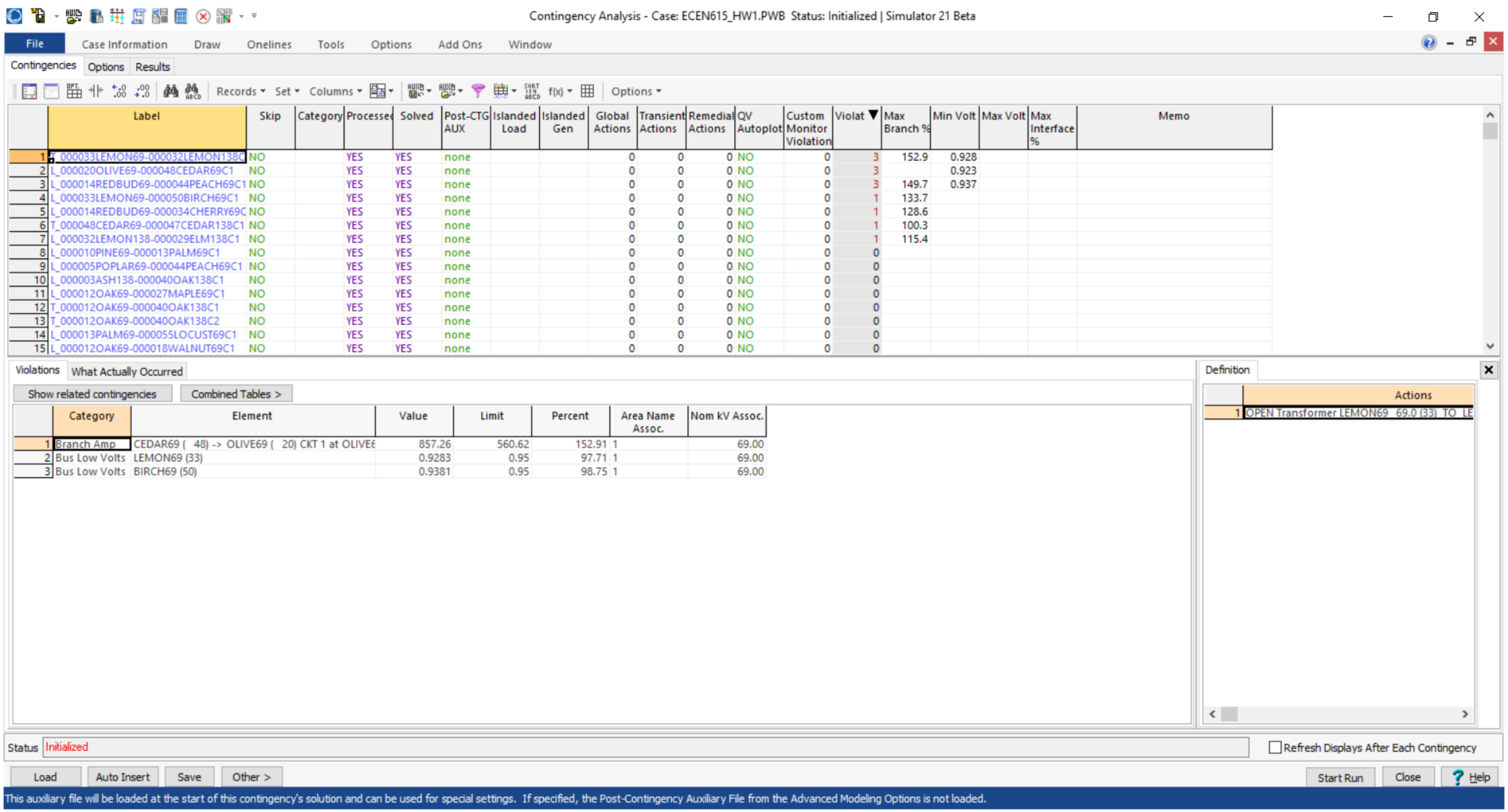
- Contingency analysis is the process of checking the impact of statistically likely contingencies
  - Example contingencies include the loss of a generator, the loss of a transmission line or the loss of all transmission lines in a common corridor
  - Statistically likely contingencies can be quite involved, and might include automatic or operator actions, such as switching load
- Reliable power system operation requires that the system be able to operate with no unacceptable violations even when these contingencies occur
  - N-1 reliable operation considers the loss of any single element

