ECEN 615
Methods of Electric Power Systems Analysis

Lecture 13: Power Flow Sensitivities

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Announcements

- Read Chapter 7 (the term reliability is now often used instead of security)
- First exam average was 86. The answers are posted.
- Homework 4 is due on Thursday October 14.

Power Flow Sensitivity Analysis

- The idea of power flow sensitivity analysis is to get an estimate of how some set of values would change with respect to a change in a set of control values
  - Need to keep in mind which control responses are implicitly modeled, such as P and Q changes at the slack, Q at PV buses
- The approach works by linearizing a system about an operating point; its usefulness depends on the validity of this approximation
- Sensitivities are widely used in power system analysis, with some algorithms doing sequential linearizations
  - They are most valid for real power, less useful for reactive power

Analysis Example: Available Transfer Capability

- The power system available transfer capability or ATC is defined as the maximum additional MW that can be transferred between two specific areas, while meeting all the specified pre- and post-contingency system conditions
- ATC impacts measurably the market outcomes and system reliability and, therefore, the ATC values impact the system and market behavior
- A useful reference on ATC is *Available Transfer Capability Definitions and Determination* from NERC, June 1996 (available online)

ATC and Its Key Components

- Total transfer capability (TTC)
  - Amount of real power that can be transmitted across an interconnected transmission network in a reliable manner, including considering contingencies
- Transmission reliability margin (TRM)
  - Amount of TTC needed to deal with uncertainties in system conditions; typically expressed as a percent of TTC
- Capacity benefit margin (CBM)
  - Amount of TTC needed by load serving entities to ensure access to generation; typically expressed as a percent of TTC

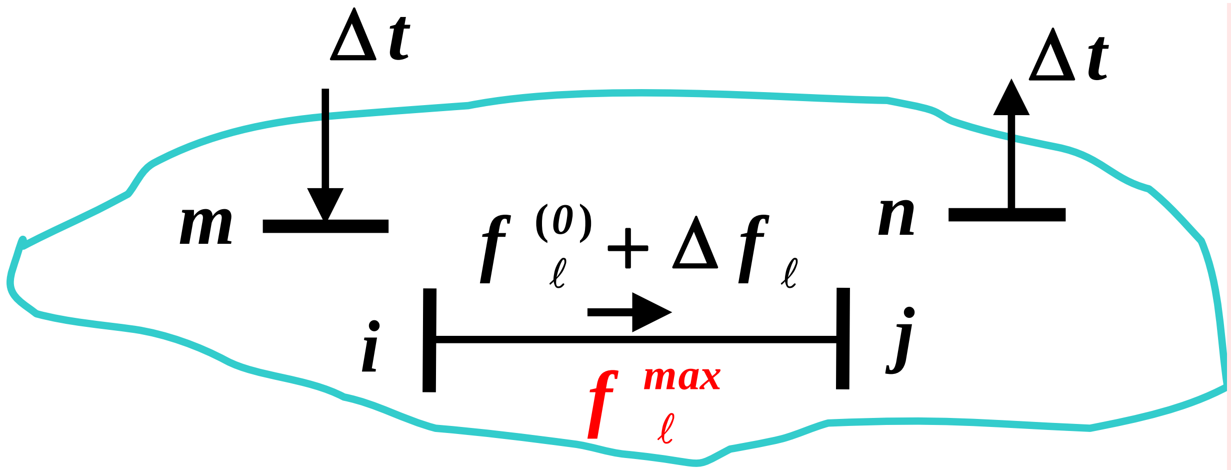
ATC and Its Key Components

- Uncommitted transfer capability (UTC)

  UTC ⍰ TTC – existing transmission commitment
- Formal definition of ATC is

  ATC ⍰ UTC – CBM – TRM
- We focus on determining $U_{m,n}$, the UTC from node m to node n
- $U_{m,n}$ is defined as the maximum additional MW that can be transferred from node m to node n without violating any limit in either the base case or in any post-contingency conditions
- Initially we'll focus on flow limits; voltage magnitude and voltage stability will be considered later

UTC (or TTC) Evaluation



Goal is to load the lines up to their limits, though only when also considering contingencies

$$U_{m,n} = \max \; \Delta t$$

s.t.

$$f_\ell^{(j)} + \Delta f_\ell \leq f_\ell^{max} \quad \forall \ell \in L$$

for the base case j = 0 and each contingency case

$j = 1,2 \ldots , J$

Conceptual Solution Algorithm

1. Solve the initial power flow, corresponding to the initial system dispatch (i.e., existing commitments); set the change in transfer $\Delta t^{(0)} = 0$, k=0; set step size δ; j is used to indicate either the base case (j=0) or a contingency, j= 1,2,3…J
2. Compute $\Delta t^{(k+1)} = \Delta t^{(k)} + \delta$
3. Solve the power flow for the new $\Delta t^{(k+1)}$
4. Check for limit violations: if violation is found set $U^{j}_{m,n} = \Delta t^{(k)}$ and stop; else set k=k+1, and goto 2

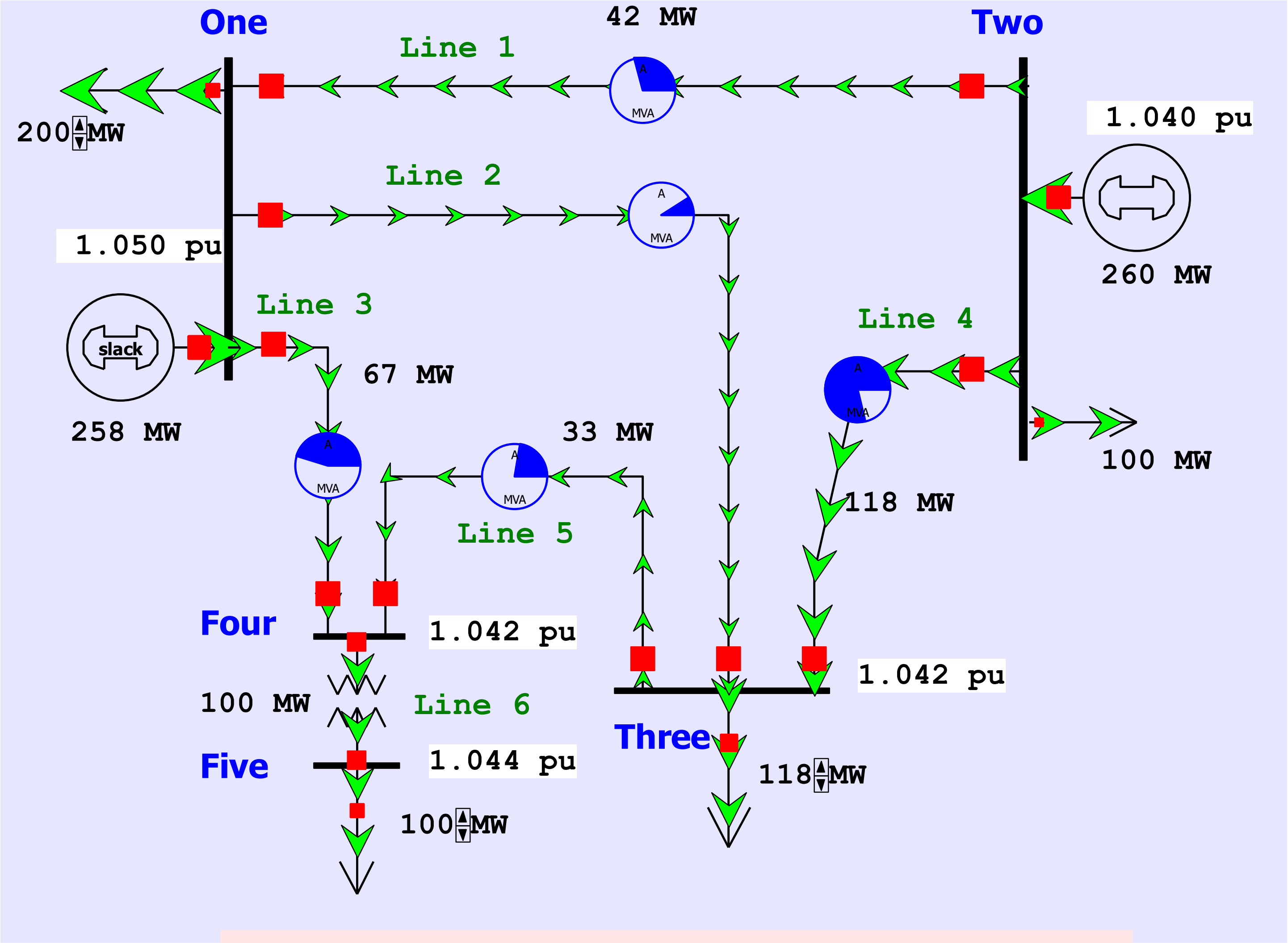
Conceptual Solution Algorithm, cont.

- This algorithm is applied for the base case (j=0) and each specified contingency case, j=1,2,..J
- The final UTC, $U_{m,n}$ is then determined by

$$U_{m,n} = \min_{0 \leq j \leq J} \left\{ U_{m,n}^{(j)} \right\}$$

- This algorithm can be easily performed on parallel processors since each contingency evaluation is independent of the others

Five Bus Example: Reference



PowerWorld Case: **B5_DistFact**

Five Bus Example: Reference

ℓ	i	j	g_ℓ	b_ℓ	f_ℓ^{max} (MW)
ℓ_1	1	2	0	6.25	150
ℓ_2	1	3	0	12.5	400
ℓ_3	1	4	0	12.5	150
ℓ_4	2	3	0	12.5	150
ℓ_5	3	4	0	12.5	150
ℓ_6	4	5	0	10	1,000