Contingency Analysis

- Of course this process can be automated with the usual approach of first defining a contingency set, and then sequentially applying the contingencies and checking for violations
  - This process can naturally be done in parallel
  - Contingency sets can get quite large, especially if one considers N-2 (outages of two elements) or N-1-1 (initial outage, followed by adjustment, then second outage
- The assumption is usually most contingencies will not cause problems, so screening methods can be used to quickly eliminate many contingencies
  - We'll cover these later

Contingency Analysis in PowerWorld

- Automated using the Contingency Analysis tool

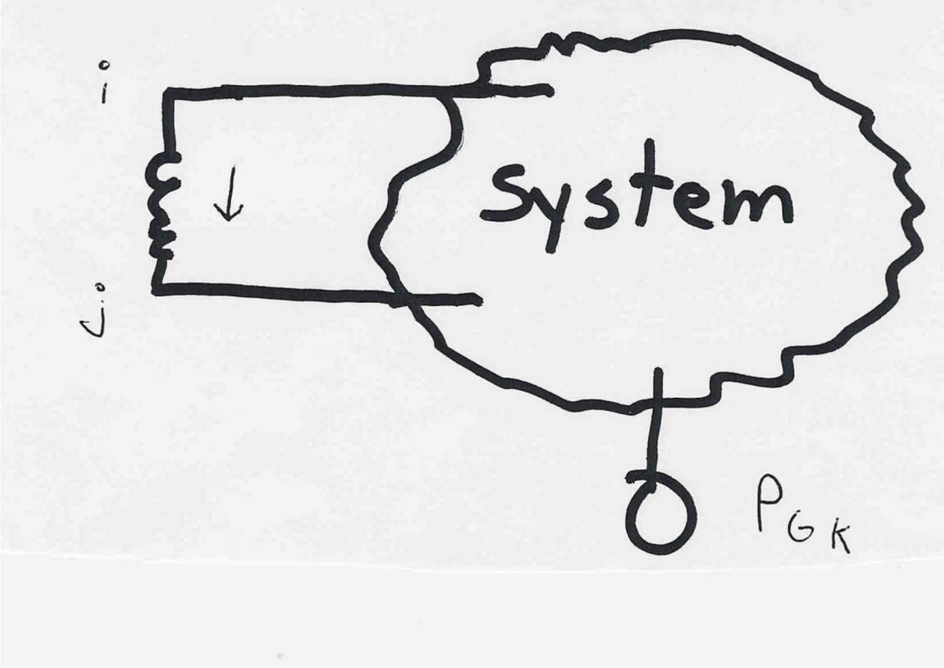


Power System Control and Sensitivities

- A major issue with power system operation is the limited capacity of the transmission system
  - lines/transformers have limits (usually thermal)
  - no direct way of controlling flow down a transmission line (e.g., there are no valves to close to limit flow)
  - open transmission system access associated with industry restructuring is stressing the system in new ways
- We need to indirectly control transmission line flow by changing the generator outputs
- Similar control issues with voltage

Indirect Transmission Line Control

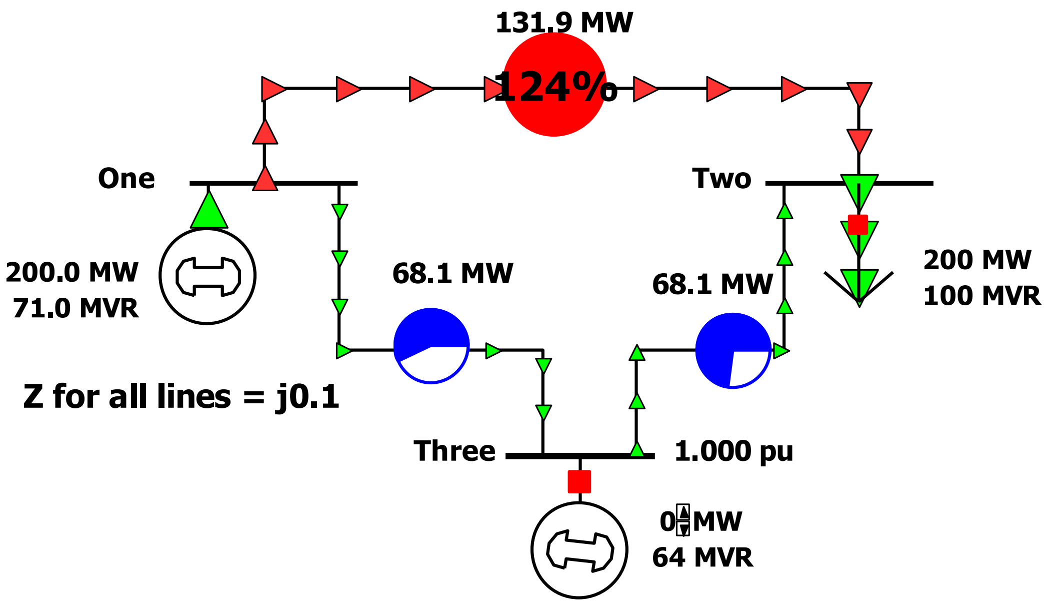
- What we would like to determine is how a change in generation at bus k affects the power flow on a line from bus i to bus j.



The assumption is that the change in generation is absorbed by the slack bus

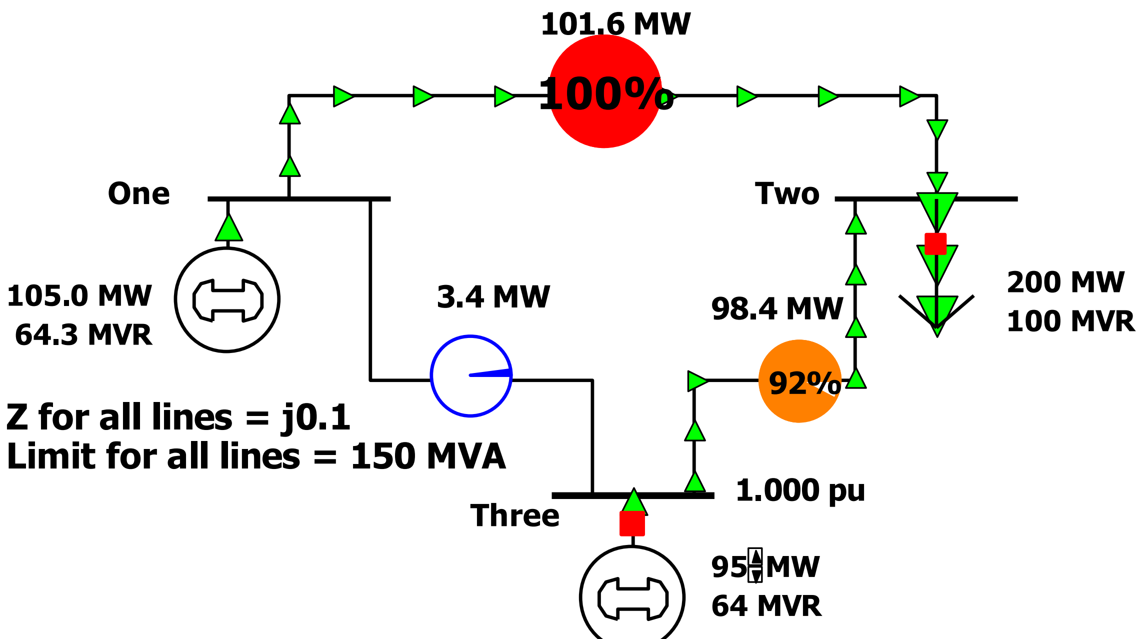
Power Flow Simulation - Before

- One way to determine the impact of a generator change is to compare a before/after power flow.
- For example below is a three bus case with an overload



Power Flow Simulation - After

- Increasing the generation at bus 3 by 95 MW (and hence decreasing it at bus 1 by a corresponding amount), results in a 30.3 MW drop in the MW flow on the line from bus 1 to 2, and a 64.7 MW drop on the flow from 1 to 3.