Five Bus Example

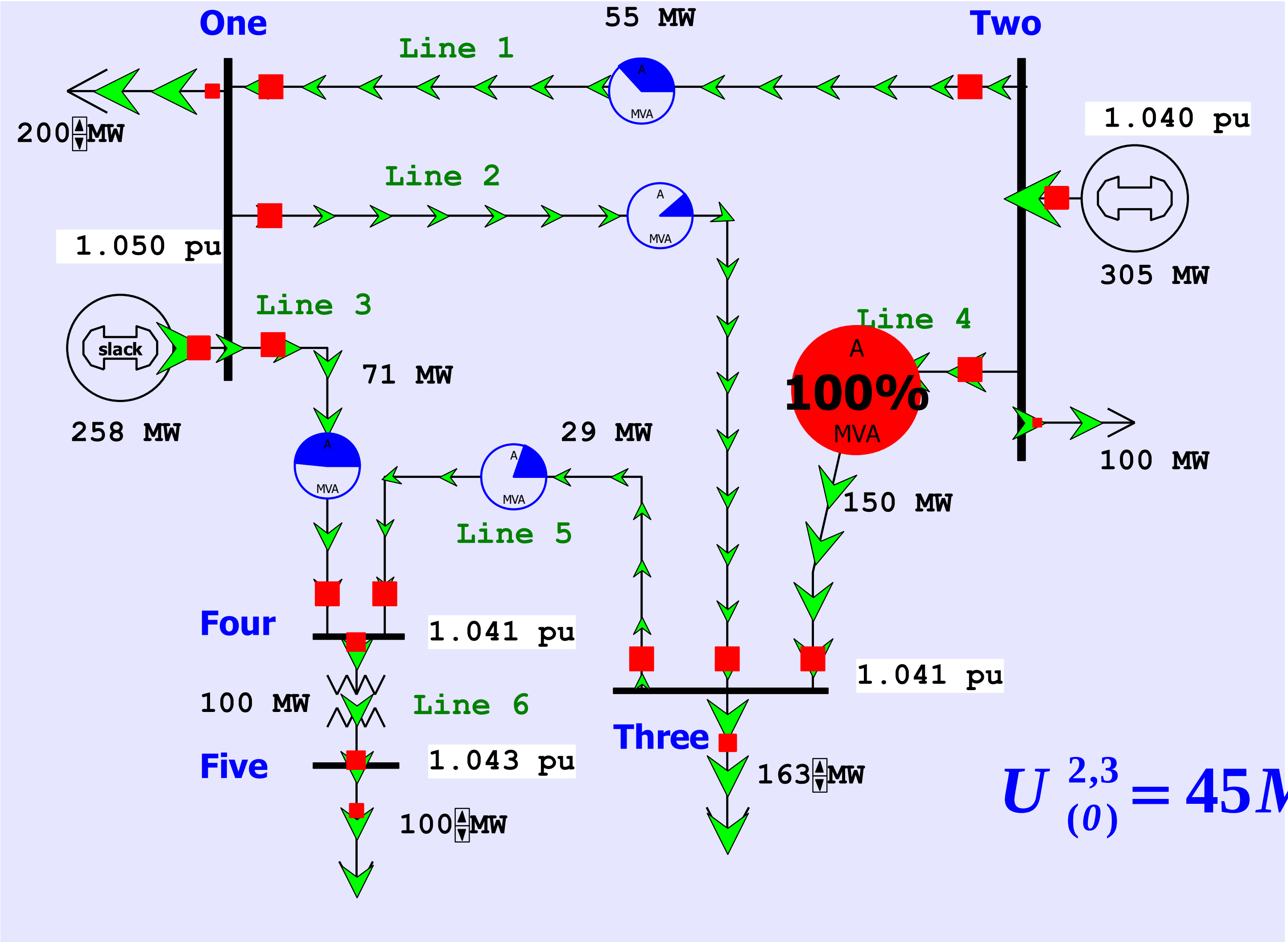
- We evaluate $U_{2,3}$ using the previous procedure
  - Gradually increase generation at Bus 2 and load at Bus 3
- We consider the base case and the single contingency with line 2 outaged (between 1 and 3): $J = 1$
- Simulation results show for the base case that

$$\boldsymbol{U_{2,3}^{(0)} = 45\ MW}$$

- And for the contingency that $\boldsymbol{U_{2,3}^{(1)} = 24\ MW}$

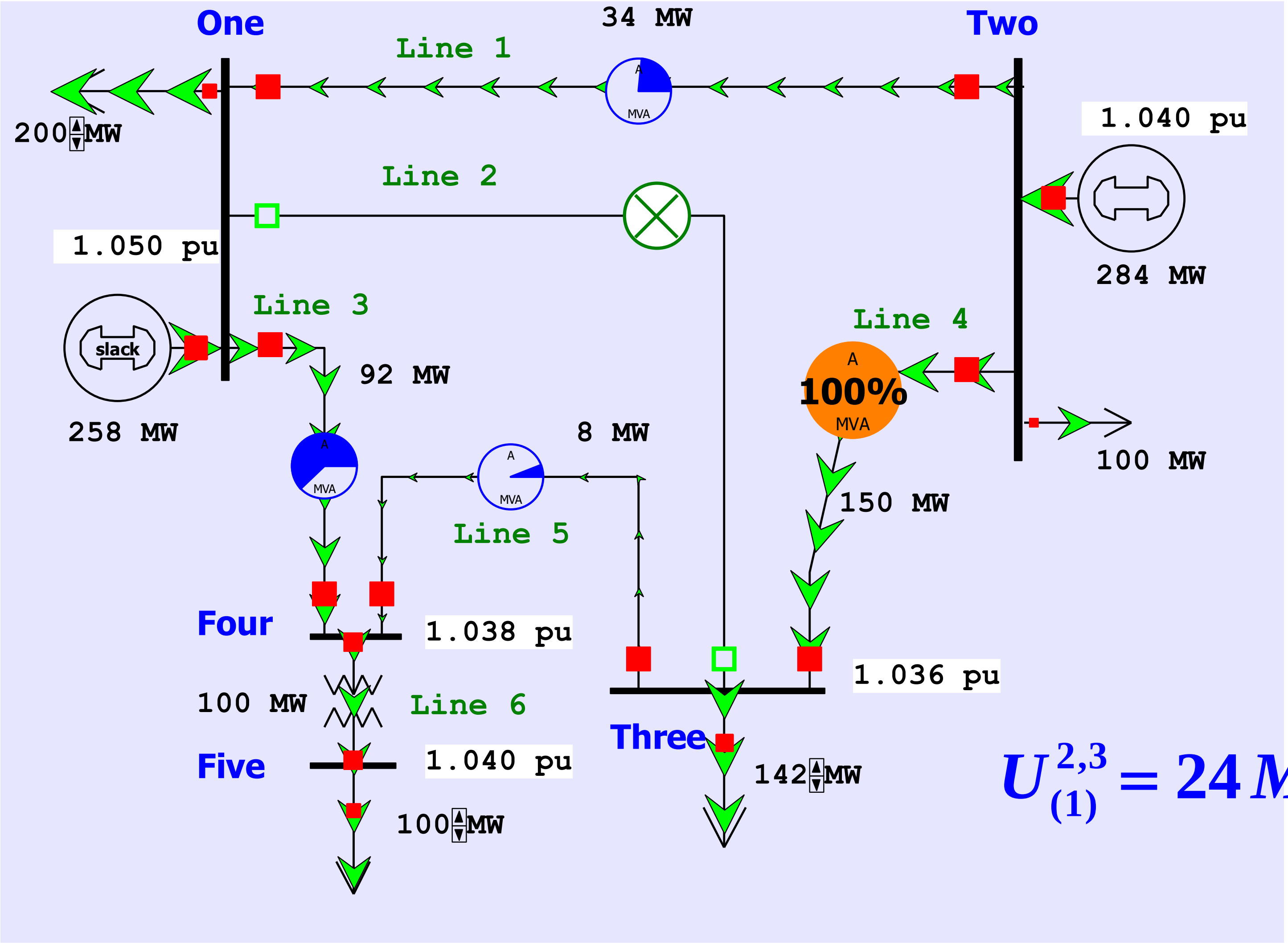
- Hence $\boldsymbol{U_{2,3} = min\left\{U_{2,3}^{(0)}, U_{2,3}^{(1)}\right\} = 24\ MW}$

Five Bus: Maximum Base Case Transfer



$$U_{(0)}^{2,3} = 45MW$$

Five Bus: Maximum Contingency Transfer



$$U_{(1)}^{2,3} = 24\,MW$$

Computational Considerations

- Obviously such a brute force approach can run into computational issues with large systems
- Consider the following situation:
  - 10 iterations for each case
  - 6,000 contingencies
  - 2 seconds to solve each power flow
- It will take over 33 hours to compute a single UTC for the specified transfer direction from m to n.
- Consequently, there is an acute need to develop fast tools that can provide satisfactory estimates

Sensitivity Problem Formulation

- Denote the system state by

$$\mathbf{x} \;@ \begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{V} \end{bmatrix} \qquad \boldsymbol{\theta} \;@\; [\theta_1, \theta_2, \cdots, \theta_N]^T \qquad \mathbf{V} \;@\; [V_1, V_2, \cdots, V_N]^T$$

The V values are the voltage magnitudes

- Denote the conditions corresponding to the existing commitment/dispatch by $\mathbf{s}^{(0)}$, $\mathbf{p}^{(0)}$ and $\mathbf{f}^{(0)}$ so that

$$\begin{cases} \mathbf{g}(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}) = \mathbf{0} \\ \mathbf{f}^{(0)} = \mathbf{h}(\mathbf{x}^{(0)}) \end{cases}$$

the power flow equations

line real power flow vector

- Define the angle difference as $\boldsymbol{\theta}_{jk} \;@\; \theta_j - \theta_k$

Sensitivity Problem Formulation

$$\mathbf{g}(\mathbf{x},\mathbf{p}) = \begin{bmatrix} \mathbf{g}^P(\mathbf{x},\mathbf{p}) \\ \mathbf{g}^Q(\mathbf{x},\mathbf{p}) \end{bmatrix}$$

g includes the real and reactive power balance equations

$$g_k^P\,(\underline{s},\underline{p}) = V_k \sum_{m=1}^{N} \left(V_m \left[G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km} \right] \right) - p_k$$

$$g_k^Q(\underline{s},\underline{p}) = V_k \sum_{m=1}^{N} \left(V_m \left[G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km} \right] \right) - q_k$$

$$h_\ell(\underline{s}) = g_\ell \left[\left(V_i \right)^2 - V_i V_j \cos\theta_{ij} \right] - b_\ell V_i V_j \sin\theta_{ij}, \ell = (i, j)$$

Sensitivity Problem Formulation

- For a small change, $\Delta\mathbf{p}$, that moves the injection from $\mathbf{p}^{(0)}$ to $\mathbf{p}^{(0)} + \Delta\mathbf{p}$, we have a corresponding change in the state $\Delta\mathbf{x}$ with

$$\mathbf{g}\,(\mathbf{x}^{(0)} + \Delta\mathbf{x}, \boldsymbol{p}^{(0)} + \Delta\mathbf{p}) = \mathbf{0}$$

- We then apply a first order Taylor's series expansion

$$\mathbf{g}\left(\mathbf{x}^{(0)} + \Delta\mathbf{x}, \mathbf{p}^{(0)} + \Delta\mathbf{p}\right) = \mathbf{g}\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right) + \left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta\mathbf{x}$$

$$+ \left.\frac{\partial \mathbf{g}}{\partial \mathbf{p}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta\boldsymbol{p} + \boldsymbol{h.o.t.}$$