Expressed as a percent, 30.3/95 =32% and 64.7/95=68%

Analytic Calculation of Sensitivities

- Calculating control sensitivities by repeat power flow solutions is tedious and would require many power flow solutions. An alternative approach is to analytically calculate these values

The power flow from bus i to bus j is

$$P_{ij} \approx \frac{|V_i||V_j|}{X_{ij}} \sin(\theta_i - \theta_j) \approx \frac{\theta_i - \theta_j}{X_{ij}}$$

So $\Delta P_{ij} \approx \dfrac{\Delta\theta_i - \Delta\theta_j}{X_{ij}}$ We just need to get $\dfrac{\Delta\theta_{ij}}{\Delta P_{Gk}}$

Analytic Sensitivities

From the fast decoupled power flow we know

$$\Delta\boldsymbol{\theta} = \mathbf{B}^{-1}\Delta\mathbf{P}(\mathbf{x})$$

So to get the change in $\Delta\boldsymbol{\theta}$ due to a change of generation at bus k, just set $\Delta\mathbf{P}(\mathbf{x})$ equal to all zeros except a minus one at position k.

$$\Delta\mathbf{P} = \begin{bmatrix} 0 \\ \vdots \\ -1 \\ 0 \\ \vdots \end{bmatrix} \leftarrow \text{Bus k}$$

Three Bus Sensitivity Example

For a three bus, three line case with $Z_{\text{line}} = j0.1$

$$\mathbf{Y}_{\text{bus}} = j\begin{bmatrix} -20 & 10 & 10 \\ 10 & -20 & 10 \\ 10 & 10 & -20 \end{bmatrix} \rightarrow \mathbf{B} = \begin{bmatrix} -20 & 10 \\ 10 & -20 \end{bmatrix}$$

Hence for a change of generation at bus 3

$$\begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \end{bmatrix} = \begin{bmatrix} -20 & 10 \\ 10 & -20 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.0333 \\ 0.0667 \end{bmatrix}$$

Then $\Delta P_{3\text{ to }1} = \dfrac{0.0667 - 0}{0.1} = 0.667 \text{ pu}$

$\Delta P_{3\text{ to }2} = 0.333 \text{ pu} \qquad \Delta P_{2\text{ to }1} = 0.333 \text{ pu}$

More General Sensitivity Analysis: Notation

- We consider a system with *n* buses and L lines given by the set given by the set $\mathscr{L} @ \{\ell_1, \ell_2, \cdots, \ell_L\}$
  - Some authors designate the slack as bus zero; an alternative approach, that is easier to implement in cases with multiple islands and hence slacks, is to allow any bus to be the slack, and just set its associated equations to trivial equations just stating that the slack bus voltage is constant
- We may denote the k^th^ transmission line or transformer in the system, ℓ_k , as

$$\ell_k @ (i_k, j_k),$$

from node

to node

Notation, cont.

- We'll denote the real power flowing on ℓ_k from bus i to bus j as f_k
- The vector of real power flows on the L lines is:

$$\mathbf{f} \; @ \, [\, f_{\ell_1}, f_{\ell_2}, \cdots , f_{\ell_L} \,]^T$$

which we simplify to $\mathbf{f} = [\, f_1, f_2, \cdots , f_L \,]^T$

- The bus real and reactive power injection vectors are

$$\mathbf{p} \; @ \; [\, p^1, p^2, \cdots , p^N \,]^T$$

$$\mathbf{q} \; @ \; [\, q^1, q^2, \cdots , q^N \,]^T$$

Notation, cont.

- The series admittance of line ℓ is $g_\ell + jb_\ell$ and we define $$\tilde{\mathbf{B}} \;@\; -diag\left\{b_1, b_2, \cdots, b_L\right\}$$

- We define the L×N incidence matrix

$$\mathbf{A} \;@\; \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_L^T \end{bmatrix}$$

The component j of $\mathbf{a}_i$ is nonzero whenever line ℓ_i is coincident with node j. Hence **A** is quite sparse, with two nonzeros per row