Sensitivity Problem Formulation

- We consider this to be a "small signal" change, so we can neglect the higher order terms (h.o.t.) in the expansion
- Hence we should still be satisfying the power balance equations with this perturbation; so

$$\left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta \mathbf{x} + \left.\frac{\partial \mathbf{g}}{\partial \mathbf{p}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta \boldsymbol{p} \approx \mathbf{0}$$

Sensitivity Problem Formulation

- Also, from the power flow equations, we obtain

$$\frac{\partial \mathbf{g}}{\partial \mathbf{p}} = \begin{bmatrix} \dfrac{\partial \mathbf{g}^P}{\partial \mathbf{p}} \\ \cdots\cdots \\ \dfrac{\partial \mathbf{g}^Q}{\partial \mathbf{p}} \end{bmatrix} = \begin{bmatrix} -\mathbf{I} \\ \cdots\cdots \\ \mathbf{0} \end{bmatrix}$$

and then just the power flow Jacobian

$$\frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{g}^P}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{g}^P}{\partial \mathbf{V}} \\ \dfrac{\partial \mathbf{g}^Q}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{g}^Q}{\partial \mathbf{V}} \end{bmatrix} = \mathbf{J}(\mathbf{x},\mathbf{p})$$

Sensitivity Problem Formulation

- With the standard assumption that the power flow Jacobian is nonsingular, then

$$\Delta \mathbf{x} \approx \left[\mathbf{J}\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right) \right]^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p}$$

- We can then compute the change in the line real power flow vector

$$\Delta \mathbf{f} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right]^{T} \Delta \mathbf{s} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right]^{T} \left[J\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right) \right]^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p}$$

Sensitivity Comments

- Sensitivities can easily be calculated even for large systems
  - If $\Delta \mathbf{p}$ is sparse (just a few injections) then we can use a fast forward; if sensitivities on a subset of lines are desired we could also use a fast backward
- Sensitivities are dependent upon the operating point
  - They also include the impact of marginal losses
- Sensitivities could easily be expanded to include additional variables in **x** (such as phase shifter angle), or additional equations, such as reactive power flow

Sensitivity Comments, cont.

- Sensitivities are used in the optimal power flow; in that context a common application is to determine the sensitivities of an overloaded line to injections at all the buses
- In the below equation, how could we quickly get these values?

$$\Delta \mathbf{f} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right]^T \Delta f \approx \left[\frac{\partial \mathbf{h}}{\partial x}\right]^T \left[J\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right)\right]^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p}$$

  - A useful reference is O. Alsac, J. Bright, M. Prais, B. Stott, "Further Developments in LP-Based Optimal Power Flow," IEEE. Trans. on Power Systems, August 1990, pp. 697-711; especially see equation 3.

Sensitivity Example in PowerWorld

- Open case **B5_DistFact** and then Select **Tools, Sensitivities, Flow and Voltage Sensitivities**
  - Select **Single Meter, Multiple Transfers**, **Buses** page
  - Select the **Device Type (Line/XFMR)**, **Flow Type (MW)**, then select the line (from Bus 2 to Bus 3)
  - Click **Calculate Sensitivities**; this shows impact of a single injection going to the slack bus (Bus 1)
  - For our example of a transfer from 2 to 3 the value is the result we get for bus 2 (0.5440) minus the result for bus 3 (-0.1808) = 0.7248
  - With a flow of 118 MW, we would hit the 150 MW limit with (150-118)/0.7248 =44.1MW, close to the limit we found of 45MW

Sensitivity Example in PowerWorld

- If we change the conditions to the anticipated maximum loading (changing the load at 2 from 118 to 118+44=162 MW) and we re-evaluate the sensitivity we note it has changed little (from -0.7248 to -0.7241)
  - Hence a linear approximation (at least for this scenario) could be justified
- With what we know so far, to handle the contingency situation, we would have to simulate the contingency, and reevaluate the sensitivity values
  - We'll be developing a quicker (but more approximate) approach next

Linearized Sensitivity Analysis

- By using the approximations from the fast decoupled power flow we can get sensitivity values that are independent of the current state. That is, by using the **B**' and **B**'' matrices
- For the real power line flow we can approximate

$$h_\ell(\underline{s}) = g_\ell\left[\left(V_i\right)^2 - V_i V_j \cos\theta_{ij}\right] - b_\ell V_i V_j \sin\theta_{ij}, \ell = \left(i, j\right)$$

By using the FDPF appxomations

$$h_\ell(\underline{s}) \approx -b_\ell \theta_{ij} = \frac{\theta_{ij}}{X_\ell}, \ \ell = \left(i, j\right)$$

Linearized Sensitivity Analysis

- Also, for each line ℓ

$$\frac{\partial \boldsymbol{h}_\ell}{\partial \boldsymbol{\theta}} \approx -\boldsymbol{b}_\ell \, \mathbf{a}_\ell \qquad\qquad \frac{\partial \boldsymbol{h}_\ell}{\partial \mathbf{V}} \approx \mathbf{0}$$

and so,

$$\frac{\partial \mathbf{h}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}} \\ \hline \dfrac{\partial \mathbf{h}}{\partial \mathbf{V}} \end{bmatrix} = -\left[\begin{array}{c|c|c} \boldsymbol{b}_{\ell_1} \mathbf{a}_1 & \cdots & \boldsymbol{b}_{\ell_L} \mathbf{a}_L \\ & \cdots & \\ \hline \mathbf{0} & & \mathbf{0} \end{array}\right] \approx \begin{bmatrix} \mathbf{A}^T \tilde{\mathbf{B}} \\ \hline \mathbf{0} \end{bmatrix}$$

Sensitivity Analysis: Recall the Matrix Notation

- The series admittance of line ℓ is $g_{\ell} + jb_{\ell}$ and we define

$$\tilde{\mathbf{B}} \;@ -diag\left\{b_1, b_2, \cdots, b_L\right\}$$

- We define the L×N incidence matrix

$$\mathbf{A} \;@ \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_L^T \end{bmatrix}$$

where the component j of $\mathbf{a}_i$ is nonzero whenever line ℓ_i is coincident with node j. Hence **A** is quite sparse, with at most two nonzeros per row

Linearized Active Power Flow Model

- Under these assumptions the change in the real power line flows are given as

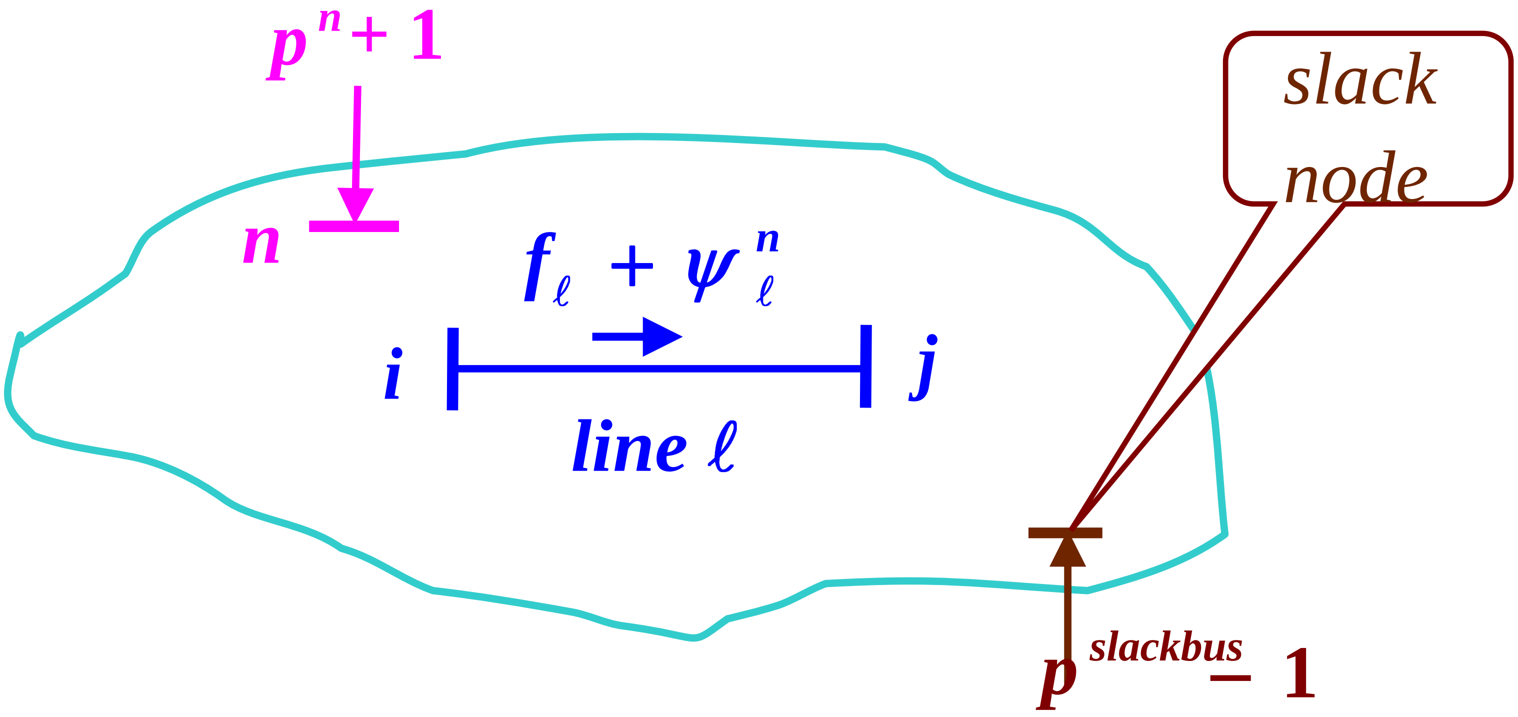
$$\Delta\mathbf{f} \approx \begin{bmatrix} \tilde{\mathbf{B}}\mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{B}' & \mathbf{0} \\ \mathbf{0} & \mathbf{B}'' \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta\mathbf{p} = \underbrace{\tilde{\mathbf{B}}\mathbf{A}\left[\mathbf{B}'\right]^{-1}}\Delta\mathbf{p} = \Psi\Delta\mathbf{p}$$

- The constant matrix $\Psi \;@\; \tilde{\mathbf{B}}\mathbf{A}\left[\mathbf{B}'\right]^{-1}$ is called the injection shift factor matrix (ISF)

Injection Shift Factors (ISFs)

- The element ψ_{ℓ}^{n} in row ⍰ and column n of ⍰ is called the injection shift factor (*ISF)* of line ⍰ with respect to the injection at node *n*
  - Absorbed at the slack bus, so it is slack bus dependent
- Terms generation shift factor (GSF) and load shift factor (LSF) are also used (such as by NERC)
  - Same concept, just a variation in the sign whether it is a generator or a load
  - Sometimes the associated element is not a single line, but rather a combination of lines (an interface)
- Terms used in North America are defined in the NERC glossary (http://www.nerc.com/files/glossary_of_terms.pdf)

ISF Interpretation



ψ_ℓ^n is the fraction of the additional 1 *MW* injection at node *n* that goes though line ℓ

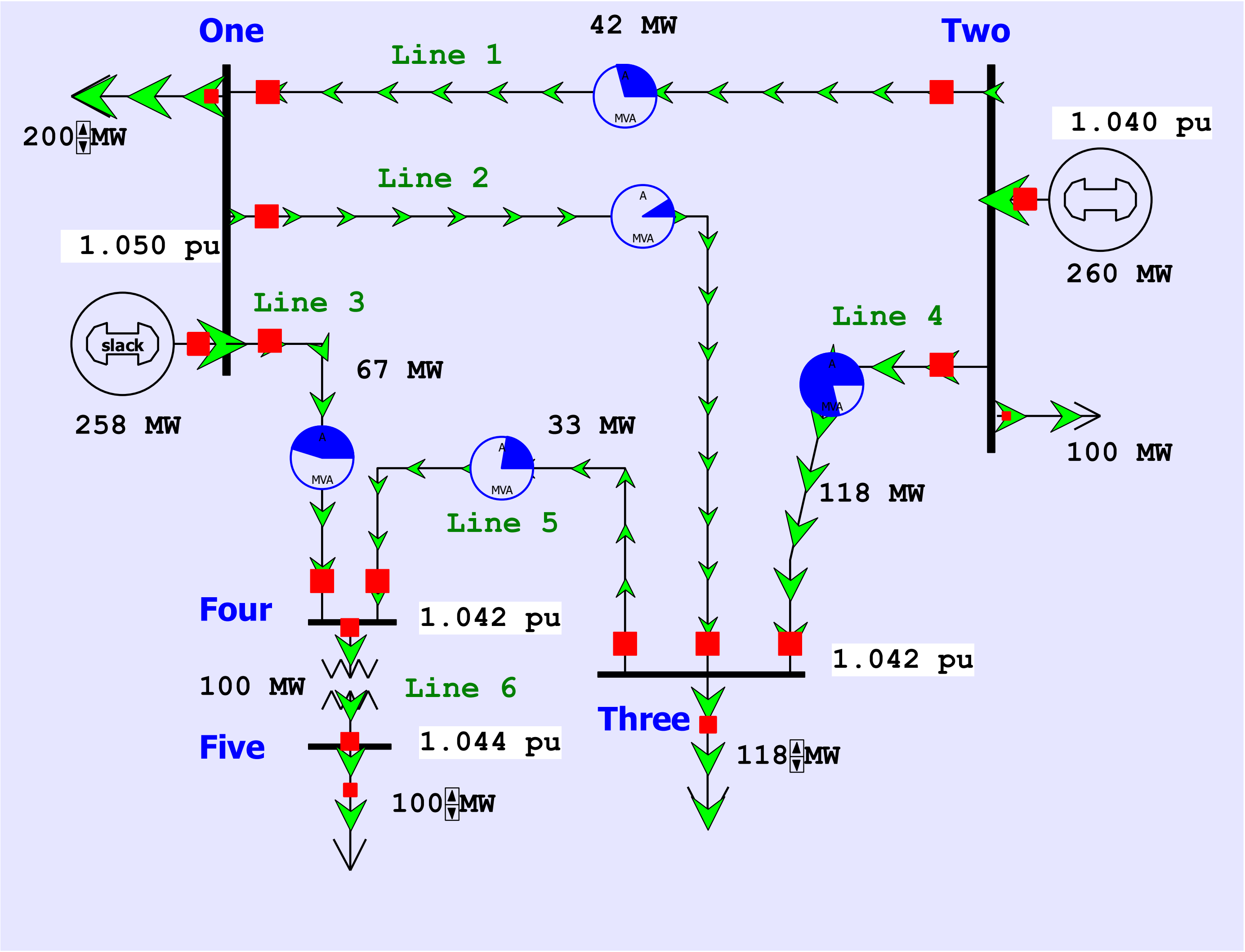
ISF Properties

- By definition, ψ_ℓ^n depends on the location of the slack bus
- By definition, $\psi_\ell^{slackbus} \equiv 0$ for $\forall \ell \in L$ since the injection and withdrawal buses are identical in this case and, consequently, no flow arises on any line ?
- The magnitude of ψ_ℓ^n is at most 1 since

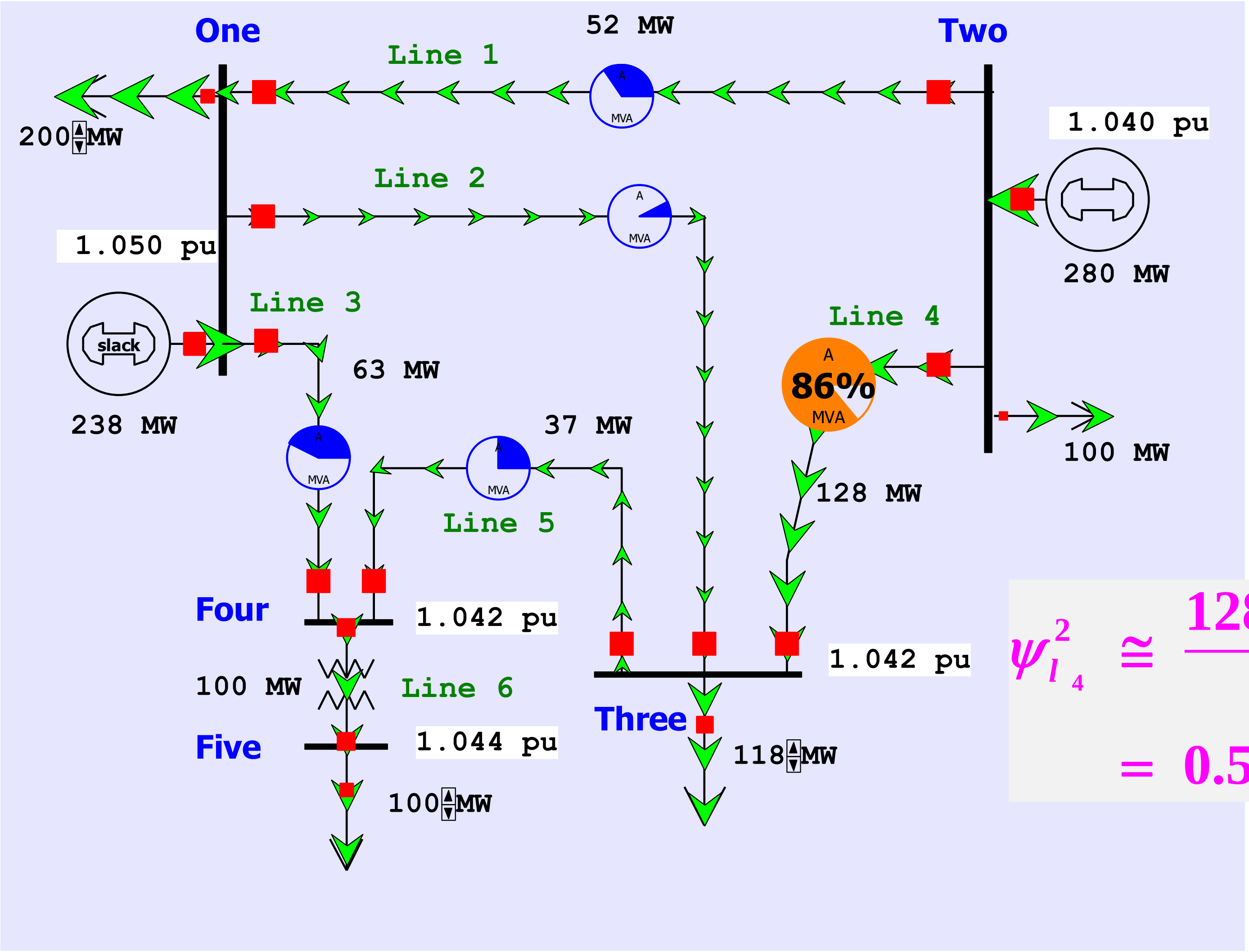
$$-1 \leq \psi_\ell^n \leq 1$$

Note, this is strictly true only for the linear (lossless) case. In the nonlinear case, it is possible that a transaction decreases losses. Hence a 1 MW injection could change a line flow by more than 1 MW.

Five Bus Example Reference



Five Bus ISF, Line 4, Bus 2 (to Slack)



$$\psi_{l_4}^{2} \cong \frac{128-118}{20} = 0.5$$

Five Bus Example

$$\tilde{\mathbf{B}} = -diag\{6.25, \quad 12.5, \quad 12.5, \quad 12.5, \quad 12.5, \quad 10\}$$

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

The row of **A** correspond to the lines and transformers, the columns correspond to the non-slack buses (buses 2 to 5); for each line there is a 1 at one end, a -1 at the other end (hence an assumed sign convention!). Here we put a 1 for the lower numbered bus, so positive flow is assumed from the lower numbered bus to the higher number

Five Bus Example

$$\mathbf{B}' = \mathbf{A}^T \tilde{\mathbf{B}} \mathbf{A} = \begin{bmatrix} -18.75 & 12.5 & 0 & 0 \\ 12.5 & -37.5 & 12.5 & 0 \\ 0 & 12.5 & -35 & 10 \\ 0 & 0 & 10 & -10 \end{bmatrix}$$

$$\underline{\Psi} = \underline{\tilde{B}}\,\underline{A}\left[\underline{B}'\right]^{-1} = \begin{bmatrix} -0.4545 & -0.1818 & -0.0909 & -0.0909 \\ -0.3636 & -0.5455 & -0.2727 & -0.2727 \\ -0.1818 & -0.2727 & -0.6364 & -0.6364 \\ 0.5455 & -0.1818 & -0.0909 & -0.0909 \\ 0.1818 & 0.2727 & -0.3636 & -0.3636 \\ 0 & 0 & 0 & -1.0000 \end{bmatrix}$$

With bus 1 as the slack, the buses (columns) go for 2 to 5

Five Bus Example Comments

- At first glance the numerically determined value of (128-118)/20=0.5 does not match closely with the analytic value of 0.5455; however, in doing the subtraction we are losing numeric accuracy
  - Adding more digits helps (128.40 – 117.55)/20 = 0.5425
- The previous matrix derivation isn't intended for actual computation; ⍰ is a full matrix so we would seldom compute all of its values
- Sparse vector methods can be used if we are only interested in the ISFs for certain lines and certain buses